

## ROTARY INVERTED PENDULUM AND CONTROL OF ROTARY INVERTED PENDULUM BY ARTIFICIAL NEURAL NETWORK

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**Abstract.** *Inverted pendulum is an unstable and highly nonlinear system. It is used as a common model for engineering applications in linear and nonlinear control. This paper presents the physical structure, the dynamic model of rotary inverted pendulum system and the method of identifying, controlling this system by an artificial neural network (ANN). This network is a mathematical model based on the structure and the function of a biological neural network. This is a state-of-the-art method of controlling which has many advantages such as the control of nonlinear object, the ability of learning and accumulating experiences and the ability of adjusting to any change of parameters in the system.*

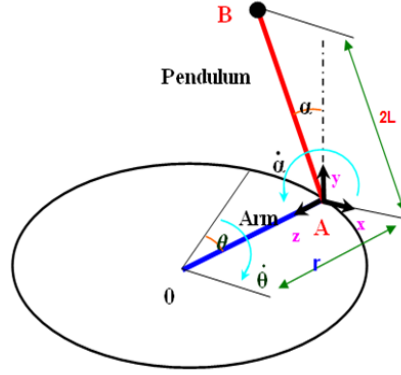
### I. INTRODUCTION

The inverted pendulum is a pendulum with its centre of gravity over its axis of rotation. The normal pendulum has its centre of gravity under its axis of rotation and therefore, it is in stable state when it directs downwards. The inverted pendulum is in unstable state because its centre of gravity is over its axis of rotation. A raised problem is how it is necessary to control the inverted pendulum so that it can keep its equilibrium state when it directs upwards.

The study of mathematical model, dynamic model and algorithms in controlling the inverted pendulum plays very important role in controlling rockets and spacecrafts and in maintaining the equilibrium state for two legs robots, skyscraping buildings, etc. At present, there are many algorithms in controlling systems of pendulum such as: PID control, slide control, fuzzy logic control and artificial neural network (ANN) control. Controllers based on their actions around a point will be unsuccessful if any change happens. An advantage of the ANN controller is that there is any change in action process of system, the ANN can adjust itself to this change and maintain its control. Other controllers cannot do that.

This paper studies the mathematical model and the dynamic model of rotary inverted pendulum and applies the algorithm of ANN control in the form of feed-forward in order to identify the system and then carry out the control of a rotary inverted pendulum by the ANN on the Matlab and the DSP TMS320 F2812.

## II. MATHEMATICAL MODEL FOR SYSTEM OF ROTARY INVERTED PENDULUM



**Figure 1.** Calculation model for system of a rotary inverted pendulum

The system of a rotary inverted pendulum shown in Figure 1 consists of a pendulum attached to a horizontal bar. The pendulum has the mass  $m$  and the length  $2L$  and can rotate freely. It sets up the angle  $\alpha$  for vertical direction. The horizontal bar has the length  $r$  and is used to move in inverse and forward directions with the angle  $\theta$ .

Supposing that the gravity of pendulum is at point  $B$ . Point  $B$  carries out a rotational motion compared with point  $A$  and the rotational velocity of point  $B$  has the following components

$$\begin{cases} \dot{x}_{AB} = -L\dot{\alpha} \cos \alpha, \\ \dot{y}_{AB} = -L\dot{\alpha} \sin \alpha. \end{cases} \quad (1)$$

Besides, the pendulum still rotates around point  $O$  with the velocity  $r\dot{\theta}$ . Therefore, the velocity of point  $B$  compared with fixed point  $O$  can be described by the equation as follows

$$\begin{cases} \dot{x}_B = r\dot{\theta} - L\dot{\alpha} \cos \alpha, \\ \dot{y}_B = -L\dot{\alpha} \sin \alpha. \end{cases} \quad (2)$$

Differentiate two sides of (2), we obtain

$$\begin{cases} \ddot{x}_B = r\ddot{\theta} + L\dot{\alpha}^2 \sin \alpha - L\ddot{\alpha} \cos \alpha, \\ \ddot{y}_B = -L\dot{\alpha}^2 \cos \alpha - L\ddot{\alpha} \sin \alpha. \end{cases} \quad (3)$$

This is the result of applying the Newton's Second Law in direction  $x$  and in direction  $y$ . Figure 2 describes forces acting on the arm and the pendulum. From that,

$$m\ddot{x}_B = \sum F_x \Rightarrow mr\ddot{\theta} + mL\dot{\alpha}^2 \sin \alpha - mL\ddot{\alpha} \cos \alpha = A_x, \quad (4)$$

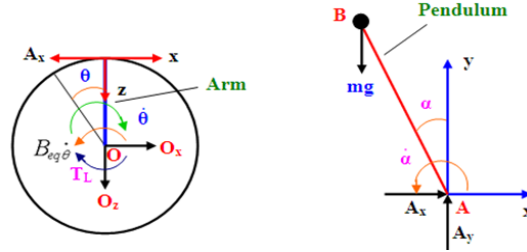
$$m\ddot{y}_B = \sum F_y \Rightarrow -mL\dot{\alpha}^2 \cos \alpha - mL\ddot{\alpha} \sin \alpha + mg = A_y. \quad (5)$$

Applying the Euler equation to the rotational motion of pendulum around point  $B$ , we obtain

$$J_B \ddot{\alpha} = \sum M_B \Rightarrow -\frac{1}{12} m (2L)^2 \ddot{\alpha} = A_x L \cos \alpha + A_y L \sin \alpha \Rightarrow \frac{1}{3} m L^2 \ddot{\alpha} = A_x L \cos \alpha + A_y L \sin \alpha. \quad (6)$$

The equation for the rotational motion of arm around point  $O$  has the form

$$J_O \ddot{\theta} = \sum M_O \Rightarrow J_{eq} \ddot{\theta} = T_L - B_{eq} \dot{\theta} - A_x r. \quad (7)$$



**Figure 2.** Forces acting on arm and pendulum

Substituting (4) and (5) into (6), we have

$$\begin{aligned} \frac{1}{3} m L^2 \ddot{\alpha} &= \left( m r \ddot{\theta} + m L \dot{\alpha} \sin \alpha - m L \ddot{\alpha} \cos \alpha \right) L \cos \alpha + (m g - m L \dot{\alpha} \cos \alpha - m L \ddot{\alpha} \sin \alpha) L \sin \alpha \\ &\Rightarrow -m L r \ddot{\theta} \cos \alpha + \frac{4}{3} m L^2 \ddot{\alpha} - m g L \sin \alpha = 0. \end{aligned} \quad (8)$$

Substituting (4) into (7), we obtain

$$\begin{aligned} J_{eq} \ddot{\theta} &= T_L - B_{eq} \dot{\theta} - \left( m r \ddot{\theta} + m L \dot{\alpha}^2 \sin \alpha - m L \ddot{\alpha} \cos \alpha \right) r \\ &\Rightarrow (J_{eq} + m r^2) \ddot{\theta} - m L r \ddot{\alpha} \cos \alpha + m L r \dot{\alpha}^2 \sin \alpha = T_L - B_{eq} \dot{\theta}, \end{aligned} \quad (9)$$

where

$$T_L = T_m - J_m \ddot{\theta} = \mu_m K_M I_m - J_m \ddot{\theta} = \mu_m K_M \frac{V_m - K_E \dot{\theta}}{R_m} - J_m \ddot{\theta}. \quad (10)$$

Substituting (9) into (10), we have

$$(J_{eq} + m r^2 + J_m) \ddot{\theta} - m L r \ddot{\alpha} \cos \alpha + m L r \dot{\alpha}^2 \sin \alpha + \left( B_{eq} + \mu_m K_M \frac{K_E}{R_m} \right) \dot{\theta} = \mu_m K_M \frac{V_m}{R_m}. \quad (11)$$

The system of equations describing the nonlinear kinetic characteristics of system has the form

$$\ddot{\theta} = \frac{1}{a} \left( b \ddot{\alpha} \cos \alpha - b \dot{\alpha}^2 \sin \alpha - e \dot{\theta} + f V_m \right), \quad \ddot{\alpha} = \frac{1}{c} \left( d \sin \alpha + b \ddot{\theta} \cos \alpha \right), \quad (12)$$

where the parameters  $a$ ,  $b$ ,  $c$ ,  $d$ ,  $e$  and  $f$  have the form

$$\begin{aligned} a &= J_{eq} + m r^2 + J_m, \quad b = m L r, \quad c = 4 m L^2 / 3, \quad d = m d l, \\ e &= B_{eq} + \mu_m K_M K_E / R_m, \quad f = \mu_m K_M / R_m \end{aligned} \quad (13)$$

For small  $\alpha$ ,  $\sin \alpha \approx \alpha$ ,  $\cos \alpha \approx 1$ , the linearization of (12) leads to the system of equations as follows

$$\ddot{\theta} = \frac{1}{a} (b\ddot{\alpha} - e\dot{\theta} + fV_m), \ddot{\alpha} = \frac{1}{c} (d\alpha + b\ddot{\theta}). \tag{14}$$

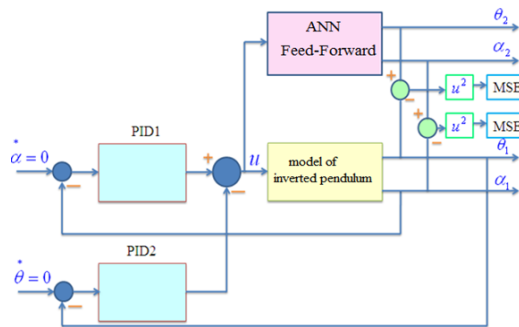
The mathematical model obtained in the system of equations (12) is used to build the model of inverted pendulum on the Simulink of Matlab in next sections.

### III. IDENTIFYING AND CONTROLLING ROTARY INVERTED PENDULUM SYSTEM BY ARTIFICIAL NEURAL NETWORK

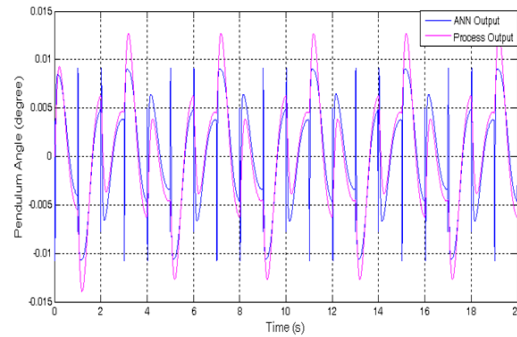
The artificial neural network (ANN) is trained to identify the system of pendulum of 1 input and 2 outputs. Supervised learning uses the input and output data to train the network and the training data is derived from the model of pendulum system with the PID controller of two variables. The input is the signal of voltage and the output is the deflection angles  $\alpha$  and  $\theta$  of the pendulum. The trained ANN model is tested qualitatively by calculating the MSE (mean square error). The MSE is a good measure in order to determine the accuracy of model. The smaller the MSE between the identifying ANN and the model of pendulum is, the better the accuracy is.

The ANN has the ability to model the pendulum. The MSE is small and the ANN model can foresee the deflection angle of pendulum. The results in Table 1 show that the more increases the number of neurons in hidden layer, the smaller the MSE between the identifying ANN and the model of pendulum is.

The quality of the model of network identifying is tested more by putting a source of random perturbation in the system with the amplitude 0.01. The result of response shows that the ANN adjusts itself to this change and identifies well with rather small error as described in Figure 4.



**Figure 3.** Identifying model of pendulum system by ANN

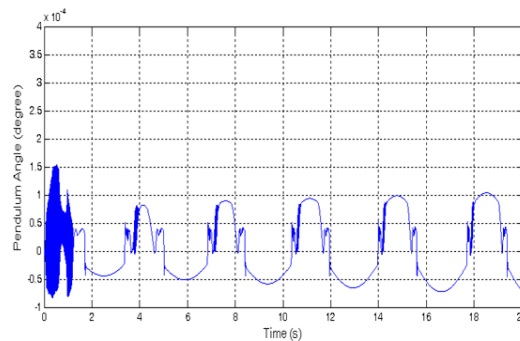


**Figure 4.** Result of identifying angle  $\alpha$  by ANN and model of system with noise

**Table 1.** Result of identifying when number of neurons changes

Form of neural network	Number of neurons in hidden layer	Number of training time	Speed of learning	Error (MSE)
FF	4	500	0.0001	$2.5509 \cdot 10^{-12}$
FF	10	500	0.0001	$1.0448 \cdot 10^{-12}$
FF	20	500	0.0001	$9.325 \cdot 10^{-13}$
FF	40	500	0.0001	$6.7929 \cdot 10^{-13}$

When the mass of pendulum is changed from 0.125 to 1 kg, the ANN controller adjusts itself to these changes so that the pendulum is in stable state as shown in Figure 5.

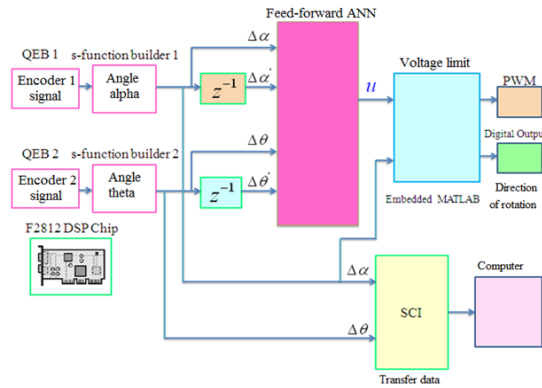


**Figure 5.** Dependence of angle  $\alpha$  on  $m$

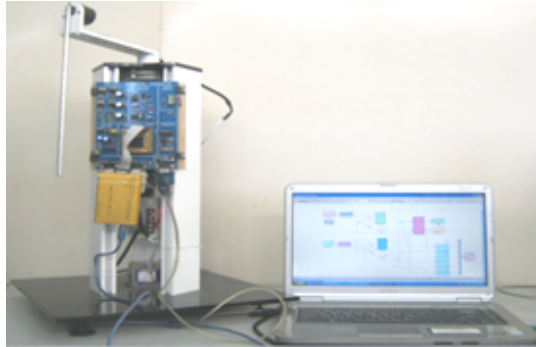
#### IV. RESULTS OF EXPERIMENT

The system of inverted pendulum is controlled by the ANN on the DSP TMS320F2812 with the sketch of control in Figure 6 and the system of experiment in Figure 7. The programme of control is written in the Matlab. The trained ANN controller keeps stable pendulum in vertical position and in upward direction. Figure 8 describes the response of pendulum angle in applying the ANN controller.

When the pendulum moves to the position corresponding to  $L = 32$  cm, the ANN controller adjusts well itself to this change and the result is shown in Figure 9. Results



**Figure 6.** System of controlling rotary inverted pendulum by ANN



**Figure 7.** System of experiment

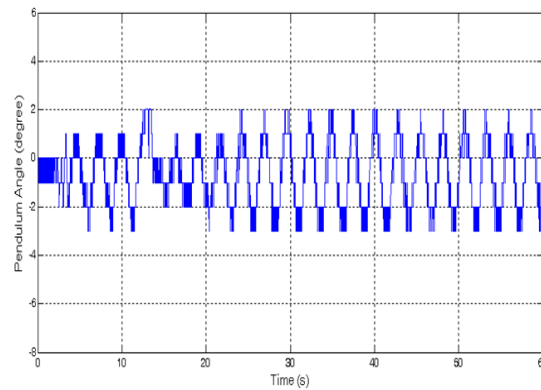
of simulation show that the identification of system the multilayer ANN gives good result of identifying with small error as presented in Table 1. The ANN controller controls successfully the pendulum system. When the parameters of system change, the ANN controller adjusts well itself to these changes and that is described in Figure 5. The ANN controller controls stably the system of inverted pendulum with small deflection angle and that is shown in Figure 8. When changes the arm ( $L$ ) of pendulum, the ANN controller gives good response and that is illuminated in Figure 8 and in Figure 9.

## V. CONCLUSION

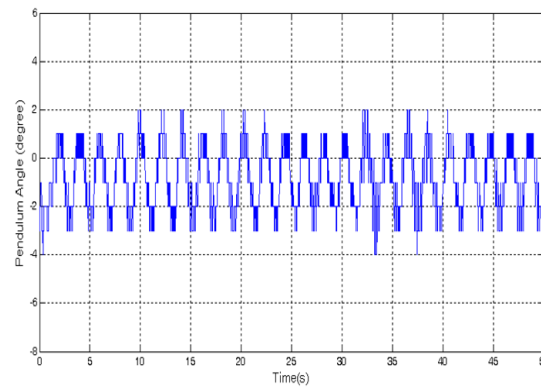
This paper presented the physical structure and the dynamic model of rotary inverted pendulum and the method of identifying the system of rotary inverted pendulum by the multilayer ANN. The ANN controller is used to control stably this system.

Obtained results of control algorithm can be applied to real physical systems with high nonlinearity and especially physical systems having the centre of gravity on the axis of rotation such as rockets, the spacecrafts, skyscraping buildings, etc.

Obtained results of identifying nonlinear system can be applied to complex physical systems and objects mathematical model of that cannot be built exactly. Then, the application of identifying method by ANN is a very good method.



**Figure 8.** Angle  $\alpha$  with neural network when  $m = 0.25$  kg ( $L = 16$  cm)



**Figure 9.** Angle  $\alpha$  with neural network when  $m = 0.25$  kg ( $L = 32$  cm)

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