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ELECTRICAL CONDUCTIVITY IN TYPE-II SUPERCONDUCTORS UNDER MAGNETIC FIELD

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Abstract. *The time-dependent Ginzburg-Landau approach in 2D is used to investigate linear response of a strongly type-II superconductor. Thermal fluctuations, represented by the Langevin white noise, are assumed to be strong enough to melt the Abrikosov vortex lattice created by the magnetic field into a moving vortex liquid and marginalize the effects of the vortex pinning by inhomogeneities. The nonlinear interaction term in dynamics is treated within self-consistent Gaussian approximation and we go beyond the often used lowest Landau level approximation to treat arbitrary magnetic fields. The results are compared to experimental data on high-*T_c *superconductor* $Bi_2Sr_2CaCuO_8$.

I. INTRODUCTION

Linear response to electric field in the mixed state of superconductors has been thoroughly explored experimentally and theoretically over the last three decades. These experiments were performed at very small voltages in order to avoid effects of nonlinearity. Magnetic field in strongly type-II superconductors create magnetic vortices, which, if not pinned by inhomogeneities, move and let the electric field to penetrate the mixed state. The dynamic properties of fluxons appearing in the bulk of a sample are strongly affected by the combined effect of thermal fluctuations, anisotropy (dimensionality) and the flux pinning [1]. Thermal fluctuations in these materials are far from negligible and in particular are responsible for existence of the first-order vortex lattice melting transition separating two thermodynamically distinct phases, the vortex solid and the vortex liquid. Magnetic field and reduced dimensionality due to pronounced layered structure (especially in materials like $Bi_2Sr_2CaCuO_{8+\delta}$) further enhance the effect of thermal fluctuations on the mesoscopic scale.

Since thermal fluctuations in the low-T_c materials are negligible compared to the inter-vortex interactions, the moving vortex matter is expected to preserve a regular lattice structure (for weak enough disorder). On the other hand, as mentioned above, the vortex lattice melts in HTSC over large portions of their phase diagram, so the moving vortex matter in the region of vortex liquid can be better described as an irregular flowing vortex liquid.

A simpler case of a zero or very small magnetic field in the case of strong thermal fluctuations was in fact comprehensively studied theoretically [6] albeit in linear response only. In any superconductor there exists a critical region around the critical temperature

$|T - T_c| \ll Gi \cdot T_c$, in which the fluctuations are strong (the Ginzburg number characterizing the strength of thermal fluctuations is just $Gi \sim 10^{-10} - 10^{-7}$ for low T_c, while $Gi \sim 10^{-5} - 10^{-1}$ for HTSC materials). Outside the critical region and for small electric fields, the fluctuation conductivity was calculated by Aslamazov and Larkin [2] by considering (noninteracting) Gaussian fluctuations within Bardeen-Cooper-Schrieffer (BCS) and within a more phenomenological Ginzburg-Landau (GL) approach. In the framework of the GL approach (restricted to the lowest Landau level approximation), Ullah and Dorsey [3] computed the Ettingshausen coefficient by using the Hartree approximation. This approach was extended to other transport phenomena like the Hall conductivity [3] and the Nernst effect [4]. The fluctuation conductivity within linear response can be applied to describe sufficiently weak electric fields, which do not perturb the fluctuations' spectrum [5, 6].

In this paper the linear electric response of the moving vortex liquid in 2D superconductor under magnetic field is studied using the time dependent GL (TDGL) approach. The TDGL approach is an ideal tool to study a combined effect of the dissipative (overdamped) flux motion and thermal fluctuations conveniently modeled by the Langevin white noise. The interaction term in dynamics is treated in self-consistent Gaussian approximation which is similar in structure to the Hartree approximation [3, 6]. A main contribution of our paper is an explicit form of the Green function incorporating all Landau levels. This allows to obtain explicit formulas for conductivity (resistivity) without need to cutoff higher Landau levels. The method is very general, and it allow us to study transport phenomena beyond linear response of type-II superconductor like the Nernst effect, Hall effect.

II. THERMAL FLUCTUATIONS IN THE TIME DEPENDENT GL MODEL IN 2D

To describe fluctuation of order parameter in thin-film superconductors, one can start with the GL free energy:

$$F_{GL} = s' \int d^2r \Big\{ \frac{\hbar^2}{2m^*} |\mathbf{D}\Psi|^2 + a|\Psi|^2 + \frac{b'}{2}|\Psi|^4 \Big\}, \tag{1}$$

where s' is the order parameter effective "thickness". For simplicity we assume $a = \alpha T_c^{mf}(t-1)$, $t \equiv T/T_c^{mf}$, although this temperature dependence can be easily modified to better describe the experimental coherence length. The "mean field" critical temperature T_c^{mf} depends on UV cutoff, τ_c, of the "mesoscopic" or "phenomenological" GL description, specified later. This temperature is higher than measured critical temperature T_c due to strong thermal fluctuations on the mesoscopic scale.

The covariant derivatives are defined by $\mathbf{D} \equiv \nabla + i(2\pi/\Phi_0)\mathbf{A}$, where the vector potential describes constant and homogeneous magnetic field $\mathbf{A} = (-By, 0)$ and $\Phi_0 = hc/e^*$ is the flux quantum with $e^* = 2|e|$. The two scales, the coherence length $\xi^2 = \hbar^2/(2m^*\alpha T_c)$, and the penetration depth, $\lambda^2 = c^2 m^* b'/(4\pi e^{*2}\alpha T_c)$ define the GL ratio $\kappa \equiv \lambda/\xi$, which is very large for HTSC. In this case of strongly type-II superconductors the magnetization is by a factor κ^2 smaller than the external field for magnetic field larger than the first critical field $H_{c1}(T)$, so that we take $B \approx H$. The electric current, $\mathbf{J} = \mathbf{J}_n + \mathbf{J}_s$, includes both the Ohmic normal part

$$\mathbf{J}_n = \sigma_n \mathbf{E}, \tag{2}$$

and the supercurrent

$$\mathbf{J}_s = \frac{ie^*\hbar}{2m^*}\left(\Psi^*\mathbf{D}\Psi - \Psi\mathbf{D}\Psi^*\right). \tag{3}$$

Since we are interested in a transport phenomenon, it is necessary to introduce a dynamics of the order parameter. The simplest one is a gauge-invariant version of the "type A" relaxational dynamics [7]. In the presence of thermal fluctuations, which on the mesoscopic scale are represented by a complex white noise [8], it reads:

$$\frac{\hbar^2\gamma'}{2m^*}D_\tau\Psi = -\frac{1}{s'}\frac{\delta F_{GL}}{\delta\Psi^*} + \zeta, \tag{4}$$

where $D_\tau \equiv \partial/\partial\tau - i(e^*/\hbar)\Phi$ is the covariant time derivative, with $\Phi = -Ey$ being the scalar electric potential describing the driving force in a purely dissipative dynamics.

Throughout most of the paper we use the coherence length ξ as a unit of length and $H_{c2} = \Phi_0/2\pi\xi^2$ as a unit of the magnetic field. In analogy to the coherence length and the penetration depth, one can define a characteristic time scale. In the superconducting phase a typical "relaxation" time is $\tau_{GL} = \gamma'\xi^2/2$. It is convenient to use the following unit of the electric field and the dimensionless field: $E_{GL} = H_{c2}\xi/c\tau_{GL}, \mathcal{E} = E/E_{GL}$. The TDGL Eq. (4) written in dimensionless units reads

$$\left(D_\tau - \frac{1}{2}D^2\right)\psi - \frac{1-t}{2}\psi + |\psi|^2\psi = \overline{\zeta}, \tag{5}$$

Here the covariant time derivative is $D_\tau = \frac{\partial}{\partial\tau} + i\mathcal{E}y$, the covariant derivatives are defined by $D_x = \frac{\partial}{\partial x} - iby$, $D_y = \frac{\partial}{\partial y}$ with $b = B/H_{c2}$, and $t = T/T_c^{mf}$. The "mean field" critical temperature T_c^{mf} depends on UV cutoff. This temperature is higher than measured critical temperature T_c due to strong thermal fluctuations on the mesoscopic scale, and it will be renormalized later. The Langevin white-noise forces $\overline{\zeta}$ are correlated through $\left\langle \overline{\zeta}^*(\mathbf{r},\tau)\overline{\zeta}(\mathbf{r}',\tau')\right\rangle = 2\eta t\delta(\mathbf{r}-\mathbf{r}')\delta(\tau-\tau')$ with $\eta = \sqrt{2Gi_{2D}}\pi$, where the Ginzburg number is defined by $Gi_{2D} = \frac{1}{2}(8e^2\kappa^2\xi^2T_c^{mf}/c^2\hbar^2s')$.

The dimensionless current density is $\mathbf{J}_s = J_{GL}\mathbf{j}_s$ where

$$\mathbf{j}_s = \frac{i}{2}\left(\psi^*\mathbf{D}\psi - \psi\mathbf{D}\psi^*\right). \tag{6}$$

with $J_{GL} = cH_{c2}/(2\pi\xi\kappa^2)$ being the unit of the current density. Consistently the conductivity will be given in units of $\sigma_{GL} = J_{GL}/E_{GL} = c^2\gamma'/(4\pi\kappa^2)$. This unit is close to the normal state conductivity σ_n in dirty limit superconductors [9]. In general there is a factor k of order one relating the two: $\sigma_n = k\sigma_{GL}$.

III. THE GREEN'S FUNCTION OF TDGL IN GAUSSIAN APPROXIMATION

As mentioned, the cubic term in the TDGL Eq. (5) will be treated in the self-consistent Gaussian approximation [10] by replacing $|\psi|^2\psi$ with a linear one $2\left\langle|\psi|^2\right\rangle\psi$

$$\left(\frac{\partial}{\partial\tau}-\frac{1}{2}D^2-\frac{b}{2}\right)\psi+\varepsilon\psi=\overline{\zeta}, \tag{7}$$

leading the "renormalized" value of the coefficient of the linear term:

$$\varepsilon=-a_h+2\left\langle|\psi|^2\right\rangle, \tag{8}$$

where the constant is defined as $a_h=(1-t-b)/2$.

The relaxational linearized TDGL equation with a Langevin noise, Eq. (7), is solved using the retarded ($G^0=0$ for $\tau<\tau'$) Green function (GF) $G^0(\mathbf{r},\tau;\mathbf{r}',\tau')$:

$$\psi(\mathbf{r},\tau)=\int d\mathbf{r}'\int d\tau' G^0(\mathbf{r},\tau;\mathbf{r}',\tau')\overline{\zeta}(\mathbf{r}',\tau'). \tag{9}$$

The GF satisfies

$$\left\{\frac{\partial}{\partial\tau}-\frac{1}{2}D^2-\frac{b}{2}+\varepsilon\right\}G^0(\mathbf{r},\mathbf{r}',\tau-\tau')=\delta(\mathbf{r}-\mathbf{r}')\delta(\tau-\tau'), \tag{10}$$

The Green function is a Gaussian

$$G^0\left(\mathbf{r},\mathbf{r}',\tau''\right)=\exp\left[\frac{ib}{2}X\left(y+y'\right)\right]g\left(X,Y,\tau''\right), \tag{11}$$

where

$$g\left(X,Y,\tau''\right)=C(\tau'')\theta\left(\tau''\right)\exp\left(-\frac{X^2+Y^2}{2\beta}\right), \tag{12}$$

with $X=x-x', Y=y-y', \tau''=\tau-\tau'$. $\theta\left(\tau''\right)$ is the Heaviside step function, C and β are coefficients.

Substituting the Ansatz (11) into Eq. (10), one obtains following conditions:

$$\varepsilon-\frac{b}{2}+\frac{1}{\beta}+\frac{\partial_\tau C}{C}=0, \tag{13}$$

$$\frac{\partial_\tau\beta}{\beta^2}-\frac{1}{\beta^2}+\frac{b^2}{4}=0. \tag{14}$$

The Eq. (14) determines β, subject to an initial condition $\beta(0)=0$,

$$\beta=\frac{2}{b}\tanh\left(b\tau''/2\right), \tag{15}$$

while Eq. (13) determines C:

$$C=\frac{b}{4\pi}\exp\left\{-\left(\varepsilon-\frac{b}{2}\right)\tau''\right\}\sinh^{-1}\left(\frac{b\tau''}{2}\right). \tag{16}$$

The normalization is dictated by the delta function term in definition of the Green's function Eq. (10).

The thermal average of the superfluid density (density of Cooper pairs) can be expressed via the Green's functions [10].

$$\begin{aligned}\left\langle |\psi(\mathbf{r},\tau)|^2\right\rangle &= 2\eta t\int d\mathbf{r}'\int d\tau''\left|G^0(\mathbf{r},\mathbf{r}',\tau'')\right| \\ &= \frac{\eta tb}{2\pi}\int_{\tau''=\tau_c}^{\infty} d\tau''\frac{\exp\left\{-\left(2\varepsilon-b\right)\tau''\right\}}{\sinh(b\tau'')}. \end{aligned} \tag{17}$$

Substituting it into the "gap equation", Eq. (8), the later takes a form

$$\varepsilon = -a_h + \frac{\eta tb}{\pi}\int_{\tau''=\tau_c}^{\infty} d\tau''\frac{\exp\left\{-\left(2\varepsilon-b\right)\tau''\right\}}{\sinh(b\tau'')}, \tag{18}$$

In order to absorb the divergence into a renormalized value a_h^r of the coefficient a_h, it is convenient to make an

$$\begin{aligned} b\int_{\tau''=\tau_c}^{\infty} d\tau''\frac{\exp\left\{-\left(2\varepsilon-b\right)\tau''\right\}}{\sinh(b\tau'')} &= -\int_0^{\infty} d\tau''\ln[\sinh(b\tau'')]\frac{d}{d\tau''}\left[\frac{\exp\left\{-\left(2\varepsilon-b\right)\tau''\right\}}{\cosh(b\tau'')}\right] \\ &- \ln(b\tau_c) \end{aligned} \tag{19}$$

Physically the renormalization corresponds to reduction in the critical temperature by the thermal fluctuations from T_c^{mf} to T_c. The thermal fluctuations occur on the mesoscopic scale. The critical temperature T_c is defined as

$$T_c = T_c^{mf}\left[1+\frac{2\eta'}{\pi}ln(\tau_c)\right]. \tag{20}$$

Then Eq. (18) can be written as

$$\varepsilon = -a_h^r - \frac{\eta' t'}{\pi}\int_0^{\infty} d\tau''\ln[\sinh(b\tau'')]\frac{d}{d\tau''}\left[\frac{\exp\left\{-\left(2\varepsilon-b\right)\tau''\right\}}{\cosh(b\tau'')}\right] - \frac{\eta t}{\pi}\ln(b), \tag{21}$$

where $a_h^r = \frac{1-b-t'}{2}$, $t' = T/T_c$ and $\eta' = \sqrt{2G'i_{2D}}\pi$, where $G'i_{2D} = \frac{1}{2}(8e^2\kappa^2\xi^2T_c/c^2\hbar^2 s')$, ($T_c^{mf}$ is now replaced by T_c after renormalization). The formula is cutoff independent.

IV. CONDUCTIVITY

IV.1. Theoretical calculation

The supercurrent density, defined by Eq. (6), can be expressed via the Green's functions as:

$$j_y^s(\tau) = i\eta t\int d\mathbf{r}'\int d\tau' G^*\left(\mathbf{r},\mathbf{r}',\tau-\tau'\right)\frac{\partial}{\partial y}G\left(\mathbf{r},\mathbf{r}',\tau-\tau'\right) + c.c. \tag{22}$$

where $G\left(\mathbf{r},\mathbf{r}',\tau-\tau'\right)$ as the Green's function of the linearized TDGL Eq. (5) in the presence of the scalar potential. One finds correction to the Green's function to linear order in the electric field

$$G(\mathbf{r},\mathbf{r}',\tau'') = G^0(\mathbf{r},\mathbf{r}',\tau'') - i\int d\mathbf{r}_1\int d\tau_1' G^0(\mathbf{r},\mathbf{r}_1,\tau_1')\mathcal{E}(\tau_1')y_1G^0(\mathbf{r}_1,\mathbf{r}',\tau_2'), \tag{23}$$

where $\mathcal{E}(\tau_1')$ are the scalar electric potential and electric field in dimensionless units respectively, $\tau_1' = \tau - \tau_1$, and $\tau_2' = \tau_1 - \tau'$.

The supercurrent density, defined by Eq. (6), can be expressed via the Green's functions as:

$$\begin{aligned} j_y^s &= i\eta t \frac{d}{s} \int_0^{2\pi/d} \frac{dk_z}{2\pi} \int_{r',\tau'} G_{k_z}^* \left(\mathbf{r} - \mathbf{r}', \tau - \tau'\right) \\ & \times \frac{\partial}{\partial y} G_{k_z} \left(\mathbf{r} - \mathbf{r}', \tau - \tau'\right) + c.c. \end{aligned} \tag{24}$$

Performing the integrals, one obtains the conductivity expression $\overline{\sigma}^s = j_y^s / \mathcal{E}$ which do match the linear response conductivity expression derived in our previous work [4].

$$\overline{\sigma}^s = \frac{\eta' t'}{4\pi s b} \left\{ 2 - \left(1 - \frac{2\varepsilon}{b}\right) \left[\psi\left(\frac{\varepsilon}{b}\right) - \psi\left(\frac{1}{2} + \frac{\varepsilon}{b}\right) \right] \right\}, \tag{25}$$

where ψ is the polygamma function.

IV.2. Comparison with experiment

Our results is compared to the experimental data on $Bi_2Sr_2CaCu_2O_{8+\delta}$ (Bi2212) [11] with $T_c = 81$ K. In order to compare the fluctuation conductivity with experimental

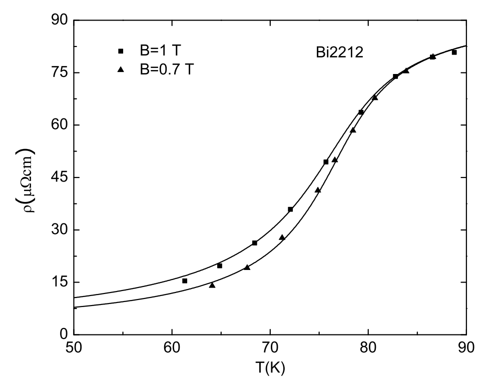

Fig. 1. Points are the resistivity for different magnetic fields of Bi2212 in Ref. [11]. The solid line is the theoretical value of the resistivity with fitting parameters (see text).

data in HTSC, one can not use the expression of relaxation time γ' in Bardeen-Cooper-Schrieffer theory which may be suitable for low-T_c superconductor. Instead of this, we use the factor k as fitting parameter. The comparison is presented in Fig. 1. The resistivity

$$\rho = \frac{1}{\sigma_s + \sigma_n}, \tag{26}$$

$$\sigma_s = \frac{\sigma_n}{k} \overline{\sigma}_s, \tag{27}$$

curve was fitted to Eq. (26) with the normal-state conductivity measured in Ref. [11] to be $\sigma_n = 1.42 \times 10^4$ $(\Omega\text{cm})^{-1}$. The best fitting parameters are: $H_{c2}(0) = 120$ T (corresponding to $\xi = 14$ Å), $\kappa = 47.8$, $s' = 4.31$ Å, $k = 0.61$ which give $Gi_{2D} = 4.5 \times 10^{-4}$. Our the resistivity results are in good agreement with experimental data on Bi2212

V. DISCUSSION AND CONCLUSION

We calculated the conductivity in a type-II superconductor in 2D under magnetic field in the presence of strong thermal fluctuations on the mesoscopic scale in linear response. Time dependent Ginzburg-Landau equations with thermal noise describing the thermal fluctuations is used to investigate the vortex-liquid regime. The nonlinear term in dynamics is treated using the renormalized Gaussian approximation. We obtained the analytically explicit expressions for the conductivity σ_s and resistivity ρ_s including all Landau levels, so that the approach is valid for arbitrary values of the magnetic field not too close to $H_{c1}(T)$.

The results were compared to the experimental data on HTSC. Our the resistivity results are in good qualitative and even quantitative agreement with experimental data on Bi2212. The thermal fluctuation was included in the present approach, so that our results should be applicable for above and below T_c.

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