

## $S_4$ FLAVOR SYMMETRY WITH SOFT-BREAKING AND PHYSICAL CONSEQUENCES

TRUONG TRONG THUC, NGUYEN THANH PHONG

*Department of Physics, College of Natural Science, Can Tho University*

**Abstract.** *We study the supersymmetric seesaw model in a  $S_4$  based flavor model. It has been shown that at the leading order, the model yields to exact tri-bimaximal pattern of the lepton mixing matrix and zero lepton-asymmetry of the decays of heavy right-handed neutrinos. By introducing a soft-breaking term in Dirac-neutrino mass matrix, a non-zero  $U_{e3}$  is generated leading to the non-zeros of mixing angle  $\theta_{13}$  and Dirac  $CP$  violating phase  $\delta_{CP}$ . We also obtained the deviations of the values  $\theta_{12}$  and  $\theta_{23}$  from their tri-bimaximal values. In addition, non-zero lepton asymmetry from the decays of right-handed neutrinos is generated, as a result, by a reasonable choice of model parameters compatible with low-energy data. The baryon asymmetry of the Universe is successful generated through flavored leptogenesis.*

### I. INTRODUCTION

Observed data from the Cosmic Microwave Background (CMB) and Big Bang Nucleosynthesis (BBN) indicate that almost no antimatter exists in our Universe and matter density is very small compared to the photon density. The baryon asymmetry of the Universe (BAU) usually is expressed as ratio of the baryon density  $n_B$  to the photon density  $n_\gamma$  of the Universe [1]

$$\eta_B = \frac{n_B}{n_\gamma} = 6.11^{+0.26}_{-0.27} \times 10^{-10}. \quad (1)$$

It needs adequately explaining since all cosmological models agree that, matter and anti-matter are generated at the same rate during the evolution of the Universe.

Besides, recent experiments of neutrino oscillations purpose determining more accurate values of mixing angles of lepton sector, and squared mass differences among neutrino masses [3]. However, properties related to the leptonic  $CP$  violation are completely unknown yet. The large mixing angles of lepton sector, which may be suggestive of a flavor symmetry, are completely different from the quark mixing ones. Therefore, it is very necessary to find a model that leads to flavor mixing model for quarks and leptons.

Based on the neutrino oscillation experimental data, Harrison et al. suggested a mixing structure called tri-bimaximal mixing (TBM) [4],  $U_{PMNS} \equiv U_{TB}P_\nu$ , where

$$U_{TB} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (2)$$

**Table 1.** Neutrino oscillation parameters from two independent global fits [2, 3]

Parameter	Reference [2]		Reference [3]	
	Best fit $1\sigma$	$3\sigma$ -interval	Best fit $1\sigma$	$3\sigma$ -interval
$\Delta m_{\text{sol}}^2 [10^{-5} \text{ eV}^2]$	$7.67^{+0.16}_{-0.19}$	7.14-8.19	$7.65^{+23}_{-0.20}$	7.05-8.34
$ \Delta m_{\text{atm}}^2  [10^{-3} \text{ eV}^2]$	$2.34^{+0.11}_{-0.80}$	2.06-2.81	$2.04^{+0.12}_{-0.11}$	2.07-2.75
$\sin^2 \theta_{12}$	$0.312^{+0.019}_{-0.018}$	0.26-0.37	$0.304^{+0.022}_{-0.016}$	0.25-0.37
$\sin^2 \theta_{23}$	$0.466^{+0.073}_{-0.058}$	0.331-0.644	$0.5^{+0.07}_{-0.06}$	0.36-0.67
$\sin^2 \theta_{13}$	$0.016^{+0.010}_{-0.010}$	$\leq 0.046$	$0.010^{+0.016}_{-0.011}$	$\leq 0.056$

and  $P_\nu$  is a diagonal matrix of Majorana  $CP$  violating phases. In this structure,  $\sin^2 \theta_{12} = \frac{1}{2}$ ,  $\sin^2 \theta_{23} = \frac{1}{3}$  and  $\sin^2 \theta_{13} = 0$ . This structure fairly agrees with experimental data, however a non-zero of  $\theta_{13}$  is a hot goal of new generation of neutrino oscillation experiments.

In recent years there have been lots of efforts in searching for models generated the TBM pattern of neutrino mixing matrix, and an absorptive way seems to be the use of some discrete non-Abelian flavor groups added to the gauge groups of the Standard Model (SM). There is a series of interesting models based on the symmetric group  $A_4$  [5],  $T'$  [6],  $S_4$  [7, 8]... The universal characteristics of this class of model are: existing at high energy scale, giving rise to the TBM structure and could not explain BAU at the leading order (LO).

In this work, we study the supersymmetric seesaw version of an  $S_4$  model. By considering a perturbation parameter in the Dirac neutrino mass matrix, we obtain non-zero  $U_{e3}$  leading to non-zero value of  $\theta_{13}$ . Besides, the values of other lepton mixing angles get small deviations compared to their TBM values. In addition, we also obtain lepton asymmetry from the out of thermal equilibrium decay of right handed neutrino. Together with a reasonable choice of parameter space of the model consistent with the experimental data at low energies, BAU is generated successfully through leptogenesis.

The rest of this work is organized as follows. Next section we review an interesting  $S_4$  model with seesaw mechanism. The effects of a soft-breaking term on the model are discussed in section 3. Section 4 is devoted for leptogenesis of the model after soft-breaking. We summary our work in the last section.

## II. $S_4$ MODEL WITH SEESAW MECHANISM

We consider the model proposed in [8], which could give rise to TBM pattern of the lepton mixing matrix at the LO by seesaw mechanism. The model is based on the flavor discrete group  $G_f = S_4 \times Z_5 \times U(1)_{FN}$  added to the gauge groups of SM. The matter fields and the flavons of the model are given in table 2. The superpotential of the model in the lepton sector reads as follows

$$w_\ell = \sum_{i=1}^4 \frac{\theta}{\Lambda} \frac{y_{e,i}}{\Lambda^3} e^{c(\ell X_i)} h_d + \frac{y_\mu}{\Lambda^2} \mu^c (\ell \psi \eta)' h_d + \frac{y_\tau}{\Lambda} \tau^c (\ell \psi) h_d + h.c. + \dots, \quad (3)$$

$$w_\nu = x(\nu^c \ell) h_u + x_d(\nu^c \nu^c \varphi) + x_t(\nu^c \nu^c \Delta) + h.c. + \dots, \quad (4)$$

**Table 2.** Transformation properties of the matter fields in the lepton sector and all the flavons of the model,  $\omega$  is the cube root of unity, i.e.  $\omega = e^{i2\pi/3}$ .

Field	$\ell$	$e^c$	$\mu^c$	$\tau^c$	$\nu^c$	$h_{u,d}$	$\theta$	$\psi$	$\eta$	$\Delta$	$\varphi$	$\xi'$
$S_4$	$3_1$	$1_2$	$1_2$	$1_1$	$3_1$	$1_1$	$1_1$	$3_1$	$2$	$3_1$	$2$	$1_2$
$Z_5$	$\omega^4$	$1$	$\omega^2$	$\omega^4$	$\omega$	$1$	$1$	$\omega^2$	$\omega^2$	$\omega^3$	$\omega^3$	$1$
$U(1)_{FN}$	$0$	$1$	$0$	$0$	$0$	$0$	$-1$	$0$	$0$	$0$	$0$	$0$

where  $X_i = \psi\psi\eta, \psi\eta\eta, \Delta\Delta\xi', \Delta\varphi\xi'$  and  $(\dots)^{(i)}$  is used to refer to the contraction in  $1_{1(2)}$  and the dots denote higher order contributions.

The alignment of the vacuum expectation values (VEVs) of flavons is given by:

$$\begin{aligned} \langle\psi\rangle &= (0 \ 1 \ 0)^T v_\psi, \quad \langle\Delta\rangle = (1 \ 1 \ 1)^T v_\Delta, \\ \langle\eta\rangle &= (0 \ 1)^T v_\eta, \quad \langle\varphi\rangle = (1 \ 1)^T v_\varphi, \quad \langle\xi'\rangle = v_{\xi'}. \end{aligned} \tag{5}$$

All the VEVs are of the same order of magnitude and for this reason these VEVs are parameterized as  $\text{VEVs}/\Lambda = u$ . The only VEV which originates from a different mechanism with respect to the others is  $v_\theta$  and we indicate the ratio  $v_\theta/\Lambda = t$ . It is shown in the reference [8] that  $u$  and  $t$  belong to a well determined range of values  $0.01 < u, t < 0.05$ .

With this setting, the mass matrix for the charged leptons is obtained as

$$m_\ell = \begin{pmatrix} y_e^{(1)}u^2t & y_e^{(2)}u^2t & y_e^{(2)}u^2t \\ 0 & y_\mu u & 0 \\ 0 & 0 & y_\tau \end{pmatrix} uv_d \tag{6}$$

where the  $y_e^{(i)}$  are the result of all the different contributions of the  $y_{e,i}$ . The neutrino mass matrices are given by

$$m_\nu^d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} xv_u, \tag{7}$$

$$M_R = \begin{pmatrix} 2c & b-c & b-c \\ b-c & b+2c & -c \\ b-c & -c & b+2c \end{pmatrix}, \tag{8}$$

where  $b = 2|x_d|v_\varphi, c = 2|x_t|v_\Delta$  are real and positive quantities and the phases  $\alpha_1, \alpha_2$  are the arguments of  $x_{d,t}$ , and  $\phi = \alpha_2 - \alpha_1$  is the only physical phase remained in  $M_R$ .

The heavy neutrino mass matrix  $M_R$  is exactly diagonalized by the TBM matrix

$$M_R^D = V_R^T M_R V_R = \text{Diag.}(M_1, M_2, M_3), \tag{9}$$

$$M_R^D = V_R^T M_R V_R = b \cdot \text{Diag.}(3ke^{i\phi} - 1, 2, 3ke^{i\phi} + 1) \tag{10}$$

$$M_1 = b|3ke^{i\phi} - 1|, \quad M_2 = 2b, \quad M_3 = b|3ke^{i\phi} + 1| \tag{11}$$

$$V_R = U_{\text{TBM}} U_P, \quad U_P = \text{Diag.}(e^{i\gamma_1/2}, 1, e^{i\gamma_2/2}), \tag{12}$$

$$\gamma_{1,2} = -\arg(3re^{i\phi} \mp 1), \quad k = c/b. \tag{13}$$

Integrating out the heavy degrees of freedom, we get the effective light neutrino mass matrix, which is given by the seesaw relation [10]  $m_{\text{eff}} = -(m_\nu^d)^T M_R^{-1} m_\nu^d$ . This mass matrix is diagonalized by the TBM matrix. We obtained the light neutrino mass eigenvalues:

$$U_\nu^T m_{\text{eff}} U_\nu = \text{Diag.}(m_1, m_2, m_3) = -\text{Diag.}\left(\frac{x^2 v_u^2}{3c-b}, \frac{x^2 v_u^2}{2b}, \frac{x^2 v_u^2}{3c+b}\right), \quad (14)$$

$$m_1 = -\frac{x^2 v_u^2}{M_1}, \quad m_2 = -\frac{x^2 v_u^2}{M_3}, \quad m_3 = -\frac{x^2 v_u^2}{M_3}, \quad (15)$$

$$U_\nu = U_{\text{TBM}} \text{Diag.}(e^{-i\gamma_1/2}, 1, e^{-i\gamma_2/2}). \quad (16)$$

The light neutrino mass eigenvalues are simply the inverse of the heavy neutrino ones, apart from a minus sign and the global factors from  $m_\nu^d$ , as can be seen in Eq. (15). The light neutrino mass spectrum can be both normal or inverted hierarchy depending on the sign of  $\cos \phi$ . If  $\cos \phi < 0$  one has normal hierarchy (NH) light neutrino mass ordering and inverted hierarchy (IH) ordering if  $\cos \phi > 0$ .

In order to find the lepton mixing matrix we need to diagonalize the charged lepton mass matrix

$$m_\ell^D = U_{\ell c}^\dagger m_\ell U_\ell = \text{Diag.}(y_e u^2 t, y_\mu u, y_\tau) u v_d, \quad (17)$$

where the unitary  $U_\ell$  results to be unity matrix. As a result we get

$$U_{\text{PMNS}} = U_\ell^\dagger U_\nu \equiv U_\nu = e^{-i\gamma_1/2} U_{\text{TBM}} \text{Diag.}(1, e^{i\beta_1}, e^{i\beta_2}), \quad (18)$$

where  $\beta_1 = \gamma_1/2$ ,  $\beta_2 = (\gamma_1 - \gamma_2)/2$  are the Majorana  $CP$  violating phases.

To determine the lepton asymmetry leading to the determination of baryon asymmetry, we need to calculate the hermitian matrix  $H = (m_\nu^d)' (m_\nu^d)'^\dagger$  in the basis where the heavy neutrino mass matrix  $M_R$  is diagonal and real, and hence Dirac mass matrix  $m_\nu^d$  gets modified to be  $m_\nu^d \rightarrow (m_\nu^d)' = V_R^T m_\nu^d$ , then

$$H = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} x^2 v_u^2. \quad (19)$$

This indicates that all off-diagonal terms,  $H_{ij}$ , ( $i \neq j$ ), vanish so the  $CP$  asymmetry could not be generated and neither leptogenesis. As a result, leptogenesis does not work in this model at the LO.

Although the model shows  $\sin \theta_{13} = 0$  is consistent with the experimental upper bound, but non-zero and complex value of  $U_{e3}$  lead to the possibility of exploring  $CP$  violation in the leptonic sector and that is the main goal of future experiments. In order to obtain non-zero  $\theta_{13}$ , low energy  $CP$  violation and leptogenesis, we must consider the next leading order (NLO) corrections [8], the renormalization process or the disturbance process. In this work, we consider the breaking the  $S_4$  symmetry through not only spontaneously, but also explicitly by introducing a soft  $S_4$  symmetry breaking term in the Lagrangian of the model in order to obtain the above goals.

### III. $S_4$ SYMMETRY WITH A SOFT BREAKING

To study the capability of generating non-zero  $U_{e3}$  and deviation of  $\theta_{12}$  and  $\theta_{23}$  mixing angles compared to their TBM values as well as leptogenesis, we introduce a soft breaking term of the  $S_4$  model through a single real dimensionless parameter  $\epsilon$  in the (31) component of  $m_\nu^d$ , while keeping  $m_\ell$  and  $M_R$  unchanged. After  $SU(2)_L \otimes U(1)_Y$  symmetry breaking, explicit form of  $m_\nu^d$  is

$$m_\nu^d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ \epsilon & 1 & 0 \end{pmatrix} x v_u. \quad (20)$$

After the seesaw with unchanged  $m_\ell$  and  $M_R$ , the effective light neutrino mass matrix  $m_{\text{eff}}$  can be diagonalized by the modified TBM matrix  $\tilde{U}_{TB}$ , up to the first order of  $\epsilon$

$$m_{\text{eff}}^D = \tilde{U}_\nu^T m_{\text{eff}} \tilde{U}_\nu = \text{Diag.}(m_1, m_2, m_3) \quad (21)$$

$$= m_0 \text{Diag.} \left( \frac{2(1 - \frac{2}{3}\epsilon)}{1 - 6k \cos \phi + 9k^2}, 1 + \frac{2}{3\epsilon}, \frac{2}{1 + 6k \cos \phi + 9k^2} \right), \quad (22)$$

$$\tilde{U}_\nu = \tilde{U}_{TB} \cdot \text{Diag.}(e^{-i\alpha_1/2}, 1, e^{-i\alpha_2/2}), \quad (23)$$

$$\alpha_{1,2} = -\arg(3ke^{i\phi} \mp 1), \quad (24)$$

where  $M_0 = b$ ,  $k = \frac{|c|}{|b|}$ ,  $m_0 = \frac{x^2 v_u^2}{2M_0}$  and  $c = ce^{i\phi}$ . The modified TBM mixing matrix  $\tilde{U}_{TB}$  is obtained, also up to the first order of  $\epsilon$

$$\tilde{U}_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} \frac{(-2+3k)}{(1+\epsilon \frac{(-2+3k)}{9(1-k)})} & \frac{1}{\sqrt{3}} (1 - 2\epsilon \frac{(-2+3k)}{9(1-k)}) & -\epsilon \frac{1-2k+3k^2}{\sqrt{2}(1-3k)} \\ -\frac{1}{\sqrt{6}} - \frac{\epsilon(17+48k+27k^2)}{18\sqrt{6}(1-k)} & \frac{1}{\sqrt{3}} - \frac{\epsilon(11-18k+9k^2)}{9\sqrt{3}(1-4k+3k^2)} & -\frac{1}{\sqrt{2}} - \frac{\epsilon(1-k)(1+3k)}{2\sqrt{2}(1-3k)} \\ -\frac{1}{\sqrt{6}} + \epsilon \frac{1-24k+27k^2}{18\sqrt{6}(1-k)} & \frac{1}{\sqrt{3}} + \epsilon \frac{7-9k^2}{9\sqrt{3}(1-4k+3k^2)} & \frac{1}{\sqrt{2}} - \epsilon \frac{(1-k)(1+k)}{2\sqrt{2}(1-3k)} \end{pmatrix}. \quad (25)$$

Lepton mixing matrix with the soft-breaking is obtained as

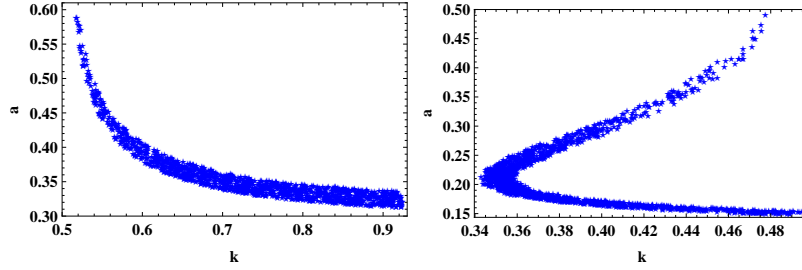
$$\tilde{U}_{\text{PMNS}} = \tilde{U}_\nu = e^{-i\alpha_1/2} \times \tilde{U}_{TB} \times \text{Diag.}(1, e^{i\tilde{\beta}_1}, e^{i\tilde{\beta}_2}), \quad (26)$$

where  $\tilde{\beta}_1 = \alpha_1/2$  and  $\tilde{\beta}_2 = (\alpha_1 - \alpha_2)/2$  are Majorana  $CP$  violating phases when there is a disturbance in the matrix  $m_\nu^d$ .

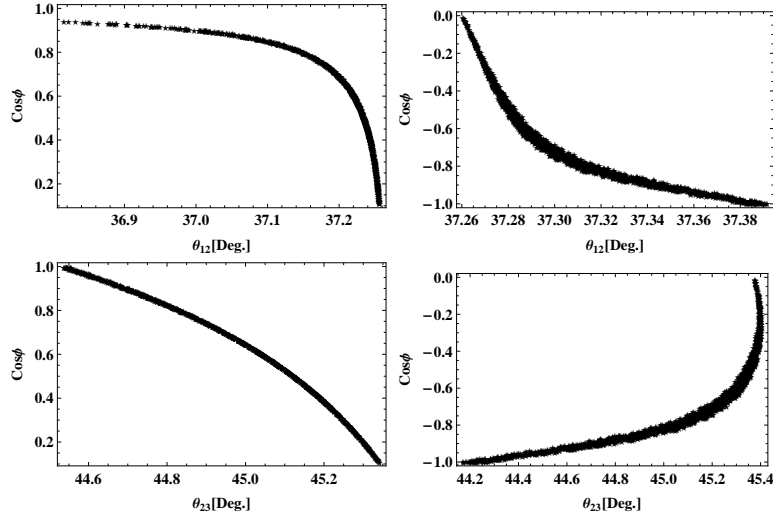
The deviation of mixing angles from their TBM values can be derived to be

$$\delta_{12} = \epsilon \frac{4(2-3k)}{27(1-k)}, \quad \delta_{23} = \epsilon \frac{(1-k)(1+3k)}{2(1-3k)}, \quad U_{e3} = -\epsilon \frac{1-2k+3k^2}{\sqrt{2}(1-3k)}, \quad (27)$$

where  $\delta_{12} = \sin^2 \theta_{12} - \frac{1}{3}$  v  $\delta_{23} = \sin^2 \theta_{23} - \frac{1}{2}$ . When there is a disturbance in the Dirac neutrino mass matrix, the considered model will generate  $\theta_{13}$  mixing angle and also will generate a non-zero lepton asymmetry. This is seen as the success of the  $S_4$  symmetry model with soft breaking. The correlations between  $b$  and  $k$  for normal mass spectrum (right plot) and inverted one (left plot) are presented in Fig. 1. Hereafter we always use the  $1\sigma$  confidence level of experimental data [3] for our numerical calculations. The lepton

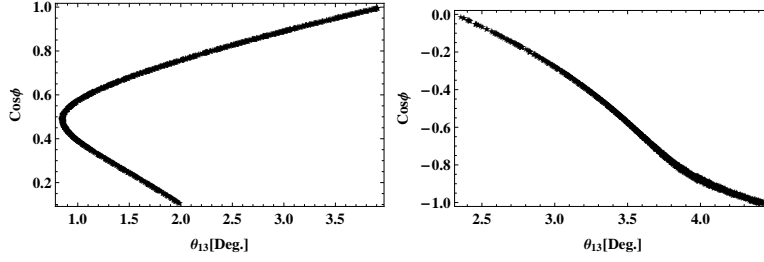


**Fig. 1.** Allowed parameter space constrained by current low energy neutrino data ( $M_0 = 10^{12}$  GeV,  $\tan \beta = 10$  and  $\epsilon = 0.1$ ). The right and left panels correspond to the NH and IH spectrum of light neutrino masses, respectively.



**Fig. 2.** The predictions of  $\theta_{12}$  and  $\theta_{23}$  with the soft breaking for NH (right panels) and IH (left panels) of neutrino mass spectrum ( $M_0 = 10^{12}$  GeV,  $\tan \beta = 10$  and  $\epsilon = 0.1$ ).

mixing angles  $\theta_{12}$  and  $\theta_{23}$  with the soft-breaking as a function of  $\cos \phi$  are shown in right panels for NH and left panels for IH of the Fig. 2. In both cases, the mixing angle  $\theta_{12}$  increases about  $2^\circ$  compared with its TBM values, whereas the mixing angle  $\theta_{23}$  oscillates with the amplitude about  $0.5^\circ$  around its TBM value. The Fig. 3 shows the non zero mixing angle  $\theta_{13}$  as a result of soft breaking for NH (right panel) and IH (left panel) of light neutrino mass spectrum. We can see that  $\theta_{13}$  is generated up to  $4.5^\circ$  and  $4^\circ$  for NH and IH, respectively; those values can be measured by new generation of neutrino oscillation experiments from which data are taken.



**Fig. 3.** The predictions of  $\theta_{12}$  after soft breaking for NH (right panel) and IH (left panel) of neutrino mass spectrum ( $M_0 = 10^{12}\text{GeV}$ ,  $\tan\beta = 10$  and  $\epsilon = 0.1$ ).

### IV. FLAVORED LEPTOGENESIS

In this section we will study the flavored leptogenesis through disturbance in the Dirac neutrino mass matrix. The lepton asymmetries which are produced by out-of-equilibrium decays of the heavy right handed neutrinos (RHN) in the early Universe, at temperatures above  $T \sim (1 + \tan^2\beta) \times 10^{12}$  GeV, do not distinguish lepton flavors (called conventional or unflavored leptogenesis). However, if the scale of the RHN masses are about  $M \leq (1 + \tan^2\beta) \times 10^{12}$  GeV, we have to take into account the lepton flavor effects and this is said as the flavored leptogenesis. In this case, the  $CP$  asymmetry generated by the decay of the RHN  $N_i$  given by [11, 12]

$$\epsilon_\alpha^i = \frac{1}{8\pi v_u^2 H_{ii}} \sum_{j \neq i} \text{Im} \left[ H_{ij} (\tilde{m}_\nu^d)_{i\alpha} (\tilde{m}_\nu^d)_{j\alpha}^* \right] g \left( \frac{M_j^2}{M_i^2} \right), \quad (28)$$

where the loop function  $g\left(\frac{M_j^2}{M_i^2}\right)$  is given by

$$g\left(\frac{M_j^2}{M_i^2}\right) \equiv g_{ij}(x) = \sqrt{x} \left[ \frac{2}{1-x} - \ln \frac{1+x}{x} \right]. \quad (29)$$

To study the leptogenesis through the decay of heavy RHN we need calculate in the basis where the RHN mass matrix is diagonal and real. In this basis, the Dirac neutrino mass matrix is modified as  $\tilde{m}_\nu^d = V_R^T m_\nu^d$ , then the hermitian matrix  $H = \tilde{m}_\nu^d (\tilde{m}_\nu^d)^\dagger$ , which is relevant to leptogenesis, is obtained as follows:

$$H = \left( \begin{array}{ccc} 1 - \frac{2\epsilon}{3} & \frac{\epsilon e^{i\gamma_1/2}}{3\sqrt{2}} & \frac{\epsilon e^{i(\gamma_1-\gamma_2)/2}}{\sqrt{3}} \\ \frac{\epsilon e^{-i\gamma_1/2}}{3\sqrt{2}} & 1 + \frac{2\epsilon}{3} & \frac{\epsilon e^{-i\gamma_2/2}}{\sqrt{6}} \\ \frac{\epsilon e^{-i(\gamma_1-\gamma_2)/2}}{\sqrt{3}} & \frac{\epsilon e^{i\gamma_2/2}}{\sqrt{6}} & 1 \end{array} \right) x^2 v_u^2. \quad (30)$$

We can see that the off-diagonal terms of the hermitian matrix is non-zero and complex, leading to leptogenesis.

In this model, the RHN masses are strongly hierarchical. For the inverted hierarchy case, the lightest RHN is of  $M_2$ . The flavored  $CP$ -asymmetries  $\varepsilon_2^\alpha$  are then obtained as

$$\varepsilon_2^e = \frac{(1 + \epsilon)m_0 M_0}{24\pi(1 + \frac{2}{3}\epsilon)v_u^2} \left[ \frac{(\epsilon^2 - 2\epsilon)}{3} \sin \gamma_1 \cdot g_{21}(x) - \epsilon^2 \sin \gamma_2 \cdot g_{23}(x) \right], \quad (31)$$

$$\varepsilon_2^\mu = \frac{\epsilon m_0 M_0}{24\pi(1 + \frac{2}{3}\epsilon)v_u^2} \left[ \frac{1}{3} \sin \gamma_1 \cdot g_{21}(x) - \sin \gamma_2 \cdot g_{23}(x) \right], \quad (32)$$

$$\varepsilon_2^\tau = \frac{\epsilon m_0 M_0}{24\pi(1 + \frac{2}{3}\epsilon)v_u^2} \left[ \frac{1}{3} \sin \gamma_1 \cdot g_{21}(x) + \sin \gamma_2 \cdot g_{23}(x) \right]. \quad (33)$$

For the normal hierarchy case, the lightest RHN is of  $M_3$ . The flavored  $CP$ -asymmetries  $\varepsilon_3^\alpha$  are obtained as

$$\varepsilon_3^e = \frac{\epsilon^2 m_0 M_0}{24\pi v_u^2} [(2 - \epsilon) \sin(\gamma_1 - \gamma_2) \cdot g_{31}(x) + (1 + \epsilon) \sin \gamma_2 \cdot g_{32}(x)], \quad (34)$$

$$\varepsilon_3^\mu = \frac{\epsilon m_0 M_0}{24\pi v_u^2} [\sin(\gamma_1 - \gamma_2) \cdot g_{31}(x) + \sin \gamma_2 \cdot g_{32}(x)], \quad (35)$$

$$\varepsilon_3^\tau = \frac{-\epsilon m_0 M_0}{24\pi v_u^2} [\sin(\gamma_1 - \gamma_2) \cdot g_{31}(x) + \sin \gamma_2 \cdot g_{32}(x)]. \quad (36)$$

Besides the  $CP$ -asymmetries, we have to calculate the washout factors due to the inverse decay of  $N_i$  into lepton flavor  $\alpha$  [13]

$$K_i^\alpha = \frac{\Gamma_i^\alpha}{H(M_i)} = \frac{\tilde{m}_i^\alpha}{m_*}, \quad (37)$$

where

$$\tilde{m}_i^\alpha = \frac{(\tilde{m}_\nu^d)_{i\alpha} (\tilde{m}_\nu^{d*})_{i\alpha}}{M_i}, \quad m_* = \frac{16\pi^{\frac{5}{2}}}{3\sqrt{5}} \sqrt{g_*} \frac{v_u^2}{M_{\text{Planck}}}, \quad (38)$$

where  $\Gamma_i^\alpha$  is partial decay rate of process  $N_i \rightarrow \ell^\alpha \varphi^\dagger$ ,  $H(M_i) \simeq \left( \frac{4\pi^3 g_*}{45} \right)^{\frac{1}{2}} \frac{M_i^2}{M_{\text{Planck}}}$  is Hubble constant at  $T = M_i$ ,  $g_* = 288.75$  is the effective number of freedom,  $v_u = v \sin \beta$ ,  $v \approx 174$  GeV,  $M_{\text{Planck}} = 1.22 \times 10^{19}$  GeV.

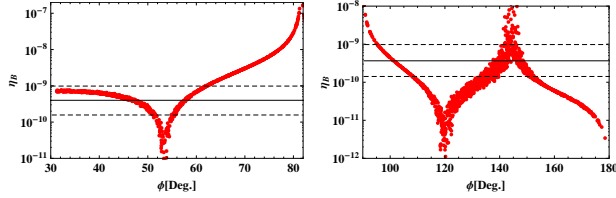
Each lepton asymmetry for a single flavor  $\varepsilon_i^\alpha$  is weighted differently by the corresponding washout parameter  $K_i^\alpha$ , and appears with different weight in the final formula for the baryon asymmetry [13],

$$\eta_B \simeq -10^{-2} \sum_{N_i} \left[ \varepsilon_i^e \kappa_i^e \left( \frac{93}{110} K_i^e \right) + \varepsilon_i^\mu \kappa_i^\mu \left( \frac{19}{30} K_i^\mu \right) + \varepsilon_i^\tau \kappa_i^\tau \left( \frac{19}{30} K_i^\tau \right) \right], \quad (39)$$

if the scale of heavy RHN masses are about  $M \leq (1 + \tan^2 \beta) \times 10^9$  GeV where the charged  $\mu$  and  $\tau$  Yukawa couplings are in equilibrium and all the flavors are to be treated separately. And

$$\eta_B \simeq -10^{-2} \sum_{N_i} \left[ \varepsilon_i^2 \kappa_i^2 \left( \frac{541}{761} K_i^2 \right) + \varepsilon_i^\tau \kappa_i^\tau \left( \frac{494}{761} K_i^\tau \right) \right], \quad (40)$$





**Fig. 4.** The prediction of flavored baryon asymmetry  $\eta_B$  as a function of  $\phi$  for the IH case (left plot) and NH case (right plot). The horizontal solid and dashed lines correspond to the experimental central value and phenomenologically-allowed region.

if  $(1 + \tan^2 \beta) \cdot 10^9 \text{ GeV} \leq M_i \leq (1 + \tan^2 \beta) \cdot 10^{12} \text{ GeV}$  where only the  $\tau$  Yukawa coupling is in equilibrium and is treated separately while the  $e$  and  $\mu$  flavors are indistinguishable,  $\varepsilon_i^2 = \varepsilon_i^e + \varepsilon_i^\mu$ ,  $K_i^2 = K_i^e + K_i^\mu$ . The wash-out factors are defined as

$$\kappa_i^\alpha \simeq \left( \frac{8.25}{K_i^\alpha} + \left( \frac{K_i^\alpha}{0.2} \right)^{1.16} \right)^{-1}. \quad (41)$$

Using Eqs. (39, 40, 41), the BAU for two cases are then obtained.

The predictions for  $\eta_B$  as a function of  $\phi$  are shown in Fig. 4 where we have used  $M_0 = 10^{12} \text{ GeV}$ , the supersymmetry parameter  $\tan \beta = 10$  and  $\epsilon = 0.1$  as inputs for all calculations. The horizontal solid and dashed lines correspond to the central value of the experiment result of BAU  $\eta_B^{\text{CMB}} = 6.1 \times 10^{-10}$  [14] and the phenomenologically allowed regions  $2 \times 10^{-10} \leq \eta_B \leq 10^{-9}$ , respectively. We can see that the model successfully explains the BAU through leptogenesis.

## V. CONCLUSION

We have studied the supersymmetric model with seesaw mechanism based on the  $S_4$  flavor symmetry group. In this model, the TBM form of the lepton mixing matrix naturally is obtained; and the combination of  $Y_\nu Y_\nu^\dagger$ , which is relevant for leptogenesis, is proportional to unity. This forbids the leptogenesis to occur. Therefore, for making leptogenesis viable we introduce a soft breaking term into the Dirac neutrino mass matrix of the model, which then naturally leads to successful leptogenesis. As a result of soft breaking, corrections to tribimaximal mixing would lead to deviations of lepton mixing angles and definite predictions for the low-energy  $CP$  violating phases. Especially, the non-zero of  $U_{e3}$  leading to the non-zero of the mixing angle  $\theta_{13}$  is predicted to be about  $\sim 4^\circ$  which can be measured by the neutrino oscillation experiments.

## REFERENCES

- [1] V.Simha and G.Steigman, [arXiv:astro-ph/0803.0586].
- [2] G.L Fogli, E. Lisi, A. Marrone, A. Palazzo, and A. M. Rotunno, in *Proceedings of NO-VE 2008, IV International Workshop on Neutrino Oscillations in Venice (Venice, Italy, April 15-18, 2008)*, edited by M. Baldo Ceolin (University of Padova, Papergraf Editions, Padova, Italy, 2008), p.21; G.L. Fogli, E. Lisi, A. Marrone, A. Palazzo, and A. M. Rotunno, *Phys. Rev. Lett.* **101**, 141801 (2008).

- [3] T. Schwetz, M. Tortola and J. W. F. Valle, *New J. Phys.* **10**, 113011 (2008) [arXiv:0808.2016 [hep-ph]]; M. Maltoni, T. Schwetz, arXiv:0812.3161 [hep-ph].
- [4] P. F. Harrison, D. H. Perkins, W. G. Scott, *Phys. Lett. B* **530**, (2002) 167 [arXiv:hep-ph/0202074]. P. F. Harrison, W. G. Scott, *Phys. Lett. B* **535**, (2002) 163 [arXiv:hep-ph/0203209]. P. F. Harrison, W. G. Scott, *Phys. Lett. B* **547**, (2002) 219. P. F. Harrison, W. G. Scott, *Phys. Lett. B* **557**, (2003) 76.
- [5] E. Ma and G. Rajasekaran, *Phys. Rev. D* **64** (2001) 113012 [arXiv:hep-ph/0106291]; K. S. Babu, E. Ma and J. W. F. Valle, *Phys. Lett. B* **552** (2003) 207 [arXiv:hep-ph/0206292]; G. Altarelli and F. Feruglio, *Nucl. Phys. B* **720** (2005) 64 [arXiv:hep-ph/0504165]; F. Bazzocchi, S. Kaneko and S. Morisi, *JHEP* **0803** (2008) 063 [arXiv:0707.3032 [hep-ph]].
- [6] F. Feruglio, C. Hagedorn, Y. Lin and L. Merlo, *Nucl. Phys. B* **775** (2007) 120 [arXiv:hep-ph/0702194]; M. C. Chen and K. T. Mahanthappa, *Phys. Lett. B* **652** (2007) 34 [arXiv:0705.0714 [hep-ph]]; P. H. Frampton and T. W. Kephart, *JHEP* **0709** (2007) 110 [arXiv:0706.1186 [hep-ph]]; G. J. Ding, arXiv:0803.2278 [hep-ph]; P. H. Frampton and S. Matsuzaki, arXiv:0902.1140 [hep-ph].
- [7] S. Pakvasa and H. Sugawara, *Phys. Lett. B* **82** (1979) 105; T. Brown, N. Deshpande, S. Pakvasa and H. Sugawara, *Phys. Lett. B* **141** (1984) 95; T. Brown, S. Pakvasa, H. Sugawara and Y. Yamanaka, *Phys. Rev. D* **30** (1984) 255; D. G. Lee and R. N. Mohapatra, *Phys. Lett. B* **329** (1994) 463 [arXiv:hep-ph/9403201]; E. Ma, *Phys. Lett. B* **632** (2006) 352 [arXiv:hep-ph/0508231]; Gui-Jun Ding, *Nucl. Phys. B* **827** (2010) 82 [arXiv:0909.2210 [hep-ph]].
- [8] Federica Bazzocchi, Luca Merlo, Stefano Morisi, *Phys. Rev. D* **80** (2009) 053003; F. Bazzocchi, L. Merlo, and S. Morisi, *Nucl. Phys.* **B816**, 204 (2009);
- [9] Tsukuba, Japan 1979, eds. A. Sawada and A. Sugamoto; R. N. Mohapatra, G. Senjanovic, *Phys. Rev. Lett.* **44**, (1980) 912. M. Fukugita and T. Yanagida, *Phys. Lett. B* **174**, (1986) 45; G. F. Giudice et al., *Nucl. Phys. B* **685** (2004) 89 [arXiv:hep-ph/0310123]; W. Buchmuller, P. Di Bari and M. Plumacher, *Annals Phys.* **315**, (2005) 305 [arXiv:hep-ph/0401240]; A. Pilaftsis and T. E. J. Underwood, *Phys. Rev. D* **72**, (2005) 113001 [arXiv:hep-ph/0506107].
- [10] P. Minkowski, *Phys. Lett. B* **67**, (1977) 421; M. Gell-Mann, P. Ramond and R. Slansky, *Proceedings of the Supergravity Stony Brook Workshop*, New York 1979, eds. P. Van Nieuwenhuizen and D. Freedman; T. Yanagida, *Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe*,
- [11] L. Covi, E. Roulet and F. Vissani, *Phys. Lett.* **B384**, (1996) 169; A. Pilaftsis, *Int. J. Mod. Phys. A* **14**, (1999) 1811 [arXiv:hep-ph/9812256].
- [12] T. Endoh, T. Morozumi, Z. Xiong, *Prog. Theor. Phys.* **111**, (2004) 123 [arXiv:hep-ph/0308276]; T. Fujihara, S. Kaneko, S. K. Kang, D. Kimura, T. Morozumi, M. Tanimoto, *Phys. Rev. D* **72**, (2005) 016006 [arXiv:hep-ph/0505076]; A. Abada, S. Davidson, A. Ibarra, F. X. Josse-Michaux, M. Losada and A. Riotto, *JHEP* **0609**, (2006) 010 [arXiv:hep-ph/0605281]; S. Blanchet and P. Di Bari, *JCAP* **0703**, (2007) 018 [arXiv:hep-ph/0607330]; S. Antusch, S. F. King and A. Riotto, *JCAP* **0611**, (2006) 011 [arXiv:hep-ph/0609038].
- [13] A. Abada, S. Davidson, F. X. Josse-Michaux, M. Losada and A. Riotto, *JCAP* **0604**, (2006) 004 [arXiv:hep-ph/0601083]; S. Antusch, S. F. King and A. Riotto, *JCAP* **0611**, (2006) 011 [arXiv:hep-ph/0609038].
- [14] WMAP Collaboration, D.N. Spergel *et al.*, *Astrophys. J. Suppl.* **148**, (2003) 175; M. Tegmark *et al.*, *Phys. Rev. D* **69**, (2004) 103501; C. L. Bennett *et al.*, *Astrophys. J. Suppl.* **148**, (2003) 1 [arXiv:astro-ph/0302207].

*Received 30-09-2012.*