

THE EFFECTS OF RENORMALIZATION EVOLUTION GROUP ON A S_4 FLAVOR SYMMETRY

DANG TRUNG SI, NGUYEN THANH PHONG

Department of Physics, College of Natural Science, Can Tho University

Abstract. *We study the supersymmetric seesaw model in a S_4 based flavor model. It has been shown that at the leading order, the model yields to exact tri-bimaximal pattern of the lepton mixing matrix, exact degenerate of the heavy right-handed neutrino (RHN) masses and zero lepton-asymmetry of the decays of RHNs. By considering the renormalization group evolution (RGE) from high energy scale (GUT scale) to low energy scale (seesaw scale), the off-diagonal terms in the combination of the Dirac Yukawa-coupling matrix can be generated and the degeneracy of heavy right-handed Majorana neutrino masses can be lifted. As a result, the flavored leptogenesis successfully realized. We also investigate the effects of RGE on the lepton mixing angles. The numerical result came out that the effects of RGE on leptonic mixing angles are negligible.*

I. INTRODUCTION

After the Big - Bang, through the mechanism of couple creation and annihilation, matter (baryon) and antimatter (anti-baryon) are formed. However, there is Baryon Asymmetry of the Universe (BAU). And the predictions of Big-Bang nucleosynthesis (BBN) and the experimental results from the Cosmic Microwave Background (CMB) showed Baryon Asymmetry of the Universe to be [1]

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} \simeq \frac{n_B}{n_\gamma} \simeq (2 - 10) \times 10^{-10} .$$

In addition, according to Standard Model (SM) of particle physics, neutrinos have no mass. However, from the results of neutrino oscillation experiments, neutrinos have mass and they are mixed. The two mentioned problems need satisfactory answers. Since the SM could not explain the BAU, and neutrinos are massless in SM, so the request is set to expand SM.

Also from the experimental data of neutrino oscillation experiments, Harrison et al. proposed the structure of lepton mixing matrix, called tri-bimaximal mixing(TBM) [2] $U_{PMNS} \equiv U_{TB}P_\nu$

$$U_{TB} = \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ 1 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{\sqrt{6}}{1} & \frac{\sqrt{3}}{1} & \frac{\sqrt{2}}{1} \\ -\frac{\sqrt{6}}{1} & \frac{\sqrt{3}}{1} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad (1)$$

where P_ν is a diagonal matrix of CP phases. In this structure, the lepton mixing angles are given as $\theta_{12} \simeq 35^0, \theta_{23} = 45^0$ and $\theta_{13} = 0$. However, the current new generation of

neutrino oscillation experiments have gone into a new phase of precise determination of mixing angles and squared-mass differences [3], where the mixing angles θ_{12}, θ_{23} have small deviations from their TBM values, and maybe the most interesting thing is the none-zero of the angle θ_{13} . Therefore, the TBM pattern needs to be modified.

The issues of neutrino mass, TBM structure, BAU can be explained by many extended SM with seesaw mechanism. It seem to be the most interesting way is to add some discrete symmetry group (flavor symmetry group) to the gauge group of SM. Among the flavor symmetry groups, the model builder recently focus on A_4, T' and S_4 groups. The common features of these models are: they exist at high energy level, they give rise to the TBM structure and they cannot explain BAU at the leading order. Therefore, in able to explain all above problems, one need to take into account the contributions of higher orders, or considering the soft breaking terms... In this work, we consider the effects of renormalization evolution group (RGE) on the lepton mixing angles and leptogenesis (to explain BAU) of a S_4 model.

The rest of this work is organized as follows. Next section we review the S_4 model. The RGE is given in section 3. Section 4 is devoted to the effects of RGE on leptogenesis and lepton mixing angles. We summarise our work in the last section.

II. MODEL S_4 AND LEPTOGENESIS

In this work, we study the S_4 flavour symmestry model which proposed in [4]. This model possesses flavor symmetry group $G_f = S_4 \times Z_3 \times Z_4$, where the three factors play different roles. The S_4 controls the mixing angles, the Z_3 guarantees the misalignment in flavor space between neutrino and charged lepton eigenstates, and the Z_4 is crucial for eliminating unwanted couplings and reproducing observed mass hierarchies. In this framework the mass hierarchies are controlled by spontaneously breaking of the flavor symmetry instead of the Froggatt-Nielsen mechanism [5]. The matter fields of lepton sector and flavons under G_f are assigned as in Table 1. The vacuum Expectation Value

Table 1. Representations of the matter fields of lepton sector and flavons under $S_4 \times Z_3 \times Z_4$.

Field	ℓ	e^c	μ^c	τ^c	ν^c	$h_{u,d}$	φ	χ	ϑ	η	ϕ	Δ
S_4	$\mathbf{3}_1$	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{1}_1$	$\mathbf{3}_1$	$\mathbf{1}_1$	$\mathbf{3}_1$	$\mathbf{3}_2$	$\mathbf{1}_2$	$\mathbf{2}$	$\mathbf{3}_1$	$\mathbf{1}_2$
Z_3	ω	ω^2	ω^2	ω^2	1	1	1	1	1	ω^2	ω^2	ω^2
Z_4	1	i	-1	-i	1	1	i	i	1	1	1	-1

(VEV) alignment of flavons are assumed as follows $\langle \varphi \rangle = (0, v_\varphi, 0)$; $\langle \chi \rangle = (0, v_\chi, 0)$; $\langle \vartheta \rangle = v_\vartheta$; $\langle \eta \rangle = (v_\eta, v_\eta)$; $\langle \phi \rangle = (v_\phi, v_\phi, v_\phi)$; $\langle \Delta \rangle = v_\Delta$.

The superpotential for the lepton sector reads

$$\begin{aligned}\omega_\ell &= \frac{y_{e1}}{\Lambda^3} e^c (\ell\varphi)_{11} (\varphi\varphi)_{11} h_d + \frac{y_{e2}}{\Lambda^3} e^c ((\ell\varphi)_2 (\varphi\varphi)_2)_{11} h_d + \frac{y_{e3}}{\Lambda^3} e^c ((\ell\varphi)_{31} (\varphi\varphi)_{31})_{11} h_d \\ &+ \frac{y_{e4}}{\Lambda^3} e^c ((\ell\chi)_2 (\chi\chi)_2)_{11} h_d + \frac{y_{e5}}{\Lambda^3} e^c ((\ell\chi)_{31} (\chi\chi)_{31})_{11} h_d + \frac{y_{e6}}{\Lambda^3} e^c (\ell\varphi)_{31} (\chi\chi)_{11} h_d \\ &+ \frac{y_{e7}}{\Lambda^3} e^c ((\ell\varphi)_2 (\chi\chi)_2)_{11} h_d + \frac{y_{e8}}{\Lambda^3} e^c ((\ell\varphi)_{31} (\chi\chi)_{31})_{11} h_d + \frac{y_{e9}}{\Lambda^3} e^c ((\ell\varphi)_2 (\varphi\varphi)_2)_{11} h_d \\ &+ \frac{y_{e10}}{\Lambda^3} e^c ((\ell\chi)_{31} (\varphi\varphi)_{31})_{11} h_d + \frac{y_\mu}{2\Lambda^2} \mu^c (\ell(\varphi\chi)_{32})_{12} h_d + \frac{y_\tau}{\Lambda} \tau^c (\ell\chi)_{11} h_d + \dots\end{aligned}\quad (2)$$

$$\omega_\nu = \frac{y_{\nu 1}}{\Lambda} (\nu^c \ell)_2 \eta)_{11} h_u + \frac{y_{\nu 2}}{\Lambda} (\nu^c \ell)_{31} \phi)_{11} h_u + \frac{1}{2} M (\nu^c \nu^c)_{11} + \dots\quad (3)$$

With this setting the mass matrix for the charged leptons is

$$m_\ell = \text{Diag.} (y_e \frac{v_\nu^3}{\Lambda^3}, y_\mu \frac{v_\varphi v_\chi}{\Lambda^2}, y_\tau \frac{v_\varphi}{\Lambda}) v_d, \quad (4)$$

where all the components are assumed to be real. The neutrino sector gives rise to the following Dirac and RH- Majorana mass matrices

$$m_\nu^d = e^{i\alpha_1} \begin{pmatrix} 2be^{i\phi} & a - be^{i\phi} & a - be^{i\phi} \\ a - be^{i\phi} & a + 2be^{i\phi} & -be^{i\phi} \\ a - be^{i\phi} & -be^{i\phi} & a + 2be^{i\phi} \end{pmatrix} v_u, \quad (5)$$

$$M_R = \begin{pmatrix} M & 0 & 0 \\ 0 & 0 & M \\ 0 & M & 0 \end{pmatrix}, \quad (6)$$

where the quantity M is also supposed to be real and positive. The phase $\phi \equiv \alpha_2 - \alpha_1$, where α_1, α_2 are denoted as the arguments of $y_{\nu 1}, y_{\nu 2}$ respectively, is the only physical phase survived because the global phase α_1 can be rotated away. The real and positive components a and b are defined as

$$a = |y_{\nu 1}| \frac{v_\eta}{\Lambda}; \quad b = |y_{\nu 2}| \frac{v_\phi}{\Lambda}; \quad v_u = v \sin \beta; \quad v = 174 \text{ GeV}.$$

After seesawing, the effective light neutrino mass matrix is obtained from seesaw formula $m_{\text{eff}} = -(m_\nu^d)^T M_R^{-1} m_\nu^d$, which can be diagonalized by the TBM matrix

$$U_\nu^T m_{\text{eff}} U_\nu = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} = \text{Diag.} (m_1, m_2, m_3) \quad (7)$$

where

$$U_\nu = e^{-i\gamma_1/2} U_{\text{TB}} \text{Diag.} (1, e^{i\beta_1}, e^{i\beta_2}), \quad (8)$$

$$\gamma_1 = \arg[-(a - 3be^{i\phi})^2], \quad \gamma_2 = \arg[(a + 3be^{i\phi})^2], \quad (9)$$

$$m_1 = m_0 [1 - 6r \cos \phi + 9r^2], \quad m_2 = 4m_0, \quad (10)$$

$$m_3 = m_0 [1 + 6r \cos \phi + 9r^2], \quad m_0 = \frac{v_u^2 a^2}{M}, \quad r = \frac{b}{a}. \quad (11)$$

There are two possible orderings in the masses of effective light neutrinos depending on the sign of $\cos \phi$: the normal hierarchy (NH) corresponding to $\cos \phi > 0$ while the inverted hierarchy (IH) corresponding to $\cos \phi < 0$. In this work we only study the NH case.

The neutrino mass spectrum for NH is shown in the figure 1. Hereafter we have used the super symmetric parameter $\tan \beta = 30$, $M = 10^6$ GeV, $\cos \phi > 0$ and the experimental results [3] at 3σ confidential level as the universal inputs for numerical calculation.

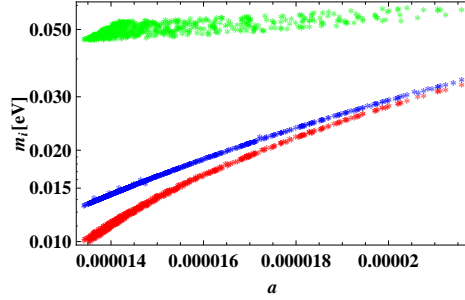


Fig. 1. Comparison of the neutrino mass in eigenstate m_1, m_2, m_3 as a function of a .

An important physical quantity is the effective mass $|\langle m_{ee} \rangle|$ in the neutrinoless double beta decay $0\nu\beta\beta$:

$$\begin{aligned} |\langle m_{ee} \rangle| &= |m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2| = \frac{1}{3} |2m_1 + m_2 e^{2i\beta_1}| \\ &= \frac{v_u^2 a^2}{M} \sqrt{1 - 4r \cos \phi + 2r^2(2 + 3 \cos 2\phi) - 12r^3 \cos \phi + 9r^4}. \end{aligned} \quad (12)$$

The prediction of $|\langle m_{ee} \rangle|$ is plotted in figure 2. We can see that $|\langle m_{ee} \rangle|$ is totally stayed in the measurable region of in running neutrinoless double beta decay experiments.

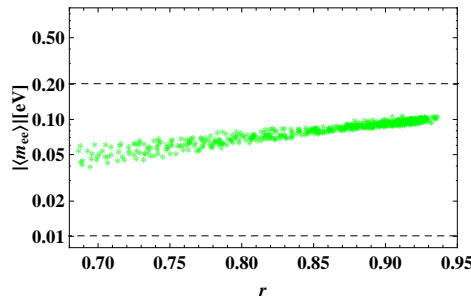


Fig. 2. Prediction of $|\langle m_{ee} \rangle|$ as a function of r .

To calculate leptogenesis, we need to go into the basis where M_R is real and diagonal. In a basis where the charged current is flavor diagonal, the right handed neutrino mass matrix M_R is diagonalized as

$$V_R^T M_R V_R = \text{Diag}(M, M, -M), \quad (13)$$

where

$$V_R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (14)$$

In this basis, the Dirac mass matrix m_ν^d gets the form

$$m_\nu^d \longrightarrow V_R^T m_\nu^d, \quad (15)$$

then the coupling of N_i with leptons and scalar, Y_ν , is given by

$$Y_\nu = \begin{pmatrix} 2be^{i\phi} & \frac{a - be^{i\phi}}{\sqrt{2}} & \frac{a - be^{i\phi}}{\sqrt{2}} \\ \sqrt{2}(a - be^{i\phi}) & \frac{(a + be^{i\phi})}{\sqrt{2}} & \frac{(a + be^{i\phi})}{\sqrt{2}} \\ 0 & \frac{-(a + 3be^{i\phi})}{\sqrt{2}} & \frac{a + 3be^{i\phi}}{\sqrt{2}} \end{pmatrix}. \quad (16)$$

Concerning with CP violation, we notice that the CP phase ϕ coming from m_ν^d obviously takes part in low-energy CP violation as the Majorana phases β_1 and β_2 which are the only sources of low-energy CP violation in the leptonic sector. On the other hand, leptogenesis is associated with both Y_ν itself and the combination of Yukawa coupling matrix, $H \equiv Y_\nu Y_\nu^\dagger$, which is given as

$$H = \begin{pmatrix} 2a^2 + 6b^2 - 4ab \cos \phi & \sqrt{2}(a^2 - 3b^2 + 2ab \cos \phi) & 0 \\ \sqrt{2}(a^2 - 3b^2 + 2a \cos \phi) & 3a^2 + 3b^2 - 2ab \cos \phi & 0 \\ 0 & 0 & a^2 + 9b^2 + 6ab \cos \phi \end{pmatrix}. \quad (17)$$

We can see that H is a real matrix, so leptogenesis without considering the contribution of lepton generations does not occur. Then, leptogenesis considering the contribution of lepton generation can work but the degeneracy $M_1 = M_2 = M_3$ must be lifted, and this is done through the process of renormalization group evolution (RGE).

III. THE RENORMALIZATION OF THE MODEL S_4

The RGE of heavy neutrino mass matrix M_R is given as [6]

$$\frac{d\mathbf{M}_R}{dt} = q[(\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger) \mathbf{M}_R + \mathbf{M}_R (\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger)^T] \quad (18)$$

where $t = \frac{1}{16\pi^2} \ln \frac{M}{\Lambda'}$, and M is an arbitrary renormalization scale. The cutoff scale Λ' can be regarded as the G_f breaking scale $\Lambda' = \Lambda$ and assumed to be of order of the GUT scale, $\Lambda' \sim 10^{16}$ GeV. It is convenient to write Eq. (18) in the basis where M_R is real and diagonal. At first we diagonalize M_R

$$V_R^T \mathbf{M}_R V_R = \text{Diag}(M_1, M_2, M_3). \quad (19)$$

Since M_R depends on energy scale so V also depends on energy scale too

$$\frac{dV_R}{dt} = V_R A; \quad \frac{dV_R^T}{dt} = A^T V_R^T, \quad (20)$$

$$A^\dagger = -A; \quad A_{ii} = 0, \quad (21)$$

A is anti-Hermitian matrix. The RGE of M_R in the new basis

$$\frac{dM_i}{dt} \delta_N^{ij} = (A^T M)_{ij} + (MA)_{ij} + 2 \left[V_R^T (\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger) \mathbf{M}_R V_R + V_R^T \mathbf{M}_R (\mathbf{Y}_\nu^* \mathbf{Y}_\nu^T) V_R \right]_{ij}, \quad (22)$$

Using

$$Y_\nu \equiv V_R^T \mathbf{Y}_\nu; \quad Y_\nu^\dagger \equiv \mathbf{Y}_\nu^\dagger V_R^*; \quad Y_\nu^T \equiv \mathbf{Y}_\nu^T V_R; \quad Y_\nu^* \equiv V_\nu^\dagger \mathbf{Y}_\nu^*, \quad (23)$$

$$\frac{dM_i}{dt} \delta_N^{ij} = A_{ij}^T M_j + M_i A_{ij} + 2 \left[(Y_\nu Y_\nu^\dagger)_{ij} M_j + M_i (Y_\nu Y_\nu^\dagger)_{ij}^* \right], \quad (24)$$

the diagonal part is obtained

$$\frac{dM_i}{dt} = 4M_i (Y_\nu Y_\nu^\dagger)_{ii}. \quad (25)$$

The heavy Majorana mass splitting generated through the relevant RG evolution is thus calculated to be

$$\delta_N^{ij} = 1 - \frac{M_j}{M_i} \simeq 4(H_{ii} - H_{jj})t. \quad (26)$$

Off-diagonal part of Eq. (24) leads to

$$A_{ij} = 2 \frac{M_i + M_j}{M_j - M_i} \text{Re}[(Y_\nu Y_\nu^\dagger)_{ij}] + 2i \frac{M_j - M_i}{M_j + M_i} \text{Im}[(Y_\nu Y_\nu^\dagger)_{ij}]. \quad (27)$$

The RG equation for Y_ν in the basis of diagonal M_R is given by

$$\frac{d\mathbf{Y}_\nu}{dt} = \mathbf{Y}_\nu \left\{ (T - 3g_2^2 - \frac{3}{5}g_1^2) + \mathbf{Y}_\ell^\dagger \mathbf{Y}_\ell + 3\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \right\} \quad (28)$$

Using (23), we have

$$\frac{dY_\nu}{dt} = A^T Y_\nu + Y_\nu [(T - 3g_2^2 - \frac{3}{5}g_1^2) + \mathbf{Y}_\ell^\dagger \mathbf{Y}_\ell + 3Y_\nu^\dagger Y_\nu], \quad (29)$$

$$\frac{dY_\nu^\dagger}{dt} = Y_\nu^\dagger A^* + [(T - 3g_2^2 - \frac{3}{5}g_1^2) + \mathbf{Y}_\ell^\dagger \mathbf{Y}_\ell + 3Y_\nu^\dagger Y_\nu] Y_\nu^\dagger. \quad (30)$$

Finally, we obtain the RG equation for H responsible for the leptogenesis:

$$\frac{dH}{dt} = 2Y_\nu (T - 3g_2^2 - \frac{3}{5}g_1^2) Y_\nu^\dagger + 2Y_\nu (\mathbf{Y}_\ell^\dagger \mathbf{Y}_\ell) Y_\nu^\dagger + 6H^2 + A^T H + HA^*. \quad (31)$$

Since the τ Yukawa coupling constant dominates the evolution of H so it implies that RG effect due to the τ -Yukawa charged-lepton contribution takes the leading order

$$H_{ij}(t) = 2y_\tau^2 (Y_\nu)_{i3} (Y_\nu)_{j3}^* \times t. \quad (32)$$

IV. LEPTOGENESIS OF THE MODEL VIA RENORMALIZATION PROCESS

When the heavy right handed neutrino (RHN) mass are almost degenerate, leptogenesis receives the contributions from the decays of all generations of HRN. The CP asymmetry generated by the decay of N_i heavy RH neutrino is given by [7]

$$\varepsilon_i^\alpha = \sum_{j \neq i} \frac{\text{Im}[H_{ij}(Y_\nu)_{i\alpha}(Y_\nu)_{j\alpha}^*]}{16\pi H_{ii}\delta_N^{ij}} \left(1 + \frac{\Gamma_j^2}{4M_j\delta_N^{ij2}}\right), \quad (33)$$

from which we can obtain explicitly of ε_i^α as

$$\begin{aligned} \varepsilon_1^e &\simeq -2\varepsilon_1^\mu = -2\varepsilon_1^\tau = \frac{-r \sin \phi}{32\pi(1+3r^2-2r \cos \phi)t}, \\ \varepsilon_2^e &\simeq -2\varepsilon_2^\mu = -2\varepsilon_2^\tau = \frac{-r \sin \phi}{16\pi(3+3r^2-2r \cos \phi)t}, \\ \varepsilon_3^e &= \varepsilon_3^\mu = \varepsilon_3^\tau \simeq 0. \end{aligned} \quad (34)$$

Once the initial values of ε_i^α are fixed, the final result of BAU, η_B , can be given by solving a set of flavor dependent Boltzmann equations including the decay, inverse decay, and scattering processes as well as the nonperturbative sphaleron interaction. In order to estimate the wash-out effects, one introduces parameters K_i^α which are the wash-out factors due to the inverse decay of Majorana neutrino N_i into the lepton flavor α . The explicit form of K_i^α is given by [8]

$$K_i^\alpha = \frac{\Gamma_i^\alpha}{H(M_i)} = (Y_\nu^\dagger)_{\alpha i}(Y_\nu)_{i\alpha} \frac{v_u^2}{m_* M_i}, \quad (35)$$

where Γ_i^α is the partial decay width of N_i into the lepton flavors and Higgs scalars; $H(M_i) = (4\pi^3 g_*/45)^{1/2} M_i^2/M_{Pl}$, with the Planck mass $M_{Pl} = 1.22 \times 10^{19}$ GeV and the effective number of degrees of freedom $g_* = 228.75$, is the Hubble parameter at temperature $T = M_i$; and the equilibrium neutrino mass $m_* \simeq 10^{-3}$.

Each lepton asymmetry for a single flavor ε_i^α is weighted differently by the corresponding washout parameter K_i^α , appearing with a different weight in the final formula for the baryon asymmetry [9]

$$\eta_B \simeq -10^{-2} \sum_{N_i} \left[\varepsilon_i^e \kappa_i^e \left(\frac{93}{110} K_i^e \right) + \varepsilon_i^\mu \kappa_i^\mu \left(\frac{19}{30} K_i^\mu \right) + \varepsilon_i^\tau \kappa_i^\tau \left(\frac{19}{30} K_i^\tau \right) \right], \quad (36)$$

provided that the scale of heavy RH neutrino masses is about $M \leq (1 + \tan^2 \beta) \times 10^9$ GeV where the μ and τ . Yukawa couplings are in equilibrium and all the flavors are to be treated separately. And

$$\eta_B \simeq -10^{-2} \sum_{N_i} \left[\varepsilon_i^2 \kappa_i^2 \left(\frac{541}{761} K_i^2 \right) + \varepsilon_i^\tau \kappa_i^\tau \left(\frac{494}{761} K_i^\tau \right) \right] \quad (37)$$

is given if $(1 + \tan^2 \beta) \times 10^9$ GeV $\leq M \leq (1 + \tan^2 \beta) \times 10^{12}$ GeV where only the τ Yukawa coupling is in equilibrium and treated separately while the e and μ flavors are

indistinguishable. Here $\varepsilon_i^2 = \varepsilon_i^e + \varepsilon_i^\mu$; $\kappa_i^2 = \kappa_i^e + \kappa_i^\mu$; $K_i^2 = K_i^e + K_i^\mu$

$$\kappa_i^\alpha \simeq \left(\frac{8.25}{K_i^\alpha} + \left(\frac{K_i^\alpha}{0.2} \right)^{1.16} \right)^{-1}. \quad (38)$$

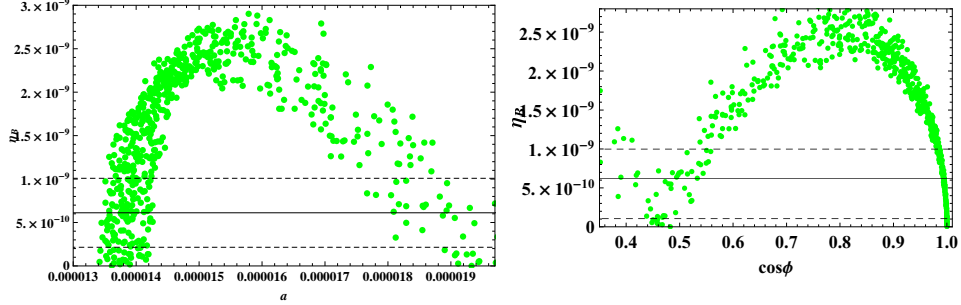


Fig. 3. Prediction of η_B as a function of a and $\cos \phi$.

The prediction of η_B is shown in figure 3 as a function of a (left panel) and of $\cos \phi$ (right panel). The solid horizontal line and the dotted horizontal lines correspond to the experimental value of baryon asymmetry, $\eta_B^{\text{CMB}} = 6.1 \times 10^{-10}$ [10], and phenomenologically allowed regions $2 \times 10^{-10} \leq \eta_B \leq 10^{-9}$. We can see that, under the effects of RGE, the BAU is successfully explained through flavored leptogenesis (leptogenesis considering the separately contributions of flavor generations).

The predictions of lepton mixing angles θ_{12} (left panel), θ_{13} (middle panel) and θ_{23} (right panel) are plotted in figures 4. The deviations of these angles from their TBM values are negligible and this agrees with recent theoretically studies of the effects of RGE on lepton mixing angles of flavor symmetry groups [11].

V. SUMMARY

We study the S_4 models in the context of a seesaw model which naturally leads to the TBM form of the lepton mixing matrix. In this model, the combination $Y_\nu Y_\nu^\dagger$ is real matrix and the heavy right-handed neutrino masses are exact degenerate, which reasons forbid the leptogenesis (both conventional and flavored) to occur. Therefore, for

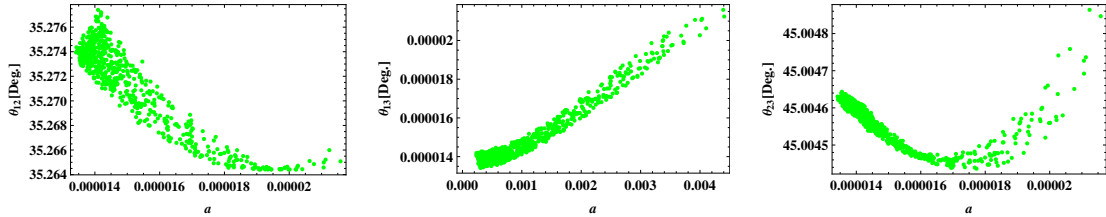


Fig. 4. Prediction of θ_{12} , θ_{13} and θ_{23} as a parameter a .

leptogenesis making viable, the imaginary parts of the off-diagonal terms of $Y_\nu Y_\nu^\dagger$ have to be generated and the degenerate have to be removed. This can be easily achieved by renormalization group effects from high energy scale to low energy scale which then naturally leads to a successful leptogenesis.

We have also studied the effects of RGE on the lepton mixing matrix with the hope that the generation of θ_{13} is large enough that it can be measured by in-running neutrino oscillation experiments. However, it came out that the effects of RGE on lepton mixing angles are negligible.

REFERENCES

- [1] E. W. Kolb and M. S. Turner, The Early Universe, Westview Press, 1994.
- [2] P. F. Harrison, D. H. Perkins, W. G. Scott, Phys. Lett. B **530**, (2002) 167 [arXiv:hep-ph/0202074]. P. F. Harrison, W. G. Scott, Phys. Lett. B **535**, (2002) 163 [arXiv:hep-ph/0203209]. P. F. Harrison, W. G. Scott, Phys. Lett. B **547**, (2002) 219. P. F. Harrison, W. G. Scott, Phys. Lett. B **557**, (2003) 76.
- [3] T. Schwetz, M. Tortola and J. W. F. Valle, New J. Phys. **10**, 113011 (2008) [arXiv:0808.2016 [hep-ph]]; M. Maltoni, T. Schwetz, arXiv:0812.3161 [hep-ph].
- [4] Gui-Jun Ding, Nucl. Phys. B **827**, 82-111 (2010) [arXiv:0909.2210[hep-ph]].
- [5] C. D. Froggatt and H. B. Nielsen, Nucl. Phys. B **147** (1979) 277.
- [6] J. A. Casas et al., Nucl. Phys. B **573**, (2000) 652 [arXiv: hep-ph/9910420]; Nucl. Phys. B **569**, (2000) 82 [arXiv: hep-ph/9905381].
- [7] S. Pascoli, S. T. Petcov and A. Riotto, Nucl. Phys. B **774**, 1 (2007) [arXiv: hep-ph/0611338].
- [8] S. Antusch, S. F. King and A. Riotto, J. Cosmol. Astropart. Phys. **11** (2006) 011.
- [9] A. Abada, S. Davidson, F. X. Josse-Michaux, M. Losada and A. Riotto, JCAP **0604**, (2006) 004 [arXiv:hep-ph/0601083]; S. Antusch, S. F. King and A. Riotto, JCAP **0611**, (2006) 011 [arXiv:hep-ph/0609038].
- [10] WMAP Collaboration, D.N. Spergel et al., Astrophys. J. Suppl. **148**, (2003) 175; M. Tegmark et al., Phys. Rev. D **69**, (2004) 103501; C. L. Bennett et al., Astrophys. J. Suppl. **148**, (2003) 1 [arXiv:astro-ph/0302207].
- [11] Gui-Jun Ding, Dong-Mei Pan. Eur.Phys.J. **C71** (2011) 1716 [arXiv: 1011.5306 [hep-ph]].

Received 30-09-2012.