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IMPACT OF THE EXTERNAL MAGNETIC FIELD AND THE CONFINEMENT OF PHONONS ON THE NONLINEAR ABSORPTION COEFFICIENT OF A STRONG ELECTROMAGNETIC WAVE BY CONFINED ELECTRONS IN COMPOSITIONAL SUPERLATTICES

Hoang Dinh Trien, Le Thai Hung, Vu Thi Hong Duyen, Nguyen Quang Bau
Department of Physics, University of Natural Sciences, Hanoi National University
Nguyen Thu Huong, Nguyen Vu Nhan
Academy of Air Defense and Air Force, Son Tay, Hanoi, Vietnam

Abstract. *Impact of the external magnetic field and the confinement of phonon on the nonlinear absorption coefficient (NAC) of a strong electromagnetic wave (EMW) by confined electrons in compositional superlattices is theoretically studied by using the quantum transport equation for electrons. The formula which shows the dependence of the NAC on the energy ($\hbar\Omega$), the intensity E_0 of EMW, the energy ($\hbar\Omega_B$) of external magnetic field and quantum number m characterizing confined phonon is obtained. The analytic expressions are numerically evaluated, plotted and discussed for a specific of the $GaAs - Al_{0.3}Ga_{0.7}As$ compositional superlattices. The results show clearly the difference in the spectrums and values of the NAC in this case from those in the case without the impact of the external magnetic field and the confinement of phonon.*

I. INTRODUCTION

Recently, there are more and more interests in studying the behavior of low-dimensional system, such as compositional superlattices, doped superlattices, compositional superlattices, quantum wires and quantum dots. The confinement of electrons and phonons in low-dimensional systems considerably enhances the electron mobility and leads to unusual behaviors under external stimuli. Many attempts have been conducted dealing with these behaviors, for examples, electron-phonon interaction effects in two-dimensional electron gases (graphene, surfaces, quantum wells) [1, 2, 3]. The dc electrical conductivity [4, 5], the electronic structure [6], the wavefunction distribution [7] and the electron subband [8] in quantum wells have been calculated and analyzed. The problems of the absorption coefficient for a weak electromagnetic wave in quantum wells [9], in doped superlattices [10] have also been investigated by using Kubo-Mori method. The nonlinear absorption of a strong electromagnetic wave in low-dimensional systems have been studied by using the quantum transport equation for electrons [11]. However, the nonlinear absorption of a strong electromagnetic wave in compositional superlattices in the presence of an external magnetic field with influences of confined phonons is still open question. In this paper, we consider quantum theories of the nonlinear absorption of a strong electromagnetic wave caused by confined electrons in the presence of an external magnetic field in low dimensional systems taking into account the effect of confined phonons. The problem is considered for the case of electron-optical phonon scattering. Analytical expressions of the nonlinear absorption coefficient of a strong electromagnetic wave caused by confined electrons in the presence of an external magnetic field in low-dimensional systems are obtained. The analytical expressions are numerically calculated and discussed to show the differences in comparison with the case of absence of an external magnetic with a specific of the $GaAs - Al_{0.3}Ga_{0.7}As$ compositional superlattices.

II. CALCULATIONS OF THE NONLINEAR ABSORPTION COEFFICIENT OF A STRONG ELECTROMAGNETIC WAVE BY CONFINED ELECTRONS IN A COMPOSITIONAL SUPERLATTICE IN THE PRESENCE OF A MAGNETIC FIELD IN CASE OF CONFINED PHONONS

It is well known that in the compositional superlattices, the motion of electrons is restricted in one dimension, so that they can flow freely in two dimensions. In this article, we assume that the quantization direction is in z direction and only consider intersubband transitions ($n \neq n'$) and intrasubband transitions (n=n'). We consider a compositional superlattice with a magnetic field applied perpendicular to its barriers. The Hamiltonian of the confined electron optical phonon system in a compositional superlattice in the presence of an external magnetic field $\vec{B}$ in the second quantization representation can be written as follows [12]:

$$H = \sum_{n,N,\vec{k}_\perp} \varepsilon_{n,N}^{H}\left(\vec{k}_\perp - \frac{e}{\hbar c}\vec{A}(t)\right) a^{+}_{n,N,\vec{k}_\perp} a_{n,N,\vec{k}_\perp} + \sum_{\vec{q}_\perp,m} \hbar\omega_{m,\vec{q}_\perp} b^{+}_{m,\vec{q}_\perp} b_{m,\vec{q}_\perp} +$$
$$+ \sum_{n,n',\vec{k}_\perp} \sum_{m,\vec{q}_\perp} C_{m,\vec{q}_\perp} I^{m}_{n,n'}(q_z)\, J_{N,N'}\left(\frac{1}{2}a_c^2 q_\perp^2\right) a^{+}_{n',N',\vec{k}'_\perp+\vec{q}_\perp} a_{n,N,\vec{k}_\perp}\left(b_{m,\vec{q}_\perp} + b^{+}_{m,-\vec{q}_\perp}\right) \quad (1)$$

where N is the Landau level index (N = 0, 1, 2 ...), n (n = 1, 2, 3, ...) denotes the quantization of the energy spectrum in the z direction, $(n, N, \vec{k}_\perp)$ and $(n', N', \vec{k}_\perp + \vec{q}_\perp)$ are electron states before and after scattering, $\vec{k}_\perp(\vec{q}_\perp)$ is the in-plane (x, y) wave vector of the electron (phonon), $a^{+}_{n,N,\vec{k}_\perp}$, $a_{n,N,\vec{k}_\perp}$ ($b^{+}_{m,\vec{q}_\perp}$, $b_{m,\vec{q}_\perp}$) are the creation and the annihilation operators of the confined electron (phonon), respectively, $\vec{A}(t)$ is the vector potential of an external electromagnetic wave $\vec{A}(t) = e\vec{E}_o \sin(\Omega t)/\Omega$ and $\hbar\omega_{m,\vec{q}_\perp}$ is the energy of a confined optical phonon. The electron energy $\varepsilon_{n,N}^{H}(\vec{k}_\perp)$ in compositional superlattices takes the simple form:

$$\varepsilon_{n,N}^{H}(\vec{k}) = \left(N + \frac{1}{2}\right)\hbar\Omega_B - \Delta_n \cos k_{||}^{n} d \quad (2)$$

where Ω_B = eB/m* is the cyclotron frequency, m* is the effective mass of electron; L is the width of compositional superlattices and $C^{m}_{\vec{q}_\perp}$ is the electron-phonon interaction factor. In case of the confined electron- confined optical phonon interaction with the quantization direction in z direction, $C^{m}_{\vec{q}_\perp}$ is:

$$\left|C_{m,\vec{q}_\perp}\right|^2 = \frac{2\pi e^2\hbar\omega_o}{\varepsilon_o V}\left(\frac{1}{\chi_\infty} - \frac{1}{\chi_o}\right)\frac{1}{q_\perp^2 + q_z^2}; q_z = \frac{m\pi}{L}; m = 1, 2, 3... \tag{3}$$

where V, e and ε_o are the normalization volume, the effective charge and the electronic constant (often one takes V=1); $\hbar\omega_o$ is the energy of a optical phonon ($\hbar\omega_{m,\vec{q}_\perp} \approx \hbar\omega_o$); m (m=1, 2, . . .), is the quantum number characterizing confined phonons, L is well's width, χ_∞ and χ_0 are the static and the high-frequency dielectric constant, respectively. The electron form factor in case of confined phonons is written as follows:

$$I_{n,n'}^m = \int_0^{S_o.d} \psi_n^*(z)\psi_{n'}(z)e^{iq_z z}dz \tag{4}$$

Here, $\psi_n(z)$ is the wave function of the n-th state in one of the one-dimensional superlattice potential wells, d is the superlattices period, S_o is the number of superlattices period. Δ_n is the width of n-th miniband.
The $J_{N,N'}(u)$ takes the simple form:

$$J_{N,N'}\left(u\right) = \int_{-\infty}^{+\infty} \varphi_{N'}\left(\vec{r}_\perp - a_c^2\left(\vec{k}_\perp - \vec{q}_\perp\right)\right)e^{i\vec{k}_\perp\vec{q}_\perp}\varphi_N\left(\vec{r}_\perp - a_c^2\vec{k}_\perp\right)dr. \tag{5}$$

$\vec{r}_\perp$ and a_c = c/eB is position and radius of electron in the (x, y) plane, c is the light velocity, u $=a_c^2 q_\perp^2/2$, $\phi N(x)$ represents the harmonic wave function.
The carrier current density $\vec{j}(t)$ and the nonlinear absorption coefficient of a strong electromagnetic wave α take the form [14]

$$\vec{j}(t) = \frac{e}{m^*}\sum_{n,N,\vec{k}_\perp}\big(\vec{p} - \frac{e}{c}\vec{A}(t)\big)n_{n,N,\vec{k}_\perp}(t);\ \alpha = \frac{8\pi}{c\sqrt{\chi_\infty}E_0^2}\langle\vec{j}(t)\vec{E}_0 sin\Omega t\rangle_t \tag{6}$$

where $n_{n,N,\vec{k}_\perp}(t)$ is electron distribution function, $\langle X\rangle_t$ means the usual thermodynamic average of X at moment t.

In order to establish analytical expressions for the nonlinear absorption coefficient of a strong EMW by confined electrons in compositional superlattices, we use the quantum kinetic equation for particle number operator of electron

$$i\frac{\partial n_{n,N,\vec{k}_\perp}(t)}{\partial t} = \langle[a_{n,N,\vec{k}_\perp}^+ a_{n,N,\vec{k}_\perp}, H]\rangle_t \tag{7}$$

From Eq.(7), using Hamiltonian in Eq.(1), we obtain quantum kinetic equation for confined electrons in superlattices. Using the first order tautology approximation method [13] to solve this equation, we obtain the expression of electron distribution function $n_{n,N,\vec{k}_\perp}(t)$. We insert the expression of $n_{n,N,\vec{k}_\perp}(t)$ into the expression of $\vec{j}(t)$ and then insert the expression of $\vec{j}(t)$ into the expression of α in Eq.(5). Using property of Bessel function $J_{k-1}(x) + J_{k+1}(x) = \frac{k}{x}J_k(x)$, we obtain the nonlinear absorption coefficient of a strong electromagnetic wave in a compositional superlattice in the presence of an external magnetic field under influence of confined phonons:

$$\alpha = \frac{e^4\Omega_B^2 k_B T n_o^*}{2c\sqrt{X_\infty}\varepsilon_o^2\pi\Omega^3 a_c^2}\left(\frac{1}{\chi_\infty}-\frac{1}{\chi_o}\right)\sum_{n,N,n',N'}\sum_m \left|I_{n,n'}^m(q_z\right|^2\left\{1+\frac{3e^2E_o^2}{16a_c^2 m^{*2}\Omega^4}(N+N'+1)\right\}$$
$$\times\left\{\exp\left[-\frac{1}{k_BT}(N+\frac{1}{2})\hbar\Omega_B-\Delta_s\cos k_{||}^n d\right]-\exp\left[-\frac{1}{k_BT}(N'+\frac{1}{2})\hbar\Omega_B-\Delta_s\cos k_{||}^{n'} d\right]\right\}$$
$$\times\frac{\hbar\sqrt{A_1|M|}}{|M|(\hbar\Omega-\hbar\omega_o+(N'-N)\hbar\Omega_B-\Delta_s(\cos k_{||}^{n'}d-\cos k_{||}^n d))+\hbar^2 A_1} \quad (8)$$

here, $M = N - N'$; $A_1 = N_o\sum_{n,n',q_z}\frac{e^2\omega_o}{2\pi L\hbar^2}\left(\frac{1}{\chi_\infty}-\frac{1}{\chi_o}\right)$; $N_o = \frac{k_BT}{\hbar\omega_o}$
In Eq.(8), it's noted that we only consider the absorption close to its threshold because in other case (the absorption far away from its threshold) α is very smaller. In this case, the condition $|g\Omega-\omega_0| << \bar{\varepsilon}$ ($\bar{\varepsilon}$ is the average energy of electron) must be satisfied [15]. The formula of the nonlinear absorption coefficient contains the quantum number m characterizing confined phonons. The Eq. (8) shows that when quantum number m reaches to zero, the expression for the nonlinear absorption coefficient turns back to case of unconfined phonon.

III. NUMERICAL RESULTS AND DISCUSSION

In order to clarify the mechanism for the NAC of a strong EMW in compositional superlattices in case of confined phonons, in this section, we will evaluate, plot and discuss the expression of the NAC for the case of the $GaAs - Al_{0.3}Ga_{0.7}As$ compositional superlattices. We use some results to make the comparision with case of unconfined phonons. The parameters used in the caculations are as follows: $\chi_o = 12.9$, $\chi_\infty = 10.9$, $n_o = 10^{23}$, $\Delta_n = 0.85meV$; $L = 118A^o$;$m = 0.067m_o$, m_o being the mass of a free electron, $\hbar\omega_o = 36.25meV$ and $\Omega = 2.10^{14}s^{-1}$, $d_A = 134.10^{-10}m$, $d_B = 16.10^{-10}m$.
Fig.1 and fig.2 show that the dependence of the absorption coefficient on the frequency Ω of an external strong electromagnetic wave and the cyclotron frequency Ω_B for the case of an external magnetic field has one main maximum and several neighboring secondary maxima. The further away from the main maximum, the secondary one is the smaller. But in the case of absence of an external magnetic field, there are only two maxima of nonlinear absorption coefficient. Another point is that the absorption coefficient in the presence of an external magnetic field is smaller than that without a field because in this case, the number of electrons joining in the absorption process is limited. This is different from that for normal bulk semiconductors (index of the Landau level that electrons can reach after the absorption process is arbitrary), therefore, the dependence of the absorption coefficient on Ω_B and Ω is not continuous. Fig. 3 shows the nonlinear and the linear absorption coefficients in compositional superlattices in the presence of an external magnetic field for the case of unconfined phonon. Fig.1 and fig.3 show that there are differences in the spectrum of absorption coefficients in two cases. The confinement of optical phonon effects much strongly on absorption coefficients. In fig.1, the density of resonance peaks is greater and the value of absorption coefficients is higher than in case

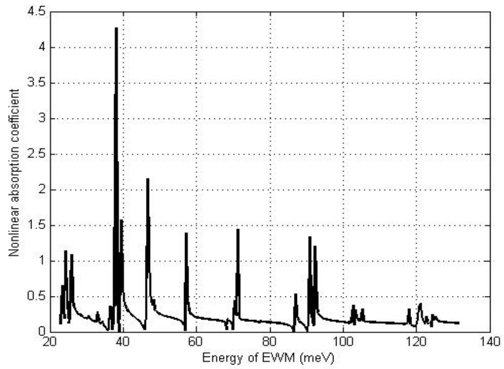

Fig. 1. The dependence of α on $\hbar\Omega$ in case of confined phonons

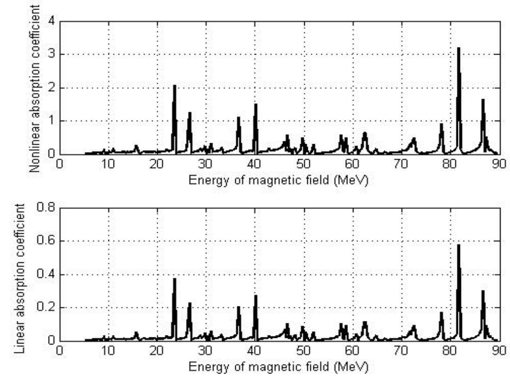

Fig. 2. The dependence of α on $\hbar\Omega_B$ in case of confined phonons

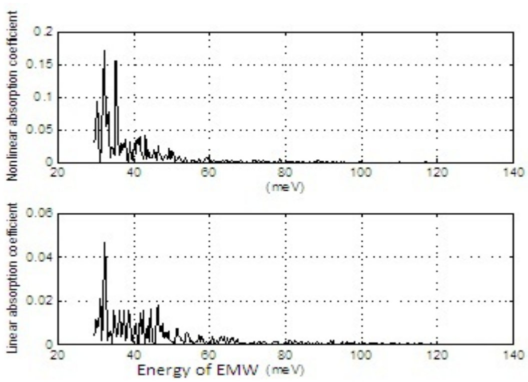

Fig. 3. The dependence of α on $\hbar\Omega$ in case of unconfined phonon

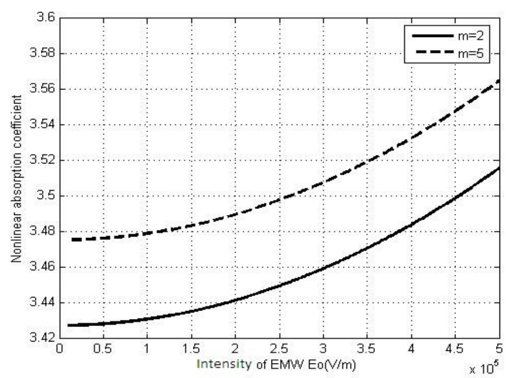

Fig. 4. The dependence of α on E_0 in case of confined phonon (m=2, m=5)

of unconfined phonon (fig.3) for both of the nonlinear and the linear absorptions. Fig.4 shows the dependence of the nonlinear absorption coefficient on intensity E_0 of EMW. It can be seen from this figure that the absorption coefficient depends much strongly on quantum number m characterizing confined phonon and it is greater when m increases.

IV. CONCLUSION

In this paper, by using the method of the quantum kinetic equation for electrons, the analytical expressions for nonlinear absorption coefficient of a strong electromagnetic wave by confined electrons in compositional superlattices in the presence of an external magnetic field for the case of confined phonon is obtained. The analytical results show that the nonlinear absorption coefficient depends on the frequency Ω of the external strong electromagnetic wave, the intensity E_0 of EMW, the cyclotron frequency, the quantum number characterizing confined phonon. This dependence is different from those obtained in case of unconfined phonon and absence of an external magnetic field. The expressions for the nonlinear absorption coefficient is the sum over the quantum number of confined electron-confined optical phonon and contains the cyclotron frequency. The expression for the nonlinear absorption coefficient turns back to the case of unconfined phonon and absence of an external magnetic field if the quantum number and the cyclotron frequency reaches to zero. The numerical results show that the phonon confinement effect and the presence of an external magnetic field in low dimensional systems change significantly the nonlinear absorption coefficient. Namely, the values of nonlinear absorption coefficient for the case of confined phonon are much higher than case of unconfined phonon. Density of nonlinear absorption coefficient peaks in the presence of an external magnetic field is bigger than the case when the external magnetic field is absent.

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