Proc. Natl. Conf. Theor. Phys. **36** (2011), pp. 283-289

# MAGNETIC FIELD DEPENDENCE OF MAGNETIC CASIMIR EFFECT

DO PHUONG LIEN
*Institute of Engineering Physics (IEP), Hanoi University of Technology (HUT), 1 Dai Co Viet, Hai Ba Trung Dist, Ha Noi*
NGUYEN ANH TUAN, NGUYEN TUAN ANH
*International Training Institute for Materials Science (ITIMS), Hanoi University of Technology (HUT), 1 Dai Co Viet, Hai Ba Trung Dist, Ha Noi*

**Abstract.** *In this work, we study the Casimir force and energy between two parallel ferromagnetic plates in presence of an external magnetic field. The magnetic field dependent dielectric permittivity of metals is described using the Drude model approache. The dependence of the Casimir force on magnetic field strength (via cyclotron frequency) is calculated with different separation distances of plates, D. Results show that the Casimir force increases as function of the cyclotron frequency with a parabolic law. We also investigate the effect of the magnetic field direction (defined by the angle, $\theta$, between the magnetic field direction and the surface normal of the plates) on the Casimir force at different distances D. It is showed that the Casimir force decreases when the angle $\theta$ increases by a Gaussian law. These behaviors are discussed.*

## I. INTRODUCTION

A long-range magnetic interaction given rise by the interplay of the Casimir effect and of the magnetic effect for two ferromagnetic layers separated by vacuum has been shown [1]. A magnetic Casimir effect given rise by interaction between two ferromagnetic plates in two different congurations, in which magnetization parallel and perpendicular to the layers, has also been studied [2]. In the previous paper [3], we presented a theoretical approach to calculate the Casimir energy and force of the interaction between two magnetic mirrors. The Drude model has been used for a general case where a number of numerical simulations have been realized and showed the dependence of the force and energy on the interplates distance and the change of sign of the interaction when this distance varies. We have also carried out numerical calculations for a real system with cobalt plates using the experimental data for the dielectric tensor of cobalt.

In this paper, we investigate the effect of the external magnetic field on the Casimir interaction between two ferromagnetic metals plates by changing the amplitude (via the cyclotron frequency) and the direction of the magnetic field towards to the plates which can be represented by an angle $\theta$ between magnetic field direction and the normal of plates. The first, we briefly introduce necessary theory for the dielectric permittivity and magnetic permeability as functions of frequency. And then we carry out numerical calculations of the Casimir energy and force per unit area as functions of separation of two parallel plates, $D$, with different values of cyclotron frequencies $\omega_c$ and different values of angles $\theta$.

## II. THEORY

### II.1. Frequency dependent magnetic permeability $\mu(\omega)$

The investigation of the influence of magnetic properties on the Casimir force requires an model for frequency dependent dielectric permittivity $\varepsilon(\omega$ and magnetic permeability magnetic permeability $\mu(\omega)$ . It was showed [4, 5], however, that for real materials $\mu$ is equal to unity in the range of frequencies which gives major contribution to the Casimir force. It is common knowledge that all materials possess diamagnetic polarization, i.e., they are magnetized in direction opposite to the applied magnetic field. For all substances the magnetic permeability is represented in the form $\mu(i\omega) = 1 + 4\pi\chi(i\omega)$, where $\chi(i\omega)$ is the magnetic susceptibility calculated along the imaginary frequency axis. The magnitude of $\chi(i\omega)$ is a monotonously decreasing function of $\omega$.

For diamagnets, because $\chi(0) < 0$, $\mu(0) < 1$, the magnetic properties cannot influence the Casimir force. Meanwhile, for materials possessing paramagnetic polarization, and if the materials are magnetized in the direction of an applied magnetic field, $\chi(0) > 0$ and $\mu(0) > 1$. Paramagnetic effects, if they are present, overpower the diamagnetic ones. However, for all paramagnets in the broad sense, with exception of ferromagnets, $\mu(0)$ remains as small as mentioned above. This leads to the conclusion that magnetic properties of paramagnets (with the single exception of ferromagnets) cannot markedly affect the Casimir force acting between macroscopic bodies.

For ferromagnets, $\mu(0) >> 1$ at $T < T_C$ where $T_C$ is the Curie temperature. There is a lot of ferromagnetic materials with various electric properties (both metals and dielectrics). They are characterized by strong interaction between constituent microscopic magnetic moments which results in large values of $\mu$ at low frequencies and in the possibility of spontaneous magnetization (hard ferromagnetic materials). It is not reasonable to consider parallel plates made of hard ferromagnetic materials because the magnetic interaction between such plates far exceeds any conceivable Casimir force. So only soft ferromagnetic materials are in interest. An important question arising in the calculation of the Casimir force between ferromagnetic plates is how quickly the initial magnetic permeability $\mu(H = 0)$ decreases with the increase of frequency. The rate of decrease of $\mu(i\omega)$ with increasing $\omega$ depends on the value of electric resistance. The lower is the resistance of a ferromagnetic material, the lower is the frequency at which $\mu(i\omega)$ drops toward unity. Thus, for ferromagnetic metals $\mu(i\omega)$ becomes equal to unity at frequencies above of order $10^5$ Hz (see, e.g., [6]). As mentionned in the paper [3], the frequencies which give major contribution to the Casimir force are larger than $10^{12}$ Hz, we can suppose always that magnetic permeability of ferromagnetic plates is equal to unity.

So, for two similar plates made of ferromagnetic metal the influence of magnetic properties on the magnitude of the Casimir force strongly depends on the model of dielectric permittivity.

### II.2. Frequency dependent electric permeability $\varepsilon(\omega)$

We remind that the expression of the dielectric tensor in the Drude model which has been described in paper [3], is written as: $\varepsilon_{xx}(i\omega) = 1 + \frac{\omega_P^2\tau}{\omega(1+\omega\tau)}$ and $\varepsilon_{xy}(i\omega) = \frac{\omega_P^2\omega_c\tau^2}{\omega(1+\omega\tau)^2}$ where the Plasma frequency $\omega_P$ is defined by $\omega_P = \frac{4\pi ne^2}{m^*}$; the cyclotron frequency is given by $\omega_c = \frac{eB}{m^*}$, $B$ is the magnetic field amplitude; $\tau$ is the relaxation time, $m^*$ is the effective mass of the electron.

## III. NUMERICAL CALCULATION RESULTS

### III.1. Dependence of the Casimir force on magnetic field amplitude

The influence of the magnetic field strength on the Casimir force are investigated by changing the cycloton frequency (see Eq.9). We have used the same inputs as in paper [3]. In our numerical calculations, we have chosen the typical values [2]: $\tau = 10^{-13}s$, $\hbar\omega_P = 9.85\ eV$. The dependence of the Casimir energy and interaction force between the two plates on the inter-plates distances $D$ (from 1 nm to 1 $\mu$m) have been presented in Fig.1a and Fig.1.b for the in-plane configuration. The curves corespond to 4 values of cyclotron frequencies of $9.10^{12}$ ($\hbar\omega_c = 5.9\ eV$) ; $4,5.10^{12}$; $2,2.10^{12}$and $1,1.10^{12} rad/s$.

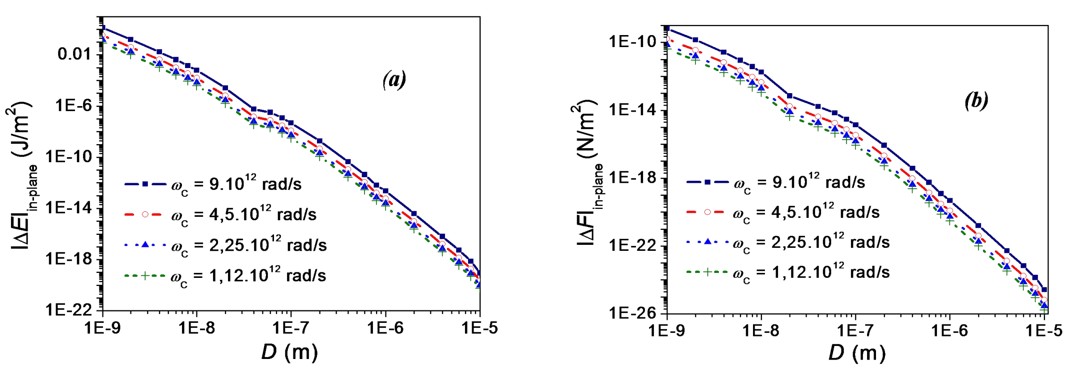


**Fig. 1.** Absolute of magnetic Casimir force (a) and energy (b) between the plates described by a Drude model for the in-plane configuration as a function of separation distances of plates at four values of the cyclotron frequencies.

The change of the sign of the energy and force happens at the distance $D \approx 30$ nm. As a result of the study, we present in Fig. 2a and Fig.2.b the energy and force as a function of the strength of magnetic field at the distance $D = 10\ nm$ and $D = 100\ nm$. The curve for the case of $D = 10\ nm$ presents absolute value of the energy because in this case the interaction energy (and force) between two plates is negative. While, this energy is positive for the case of $D = 100$ nm.

### III.2. Dependence of the Casimir force on direction of magnetic field

To study the influence of the direction on the magnetic field on the Casimir effect, we changed the angle $\theta$ between the direction of the magnetization in two plates and the normal of the plates which varies from 0 to $\pi/2$. In this study, the amplitude of the field is kept constant ($\hbar\omega_c = 5.9\ meV$).

In the last paper [3], we calculated the Casimir energy and force of the interaction between two magnetic mirrors when the magnetizations in plates are in plane and perpendicular to the plates. In this paper, we would like to study the case of the arbitrary

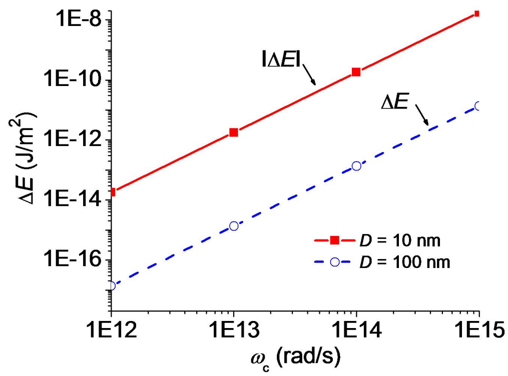


**Fig. 2.** Casimir energy between the plates described by a Drude model for the in-plane configuration as a function of cyclotron frequencies for the cases and.

magnetization direction. For this, the angle $\theta$ between the magnetization direction and the normal of the plates takes 4 values from 0 to $\pi/2$: $\theta = 0^o$, $\theta = 30^o$, $\theta = 60^o$ and $\theta = 90^o$. The amplitude of the magnetic field is kept constant ($\hbar\omega_c = 5.9\ meV$). The magnetic field vector has two components, parallel and perpendicular to the plates, their magnitudes are $B\cos\theta$ and $B\sin\theta$ respectively. So in order to use the Eqn.6 and Eqn.7 [3] for calculating the Casimir energy and force, we have to replace $\omega_c$ in these equations by $\omega_c\cos\theta$ and $\omega_c\sin\theta$ for the polar configuration and the in-plane configuration repectively. By consequence, the dielectric tensor can be expressed as:

$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_{xx}(\omega) & \varepsilon_{xy}(\omega)\sin\theta \\ -\varepsilon_{xy}(\omega)\sin\theta & \varepsilon_{xx}(\omega) \end{pmatrix}$$

The energy and force for two configurations are calculated using the same formula as in the work [3]. The total energy of the system is the sum of the two components coming from polar and in-plane configurations. We present in the Fig.3. the total energy as function of the distance $D$ between two plates for 4 values of $\theta = 0^o$, $\theta = 30^o$, $\theta = 60^o$ and $\theta = 90^o$). As we used the logarith scale, all the negative values of the energies can not be showed. We can observe the points where the total energy changes its sign (from negative value to positive value) for each angle $\theta$. It is clear from Fig. 3 that:

- there is no change of sign of the energy when $\theta = 0^o$, eg for the polar configuration

- the points where the total energies change their sign move towards to greater distances $D$ when $\theta$ increases, they correspond to distances of $D$ = 2 nm, 6 nm and 40 nm for $\theta = 30^o$, $\theta = 60^o$ and $\theta = 90^o$ respectively.

- at a given distance $D$, the total energy reduces when $\theta$ increases.

For a further study of the influence of the magnetic field direction on the Casimir energy, we present the contribution of the polar and in-plane configuration energies in the total energy in Fig.4 for the case of $\theta = 30^o$ and in Fig.5 for $\theta = 60^o$. As seen in the Fig.3, the energies change their sign at the distances lower than $1.10^{-8}m$ for these two cases. So, in order to observe how the change of sign happens, we present in Fig.4a,

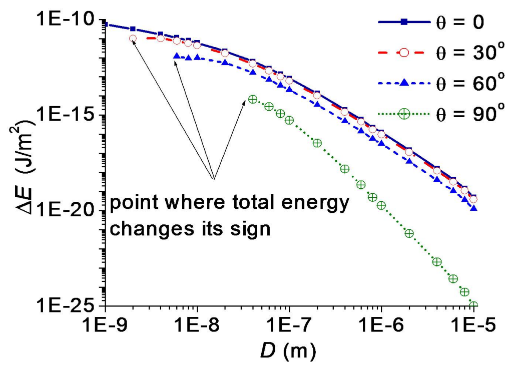


**Fig. 3.** Magnetic Casimir energy between the plates as a function of separation distances of plates $D$ at four values of $\theta$. The total energy (and force) is negative at the distance $D$ below the point where it changes sign.

5a the total energy at small distances $D$ $(D < 2.10^{-8}m)$ and in order to observe clearly which configuration, polar configuration or in-plane configuration, contributes mainly to the total energy, we show the energies for greater distances $D$ $(2.10^{-8}m < D < 2.10^{-7}m)$ in Fig.4b, 5b.

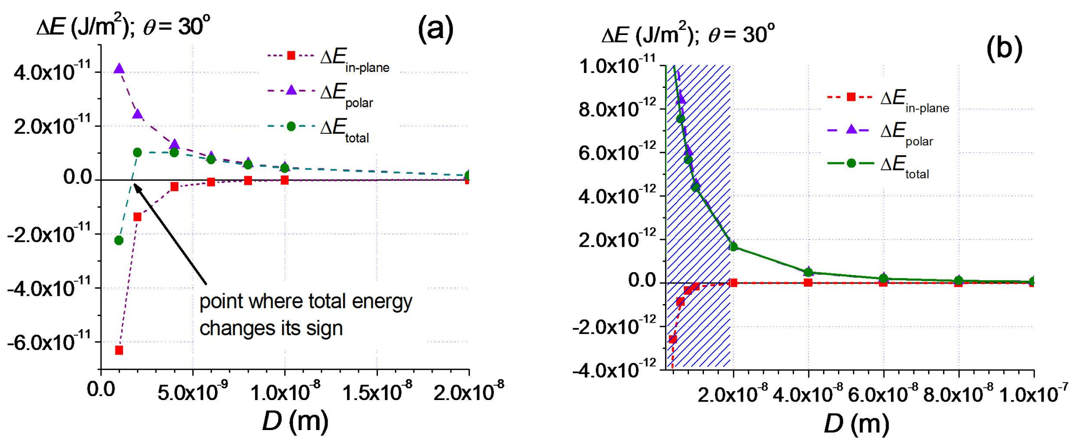


**Fig. 4.** Magnetic Casimir energy between the plates for the case $\theta = 30^o$ and at distances of plates $D < 2.10^{-8}m$ *(a), and*$D > 2.10^{-8}m$ *(b).*

We have here some remarks below:

- Fig. 4a and Fig. 5a indicate that the total energy changes from negative value to positive value at the distance about 2 nm and 6 nm respectively for $\theta = 30^o$ and $\theta = 60^o$. So at these distances the force between two plates changes from attractive to repulsive.

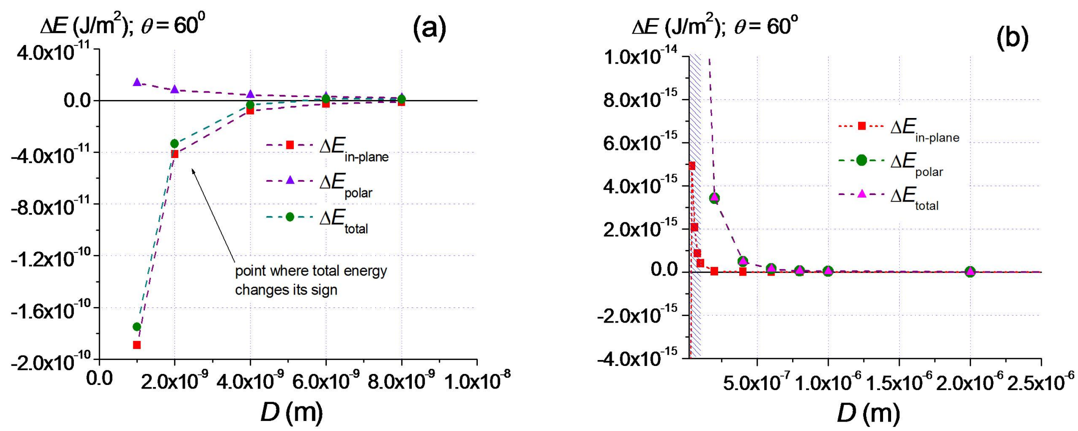


**Fig. 5.** Magnetic Casimir energy between the plates for the case $\theta = 60^o$ and at distances of plates $D < 2.10^{-8}m$ *(a), and* $D > 2.10^{-8}m$ *(b).*

- Fig. 4b and Fig. 5b show that the contribution of the polar configuration is exclusive in the total energy. We can also see, as expected, that the contribution of the polar configuration energy (dotted lines) in the total energy increases and of the in-plane configuration decreases (dashed lines) when $\theta$ increases.

We have also investigated how the Casimir energy and force change when $\theta$ varies at a fixed distance $D$, for example at $D = 10$ nm. We present the dependence of energy anf force on $\theta$ in Fig.6a and Fig.6b respectively. It can be clearly seen that the Casimir energy and force reduce when $\theta$ increases and they become zero when $\theta = 90^o$, so corresponding to the in-plane magnetization. As two plates are equivalent, the results on energy and force for the case where $\theta$ varies from $90^o$ to $180^o$ are the same as of the case where $\theta$ varies from $90^o$ to $0^o$.

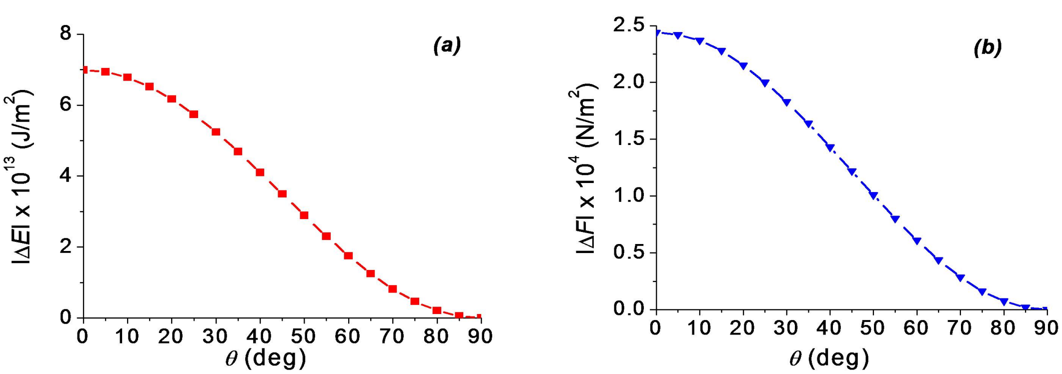


**Fig. 6.** Magnetic Casimir energy (a) and force (b) between the plates for the case of $D$=10 nm.

## IV. CONCLUSIONS

We have studied the Casimir effect between two parallel ferromagnetic plates in presence of an external magnetic field. The magnetic field dependent dielectric permittivity of metals is described using the Drude model approach. The dependence of the Casimir force on magnetic field strength (via cyclotron frequency) is calculated for different separation distances of plates, $D$. Results show that the Casimir force increases as an linear function of frequencies. We also investigated the effect of the magnetic field direction (defined by the angle, $\theta$, between the magnetic field direction and the surface normal of the plates) on the Casimir force at different distances $D$. It is showed that the Casimir force decreases when $\theta$ increases by a Gaussian law.

## ACKNOWLEDGMENT

This work was supported by the National Foundation for Science and Technology Development (NAFOSTED) under Project Code No. 103.02.50.09.

## REFERENCES

[1] P. Bruno, *Phys. Rev. Lett.* **88** (2002) 240401.
[2] G. Metalidis, P. Bruno, *Phys. Rev. A* **66** (2002) 062102.
[3] Do Phuong Lien, Nguyen Anh Tuan, Nguyen Tuan Anh, *Proc. Nat. Conf. Theor. Phys.* **35** (2010) 189-196 (to be printed in Comm. Phys. 6/2012).
[4] D. Iannuzzi, F. Capasso, *Phys. Rev. Lett.* **91** (2003) 029101.
[5] S. V. Vonsovskii, *Magnetism*, J. Wiley (1974), New York.
[6] R. S. Turtelli, R. Grössinger, C. Kussbach, *J. Appl. Phys.* **83** (1998) 1581.

*Received 30-09-2011.*