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PARAMETRIC RESONANCE OF ACOUSTIC AND OPTICAL PHONONS IN A COMPOSITIONAL SEMICONDUCTOR SUPERLATTICE

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Abstract. *The parametric resonance of acoustic and optical phonons in a Compositional Semiconductor superlattice with non-degenerative electron gas in the presence of a laser field is theoretically predicted by using a set of quantum transport equations for the phonons. Dispersions of the resonant phonon frequency and the threshold amplitude of the field for parametric amplification of the acoustic phonons are obtained. If they are obtained, then they are also estimated for realistic semiconductor models.*

I. INTRODUCTION

It is well known that in the presence of an external electromagnetic field (EEF), an electron gas becomes non-stationary. When the conditions of parametric resonance (PR) are satisfied, parametric interactions and transformations (PIT) of the same kind of excitations, such as phonon-phonon, plasmon-plasmon, or of different kinds of excitations, such as plasmon-phonon, will arise; i.e., energy exchange processes between these excitations will occur [1]. The PIT of acoustic and optical phonons has been considered in bulk semiconductors and in quantum wells [2, 3]. The physical picture can be described as follows: due to the electron-phonon interaction, propagation of an acoustic phonon with a frequency $\omega_{\vec{q}}$ is accompanied by a density wave with the same frequency. When an EEF with frequency is presented, a charge density waves (CDW) with a combination frequency $\omega_{\vec{q}} \pm \ell\Omega$ $(\ell = 1, 2, 3...)$ will appear. If among the CDW there exists a certain wave having a frequency which coincides, or approximately coincides, with the frequency of the optical phonon, $\nu_{\vec{q}}$, optical phonons will appear. These optical phonons cause a CDW with a combination frequency of $\nu_{\vec{q}} \pm \ell\Omega$, and when $\nu_{\vec{q}} \pm \ell\Omega \cong \omega_{\vec{q}}$, a certain CDW causes the acoustic phonons mentioned above. The result of the study shows that the PIT can speed up the damping process for one excitation and the amplification process for another excitation, namely acoustic phonons are amplified while optical phonon are declined or it can be on the contrary. For low-dimensional semiconductors, there have been several works on the generation and amplification of acoustic phonons [5]. However, in our opinion, the energy exchange processes between two different kinds of phonons in superlattices, which are driven by a PR of a two-phonon kind, have not yet been reported. It should be noted that the mechanism for PIT is different from that for phonon amplification under a laser field [6] and from PR of a defect mode [7].

In Ref. 3 and 4, we have studied the PIT in a quantum well and in a doper Compositional superlattice (DSSL) with non-degenerative electron gases. In order to continue the ideas of Refs. 2 and 3, the purpose of this paper is to also study the parametric resonance of acoustic and optical phonons, but in a Compositional Semiconductor superlattice (CSSL). The electron gas is assumed to be non-degenerate. Because the analytic calculation process in the present paper is similar that in Ref. 3 and the main differences are expressions of form factor and energy spectrum of electron in the models, only a brief description of the calculation will be given in this paper. In Sec. II, we introduce the dispersion equation obtained from the quantum transport equations for phonons. In Sec. III, we present results of an analytical approximation for the resonant acoustic phonon frequency and the threshold amplitude of the field for parametric amplification of acoustic phonons. Conclusions are shown in Sec. IV.

II. GENERAL DISPERSION EQUATION

The superlattice potential in CSSLs is created solely by the spatial distribution of the charge. The substantial improvement in the spatial (in an atomic scale) monitoring of the doping during film growth by means of molecular-beam epitaxy enabled growing Compositional Semiconductor superlattices-periodic alternation of thin ($\sim 1-2$ nm) layers of ($GaAs - Al_xGa_{1-x}As$). We consider a CSSL, in which the electron gas is confined by a superlattice potential along the z direction (the axis of the superlattice) and electrons are free on the $x - y$ plane. It is well known that the motion of an electron is confined in each layer of the CSSL and that its energy spectrum is quantized into discrete levels in the z direction. The electron state, α, is defined by the quantum number n in the z direction and the wave vector $\vec{k}_\perp$ on the $x - y$ plane perpendicular to z-axis, $\alpha = (n, \vec{k}_\perp)$, $\vec{k}^2 = \vec{k}_\perp^2 + k_z^2$.

A laser field irradiates the sample in the z direction, the electric field of the laser wave polarized in the $x-y$ plane, $\vec{E} = \vec{E}_0 \sin \Omega t$ ($\vec{E}_0$ and Ω are the amplitude and the frequency of the laser field, respectively). The vector potential of the field is $\vec{A}(t) = \vec{A}_0 \cos \Omega t$. If the Frohlich electron-acoustic and optical phonon interaction potential is used, the Hamiltonian for the system of the electrons and the acoustic and optical phonons in the laser field is: $H(t) = H_0(t) + H_{e-ph}$, in which:

$$H_0(t) = \sum_{\alpha} \varepsilon_\alpha(t) a_\alpha^+ a_\alpha + \sum_{\vec{q}} \hbar\omega_{\vec{q}} b_{\vec{q}}^+ b_{\vec{q}} + \sum_{\vec{q}} \hbar\nu_{\vec{q}} c_{\vec{q}}^+ c_{\vec{q}}, \quad (1)$$

$$H_{e-ph} = \sum_{\vec{q}} \sum_{\alpha',\alpha} G_{nn'}(\vec{q}) a_{\alpha'}^+ a_\alpha (b_{\vec{q}} + b_{-\vec{q}}^+) + \sum_{\vec{q}} \sum_{\alpha',\alpha} D_{nn'}(\vec{q}) a_{\alpha'}^+ a_\alpha (c_{\vec{q}} + c_{-\vec{q}}^+). \quad (2)$$

where $\varepsilon_\alpha(t) \equiv \varepsilon_n \left(\vec{k}_\perp - (e/c\hbar)\vec{A}(t)\right)$, $\varepsilon_n(\vec{k}_\perp) \equiv \varepsilon_\alpha$, ε_α and a_α^+ are the energy spectrum and the creation operator of an electron for state α, $b_{\vec{q}}^+$ ($c_{\vec{q}}^+$) is the creation operator of an acoustic (optical) phonon for energy $\hbar\omega_{\vec{q}}$ ($\hbar\nu_{\vec{q}}$). In this paper, we will deal with bulk (3-dimensional) phonons; therefore, the electron-acoustic and -optical phonon interaction constants take the forms $G_{nn'}(\vec{q}) = G_{\vec{q}}M_{nn'}(q_z)$, $D_{nn'}(\vec{q}) = D_{\vec{q}}M_{nn'}(q_z)$, where [10]

$$|G_{\vec{q}}|^2 = \frac{\hbar q \xi^2}{2\rho v_a V} \quad , \quad |D_{\vec{q}}|^2 = \frac{e^2 \hbar \nu_{\vec{q}}}{2Vq^2}\left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0}\right). \tag{3}$$

Here, V, ρ, v_a, and ξ are the volume, the density, the acoustic velocity, and the deformation potential constant, respectively; χ_0 and χ_∞ are the static and high-frequency dielectric constants, respectively. The electron form factor, $M_{nn'}(q_z)$, is written as [11]

$$M_{nn'}(q_z) = \sum_{j=1}^{s_0} \int_0^d e^{iq_z z} \Phi_n(z - jd)\Phi_{n'}(z - jd)\mathrm{d}z, \tag{4}$$

where d is the period of CSSL and s_0 is the number of period of CSSL, $\Phi_n(z)$ is the eigenfunction of the electron for an individual potential well.

In most cases, the interaction between the neighboring quantum wells in the CSSL can be neglected, i.e., the dependence of the energy on the wave vector k_z can be neglected. The energy spectrum of an electron in the CSSL for the state α takes the form [12, 13]

$$\varepsilon_n(\vec{k}_\perp) = \frac{\hbar^2 k_\perp^2}{2m} + \frac{\hbar^2 \pi^2 n^2}{2md^2} - \Delta_n cos(k_z d) = \frac{\hbar^2 k_\perp^2}{2m} + \varepsilon_n(k_z), \tag{5}$$

where m and e are the effective mass and the charge of the electron, respectively, and ε_n are the energy levels of an individual well.

In order to establish a set of quantum transport equations for acoustic and optical phonons, we use the general quantum distribution functions for the phonons, $\langle b_{\vec{q}} \rangle_t$ and $\langle c_{\vec{q}} \rangle_t$, where $\langle \psi \rangle_t$ denotes a statistical average at the moment t: $\langle \psi \rangle_t = Tr(\widehat{W}\widehat{\psi})$ ($\widehat{W}$ is the density matrix operator, Tr denotes the trace). Using Hamiltonian $H(t)$ and realizing operator algebraic calculations as in Ref. 3, we obtain a set of coupled quantum transport equations. The equation for the acoustic phonons is

$$\begin{aligned} \frac{\partial}{\partial t}\langle b_{\vec{q}} \rangle_t + i\omega_{\vec{q}}\langle b_{\vec{q}} \rangle_t = \frac{1}{\hbar^2} \sum_{n,n',\vec{k}_\perp} \sum_{s,s'=-\infty}^{+\infty} J_s(\lambda) J_{s'}(\lambda) e^{i(s-s')\Omega t} \left[f_n(\vec{k}_\perp - \vec{q}) - f_{n'}(\vec{k}_\perp) \right] \\ \times \int_{-\infty}^{t} \left(|G_{nn'}(\vec{q})|^2 \left[\langle b_{\vec{q}} \rangle_{t'} + \langle b^+_{-\vec{q}} \rangle_{t'} \right] + G_{nn'}(-\vec{q}) D_{nn'}(\vec{q}) \left[\langle c_{\vec{q}} \rangle_{t'} + \langle c^+_{-\vec{q}} \rangle_{t'} \right] \right) \\ \times \exp\left(\frac{i}{\hbar} \left[\varepsilon_n(\vec{k}_\perp - \vec{q}) - \varepsilon_{n'}(\vec{k}_\perp) + s\hbar\Omega \right] (t' - t) \right) \mathrm{d}t'. \end{aligned} \tag{6}$$

Here $f_n(\vec{k}_\perp)$ is the distribution function of electrons in the state $(n, \vec{k}_\perp)$, $J_s(\lambda)$ is the Bessel function, and $\lambda = e(\vec{q}_\perp.\vec{E}_0)/(m\Omega^2)$.

From Eq. (6) we can obtain an equation for the Fourier transformation $B_{\vec{q}}(\omega)$ of $\langle b_{\vec{q}}\rangle_t$:

$$\begin{aligned}(\omega-\omega_{\vec{q}})B_{\vec{q}}(\omega) &= \frac{2}{\hbar^2}\sum_{n,n'}\sum_{\ell=-\infty}^{\infty}|G_{nn'}(\vec{q})|^2\frac{\omega_{\vec{q}}B_{\vec{q}}(\omega-\ell\Omega)}{\omega-\ell\Omega+\omega_{\vec{q}}}P_\ell(\vec{q},\omega)\\ &+ \frac{2}{\hbar^2}\sum_{n,n'}\sum_{\ell=-\infty}^{\infty}G_{nn'}(-\vec{q})D_{nn'}(\vec{q})\frac{\nu_{\vec{q}}B_{\vec{q}}(\omega-\ell\Omega)}{\omega-\ell\Omega+\nu_{\vec{q}}}P_\ell(\vec{q},\omega),\end{aligned} \quad (7)$$

where we have put

$$P_\ell(\vec{q},\omega)=\sum_{s=-\infty}^{\infty}J_s(\lambda)J_{s+\ell}(\lambda)\Gamma_{\vec{q}}(\omega+s\Omega), \quad (8)$$

$$\Gamma_{\vec{q}}(\omega+s\Omega)=\sum_{\vec{k}_\perp}\frac{\hbar\left[f_{n'}(\vec{k}_\perp)-f_n(\vec{k}_\perp-\vec{q})\right]}{\varepsilon_{n'}(\vec{k}_\perp)-\varepsilon_n(\vec{k}_\perp-\vec{q})-\hbar(\omega+s\Omega)-i\delta}. \quad (9)$$

It can be noted that $\Gamma_{\vec{q}}(\omega+s\Omega)$ is the polarization operator of the electron distribution function in the n-th miniband [14] and the quantity δ is infinitesimal and appears due to the assumption of an adiabatic interaction of the EEF.

Repeating above proses we can also obtain an equation for the Fourier transformation $B^+_{-\vec{q}}(\omega)$ of $\langle b^+_{-\vec{q}}\rangle_t$ and relative expression between $B_{\vec{q}}(\omega)$ and $B^+_{-\vec{q}}(\omega)$. In the same way, but for optical phonons, we obtain a similar equation in which $\omega_{\vec{q}}$, $B_{\vec{q}}(\omega)$, $B_{\vec{q}}(\omega-\ell\Omega)$, $G_{nn'}(\vec{q})$, $D_{nn'}(\vec{q})$, and $\nu_{\vec{q}}$ are replaced with $\nu_{\vec{q}}$, $C_{\vec{q}}(\omega)$, $C_{\vec{q}}(\omega-\ell\Omega)$, $D_{nn'}(\vec{q})$, $G_{nn'}(\vec{q})$, and $\omega_{\vec{q}}$, respectively. In the equations, $B_{\vec{q}}(\omega)$ and $C_{\vec{q}}(\omega)$ are the Fourier transformations of $\langle b_{\vec{q}}\rangle_t$ and $\langle c_{\vec{q}}\rangle_t$, respectively. In these coupled equations, the first terms describe the interaction between phonons that belong to the same kind (acoustic-acoustic or optical-optical phonons) while the second terms describe interaction between phonons that belong to different kinds (acoustic-optical phonon). We can put $\ell=0$ in the first terms of the coupled equations because we are now focusing on the PIT of the acoustic and optical phonons. Solving the set, we obtain a general dispersion equation for the PIT of the acoustic and optical phonons:

$$\begin{aligned}&\left[\omega^2-\omega_{\vec{q}}^2-\frac{2}{\hbar^2}\sum_{n,n'}\left|G_{nn'}(\vec{q})\right|^2\omega_{\vec{q}}P_0(\vec{q},\omega)\right]\\ \times\ &\left[(\omega-\ell\Omega)^2-\nu_{\vec{q}}^2-\frac{2}{\hbar^2}\sum_{n,n'}\left|D_{nn'}(\vec{q})\right|^2\nu_{\vec{q}}P_0(\vec{q},\omega-\ell\Omega)\right]\\ =\ &\frac{4}{\hbar^4}\sum_{n,n'}\sum_{\ell=-\infty}^{+\infty}\left|G_{nn'}(\vec{q})\right|^2\left|D_{nn'}(\vec{q})\right|^2\omega_{\vec{q}}\nu_{\vec{q}}P_\ell(\vec{q},\omega)P_\ell(\vec{q},\omega-\ell\Omega)\end{aligned} \quad (10)$$

III. CONDITION FOR PARAMETRIC AMPLIFICATION

The solution to the general dispersion equation, Eq. (10), is complicated; therefore, we limit our calculation to the case of the first order resonance ($\ell=1$), in which $\omega_{\vec{q}}\pm$

$\nu_{\vec{q}} = \Omega$. We also assume that the electron-phonon interactions satisfy the condition $|G_{nn'}(\vec{q})|^2 \, |D_{nn'}(\vec{q})|^2 << 1$. In these limitations, if we write the dispersion relations for acoustic and optical phonons as $\omega_{ac}(\vec{q}) = \omega_a + i\tau_a$ and $\omega_{op}(\vec{q}) = \omega_0 + i\tau_0$, we obtain the resonant acoustic phonon modes

$$\omega_{\pm}^{(\pm)} = \omega_a + \frac{1}{2}\left[(v_a \pm v_0)\Delta q - i(\tau_a + \tau_0) \pm \sqrt{[(v_a \mp v_0)\Delta q - i(\tau_a - \tau_0)]^2 \pm \Lambda^2}\right] \qquad (11)$$

where v_a and ω_a (v_0 and ω_0) are the group velocity and the renormalization (by the electron-phonon interaction) frequency of the acoustic (optical) phonon, respectively, $\Delta q = q - q_0$, q_0 being the wave number for which the resonance is maximal, and

$$\Lambda = \frac{2}{\hbar^2}\sum_{n,n'} |G_{nn'}(\vec{q})| \, |D_{nn'}(\vec{q})| \, P_1(\vec{q}, \omega_{\vec{q}}). \qquad (12)$$

In Eq. (11), the signs ($\pm$) in the sub-script of $\omega_{\pm}^{(\pm)}$ correspond to the signs ($\pm$) in front of the root and the signs ($\pm$) in the superscript of $\omega_{\pm}^{(\pm)}$ correspond to the other sign pairs. These signs depend on the resonance condition $\omega_{\vec{q}} \pm \nu_{\vec{q}} = \Omega$. For instance, the existence of a positive imaginary part of $\omega_{+}^{(-)}$ implies a parametric amplification of the acoustic phonon. In such cases that $\lambda << 1$, the maximal resonance, and $q = q_{\perp}$, $q_z = 0$, we obtain

$$F = \mathtt{Im}[\omega_{+}^{(-)}] = \frac{1}{2}\left[-(\tau_a + \tau_0) + \sqrt{(\tau_a - \tau_0)^2 + |\Lambda|^2}\right], \qquad (13)$$

where τ_a and τ_0 are imaginary parts of frequencies of acoustic and optical phonons, they take forms

$$\tau_a = -\frac{1}{\hbar^2}\sum_{nn'} |G_{nn'}(\vec{q})|^2 \, \gamma(\omega_{\vec{q}}) \qquad (14)$$

$$\tau_0 = -\frac{1}{\hbar^2}\sum_{nn'} |D_{nn'}(\vec{q})|^2 \, \gamma(\nu_{\vec{q}}), \qquad (15)$$

$$|\Lambda| = \frac{\lambda}{\hbar^2}\sum_{nn'} |G_{nn'}(\vec{q})| \, |D_{nn'}(\vec{q})| \left\{[\theta(\omega_{\vec{q}}) - \theta(\omega_{\vec{q}} - \Omega)]^2 + [\gamma(\omega_{\vec{q}}) - \gamma(\omega_{\vec{q}} - \Omega)]^2\right\}^{1/2} \qquad (16)$$

with

$$\gamma(\omega) = \frac{C_1.S.d.m^{3/2}}{4\pi\sqrt{2\pi\beta}\hbar^2 q} e^{\beta(\varepsilon_F - \varepsilon_{n'})}\big[1 - e^{\beta\hbar\omega}\big] \exp\Big[-\frac{m\beta}{2\hbar^2 q^2}[\varepsilon_{n'n}(\omega)]^2\Big], \qquad (17)$$

$$\theta(\omega) = \frac{Sme^{\beta\varepsilon_F}}{2\pi\beta\hbar}\frac{e^{-\beta\varepsilon_{n'}} - e^{-\beta\varepsilon_n}}{\varepsilon_{n'n}(\omega)}. \qquad (18)$$

where $\beta = 1/(k_B T)$, k_B being the Boltzmann constant and T the temperature of the system, ε_F is the Fermi level, S is the area of the sample, and

$$\varepsilon_{n'n}(\omega) = \varepsilon_0(n' - n) - \hbar\omega - \hbar^2 q^2/(2m); \quad \mathrm{C}_1 = \int_{-\frac{\pi}{d}}^{\frac{\pi}{d}} e^{\beta\Delta_n \cos k_z d} dk_z. \qquad (19)$$

From Eq. (13), the condition for the resonant acoustic phonon modes to have a positive imaginary part leads to $|\Lambda|^2 > 4\tau_a\tau_0$. Using these conditions and Eqs. (15)-(16) yields the threshold amplitude for the EEF for non-degenerate electron gas:

$$E_0 \quad > \quad E_{th} = \frac{2m\Omega^2}{eq} \frac{\sqrt{\gamma(\omega_{\vec{q}})\gamma(\nu_{\vec{q}})}}{\sqrt{[\theta(\omega_{\vec{q}}) - \theta(\omega_{\vec{q}} - \Omega)]^2 + [\gamma(\omega_{\vec{q}}) - \gamma(\omega_{\vec{q}} - \Omega)]^2}}. \tag{20}$$

Equation (20) means that the parametric amplification of the acoustic phonons is achieved when the amplitude of the EEF is higher than some threshold amplitude.

To numerically estimate the threshold amplitude E_{th} for the parametric amplification of acoustic phonons we use the superlattice $GaAs - Al_xGa_{1-x}As$: Be with the parameters as follows [10, 12]: $\xi = 13.5$ eV, $\rho = 5.32$ gcm^{-3}, $v_a = 5370$ ms^{-1}, $\varepsilon_F = 50$ meV, $s_0 = 100$, $d = 40$ nm, $\chi_\infty = 10.9$, $\chi_0 = 12.9$, $\Delta = 1.3$ meV, $m = 0.067m_0$, m_0 being the mass of free electron, and $\hbar\nu_{\vec{q}} \simeq \hbar\nu_0 = 36.25$ meV.

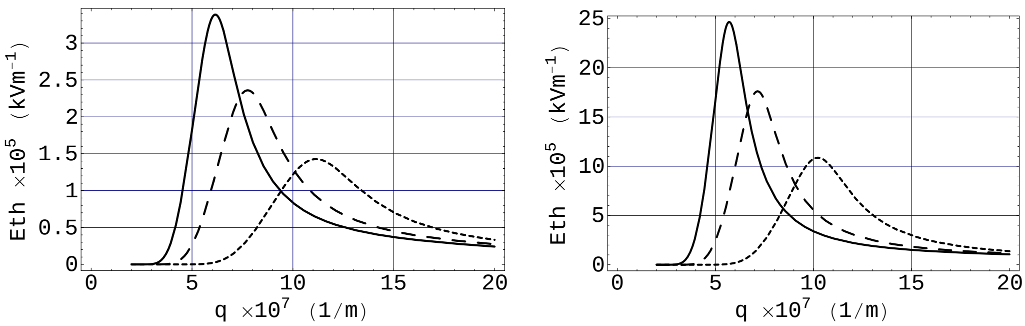

Fig. 1. Threshold amplitude (kV.m^{-1}) as a function of the wave number at temperature of 100 K (dot line), 200 K (dashed line), and 300 K (solid line). Here, the laser field frequency is $\Omega = 2 \times 10^{14}$ Hz (on the left) and $\Omega = 4.0 \times 10^{14}$ Hz (on the right).

In Fig. 1, we show threshold amplitude, E_{th}, as a function of the wave number, for three different temperatures. The figure shows that the curves have maximal values and are non-symmetric around the maxima. This is due to the fact that a fixed EEF, with an amplitude greater than the corresponding threshold amplitude, can induce parametric amplification for acoustic phonons in two regions of the wave number corresponding to the two signs in $\omega_{\vec{q}} \pm \nu_{\vec{q}} = \Omega$. The maxima increases as the temperature increases. A consequence of the non-symmetric behavior of the curves is that at fixed temperature (for example, 77 K) an EEF having a small amplitude (for instance, smaller than 10 kVcm^{-1}) can amplify only acoustic phonons with wave numbers that are smaller than 0.85×10^6 cm^{-1}, while an EEF having a large amplitude (for instance, large than 15 kVcm^{-1}) can amplify acoustic phonons with wave numbers that are either smaller than 0.9×10^6 cm^{-1} or greater than 2.0×10^6 cm^{-1}. These characteristics are similar as in quantum well [3].

The dependence of the threshold amplitude on the temperature is presented in figure 2. When the temperature is decreased, the threshold amplitude for parametric amplification of acoustic phonons in which $\omega_{\vec{q}}+\nu_{\vec{q}} = \Omega$ decreases; the threshold amplitude,

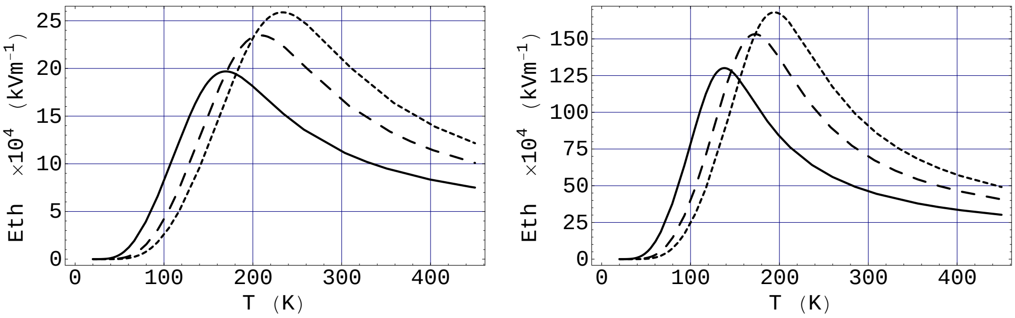

Fig. 2. Threshold amplitude (kVm^{-1}) as a function of the temperature at the wave number of 0.5×10^8 m^{-1} (dot line), 0.75×10^8 m^{-1} (dashed line), and 0.9×10^8 m^{-1} (solid line). Here, the laser field frequency is $\Omega = 2\times10^{14}$ Hz (on the left) and $\Omega = 4.0\times10^{14}$ Hz (on the right).

however, increases for the case of $\omega_{\vec{q}} - \nu_{\vec{q}} = \Omega$. We can see that the threshold amplitude is sensitive to the temperature change and it is more sensitive to the temperature change for the case in which the resonant frequency is smaller than it is for the case in which the resonant frequency is larger (in Fig. 1, in the region to the left of the maximum, E_{th} is more sensitive to temperature than it is in the region to the right of the maximum). We can also realize that the threshold amplitude is saturable as the temperature increases. This characteristic is also manifested in fig. 1 in which three lines for three different temperatures are coincident as the wave number increases. The sensitivity of E_{th} to temperature change, which is a behavior of acoustic phonons, is clearly present in this mechanism. Saturability of E_{th} to temperature change in region of high temperature, which is a behavior of optical phonons, can be explained by non-dispersion of optical phonons.

IV. CONCLUSION

In this paper, we analytically investigate the possibility of parametric resonance of acoustic and optical phonons in CSSL. We have obtained a general dispersion equation for parametric amplification and transformation of phonons. However, an analytical solution to the equation can only be obtained within some limitations. Using these limitations for simplicity, we obtain dispersions of the resonant acoustic phonon modes and the threshold amplitude of the field for acoustic phonon parametric amplification. Similarly to the mechanism pointed out in previous papers for bulk semiconductors and for quantum wells, parametric amplification for acoustic phonons in a doped superlattice can occur under the condition that the amplitude of the external electromagnetic field is higher than some threshold amplitude. Analytical expressions show that the threshold amplitude depends on parameters of the field, material, and physical conditions.

Numerical results for the superlattice $GaAs - Al_xGa_{1-x}As$:Be clearly show the predicted mechanism. Parametric amplification for acoustic phonons and the threshold amplitude depend on the physical parameters of the system and are sensitive to the temperature at the region of low-temperature but having saturable characteristic at the region of high-temperature. These characteristics are similar as in quantum wells and maybe they are common properties of quasi-two-dimensional systems.

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