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CP VIOLATION

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Abstract. *We study the CP violation in the electroweak decay. We show how the CP violation of asymmetry depends on the weak phase and strong phase of form factors. In framework of the resonance effective theory of QCD with external tensor source, the direct CP violation in the semileptonic decay is considered. The maximal value of the CP asymmetry is approximately twenty percents at* $s = M_\rho^2$.

I. INTRODUCTION

CPT theorem is one of the most important and generally valid theorems in quantum field theory. All interactions are invariant under combined C, P and T. It implies particle and anti-particle have equal masses and lifetimes. However, CP violation was found by Cronic and Fitch in 1964 ([2])in decay of neutral kaon. The discovery of CP violation implies that there is the difference between particle and anti-particle.

In the Standard Model (SM), all the observable which violate CP depend essentially on one parameter (the phase of the Cabibbo-Kobayashi- Maskawa matrix). The measurements of CP violation in B and K decays are consistent with the prediction of CKM mechanism of the standard model predicts. If quarks violate CP why not CP violation with leptons. On the other hand,the CP violation that we see in outer space is much larger than the CP violation which we see with quarks. In the SM, the lepton family number and the total lepton number are strictly conserved. However, neutrino experiments have established the existence of the lepton flavor violation (LFV) in the neutral lepton sector. Hence, we expect this phenomenon can occurs in the charged lepton sector even decays involving charged LFV have not yet been observed. Many extensions of the SM predict measurable rates for LFV processes with the branching ratios for charged lepton LFV are much suppressed, specially LFV decays of charged leptons occur just below the current experimental bounds given in the Belle experiment [13].

The search for LFV decays of the τ charged lepton is particularly interesting since LFV sources involving the third generation can decay into particles belonging to the first and the second ones. Moreover, the τ lepton is the only known lepton massive enough to decay into hadronic; its semileptonic decay is an ideal tool for studying strong interaction effects in very clean conditions. Thus, in the framework of some popular models beyond the SM, studying the semileptonic τ decays related to LFV and CP violation is very interesting and needed. In fact, the effects of new physics on LFV semileptonic τ decays [14]. They have shown that these decay processes are very sensitive to the new physics effects and the constraint on the free parameters of some beyond SM models have obtained.

Furthermore, the BELLE and BABAR collaborations have published many reported sensitivity [15]recently that are more competitive with the LFV τ leptonic decays.On the other hand, the coupling constants of τ lepton with the new particles are expected to be larger than these of muon and electron, the people not only expected to search for LFV in τ decay but also expected to search for CP violation in τ decay. The searches for CP violation in τ lepton decays have been proposed in [16],as the observation of non-zero CP asymmetry in the τ decay into neutrino modes would be a clear and unambiguous signature for new physics.

At the low energy, interactions of light quarks with gluon are strong. Therefore, semileptonic τ decay processes allow us to study the properties of bilinear QCD currents, and provide relevant information on dynamics of the resonances entering into the processes. The hadronization of these currents involves the strong interaction at the low energy and, therefore, nonperturbative features of QCD have to be implemented properly into their evaluation. The effective theory frame work has a long story of successful achievements in this task. One of successful theory is the Chiral Perturbation Theory (χPT)[4]. At the energies $E \sim M_\rho$, (M_ρ is the mass of the $\rho(770)$, the lightest hadron resonance), the resonance meson are active degrees of freedom that can not be integrated out by χPT, and they have to be properly included into the relevant Lagrangian [5]. The procedure is ruled by the chiral symmetry under $SU(3)_L \times SU(3)_R$, that drives the interaction of Goldstone bosons (the lightest octet of pseudoscalar mesons), and the $SU(3)_V$ assignments of the resonance multiples. Its systematic arrangement has been done in [6] as the Resonance Chiral theory ($R\chi T$). Basing on the context of (χPT) and ($R\chi T$), they accounted for the set of the form factors for semileptonic decays of the Dirac bilinear $\overline{\psi}\gamma_\mu\psi, \overline{\psi}\gamma_\mu\gamma_5\psi, \overline{\psi}\psi$ and $\overline{\psi}i\gamma_5\psi$. In particular, no systematic introduction of the tensor Dirac bilinear $\overline{\psi}\sigma_{\mu\nu}\psi$ has been studied. Our calculations can be parameterized by introducing form factors base on the context of (χPT) and ($R\chi T$).

The article is organized as follows: In Sec. II we study the effective theory base on the context of (χPT) and ($R\chi T$) with tensor source. The bilinear currents form factors at the tree level in $\tau^+ \to \mu^+ PP$ with $PP = \pi^+\pi^-, \pi^o\pi^o, \eta\eta, \eta'\eta', \eta\eta', K^oK^o, K^+K^-$ are given in Sec. II. The Sec. III is devoted the analytical results of the lepton flavor violation in $\tau^+ \to P\mu^+$, $V\mu^+$, $PP\mu^+$ processes. It includes the information of CP odd and T odd quantities. The numerical results are given by consider the special mode $\tau \to \pi^+\pi^-\mu^+$ in sec. IV. Finally, in sec. V, we sketch our conclusions.

II. THE CHIRAL PERTURBATION THEORY WITH TENSOR SOURCE AND RESONANCE EFFECTIVE THEORY

The general formalism to hadronize the bilinear quark currents was studied in [7] by using the (χPT) and ($R\chi T$). Basing on these theories, there is no way to calculate the form factor for tensor quark Dirac current. As mentioned, the physics beyond SM can provide the tensor quark Dirac currents. Hence, studying the chiral perturbation theory with tensor source is needed. The chiral Lagrangian with the external tensor source is given as follows

$$L_{QCD} = L^o_{QCD} + \overline{q}\left(\gamma_\mu(v^\mu + \gamma_5 a^\mu) - (s - ip\gamma_5)\right)q + q\sigma^{\mu\nu}\overline{t}_{\mu\nu}q. \tag{1}$$

where v^μ, a^μ, s, p and $t^{\mu\nu}$ are the vector, axial-vector, scalar, pseudo-scalar and tensor external currents, respectively. They are matrices in the flavor space and extended as follows: $v_\mu = \frac{\lambda_i}{2}v^i_\mu$, $a_\mu = \frac{\lambda_i}{2}a^i_\mu$, $s = \lambda_i s^i$,$p = \lambda_i p^i$ and $\overline{t}^{\mu\nu} = \frac{\lambda^i}{2}\overline{t}^i_{\mu\nu}$. L^o_{QCD} is the massless QCD Lagrangian. From Lagrangian given in 1, we can construct the QCD generating function as follows

$$e^{iZ_{QCD}[v,a,s,p,t_{\mu\nu}]} = \int [DG_\mu][Dq][D\overline{q}]e^{i\int d^4x L_{QCD}[q,\overline{q},v_\mu,a_\mu,s,p,\overline{t}_{\mu\nu}]} \tag{2}$$

The very low-energy strong interaction in the light quark sector is known to be ruled out by the $SU(3)_L \times SU(3)_R$ chiral symmetry of massless QCD implemented in χPT. The leading $O(p^2)$, the only external scalar and pseudo-scalar are switched on, the Lagrangian is

$$L^\chi_2 = \frac{F^2}{4}\left\langle u_\mu u^\mu + \chi_+\right\rangle, \tag{3}$$

where

$$\begin{aligned} u_\mu &= i\left[u^+(\partial_\mu - ir_\mu)u - u(\partial_\mu - il_\mu)u^+\right], \\ \chi_\pm &= u^+\chi u^+ \pm u\chi^+ u, \qquad \chi = 2B_o(s + ip), \end{aligned} \tag{4}$$

and $<>$ is short for a trace in the flavor space.The lightest $U(3)$ notet of pseudoscalar mesons:

$$\begin{aligned} \Phi &= \frac{1}{\sqrt{2}}\sum_{a=0}^{8}\lambda_a\varphi_a \\ &= \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^o + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_o & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^o + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_o & K^o \\ K^- & \overline{K^o} & -\frac{2}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_o \end{pmatrix} \end{aligned} \tag{5}$$

is realized nonlinearly into the unitary matrix in the flavor space

$$u(\varphi) = \exp\left[i\frac{\Phi}{\sqrt{2}F}\right], \tag{6}$$

that transforms as

$$u(\varphi) \to g_R u(\varphi)h(g,\varphi)^+ = h(g,\varphi)u(\varphi)g^+_L \tag{7}$$

with $g \equiv (g_L, g_R) \in SU(3)_L \times SU(3)_R$ and $h(g,\varphi) \in SU(3)_V$, the external Hermitian matrix fields r_μ, l_μ, s and p promote the global $SU(3)_L \times SU(3)_R$ symmetry to a local one. Interactions with electroweak bosons can be accommodated through the vector $v_\mu = (r_\mu + l_\mu)/2$ fields and axial-vector $a_\mu = (r_\mu - l_\mu)/2$ fields. The scalar field s incorporates explicit chiral symmetry breaking through the quark masses $s = M + ...$, with $M = diag(m_u, m_d, m_s)$ and, finally, $F = F_\pi \cong 92.4$ GeV is the pion decay constant and $B_o F^2 = -\left\langle\overline{\psi}\psi\right\rangle_o$ in the chiral limit.

On the other hand, if we consider the resonances participate in the dynamic of the process, such as $\tau \to PP\mu; \tau \to V\mu$, it is necessary to include the resonance as the active degree of freedom into Lagrangian and, therefore, to consider the RχT. As concluded in [18], the contribution of tensor resonance fields exchange to the low energy constants of $O(p^4)$ is much suppressed. Because of the strong coupling of the vector mesons to pseudoscalars and the comparatively low masses of the lowest-lying vector meson nonet, the chiral vector meson can be dominated. Hence, it is a good approximate to consider the theory with only including the role of vector resonance. In [6], the simplest couplings among the lightest notet of pseudoscalar mesons with the resonances were introduced by using the antisymmetric tensor formulation to describe the the vector and axial-vector resonance. The notet of resonance fields $V_{\mu\nu}$ is given by

$$V_{\mu\nu} = \begin{pmatrix} \frac{\rho_o}{\sqrt{2}} + \frac{\omega_8}{\sqrt{6}} + \frac{\omega_1}{\sqrt{3}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho_o}{\sqrt{2}} + \frac{\omega_8}{\sqrt{6}} + \frac{\omega_1}{\sqrt{3}} & K^{*o} \\ K^{*-} & \overline{K}^{*o} & -\frac{2\omega_8}{\sqrt{6}} + \frac{\omega_1}{\sqrt{3}} \end{pmatrix} \tag{8}$$

In the antisymmetric tensor formulation, the kinematic term for the spin-1 resonances in Lagrangian read

$$L^V_{kin} = -\frac{1}{2}\left\langle \nabla^\lambda V_{\lambda\mu} \nabla_\nu V^{\nu\mu} \right\rangle + \frac{M_V^2}{4} \left\langle V_{\nu\mu} V^{\nu\mu} \right\rangle \tag{9}$$

with M_V is mass of the notet vector. The lowest order interaction Lagrangian is given as

$$L^V_{(2)} = \frac{F_V}{2\sqrt{2}} \left\langle V_{\mu\nu} f_+^{\mu\nu} \right\rangle + i\frac{G_V}{\sqrt{2}} \left\langle V_{\mu\nu} u^\mu u^\nu \right\rangle \tag{10}$$

where

$$\begin{aligned} \nabla_\mu X &\equiv \partial_\mu X + [\Gamma, X], \\ \Gamma_\mu &= \frac{1}{2}\left[u^+(\partial_\mu - ir_\mu)u + u(\partial_\mu - il_\mu)u^+\right], \\ f_+^{\mu\nu} &= uF_L^{\mu\nu}u^+ + u^+F_R^{\mu\nu}u, \end{aligned} \tag{11}$$

and F_L, F_R are the field strength tensor associated with the external right and left fields.

Let us remaind that at the next to leading order$O(p^4)$, the external tensor source is started to switch on. The Lagrangian containing the external tensor source and including resonances files given as follows

$$\begin{aligned} L_4^{\chi PT} &= \Lambda_1 < t_+^{\mu\nu} f_{+\mu\nu} > -i\Lambda_2 < t_+^{\mu\nu} u_\mu u_\nu > \\ &+ \Lambda_3 < t_+^{\mu\nu} t^+_{\mu\nu} > +\Lambda_4 < t_+^{\mu\nu} >^2 + \sqrt{2} F_{VT} M_V < V_{\mu\nu} t_+^{\mu\nu} > \end{aligned} \tag{12}$$

with $t_\pm^{\mu\nu}$ are defined as

$$t_\pm^{\mu\nu} = u^\dagger t^{\mu\nu} u^\dagger \pm u t^{\mu\nu\dagger} u. \tag{13}$$

The relevant resonance chiral theory with three flavors and including the tensor current source is described by the Lagrangian given as follows

$$L_{R\chi T} = L_2^\chi + L^V_{kin} + L_2^V + L_4^{\chi PT}, \tag{14}$$

From Eq.(2), the hadronisation of the bilinear quark currents is studied by taking the partial derivatives of the functional action with respect to the external auxiliary fields.

The detailed results are given as

$$\begin{aligned}
V_\mu^i &= \overline{q}\gamma_\mu \frac{\lambda^i}{2} q = \frac{\partial L_{R\chi T}}{\partial v_i^\mu}|_{j=0}, \quad A_\mu^i = \overline{q}\gamma_\mu\gamma_5 \frac{\lambda^i}{2} q = \frac{\partial L_{R\chi T}}{\partial a_i^\mu}|_{j=0}, \\
S^i &= -\overline{q}\lambda^i q = \frac{L_{R\chi T}}{\partial s_i}|_{j=0}, \quad P^i = \overline{q} i\gamma_5 \lambda^i q = \frac{L_{R\chi T}}{\partial p_i}|_{j=0}, \\
T_i^{\mu\nu} &= \overline{q}\sigma_{\mu\nu}\lambda_i q = \frac{\partial L_{R\chi T}}{\partial t^{\mu\nu}} + \frac{\partial L_{R\chi T}}{\partial t^{\mu\nu\dagger}}
\end{aligned} \tag{15}$$

In order to hadronise the final states in $\tau^+ \to \mu^+ PP$; $\tau^+ \to \mu^+ P$ and $\tau^+ \to \mu^+ V$, these expressions can be written as:

$$V_\mu^i = \frac{F^2}{4}\left\langle \lambda^i \left(u u_\mu u^+ - u^+ u_\mu u\right)\right\rangle - \frac{F_V}{2\sqrt{2}}\left\langle \lambda^i \partial^\nu \left(u^+ V_{\nu\mu} u + u V_{\nu\mu} u^+\right)\right\rangle |_{j=0}. \tag{16}$$

$$A_\mu^i = \frac{F^2}{4}\left\langle \lambda^i \left(u u_\mu u^+ + u^+ u_\mu u\right)\right\rangle |_{j=0}, \tag{17}$$

$$S^i = -\frac{L_{R\chi T}}{\partial s_i}|_{j=0} = \frac{1}{2} B_o F^2 \left\langle \lambda^i (u^+ u^+ + uu)\right\rangle |_{j=0}, \tag{18}$$

$$P^i = \overline{q} i\gamma_5 \lambda^i q = \frac{L_{R\chi T}}{\partial p_i}|_{j=0} = \frac{i}{2} B_o F^2 \left\langle \lambda^i (u^+ u^+ - uu)\right\rangle |_{j=0}. \tag{19}$$

$$\begin{aligned}
T_i^{\mu\nu} &= -i\Lambda_2 \left\langle \lambda_i (P^{L\mu\nu\rho} u^\dagger u_\rho u_\lambda u^\dagger + P^{R\mu\nu\rho} u u_\rho u_\lambda u)\right\rangle \\
&+ \sqrt{2} F_{VT} M_V \left\langle \lambda_i (P^{L\mu\nu\rho} u^\dagger V_{\rho\lambda} u^\dagger + P^{R\mu\nu\rho} u V_{\rho\lambda} u)\right\rangle |_{j=0}
\end{aligned} \tag{20}$$

with J indicates the external currents and $P^{L\mu\nu\rho}, P^{R\mu\nu\rho}$ are given in [10]. We will use the Eqs.(16), (17), (18)and (19), (20) as the powerful tool to evaluate the form factor in $\tau^+ \to P\mu^+$, $\tau^+ \to PP\mu^+$, $\tau^+ \to V\mu^+$ semileptonic decay.

III. CONSIDERING THE $\tau^+ \to P1P2\mu^+$ PROCESSES

Let us start from the effective Lagrangian.

$$\begin{aligned}
L_{eff} &= -\frac{4G_F}{\sqrt{2}}\Big[M_\tau A_R \overline{\tau}\sigma^{\mu\nu}P_L\mu F_{\mu\nu} + M_\tau A_L \overline{\tau}\sigma^{\mu\nu}P_R\mu F_{\mu\nu} \\
&+ \sum_{i=u,d,s} g_1^i(\overline{\tau}P_L\mu)(\overline{q_i}P_L q_i) + \sum_{i=u,d,s} g_2^i(\overline{\tau}P_L\mu)(\overline{q_i}P_R q_i) \\
&+ \sum_{i=u,d,s} g_3^i(\overline{\tau}P_R\mu)(\overline{q_i}P_R q_i) + \sum_{i=u,d,s} g_4^i(\overline{\tau}P_R\mu)(\overline{q_i}P_L q_i) \\
&+ \sum_{i=u,d,s} g_5^i(\overline{\tau}\gamma^\mu P_R\mu)(\overline{q_i}\gamma^\mu P_R q_i) + \sum_{i=u,d,s} g_6^i(\overline{\tau}\gamma^\mu P_L\mu)(\overline{q_i}\gamma^\mu P_L q_i) \\
&+ \sum_{i=u,d,s} g_7^i(\overline{\tau}\gamma^\mu P_R\mu)(\overline{q_i}\gamma^\mu P_L q_i) + \sum_{i=u,d,s} g_8^i(\overline{\tau}\gamma^\mu P_L\mu)(\overline{q_i}\gamma^\mu P_R q_i) \\
&+ \sum_{i=u,d,s} g_9^i(\overline{\tau}\sigma^{\mu\nu} P_R\mu)(\overline{q_i}\sigma_{\mu\nu} q_i) + \sum_{i=u,d,s} g_{10}^i(\overline{\tau}\sigma^{\mu\nu} P_L\mu)(\overline{q_i}\sigma_{\mu\nu} q_i) + H.c.\Big]
\end{aligned} \tag{21}$$

There is no appearance of the the axial-vector and pseudoscalar quark currents on the $\tau^+ \to P_1P_2\mu^+$ processes, the total amplitude is obtained as

$$\begin{aligned}
M_{\tau^+\to P1P2\mu^+} &= \frac{4G_F}{\sqrt{2}}\Bigg(\sum_{i=u,d,s}\frac{g_3^i+g_4^i}{2}(\overline{v}_\mu P_L v_\tau)(\overline{u}_{q_i}u_{q_i}) + \sum_{i=u,d,s}\frac{g_1^i+g_2^i}{2}(\overline{v}_\mu P_R v_\tau)(\overline{u}_{q_i}u_{q_i}) \\
&+ \sum_{i=u,d,s}\frac{g_5^i+g_7^i}{2}(\overline{v}_\mu P_L\gamma_\alpha v_\tau)(\overline{u}_{q_i}\gamma^\alpha u_{q_i}) + \sum_{i=u,d,s}\frac{g_6^i+g_8^i}{2}(\overline{v}_\mu P_R\gamma_\alpha v_\tau)(\overline{u}_{q_i}\gamma^\alpha u_{q_i}) \\
&+ \sum_{i=u,d,s} g_9^i(\overline{v}_\mu P_L\sigma_{\alpha\beta}v_\tau)(\overline{u}_{q_i}\sigma^{\alpha\beta}u_{q_i}) + \sum_{i=u,d,s} g_{10}^i(\overline{v}_\mu P_L\sigma_{\alpha\beta}v_\tau)(\overline{u}_{q_i}\sigma^{\alpha\beta}u_{q_i}) \\
&+ iM_\tau\overline{v}_\mu\sigma_{\alpha\beta}q^\beta(A_RP_R + A_LP_L)v_\tau\frac{e^2}{q^2}\sum_{i=u,d,s}Q^i\overline{u}_{q_i}\gamma^\alpha u_{q_i}\Bigg)
\end{aligned} \tag{22}$$

After parameterizing by introducing the form factor, the total amplitude can be described as

$$\begin{aligned}
M_{\tau^+\to P1P2\mu^+} &= \frac{4G_F}{\sqrt{2}}\big(D_{1L}\overline{v}_\mu P_L v_\tau + D_{1R}\overline{v}_\mu P_R v_\tau \\
&+ \frac{1}{q^2}(\overline{v}_\mu iM_\tau\sigma_{\alpha\beta}q^\beta(D_{2L}P_L + D_{2R}P_R)v_\tau)(p_1^\alpha - p_2^\alpha) \\
&+ D_{3L}(\overline{v}_\mu P_L\gamma_\alpha v_\tau)(p_1^\alpha - p_2^\alpha) + D_{3R}(\overline{v}_\mu P_R\gamma_\alpha v_\tau)(p_1^\alpha - p_2^\alpha) \\
&+ (\overline{v}_\mu(D_{4L}P_L + D_{4R}P_R)\sigma_{\alpha\beta}v_\tau)(p_1^\alpha p_2^\beta - p_1^\beta p_2^\alpha)\big)
\end{aligned} \tag{23}$$

with p_1,p_2 are the four momentums of the P_1, P_2 mesons, respectively and the D_{iL} functions are given as

$$\begin{aligned}
D_{1L} &= \frac{1}{2}(C^{g_3}_{P_1P_2} + C^{g_4}_{P_1P_2}); \quad D_{1R} = \frac{1}{2}(C^{g_1}_{P_1P_2} + C^{g_2}_{P_1P_2}), \\
D_{2L} &= e^2 A_L F^{P_1P_2}; \quad D_{2R} = e^2 A_R F^{P_1P_2}, \\
D_{3L} &= \sum_{i=u,d,s} \frac{g_3^i + g_5^i}{2} F_{q_i}^{P_1P_2} \quad D_{3R} = \sum_{i=u,d,s} \frac{g_4^i + g_6^i}{2} F_{q_i}^{P_1P_2}, \\
D_{4L} &= C^{Tg_9}_{P_1P_2}; \quad D_{4L} = C^{Tg_{10}}_{P_1P_2}
\end{aligned} \tag{24}$$

with the $C^{g_i}_{P_1P_2}$ functions are given in ([1]) but the Y charges are replaced by the g_i charges. In order to calculate the dB and dR, we have to define the Lorentz frame (frame 4) for the three body decays [9]. This frame is the rest frame of τ^+ and the z direction is the μ^+ moving direction. The decay plane is the xz plane where the x positive direction is determined by the x component of the P_1 mesons momentum with the larger energy.We also define the energy variables $x_1 = 2E_1/M$ and $x_2 = 2E_2/M$, with E_1, E_2 are the energy of the P_1, P_2 mesons, respectively. On the basis of these energy variables and the rotational angles (θ, ϕ, ψ), we obtain the branching and spin dependence terms as follows

$$dB^{\tau^+ \to P1P2\mu^+} = \frac{1}{\Gamma} \frac{M_\tau^5 G_F^2}{256\pi^5} dx_1 dx_2 d\cos\theta d\phi d\psi X \tag{25}$$

where s_i, c_i are presented $\sin i, \cos i$ with $(i = \theta, \phi, \psi)$, respectively. The X, Y, Z functions are given as

$$\begin{aligned}
X &= \frac{1}{2} \left[\left(|D'_{1L}|^2 + |D'_{1R}|^2\right) \alpha_1 + \left(|D_{2L}|^2 + |D_{2R}|^2\right) \alpha_2 \right. \\
&+ \left(|D_{3L}|^2 + |D_{3R}|^2\right) \alpha_3 + \left(|D'_{4L}|^2 + |D'_{4R}|^2\right) \alpha_4 \\
&+ \left(D'^*_{4L} D_{3L} + D'^*_{4R} D_{3R} + H.c\right) \alpha_5 + \left(D'^*_{4L} D_{2L} + D'^*_{4R} D_{2R} + H.c\right) \alpha_6 \\
&+ \left(D'^*_{4L} D'_{1L} + D'^*_{4R} D'_{1R} + H.c\right) \alpha_7 + (D^*_{3R} D_{2R} + D^*_{3L} D_{2L} + H.c)\alpha_1 \\
&+ \left. (D^*_{2R} D'_{1R} + D^*_{2L} D'_{1L} + H.c)\alpha_8 - (D^*_{3R} D'_{1R} + D^*_{3L} D'_{1L} + H.c)\alpha_8 \right]
\end{aligned} \tag{26}$$

where $D'_{1L,R} = \frac{D_{1L,R}}{M_\tau}$, $D'_{4L,R} = M_\tau D_{4L,R}$ and the α_{1-8}, are given in the appendix (A). There are eight classes of terms in our results which arise from: (i)the four fermion coupling constants $(|D'_{1L,R}|^2, |D_{3L,R}|^2, |D'_{4L,R}|^2)$, (ii) the photon-penguin coupling constants $(|D_{2L,R}|^2)$, (iii) the interferences between the scalar type four fermion couplings and photon-penguin couplings $(D^*_{2L,R} D'_{1LR})$, (vi)the interferences between the vector type four fermion couplings and photon-penguin couplings$(D^*_{2L,R} D_{3L,R})$, (v) the interferences between the scalar type and vector type four fermion couplings $(D^*_{3L,R} D'_{1L,R})$, (vi) the interferences between the scalar type and tensor type four fermion couplings $(D'^*_{4L,R} D'_{1L,R})$, (vii) the interferences between the tensor type four fermion couplings and photon-penguin couplings ($D'^*_{4L,R}D'_{2L,R}$), (viii) the interferences between the tensor type and the vector type four fermion couplings ($D'^*_{4L,R}D'_{3L,R}$).

Let us start from analyzing the $D_{iL,R}$ functions as follows

$$\begin{aligned} D'_{1L,R} &\propto Y^{qs}_{1L,R}X^{ls}_{1L,R}f_s \equiv W^s_{1L,R}f_s; \quad D_{2,3L,R} \propto Y^{qv}_{2,3L,R}X^{lv}_{2,3,L,R}f_v \equiv W^v_{2,3L,R}f_v \\ D'_{4L,R} &\propto Y^{qt}_{4L,R}X^{lt}_{4L,R}f_t \equiv W^t_{4,L,R}f_t \end{aligned} \tag{27}$$

where $Y^{qa}_{iL,R}, i = 1...4, a = s, v, t$ are the complex weak coupling constants of scalar, vector and tensor quark currents with scalar, vector and tensor fields in beyond SM and $X^{la}_{iL,R}, i = 1...4, a = s, v, t$ are these of lepton currents. The product of $Y^{qa}_{iL,R}$ and $X^{la}_{iL,R}$ coupling constants are denoted as follows

$$W^a_{i,L,R} = Y^{qa}_{iL,R}X^{la}_{iL,R} = |W^a_{i,L,R}|e^{i\phi_a} \tag{28}$$

and $f_a, a = s, v, t$ are the scalar, vector and tensor form factors, respectively. In general case,the form factor can be written as:

$$f_v = |f_v|e^{i\delta^v}; \quad f_s = |f_s|e^{i\delta^s}; \quad f_t = |f_t|e^{i\delta^t} \tag{29}$$

where $\delta^a, a = s, v, t$ are strong phases associated with the hadronic form factors of scalar, vector, tensor quark currents, respectively. In language of strong phase, the interference terms can be written as

$$\begin{aligned} &D^*_{iL,R}D_{jL,R} + H.c \propto \\ &\propto 2|W^a_{i,L,R}||W^b_{j,L,R}||f_a||f_b| \left(\cos(\phi_b - \phi_a)\cos(\delta_b - \delta_a) - \underbrace{\sin(\phi_b - \phi_a)\sin(\delta_b - \delta_a)}_{CPodd} \right) \end{aligned} \tag{30}$$

and

$$\begin{aligned} &D^*_{iL,R}D_{jL,R} - H.c \propto \\ &\propto 2i|W^a_{i,L,R}||W^b_{j,L,R}||f_a||f_b| \left(\cos(\phi_b - \phi_a)\sin(\delta_b - \delta_a) + \underbrace{\sin(\phi_b - \phi_a)\cos(\delta_b - \delta_a)}_{CPodd} \right) \end{aligned} \tag{31}$$

Because of invariance of the strong interaction under charge conjugation, the differences between the strong phase ($\delta_a - \delta_b$) is uncharged under CP conjugation. However, the phases of weak coupling constants change sign under CP conjugation. It leads to the underbrace terms in Eqs. (30) and (31) are CP odd terms. Hence, the branching can be separated into the CP odd and CP even part, specially the CP odd part depends on the imaginary part of the interference between two types of four fermion couplings, while that even part depends on the real part of these.

IV. CONSIDERING $\tau^+ \to \pi^+\pi^-\mu^+$ PROCESS

Before considering detail $\tau^+ \to \pi^+\pi^-\mu^+$ process, let us consider the pion invariant mass available defined by $s = (p_1^\mu + p_2^\mu)^2$, the values of this available are shown in the Fig. (1). The small values ($s < 4m_\pi^2$) are nearly the edge of $x_1 + x_2 \simeq 1$ of the kinematical allowed region. In this region, the chiral perturbation theory is very useful to describe the form factor. Hence, in the next considering, we are going to consider the $\tau^+ \to \pi^+\pi^-\mu^+$ process by using the chiral perturbation theory.

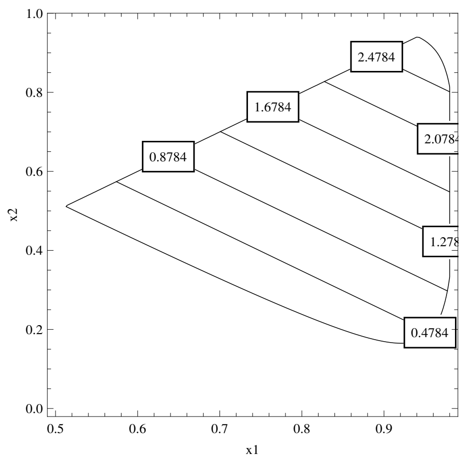

Fig. 1. The values of the s available are normalized by constants in GeV unit.

Base on the previous consideration, the total contribution to the $\tau^+ \to \pi^+\pi^-\mu^+$ process depends on by the photon penguin coupling and scalar, vector, tensor currents. Because, the vector contribution is expected to be much smaller than that of the photon penguin coupling[7], we will ignore the vector contribution and keep all of the other ones.

The pion scalar form factor at the next to leading order was performed in [4] with the result as follows

$$\begin{aligned} f_s^{\chi PT}(s) &= f_s(0)\left(1 + \frac{1}{16\pi^2 F^2}(\bar{l}_4 - \frac{13}{12})s + \Delta_{o,2} + O(E^4)\right) \\ f_s(0) &= m_\pi^2\left(1 - \frac{m_\pi^2}{32\pi^2 F^2}(\bar{l}_3 - 1) + O(m_\pi^4)\right) \end{aligned} \quad (32)$$

with

$$\begin{aligned} \Delta_{o,2} &= \frac{1}{2F^2}\left((2s - m_\pi^2)\overline{J}(s) + \frac{s}{96\pi^2}\right) \\ \overline{J}(s) &= \frac{1}{16\pi^2}\left[\sigma \ln\frac{\sigma - 1}{\sigma + 1} + 2\right] \end{aligned} \quad (33)$$

$\bar{l}_3, \bar{l}_4$ are two low-energy constants which are constrained in [4] $\bar{l}_3 = 2.9 \pm 2.4, \bar{l}_4 = 4.3 \pm 0.9$. At the one loop correction, $f_s(0) = (0.99 \pm 0.02)m_\pi^2$ is very close to the tree result $f_s^{tree} = 1$

In order to present the numerical results, we would like to remaind that the expressions of $D'^s_{L,R}$ are written as follows

$$D'^s_{L,R} = \frac{W^s_{L,R}}{M_\tau} f_s = \frac{W^s_{L,R}}{M_\tau}\frac{2f^{\chi PT}_s(s)}{m_u + m_d} = \frac{2|W^s_{L,R}|m^2_\pi}{(m_u + m_d)M_\tau} e^{i\phi_s} f'_s \tag{34}$$

with $f'_s = \big(1 + \frac{1}{16\pi^2 F^2}(\bar{l}_4 - \frac{13}{12})s + \Delta_{o,2} + O(E^4)\big)$.

Next, we consider the effects of the vector quark current on the $\tau^+ \to \pi^+\pi^-\mu^+$ process. The vector form factor at the next to leading order is given in [4] as follows

$$\begin{aligned} F_v(s) &= 1 + \frac{1}{6} < r^2 >^\pi_v + \Delta_{1,2}(s) + O(E^4) \\ < r^2 >^\pi_v &= \frac{1}{16\pi^2 F^2_\pi}(\bar{l}_6 - 1) + O(M^2_\pi) \end{aligned} \tag{35}$$

with

$$\Delta_{1,2}(s) = \frac{1}{6F^2_\pi}\left[(s - 4m^2_\pi)J(s) + \frac{s}{24\pi^2}\right] \tag{36}$$

The constant $\bar{l}_6 = 16.5 \pm 1.1$ is one of the low-energy parameter which is given in the effective Lagrangian at order E^4.

Now, we assume that the effects of vector current and scalar current on the $\frac{d^2 B^{\tau^+\to\pi^+\pi^-\mu^+}}{dx_1 dx_2}$ are dominated and skip the effects of the other currents. The total the differential branching ratio of this decay is obtained by considering both of contribution of vector and scalar currents is shown in Figs.(2). These results show the values of the $\frac{d^2 B^{\tau^+\to\pi^+\pi^-\mu^+}}{dx_1 dx_2}$ reach to maximal value at the $x_1 + x_2 \simeq 1$ and decrease when x_1 turns to be larger than 0.85. The interference between the scalar and vector currents doesn't effect on the differential branching ratio of this decay at the low value of the pion invariant mass.

On the other hand, the results displayed Figs.(3) show the allowed invariant mass of the pions can not exceed 1.75 GeV and the contributions at the lower energy are dominated.

Let us consider the contribution of CP odd terms to the $\frac{d^2 B^{\tau^+\to\pi^+\pi^-\mu^+}}{dx_1 dx_2}$. The CP asymmetry A_{CP} is defined as follows

$$A_{CP} = \frac{\frac{d^2 B^{\tau^+\to\pi^+\pi^-\mu^+}}{dx_1 dx_2} - \frac{d^2 B^{\tau^-\to\pi^+\pi^-\mu^-}}{dx_1 dx_2}}{\frac{d^2 B^{\tau^+\to\pi^+\pi^-\mu^+}}{dx_1 dx_2} + \frac{d^2 B^{\tau^-\to\pi^+\pi^-\mu^-}}{dx_1 dx_2}} \tag{37}$$

The results displayed in Fig.(4) show the value of CP asymmetry increases if the energy of pion with larger energy increases and maximal value of CP asymmetry is obtained at the maximal value of x_1. However, in this kinematical region, the χPT predicts the differential branching ratio decreases. Thus, it is hard to obtain the couple of pion emitted in kinematical region which allows the maximal value of CP asymmetry. On the other hand, the weak phases are model dependent and the value of $\sin(\phi_a - \phi_b)$ is smaller than one unit. Hence, the value of CP asymmetry in the $\tau \to \pi^+\pi^-\mu^+$ is smaller than our predicted values given in Fig.(4). For more detail, we consider the effects of the interference

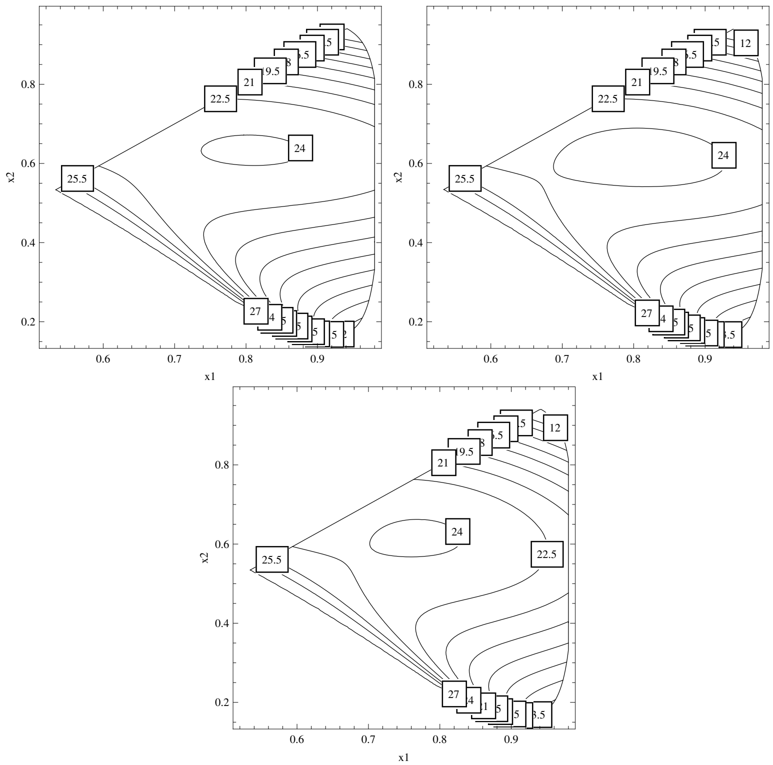

Fig. 2. The total contributions of the vector and scalar currents to $\frac{d^2B^{\tau^+\to\pi^+\pi^-\mu^+}}{dx_1dx_2}$ normalized by constants. The figures from left to right are the numerical results by taking $\sin(\phi_s-\phi_v)=1;0.5;0.01$, respectively.

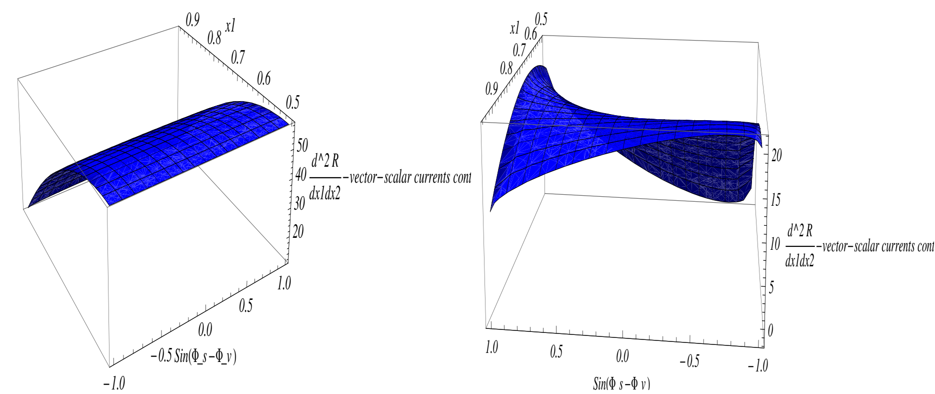

Fig. 3. The left and right figures represent the total contribution of the vector current and scalar to $\frac{d^2B^{\tau^+\to\pi^+\pi^-\mu^+}}{dx_1dx_2}$ at the $s=4m_\pi^2$ and $s\simeq 1.75GeV$, respectively.

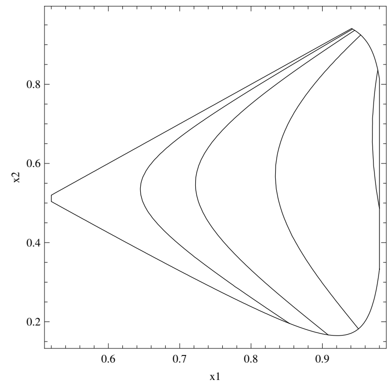

Fig. 4. The CP asymmetry normalized by constants and taking $\sin(\phi_s - \phi_v) = 1$. The curves from left to right represent the CP asymmetry normalized by $10^{-\frac{n}{2}}, n = 1, 2, 3, 4$, respectively.

between the scalar and vector currents on the CP asymmetry by fixing the value of the pion invariant mass. At the low energy $s \leq 4m_\pi^2$, the CP asymmetry given in the Fig. (5) is approximately zero. On the other hand, by taking $\sin(\phi_s - \phi_v) = 6 \times 10^{-3}$, we obtain

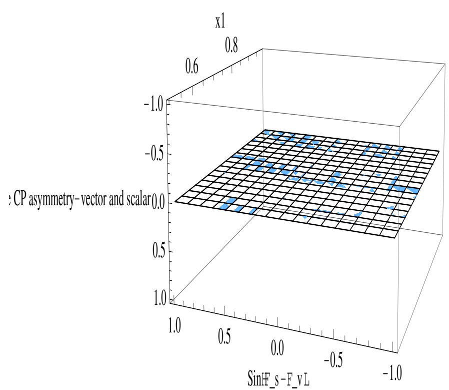

Fig. 5. The CP asymmetry at the $s = 4m_\pi^2$.

the total contribution of vector and scalar currents to $\frac{d^2B^{\tau^+\to\pi^+\pi^-\mu^+}}{dx_1dx_2}$ at the low energy $s = 4m_\pi^2$ and $s \simeq 0.6GeV$ given in the Fig. (6). These results show that the probability of the pair of pions emitted at the lower energy are larger than that at higher energy. This is very nice information to confirm that the chiral perturbation theory is useful to describe the pion form factor in the τ decays to $\mu^+\pi^+\pi^-$ at the low invariant mass of pion.

As far as we know the chiral perturbation theory reproduces the experimental data for the strong phase in the low energy region, however fails above $\sqrt{s} \sim 500$ MeV, where the $\rho-$ resonance starts to dominate. In order to account for this, we need to use an effective theory with explicit resonance fields as degrees of freedom, namely resonance

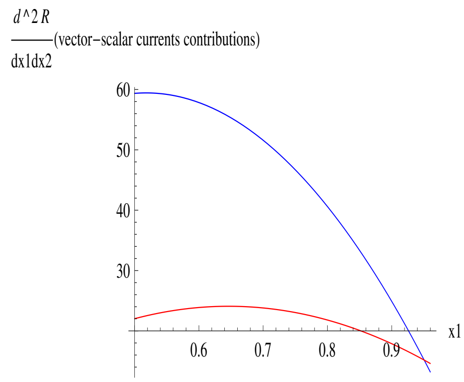

Fig. 6. The blue and red curves represent the total contribution of the vector and scalar currents to $\frac{d^2B^{\tau^+\to\pi^+\pi^-\mu^+}}{dx_1dx_2}$ by taking $\sin(\phi_s-\phi_v)=0.6\times10^{-3}$ at the low energy $s=4m_\pi^2$ and $s\simeq 0.6GeV$, respectively.

chiral effective theory. In this theory, the vector pion form factor is given in [12] as follows

$$F_v(s) = \frac{M_\rho^2}{M_\rho^2-s-iM_\rho\Gamma_\rho(s)} \times \exp -\frac{s}{96\pi^2F^2}\left(ReA(\frac{m_\pi^2}{s},\frac{m_\pi^2}{M_\rho^2})+\frac{1}{2}ReA(\frac{m_K^2}{s},\frac{m_K}{M_\rho^2})\right) \quad (38)$$

where $A(\frac{m_p^2}{s},\frac{m_p^2}{M_\rho^2})$ is given in [12]. With choice of vector form factor given in (38), we obtain a distribution of the vector quark current including role of the rho meson to the $\frac{d^2B^{\tau^+\to\pi^+\pi^-\mu^+}}{dx_1dx_2}$ described in the Fig.(7). It is clearly to see the maximal contribution of vector current obtained at the $\sqrt{s}=M_\rho$. For $\sqrt{s}>M_\rho$, this contribution decreases very quickly but for $\sqrt{s}<M_\rho$, it decreases inch by inch and these values are the same order as these of the chiral perturbation theory.

The total contribution of the scalar and vector currents including role of the rho meson is shown in the Fig. (8). The interference term does not effect much on the $\frac{d^2B^{\tau^+\to\pi^+\pi^-\mu^+}}{dx_1dx_2}$ in the vector dominated region but it's effect is more clearly in the scalar dominated region.

The CP asymmetry including role of rho meson are displayed in the Fig.(9). If the weak coupling constants are pure imaginary, the maximal contribution of CP odd term can be larger than ten percents of the total contribution. By fixing the invariant mass $s=M_\rho^2$, we obtain the value of the CP asymmetry given in the Fig. 10. The CP asymmetry closes to it's maximal value at the largest value of x_1. This maximal values can be reached to 0.2 if the weak phase is taken by $\sin(\phi_s-\phi_v)=1$.

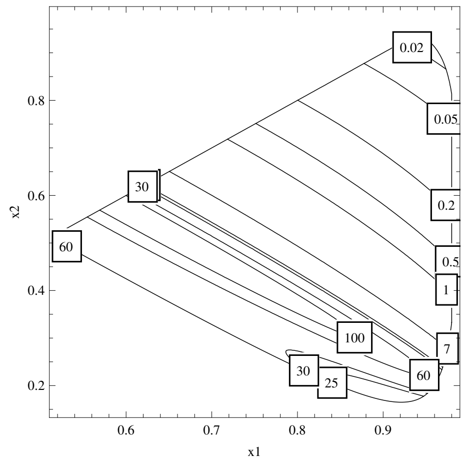

Fig. 7. The contribution of the vector current including role of the rho meson to $\frac{d^2B^{\tau^+\to\pi^+\pi^-\mu^+}}{dx_1dx_2}$ normalized by constants. The left and right figures correspond to $\sin(\phi_s-\phi_v)=1; 0.5$, respectively

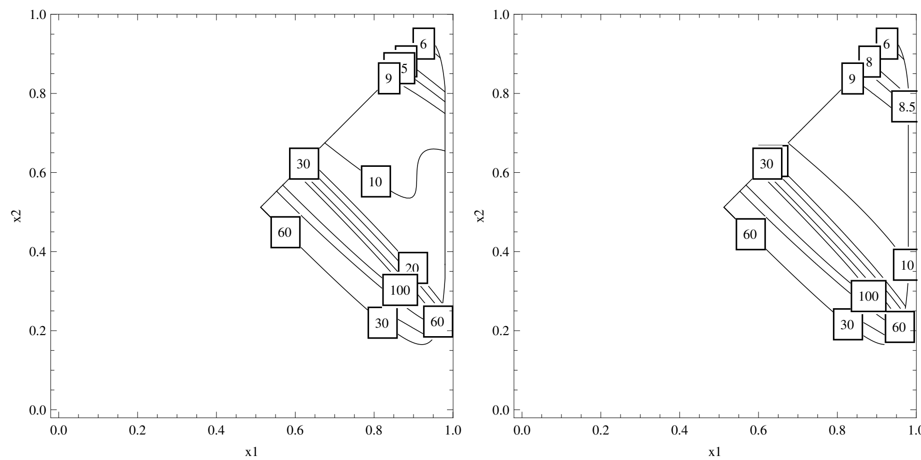

Fig. 8. The contribution of the vector current including role of the rho meson to $\frac{d^2B^{\tau^+\to\pi^+\pi^-\mu^+}}{dx_1dx_2}$ normalized by constants. The left and right figures correspond to $\sin(\phi_s-\phi_v)=1; 0.5$, respectively.

V. CONCLUSIONS

In this work, the general differential cross sections for $t^+t^-\to f_Af_B$ with $f_B=\nu\pi^-$ and $f_A=\mu^+PP$ are presented. For $\tau^+\to\mu^+PP$, we obtain the P and T odd asymmetries of three body decays as being done in [3]. However, we show the existence of both of CP odd and CP even parts in each of P and T odd asymmetries. The CP odd terms are given

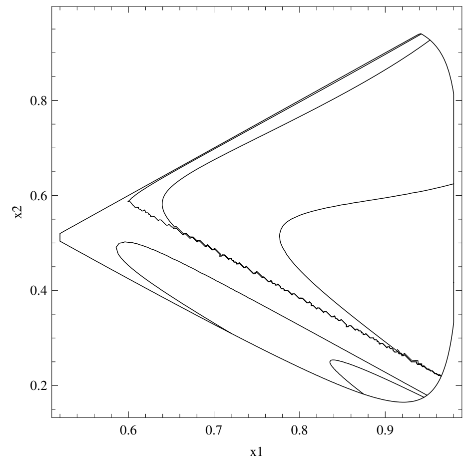

Fig. 9. The CP asymmetry including role of the rho meson normalized by constants at $\sin(\phi_s - \phi_v) = 1$. The top part of the figure, from left to right, the curves are normalized by $10^{-3}, 10^{-2}, 10^{-1}$, respectively. The below part of the figure, from left to right, the curves are normalized by $10^{-3}, 10^{-2}$, respectively

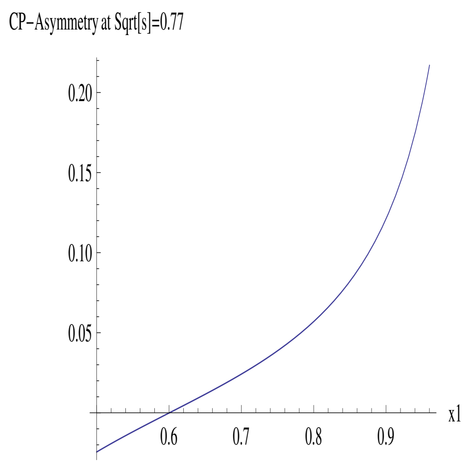

Fig. 10. The CP asymmetry including role of the rho meson normalized by constants at $\sin(\phi_s - \phi_v) = 1$ and $\sqrt{s} = M_\rho$

by interference between two kinds of quark currents and they depend on the imaginary of weak coupling constants.

We also considered very detail of $\tau^+ \to \mu^+\pi^+\pi^-$ process by assuming the dominance of resonance vector meson. We show that chiral perturbation theory is very well to consider this semilepton τ decay up to $s < 4m_\pi^2$. At the large invariant mass values, the role of the ρ meson becomes important. The Dalitz plot not only displayed the kinematical region where the contribution of vector current dominates but also shown the kinematical region where that of the vector current dominates. As large as s-values, the dominance contribution of scalar current is more precisely. The maximal value of the CP asymmetry is approximately twenty percents at $s = M_\rho^2$.

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Appendix A. THE KINEMATICAL FUNCTIONS

$$
\begin{aligned}
\alpha_1 &= 2 - x_1 - x_2; \quad \alpha_2 = -\frac{(x_1 - x_2)^2 + x_1 + x_2 - 2}{x_1 + x_2 - 1} \\
\alpha_3 &= x_1(3 - 4x_2) + 3x_2 - 2; \quad \alpha_4 = (1 - x_1 - x_2)\left((x_1 - x_2)^2 + x_1 + x_2 - 2\right) \\
\alpha_5 &= (x_1 + x_2 - 2)(x_1 + x_2 - 1); \quad \alpha_6 = \left((x_1 - x_2)^2 + x_1 + x_2 - 2\right) \\
\alpha_7 &= (x_1 - x_2)(x_1 + x_2 - 1); \quad \alpha_8 = x_1 - x_2
\end{aligned} \tag{39}
$$

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