Proc. Natl. Conf. Theor. Phys. **36** (2011), pp. 175-181

CYCLOTRON RESONANCE LINE-WIDTH DUE TO INTERACTION OF ELECTRON AND LO-PHONON IN RECTANGULAR QUANTUM WIRE

HUYNH VINH PHUC
Department of Physics, Dong Thap University, 783 Pham Huu Lau, Cao Lanh, Dong Thap
LE DINH and TRAN CONG PHONG
Department of Physics, Hue University's College of Education, 32 Le Loi, Hue

Abstract. *In this paper, the dependence of cyclotron resonance line-width (CRLW) on the magnetic field, the wire's size and the temperature are theoretically considered by computational method for a rectangular quantum wire (RQW) in the presence of an external static magnetic field. It is shown that in intense magnetic field CRLW depends strongly on the magnetic field strength whereas the behavior of CRLW is determined primarily by the wire's size in the weak magnetic field. In both cases of intense and weak magnetic field, CRLW increases with temperature and decreases with the wire's size.*

I. INTRODUCTION

The study of magneto-optical transitions, including CRLW, is known as a good tool for investigating transport behavior of electrons in semiconductor material. CRLW has been studied both theoretically [1-7] and experimentally [8]. However, most of these works is merely focused on the quasi 2 or 3-dimensional electron systems. Therefore, CRLW in 1D semiconductors, such as RQW is needed for studying.

Up to date, there have been many works studying CRLW using different methods. In the work of Mayer [6], Suzuki [7] and Kobori [8], the absorption power $P(\omega)$ of the incident electromagnetic wave of frequency ω is given by [9]

$$P(\omega) = \int_{-\infty}^{+\infty} dk_z A(\omega, k_z) \frac{\Gamma(\omega, k_z)}{(\omega-\omega_c)^2 + [\Gamma(\omega, k_z)]^2}, \tag{1}$$

where ω_c is the cyclotron frequency, $A(\omega, k_z)$ is the function of ω and the component of the wave vector $\vec{k}$ along z-direction (k_z), and $\Gamma(\omega, k_z)$ is the CRLW. However, in one of their papers [9], Cho and Choi termed $\Gamma(\omega, k_z)$ as the energy-dependent relaxation rate but not the line-widths. According to these authors, the line-widths can be obtained if $P(\omega)$ can be plotted.

Recently, our group has proposed a method to obtain the line-widths from graphs of $P(\omega)$ [10]. In this paper, we use this method to determine CRLW in RQW, one of the quasi one-dimensional electron systems. We study the dependence of CRLW on the magnetic field induction B, the wire's size L_x and the temperature T. The paper is organized as follows: The calculation of analytic expression of the absorption power $P(\omega)$ in a specific GaAs/AlAs RQW in the presence of a magnetic field is presented in section II. Section

III shows the graphic dependence of $P(\omega)$ on the photon energy. From this we obtain the CRLW and examine the dependence of it on B, T, and L_x. A conclusion is introduced in Section IV.

II. ABSORPTION POWER IN RECTANGULAR QUANTUM WIRES

We consider a RQW semiconductor model, where the conduction electrons are free along the z-direction and confined in the (x, y) plane with confined potentials are given by

$$V_1(x) = \begin{cases} 0 & 0 \leqslant x \leqslant L_x, \\ \infty & x < 0, \quad x > L_x, \end{cases} \quad V_2(y) = \begin{cases} 0 & 0 \leqslant y \leqslant L_y, \\ \infty & y < 0, \quad y > L_y. \end{cases} \tag{2}$$

We assume that a static magnetic $\vec{B} \| \hat{\vec{z}}$ is applied along the z-direction of the wire. The one-particle Hamiltonian H_e, the normalized eigenfunctions $|\lambda\rangle$, and the eigenvalues E_λ in the Landau gauge of vector potential $\vec{A} = (-By, 0, 0)$ for confined electrons are obtained using the effective-mass approximation [13]

$$H_e = (\vec{p} - e\vec{A})^2/2m^* + V_1(x) + V_2(y), \tag{3}$$

$$|\lambda\rangle \equiv |N, n, k_z\rangle = \Phi_N(y - y_\lambda)\sqrt{\frac{2}{L_x}} \sin\frac{n\pi x}{L_x} \exp(ik_z z)/\sqrt{L_z}, \tag{4}$$

$$E_\lambda \equiv E(N, n, k_z) = (N + 1/2)\hbar\omega_c + n^2 E_0 + \hbar^2 k_z^2/(2m^*), \tag{5}$$

where $\vec{p}$ and e are the momentum operator and charge of a conduction electron, respectively, $N = 0, 1, 2, \ldots$ and $n = 1, 2, 3, \ldots$ denote the Landau-level index and subband indices, respectively, m^* is the effective mass of electron, $\omega_c = eB/m^*$ is cyclotron frequency, H_N is a Hermite polynomial, $a_c = (\hbar/m^*\omega_c)^{1/2}$ is the cyclotron radius, energy $E_0 = \hbar^2\pi^2/2m^* L_x^2$. The function $\Phi_N(y - y_\lambda)$ in Eq. (4) represents harmonic oscillator, centered at $y_\lambda = -\hbar k_z/(m^*\omega_c)$ and can be written as

$$\Phi_N(y - y_\lambda) = \frac{1}{(\sqrt{\pi}2^N N! a_c)^{1/2}} \exp\left(-\frac{(y - y_\lambda)^2}{2a_c^2}\right) H_N\left(\frac{y - y_\lambda}{a_c}\right). \tag{6}$$

For calculating the absorption power of electromagnetic wave in RQW we use the following matrix elements [14]

$$|\langle\lambda|\mathrm{e}^{\pm i\vec{q}\cdot\vec{r}}|\lambda'\rangle|^2 = [J_{nn'}(\pm q_x)]^2 |J_{N,N'}(u)|^2 \delta_{k_z', k_z \pm q_z}, \tag{7}$$

$$J_{nn'}(\pm q_x) = \int_0^{L_x} \sin\frac{n\pi x}{L_x} \mathrm{e}^{\pm i\vec{q}\cdot\vec{r}} \sin\frac{n'\pi x}{L_x} dx, \tag{8}$$

$$|J_{N,N'}(u)|^2 = \frac{N!}{N'!} \mathrm{e}^{-u} u^{N'-N} [L_N^{N'-N}(u)]^2, \; N \leq N', \tag{9}$$

$$\int_{-\infty}^{+\infty} |J_{nn'}(\pm q_x)|^2 dq_x = (\pi/L_x)(2 + \delta_{nn'}), \tag{10}$$

where $\vec{q}$ is the wave vector of phonon, $L_N^{N'-N}(u)$ is a Laguerre polynomial of variable $u = a_c^2(q_y^2 + q_z^2)/2$. Phonons under consideration are assumed to be dispersionless (i.e. $\hbar\omega_q \approx \hbar\omega_{LO} = \text{const}$, with ω_{LO} is the LO-phonon frequency). We now apply the general expression for the absorption power in bulk semiconductors, presented in Ref. 5, to RQW.

Considering transitions between two lowest Landau levels with $N = 0$ and $N' = 1$, and supposing that the scattering process occurs at the boundary of Brillouin zone, we obtain the absorption power in RQW

$$P(\omega) = \frac{eE_{0\omega}^2}{m^*\omega_c}\sum_{n,n'}\frac{\{f[E(0,n,0)] - f[E(1,n',0)]\}\gamma(\omega)}{(\omega-\omega_c)^2 + [\gamma(\omega)]^2}, \tag{11}$$

where $E_{0\omega}$ is the intensity of electromagnetic wave and

$$\hbar\gamma(\omega) = \pi\sum_{n",\vec{q}}|V_q|^2[J_{nn"}(\pm q_x)]^2[(1+N_q)X_1 + N_q X_2], \tag{12}$$

with $N_q = [\exp(\hbar\omega_{LO}/(k_BT)) - 1]^{-1}$, is the distribution function for LO-phonons, k_B being the Boltzmann constant. In Eq. (12), the quantity $J_{nn"}(\pm q_x)$ is given in Eq. (8), X_1 and X_2 are defined as follows from Ref. 5

$$\begin{aligned} X_1 \;=\; & \sum_{N"\neq 1}\delta[\hbar\omega + E(0,n,0) - E(N",n",0) - \hbar\omega_{LO}]|J_{0,N"}(u)|^2 \\ & + \sum_{N"\neq 0}\delta[\hbar\omega - E(1,n',0) + E(N",n",0) + \hbar\omega_{LO}]|J_{1,N"}(u)|^2, \end{aligned} \tag{13}$$

$$\begin{aligned} X_2 \;=\; & \sum_{N"\neq 1}\delta[\hbar\omega + E(0,n,0) - E(N",n",0) + \hbar\omega_{LO}]|J_{0,N"}(u)|^2 \\ & + \sum_{N"\neq 0}\delta[\hbar\omega - E(1,n',0) + E(N",n",0) - \hbar\omega_{LO}]|J_{1,N"}(u)|^2. \end{aligned} \tag{14}$$

Here the coupling factor expressed is given by [15]

$$|V_q|^2 = \frac{2\pi e^2\hbar\omega_{LO}}{\varepsilon_0 V_0}\left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0}\right)\frac{1}{q^2} \approx \frac{D}{q_\perp}, \quad D = \frac{2\pi e^2\hbar\omega_{LO}}{\varepsilon_0 V_0}\left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0}\right), \tag{15}$$

where ε_0 is the permittivity of free space, χ_∞ and χ_0 are the high and low frequency dielectric constant, respectively. V_0 is the volume of the crystal.

For calculating the absorption power in Eq. (11), we need to calculate $\gamma(\omega)$ in Eq. (12). The sum over $\vec{q}$ will be transformed into the integral, the integral over q_x is given by Eq. (10). Using Eq. (A4) from Ref. [14] to calculate the integral over $q_\perp$, we obtain

$$\begin{aligned} \hbar\gamma(\omega) = \sum_{n"}\frac{DV_0}{8L_x}(2+\delta_{nn'})\Bigg\{ & N_q\Bigg[\sum_{N"\neq 1}\frac{\delta(Y_1^-)}{N"-1} + \sum_{N"\neq 0}\frac{\delta(Y_2^+)}{N"}\Bigg] \\ & + (1+N_q)\Bigg[\sum_{N"\neq 1}\frac{\delta(Y_1^+)}{N"-1} + \sum_{N"\neq 0}\frac{\delta(Y_2^-)}{N"}\Bigg]\Bigg\}, \end{aligned} \tag{16}$$

where we have denoted

$$Y_1^{\pm} = \hbar\omega \mp P\hbar\omega_c + (n^2 - n"^2)E_0 \pm \hbar\omega_{LO}, \tag{17}$$

$$Y_2^{\pm} = \hbar\omega \mp P\hbar\omega_c + (n"^2 - n'^2)E_0 \pm \hbar\omega_{LO}. \tag{18}$$

Here we have set $N" - N = -P$ in the emission term and $N" - N = P$ in the absorption term ($P = 1, 2, 3, \ldots$) [14]. Here $N = 0$ in $Y_1^{\pm}$ and $N = 1$ in $Y_2^{\pm}$. Inserting Eq. (16)

into Eq. (11), we obtain the analytical expression of the absorption power in a RQW. However, delta functions in the expression for $\gamma(\omega)$ result in the divergence of $\gamma(\omega)$ when $Y_1^{\pm} = 0$ or $Y_2^{\pm} = 0$. To avoid this we shall replace the delta functions by Lorentzians [16]

$$\delta[Y_{1,2}^{\pm}] = \frac{1}{\pi} \frac{\hbar\Gamma_{N,N'}^{\pm}}{(Y_{1,2}^{\pm})^2 + \hbar^2(\Gamma_{N,N'}^{\pm})^2}, \tag{19}$$

where $\Gamma_{NN"}^{\pm}$, the inverse relaxation time, is called the width of a Landau level. Using Eq. (A6) from Ref. 16, we have

$$(\Gamma_{N,N'}^{\pm})^2 = \sum_{N",n"} \frac{V_0 D}{\hbar^2 8\pi L_x} \frac{1}{N" - N}(2 + \delta_{nn"})(N_q + 1/2 \pm 1/2). \tag{20}$$

III. NUMERICAL RESULT AND DISCUSSION

The obtained results can be clarified by numerically calculation the absorption power $P(\omega)$ in Eq. (11) for a specific GaAs/AlAs RQW. From the graphs of $P(\omega)$, we identify the position of cyclotron resonance peaks and obtain CRLW as profiles of the curves. The dependence of CRLW on the magnetic field induction, wire's size and temperature are discussed. Parameters used in the numerical calculation are [15]: $\varepsilon_0 = 12.5$, $\chi_\infty = 10.9$, $\chi_0 = 13.1$, $m^* = 0.067m_0$ (m_0 being the mass of free electron), $\hbar\omega_{LO} = 36.25$ meV.

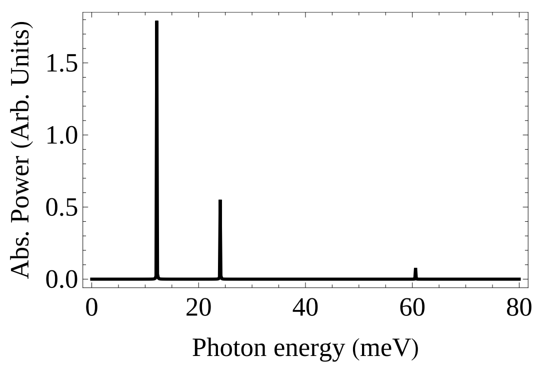

Fig. 1. Absorption power as function of the photon energy at $B = 7.0$ T, $L_x = 20$ nm, $T = 250$ K. Here, $N = 0, N' = 1$; $n, n', n" = 1 \div 2$.

Figure 1 describes the dependence of absorption power on the photon energy at $B = 7.0$ T, corresponding to cyclotron energy $\hbar\omega_c = \hbar eB/m^* = 12.19$ meV. The graph has three peaks, each of which describes a specific resonance. The first peak corresponds to the value $\hbar\omega = 12.19$ meV, which satisfies the condition $\hbar\omega = \hbar\omega_c$. Therefore, this peak is called the cyclotron resonance one. The second peak corresponds to the value $\hbar\omega = 24.06$ meV, satisfying the condition $\hbar\omega = \hbar\omega_{LO} - \hbar\omega_c = 36.25 - 12.19$ meV. This is the condition for optically detected magneto-phonon resonance (ODMPR) [17, 18] with $P = 1$. The third peak corresponds to the value $\hbar\omega = 60.63$ meV, satisfying the condition for ODMPR, $\hbar\omega = 2\hbar\omega_c + \hbar\omega_{LO} = 2 \times 12.19 + 36.25$ meV, with $P = 2$. We can see from the figure that the cyclotron resonance peak (first one) has the greatest value. This mean that the cyclotron resonance transition is dominant. In the following, we use this peak to investigate the CRLW.

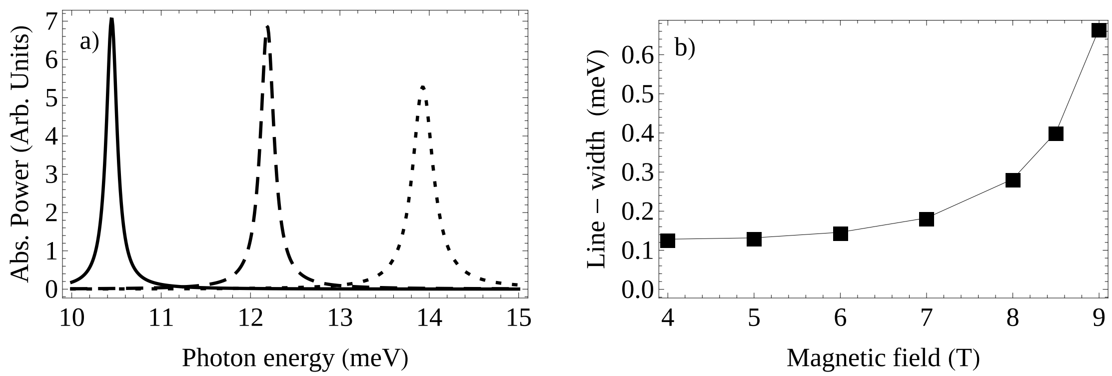

Fig. 2. a) Absorption power as function of the photon energy with different values of magnetic fields. The solid, dashed, and dotted lines correspond to $B = 6.0$ T, 7.0 T, and 8.0 T. b) Magnetic field dependence of CRLW. Here, $L_x = 20$ nm, $T = 250$ K, $N = 0, N' = 1$; $n, n', n" = 1 \div 2$.

Fig. 2a) describes the dependence of the absorption power on the photon energy with different values of B in the case of $T = 250$ K, $L_x = 20$ nm. From the figure, we can see that with $B = 6.0$ T, 7.0 T, and 8.0 T, the photon energy values corresponding to resonance peaks are, respectively, $\hbar\omega = 10.45$ meV, 1219 meV, and 13.93 meV. Consequently, the shapes of the absorption power curves have peaks at the cyclotron energy, $\hbar\omega = \hbar\omega_c$. From these curves, we obtain the magnetic field induction dependence of the CRLW as shown in Fig. 2b). It can be seen that CRLW increases with B. This result is in good agreement with those obtained by the other authors [3, 5, 8]. This can be explained that as B increases, the cyclotron frequency ω_c increases, the cyclotron radius $a_c = (\hbar/(m^*\omega_c))^{1/2}$ reduces, the confinement of electron increases, the probability of electron-phonon scattering increases, so that CRLW rises. We also see from the figure that CRLW depends strongly on the B in the region of strong magnetic field whereas in region of the weak magnetic field the influence of magnetic field on CRLW is negligible. This can be explained that in the range of a weak magnetic field, the cyclotron radius a_c is greater compared to the wire's size, so that the effect of confined electrons is determined primarily by the size of quantum wires. The influence of magnetic field on CRLW is strong in the case of $a_c \leqslant L_x/2$, with $L_x = 20$ nm. The magnetic field induction satisfies this condition is $B \geqslant 6.58$ T. Figure 2b) also shows that CRLW strongly increases when $B > 7.0$ T.

In comparison to experimental results of Kobori [8], we see that CRLW in RQW has greater value than that in normal 3D materials. This reason can be explained in the following when we investigate the dependence of CRLW on the wire's size.

Figure 3a) describes the dependence of absorption power on the photon energy with different values of wire's size. From the figure we can see that the cyclotron resonance peaks of the absorption power curves locate at the same position, corresponding to the cyclotron resonance condition, $\hbar\omega = \hbar\omega_c$, and is independent of L_x. From these curves, we obtain

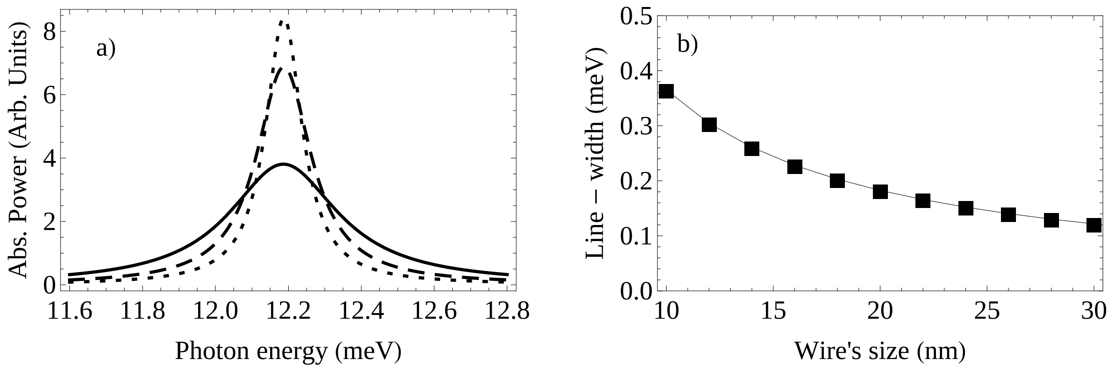

Fig. 3. a) Absorption power as function of the photon energy with different values of wire's size. The solid, dashed, and dotted lines correspond to $L_x = 10$ nm, 20 nm, and 30 nm. b) Wires's size dependence of CRLW. Here, $T = 250$ K, $B = 7.0$ Tesla, $N = 0, N' = 1$; $n, n', n" = 1 \div 2$.

the wire's size dependence of the CRLW's as shown in Fig. 3b). The figure shows that CRLW decreases with L_x. The reason for this is that as L_x increases, the confinement of electrons decreases, the probability of electron-LO-phonon scattering drops, so that CRWL decreases.

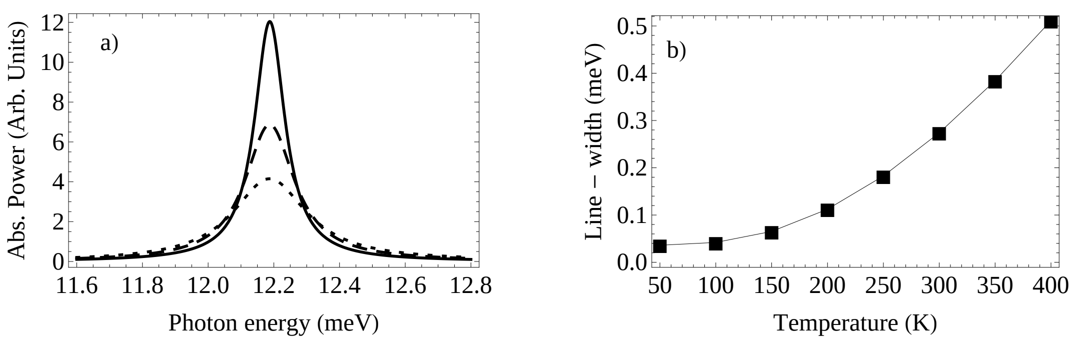

Fig. 4. a) Absorption power as function of photon energy with different values of temperatures. The solid, dashed, and dotted lines correspond to $T = 200$ K, 250 K, and 300 K. b) Temperature dependence of CRLW. Here, $L_x = 20$ nm, $B = 7.0$ T, $N = 0, N' = 1$; $n, n', n" = 1 \div 2$.

Fig. 4a) describes the dependence of the absorption power on photon energy with different values of temperature. From the figure we can see that the cyclotron resonance peaks locate at the same position $\hbar\omega = 12.19$ meV, corresponding to the cyclotron resonance's condition, $\hbar\omega = \hbar\omega_c$, and is independent of T. From these curves, we obtain the temperature dependence of the CRLW's as shown in Fig. 4b). The figure shows that CRLW increases with temperature. The result is consistent with that shown in Refs. 1-8. The reason for this is that as the temperature increases, the probability of electron-LO-phonon scattering rises.

IV. CONCLUSION

We have derived the analytical expression of the absorption power of an intensity electromagnetic wave in RQW with the presence of a static magnetic field. We have done the numerical calculation of the absorption power for GaAs/AlAs RQW and plotted graphs to clarify the theoretical results. We have obtained CRLW as profiles of the curves of the graphs. Numerical results for this RQW show clearly the dependence of the CRLW on the magnetic field induction, the wire's size and the temperature.

Computational results show that CRLW depends strongly on the magnetic field induction in the region of strong magnetic field whereas in the weak region the influence of magnetic field on CRLW is negligible. The behavior of CRLW is determined primarily by the size of quantum wires in the weak magnetic field. In both cases of strong and weak magnetic field, CRLW increases with temperature and decreases with the wire's size. The results are in good agreement with experimental data of Kobori and other theoretical results.

ACKNOWLEDGMENT

This work was supported by the National Foundation for Science and Technology Development – NAFOSTED of Vietnam (Grant No. 103.01.23.09), and MOET-Vietnam in the scope research project coded of B2010-DHH 03-60.

REFERENCES

[1] C. K. Sarkar, P. Banerji, *J. Phys. Chem. Solids* **53** (1992) 713.
[2] C. K. Sarkar, P. K. Basu, *Solid State Commun.* **60** (1986) 525.
[3] J. Y. Sug, S. G. Jo, J. Kim, J. H. Lee, S. D. Choi, *Phys. Rev. B* **64** (2001) 235210.
[4] R. Lassing, E. Gornik, *Solid State Commun.* **47** (1983) 959.
[5] N. L. Kang , K. S. Bae , C. H. Choi , Y. J. Lee , J. Y. Sug , J. H. Kim, S. D. Choi, *J. Phys.: Condens. Matter* **7** (1995) 8629.
[6] H. J. G. Meyer, *Phys. Lett.* **2** (1962) 259.
[7] A. Suzuki, S. D. Choi, S. Fujita, *J. Phys. Chem. Solids* **41** (1980) 735.
[8] H. Kobori, T. Ohyama, E. Otsuka, *J. Phys. Soc. Jpn.* **59** (1995) 2141.
[9] Y. J. Cho, S. D. Choi, *Phys. Rev. B* **49** (1994) 14301.
[10] T. C. Phong, H. V. Phuc, *Modern Phys. Lett. B* **25** (2011) 1003.
[11] P. Vasilopoulos, P. Warmenbol, F. M. Peeters, J. T. Devreese, *Phys. Rev. B* **40** (1989) 1810.
[12] M. Charbonneau, K. M. Van Vliet, P. Vasilopoulos, *J. Math. Phys.* **23** (1982) 318.
[13] M. A. Ruvinskii, B. M Ruvinskii, *Phys. Chem. Solid State*, **8** (2007) 667.
[14] P. Vasilopoulos, M. Charbonneau, K. M. Van Vliet, *Phys. Rev. B* **35** (1987) 1334.
[15] M. Masale, N. C. Constantinou, *Phys. Rev. B* **48** (1993) 11128.
[16] M. P. Chaubey, C. M. V. Viliet, *Phys. Rev. B* **33** (1986) 5617.
[17] S. C. Lee, Y. B. Kang, D. C. Kim, J. Y. Ryu, N. L. Kang, S. D. Choi, *Phys. Rev. B* **55** (1997) 6719.
[18] D. J. Barnes, R. J. Nicholas, F. M. Peeters, X. G. Wu, J. T. Devreese, J. Singleton, C. J. G. M. Langerak, J. J. Harris, C. T. Foxon, *Phys. Rev. Lett.* **66** (1991) 794.

Received 30-09-2011.