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FLAVOR SYMMETRY AND NEUTRINO MIXING

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Abstract. *We give a review of flavor symmetries recently proposed as a leading candidate in solving the tribimaximal neutrino-mixing form. We show how these symmetries work by taking concrete examples:* A_4 *symmetry in the standard model and* $3-3-1$ *model.*

I. WHY FLAVOR SYMMETRY?

Neutrinos Come in at Least Three Flavors

The known neutrino flavors	ν_e	$\longleftarrow - - - - - - - - \longrightarrow$	e	The corresponding charged-lepton flavors
	ν_μ	$\longleftarrow - - - - - - - - \longrightarrow$	μ	
	ν_τ	$\longleftarrow - - - - - - - - \longrightarrow$	τ	

The Neutrino Revolution (1998 – $\cdots$)

An sample of neutrino oscillation (flavor changing) is $\nu_\mu \longrightarrow \nu_\tau$ in atmosphere. *Remark*: Neutrinos have nonzero masses and mixing!

Neutrino Mixing

When $W^+ \longrightarrow l^+_\alpha + \nu_\alpha$ ($l_\alpha \equiv e,\ \mu,$ or τ, and $\alpha \equiv e,\ \mu,$ or τ), the produced neutrino field (ν_α—neutrino of flavor α) is $\nu_\alpha = \sum_i U_{\alpha i}\ \nu_i$, where ν_i is neutrino of definite mass m_i ($i = 1, 2, 3$). The neutrino mixing matrix $U \equiv (U_{lL})^\dagger U_{\nu L} = O_{23} \times O_{31} \times O_{12}$ is given in terms of Euler-angles parametrization.

The Current Experiment [PDG2010]

$$\triangle m^2_{21} = (8.0 \pm 0.3) \times 10^{-5}\ \text{eV}^2, \qquad |\triangle m^2_{32}| = 1.9 \text{ to } 3.0 \times 10^{-3}\ \text{eV}^2$$
$$\sin^2(2\theta_{12}) = 0.86(+/-)(0.03/0.04), \qquad \theta_{12} \simeq 34^o$$
$$\sin^2(2\theta_{23}) > 0.92, \qquad \text{best fit } \theta_{23} \simeq 45^o$$
$$\sin^2(2\theta_{13}) < 0.19$$

There are two kinds of hierarchies, "normal" or "inverted", depending on the sign of $\triangle m^2_{32}$ positive or negative, respectively.

Tribimaximal Mixing [Harrison-Perkins-Scott2002]

$$U_{\rm HPS} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

(1) This form is strongly supported by the experiment because almost its values are the best fits from the current data.
(2) In the last decade a large portion of the neutrino theories has been devoted to derive it, but **how**?

Ma Connection

$$U_{\rm HPS} \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}^\dagger \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

The first factor is Cabibbo-Wolfenstein (CW) matrix ($\omega = e^{2\pi i/3}$); the second one is Ma connection.

(1) The CW matrix contains a residual symmetry Z_3, while the Ma connection term has 2-3 reflection symmetry Z_2 with zero 1-2 and 1-3 mixing.
(2) The $U_{\rm HPS}$ can be obtained if there is an appropriate symmetry among flavors containing the residual subgroups Z_2, Z_3 and non-Abelian.

II. NON-ABELIAN DISCRETE SYMMETRIES

Flavor Symmetry—Group S_3

The simplest group (but fails) is S_3—the symmetry group of an equilateral triangle, which is also the permutation group of 3 objects.

Flavor Symmetry—Group A_4

If the underline symmetry contains an $\underline{3}$ irreducible rep. responsible for three families, the simplest of which (successful) is A_4— the symmetry group of a tetrahedron, which is also the group of even-permutations of 4 objects.

Flavor Symmetry—Group S_4

In some models, S_4—the symmetry group of a cube, which is also the permutation group of 4 objects, is required.

III. SOME MODELS WITH S_3, A_4

S_3 **Model**

The S_3 is the smallest non-Abelian discrete group. It has 6 elements in 3 equivalence classes, with the irreducible representations $\underline{1}$, $\underline{1}'$, and $\underline{2}$. Class $[C_1]$: (1)(2)(3); $[C_2]$: (123), (321); $[C_3]$: (1)(23), (2)(13), (3)(12). The fundamental multiplication rule is

$$\underline{2} \otimes \underline{2} = \underline{1}(12 + 21) \oplus \underline{1}'(12 - 21) \oplus \underline{2}(22, 11).$$

Let $(\nu_i, l_i) \sim \underline{2}$, $l_i^c \sim \underline{2}$, $(\phi_1^0, \phi_1^-) \sim \underline{1}$, $(\phi_2^0, \phi_2^-) \sim \underline{1}'$, then

$$M_l = \begin{pmatrix} 0 & fv_1 + f'v_2 \\ fv_1 - f'v_2 & 0 \end{pmatrix} = \begin{pmatrix} m_\mu & 0 \\ 0 & m_\tau \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Let $\xi_i = (\xi_i^{++}, \xi_i^{+}, \xi_i^{0}) \sim \underline{2}$ (with $u_1 = u_2$) and $\xi_0 = (\xi_0^{++}, \xi_0^{+}, \xi_0^{0}) \sim \underline{1}$,

$$\begin{aligned}\mathcal{M}_\nu = \begin{pmatrix} hu_1 & h_0u_0 \\ h_0u_0 & hu_2 \end{pmatrix} &= \begin{pmatrix} a & b \\ b & a \end{pmatrix} \\ &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} a+b & 0 \\ 0 & a-b \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}.\end{aligned}$$

Thus

$$U = (U_{lL})^\dagger U_{\nu L} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

i.e. maximal $\nu_\mu - \nu_\tau$ mixing responsible for the atmospheric neutrinos may be achieved, despite having a diagonal $\mathcal{M}_l$ with $m_\mu \neq m_\tau$.

A_4 **Model** [Ma2001,2009]

The A_4 has 12 elements in 4 equivalence classes, with the irreducible representations $\underline{1}$, $\underline{1}'$, $\underline{1}''$, and $\underline{3}$. Class $[C_1]$: (1)(2)(3)(4); $[C_2]$: (1)(234), (2)(143), (3)(124), (4)(132); $[C_3]$: (1)(432), (2)(341), (3)(421), (4)(231); $[C_4]$: (12)(34), (13)(24), (14)(23). Let $\omega = \exp \frac{2\pi i}{3}$, the fundamental multiplication rule is

$$\begin{aligned}\underline{3} \otimes \underline{3} &= \underline{1}(11 + 22 + 33) \oplus \underline{1}'(11 + \omega^2 22 + \omega 33) \oplus \underline{1}''(11 + \omega 22 + \omega^2 33) \\ &\oplus \underline{3}(23, 31, 12) \oplus \underline{3}(32, 13, 21)\end{aligned}$$

Let $(\nu_i, l_i) \sim \underline{3}$, $l_i^c \sim \underline{1}, \underline{1}', \underline{1}''$, and $(\phi_i^0, \phi_i^-) \sim \underline{3}$ with $v_1 = v_2 = v_3$, then

$$\mathcal{M}_l = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}.$$

Let $\xi_0 = (\xi_0^{++}, \xi_0^+, \xi_0^0) \sim \underline{1}$ and $\xi_i = (\xi_i^{++}, \xi_i^+, \xi_i^0) \sim \underline{3}$ with $u_2 = u_3 = 0$,

$$\mathcal{M}_\nu = \begin{pmatrix} a & 0 & 0 \\ 0 & a & d \\ 0 & d & a \end{pmatrix} = U_\nu \begin{pmatrix} a+d & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & -a+d \end{pmatrix} U_\nu^T,$$

where

$$U_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & i \end{pmatrix}$$

The neutrino mixing matrix is then

$$U = (U_{lL})^\dagger U_{\nu L} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

i.e. tribimaximal mixing. This is the simplest such realization, which is consistent with only the normal hierarchy of neutrino masses ($m_1 < m_2 < m_3$).

A_4 **3-3-1 Model** [Dong-Long-Soa-Hue2010]

Let $(\nu_i, l_i, N_i^c) \sim \underline{3}$ (with $L(N) = 0$), $l_i^c \sim \underline{1}, \underline{1}', \underline{1}''$, $(\phi_i^+, \phi_i^0, \phi_i^+) \sim \underline{3}$ with $v_1 = v_2 = v_3$, we get then

$$\mathcal{M}_l = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}.$$

Let the sextets $\sigma_0 \sim \underline{1}$ and $\sigma_i \sim \underline{3}$ with $u_2 = u_3 = 0$, the active neutrinos gain mass via a seesaw:

$$\mathcal{M}_\nu^{\text{eff}} = \begin{pmatrix} b & 0 & 0 \\ 0 & a & d \\ 0 & d & a \end{pmatrix} = U_\nu \begin{pmatrix} a+d & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & -a+d \end{pmatrix} U_\nu^T,$$

where

$$U_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & i \end{pmatrix}$$

Again, the tribimaximal mixing is obtained. This realization is consistent with arbitrary hierarchy of neutrino masses, including normal or inverted.

IV. CONCLUDING REMARKS

With the application of the non-Abelian discrete symmetries such as A_4, a plausible theoretical understanding of the tribimaximal form of the neutrino mixing matrix has been achieved.

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