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THE NONLINEAR ACOUSTOELECTRIC EFFECT IN A CYLINDRICAL QUANTUM WIRE WITH AN INFINITE POTENTIAL

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Abstract. *The nonlinear acoustoelectric effect in a cylindrical quantum wire with an infinite potential is investigated by using Boltzmann kinetic equation for an acoustic wave whose wavelength $\lambda = \frac{2\pi}{q}$ is smaller than the mean free path l of the electrons and hypersound in the region $ql \gg 1$, (where q is the acoustic wave number). The analytic expression for the acoustoelectric current I^{ac} is calculated in the case: relaxation time of momentum τ is constant approximation and degenerates electrons gas. The nonlinear dependence of the expression for the acoustoelectric current I^{ac} on the acoustic wave numbers q and on the intensity of constant electric field E are obtained. Numerical computations are performed for AlGaAs/GaAs cylindrical quantum wire with an infinite potential. The results are compared with the normal bulk semiconductors and the superlattices to show the values of the acoustoelectric current I^{ac} in the cylindrical quantum wire are different than they are in the normal bulk semiconductors and the superlattices.*

I. INTRODUCTION

When an acoustic wave is absorbed by a conductor, the transfer of the momentum from the acoustic wave to the conduction electron may give rise to a current usually called the acoustoelectric current, I^{ac}, in the case of an open circuit, a constant electric field. The study of acoustoelectric effect in bulk materials have received a lot of attention [1-5]. Recently, there have been a growing interest in observing this effect in mesoscopic structures [6-8]. The interaction between surface acoustic wave (SAW) and mobile charges in semiconductor layered structures and quantum wells is an important method to study the dynamic properties of low-dimensional systems. The SAW method was applied to study the quantum Hall effects [9-11], the fractional quantum Hall effect [12], and the electron transport through a quantum point contact [13, 14]. It has also been noted that the transverse acoustoelectric voltage (TAV) is sensitive to the mobility and to the carrier concentration in the semiconductor, thus it has been used to provide a characterization of electric properties of semiconductors [15]. Especially, in recent time the acoustoelectric effect was studied in both a one-dimensional channel [16] and in a finite-length ballistic quantum channel [17, 18, 19]. In addition, the acoustoelectric effect was measured by an experiment in a submicron-separated quantum wire [20], in a carbon nanotube [21], in an InGaAs quantum well [22]. The SAW method was also applied to the study acoustoelectric effect and acoustomagnetoelectric effect [23, 24, 25].

However, the acoustoelectric effect in the quantum wire still opens for studying, in this paper, we examine this effect in a cylindrical quantum wire with an infinite potential for the case of electron relaxation time is not dependent on the energy and degenerate electron gas. Furthermore, we think the research of this effect may help us to understand the properties of quantum wire material. We have obtained the acoustoelectric current I^{ac} in the cylindrical quantum wire. The nonlinear dependence of the expression for the acoustoelectric current I^{ac} on acoustic wave numbers q has been shown. Numerical calculations are carried out with a specific $AlGaAs/GaAs$ quantum wire to clarify our results.

II. ACOUSTOELECTRIC CURRENT

By using the classical Boltzmann kinetic equation method in [23, 24, 25], we calculated the acoustoelectric current in quantum wire. The acoustic wave is considered a hypersould in the region $ql \gg 1$ (l is the electron mean free path, q is the acoustic wave number). Under such circumstances, the acoustic wave can be interpreted as monochromatic phonons having the 3D phonon distribution function $N(\vec{k})$, and this function can be presented in the form [25]

$$N(\vec{k}) = \frac{(2\pi)^3}{\hbar\omega_{\vec{q}} v_s}\phi\delta(\vec{k}-\vec{q}), \tag{1}$$

where $\hbar = 1$, $\vec{k}$ is the current phonon wave vector, ϕ is the sound flux density, $\omega_{\vec{q}}$ and v_s are the frequency and the group velocity of sound wave with the wave vector $\vec{q}$, respectively.

It is assumed that the sound wave and the applied electric field $\vec{E}$ propagates along the axis of the quantum wire. The problem was solved in the quasi-classical case, i.e., $2\delta \gg \tau^{-1}$, (τ is the relaxation time). The density of the acoustoelectric current can be written in the form [26]

$$j^{ac} = \frac{2e}{(2\pi)^3}\int U^{ac}\psi_i d^3p, \tag{2}$$

with

$$\begin{aligned} U^{ac} =& \frac{2\pi\phi}{\omega_{\vec{q}} v_s}\{|G_{\vec{p}-\vec{q},\vec{p}}|^2[f(\varepsilon_{\vec{p}-\vec{q}}) - f(\varepsilon_{\vec{p}})]\delta(\varepsilon_{\vec{p}-\vec{q}} - \varepsilon_{\vec{p}} + \omega_{\vec{q}}) \\ &+ |G_{\vec{p}+\vec{q},\vec{p}}|^2[f(\varepsilon_{\vec{p}+\vec{q}}) - f(\varepsilon_{\vec{p}})]\delta(\varepsilon_{\vec{p}+\vec{q}} - \varepsilon_{\vec{p}} - \omega_{\vec{q}})\}. \end{aligned} \tag{3}$$

Here $\vec{p}$ is the electron momentum vector, $f(\varepsilon_{\vec{p}})$ is the distribution function, $G_{\vec{p}-\vec{q},\vec{p}}$ is the matrix element of the electron-phonon interaction and ψ_i ($i = x, y, z$) is the root of the kinetic equation given by [28]

$$\frac{e}{c}(V\times H)\frac{\partial\psi_i}{\partial p} + \widehat{W}_{\vec{p}}\{\psi_i\} = V_i, \tag{4}$$

here V_i is the electron velocity, V is the average drift velocity of the moving charges and $\widehat{W}_{\vec{p}}\{...\} = (\partial f/\partial\varepsilon)^{-1}\widehat{W}\{(\partial f/\partial\varepsilon)...\}$. The operator $\widehat{W}$ is assumed to be Hermitian [26]. In the τ approximation, $\widehat{W}_{\vec{p}} = 1/\tau$. Furthermore, $\tau = constant$, we shall seek the solution of Eq.(4) as

$$\psi_i = \psi_i^{(0)} + \psi_i^{(1)} + ... \tag{5}$$

Substituting Eq.(5) into Eq.(4) and solving by the method of iteration, we get for the zero and the first approximation. Inserting into Eq.(2) and taking into account the fact that

$$|G_{\vec{p},\vec{p}'}|^2 = |G_{\vec{p}',\vec{p}}|^2. \tag{6}$$

We obtain for the density of the acoustoelectric current the expression

$$\begin{aligned} j_i^{ac} = &- \frac{e\phi}{2\pi^2 v_s \omega_{\vec{q}}} \int |G_{\vec{p}+\vec{q},\vec{p}}|^2 [f(\varepsilon_{\vec{p}+\vec{q}}) - f(\varepsilon_{\vec{p}})] \times \\ &\times [V_i(\vec{p}+\vec{q})\tau - V_i(\vec{p})\tau] \delta(\varepsilon_{\vec{p}+\vec{q}} - \varepsilon_{\vec{p}} - \omega_{\vec{q}}) d^3p - \\ &- \frac{e^2\phi\tau^2}{2\pi^2 mcv_s \omega_{\vec{q}}} \int |G_{\vec{p}+\vec{q},\vec{p}}|^2 [f(\varepsilon_{\vec{p}+\vec{q}}) - f(\varepsilon_{\vec{p}})] \times \\ &\times [(\vec{V}(\vec{p}+\vec{q}) \times \vec{H})_i - (\vec{V}(\vec{p}) \times \vec{H})_i] \delta(\varepsilon_{\vec{p}+\vec{q}} - \varepsilon_{\vec{p}} - \omega_{\vec{q}}) d^3p. \end{aligned} \tag{7}$$

The matrix element of the electron-phonon interaction [23, 28] is given

$$|G_{\vec{p},\vec{q}}|^2 = \frac{|\Lambda|^2 |\vec{q}|^2}{2\rho\omega_{\vec{q}}}. \tag{8}$$

Where Λ is the deformation potential constant and ρ is the crystal density of the quantum wire.

In solving Eq.(7) we shall consider a situation whereby the sound is propagating along the quantum wire axis (Oz). Under such orientation the second term in Eq.(7) is responsible for the density of the acoustomagnetoelectric current and the first term in Eq.(7) is the density of the acoustoelectric current. Thus the density of the acoustoelectric current in Eq.(7) in the direction of the quantum wire axis becomes

$$j_i^{ac} = -\frac{e\phi\vec{q}^2\tau|\Lambda|^2}{4\pi^2 v_s \omega_{\vec{q}}^2 \rho} \int [f(\varepsilon_{\vec{p}+\vec{q}}) - f(\varepsilon_{\vec{p}})][V_z(\vec{p}+\vec{q}) - V_z(\vec{p})] \delta(\varepsilon_{\vec{p}+\vec{q}} - \varepsilon_{\vec{p}} - \omega_{\vec{q}}) d^3p, \tag{9}$$

the distribution function $f(\varepsilon_{\vec{p}})$ in the presence of the applied constant field $\vec{E}$ is obtained by solving the Boltzmann equation in the τ approximation. This function is given

$$f(\varepsilon_{\vec{p}}) = \int_0^\infty \frac{dt}{\tau} exp(-\frac{t}{\tau}) f_0(\varepsilon_{\vec{p}}). \tag{10}$$

In the case degenerate electrons gas is given by

$$f_0(\varepsilon_{\vec{p}}) = \theta(\varepsilon_F - \varepsilon_{\vec{p}}) = \begin{cases} 0 & \varepsilon_{\vec{p}} > \varepsilon_F \\ 1 & \varepsilon_{\vec{p}} \leq \varepsilon_F \end{cases} \tag{11}$$

Where ε_F is the Fermi energy, the energy $\varepsilon_{\vec{p}}$ of the cylindrical quantum wire with an infinite potential in the lowest miniband is given by [27]

$$\varepsilon_{\vec{p}} = \frac{\hbar^2 \vec{p}_z^2}{2m} + \frac{\hbar^2 A_{n,l}^2}{2mR^2}. \tag{12}$$

Where $l = 1, 2, 3, ...$ is the radial quantum number, $n = 0, \pm 1, \pm 2, ...$ is the azimuth quantum number, m is the electron effective mass, R is the radius of the quantum wire, p_z is the longitudinal (relative to the quantum wire axis) component of the quasi-momentum and $A_{n,l}$ is the l level root of Bessel function of the order n.

Hence

$$V_z(\vec{p}) = \frac{\partial \varepsilon_{\vec{p}}}{\partial p} = \frac{\hbar^2 p_z}{m} + \frac{\hbar^2}{2mR^2}\frac{\partial A_{n,l}^2}{\partial p}. \tag{13}$$

Substituting Eqs.(11), (12) and (13) into Eq.(9), we obtain for the acoustoelectric current with the condition is satisfied then:

$$\varepsilon_F > \frac{\hbar^2 \vec{p}_z^2}{2m} + \frac{\hbar^2}{2mR^2}A_{n,l}^2 + \hbar\omega_{\vec{q}}. \tag{14}$$

The inequalities in Eq.(14) is condition acoustic wave vector $\vec{q}$ to the acoustoelectric effect exists. Therefor, we have obtained the expression density of the acoustoelectric current

$$j_z^{ac} = \frac{e\phi\tau|\Lambda|^2 q^3}{4\pi\rho v_s \omega_{\vec{q}}^2}\int_0^\infty \frac{dt}{\tau}exp(-\frac{t}{\tau})\Big[\hbar q - 2eEt - 2\sqrt{2m\varepsilon_F - \frac{\hbar^2 A_{n,l}^2}{R^2}}\Big]. \tag{15}$$

Thus, the analytic expression for the acoustoelectric current I^{ac} in the cylindrical quantum wire with an infinite potential can be written in the form

$$I^{ac} = \frac{e\phi\tau|\Lambda|^2 R^2 q^3}{4\rho v_s \omega_{\vec{q}}^2}\int_0^\infty \frac{dt}{\tau}exp(-\frac{t}{\tau})\Big[\hbar q - 2eEt - 2\sqrt{2m\varepsilon_F - \frac{\hbar^2 A_{n,l}^2}{R^2}}\Big]. \tag{16}$$

The Eq.(16) is the acoustoelectric current in the cylindrical quantum wire with an infinite potential in the case degenerate electron gas, the expression only obtained if the condition in Eq.(14) is satisfied.

III. NUMERICAL RESULTS

In this situation Eq.(16) was solved analytically and the result were given as

$$I^{ac} = \frac{e\phi\tau|\Lambda|^2 R^2 q^3}{4\rho v_s \omega_{\vec{q}}^2}\Big[\hbar q - 2eE\tau - 2\sqrt{2m\varepsilon_F - \frac{\hbar^2 A_{n,l}^2}{R^2}}\Big]. \tag{17}$$

Eq.(17) is the acoustoelectric current in the cylindrical quantum wire with an infinite potential in the case degenerate electron gas. The dependences of the expression for the acoustoelectric current I^{ac} on the intensity of the electric field E, the frequency $\omega_{\vec{q}}$ of the acoustic wave, the acoustic wave numbers q and the radius R of the quantum wire are obtained.

In the paper, we consider a $AlGaAs/GaAs$ cylinder quantum wire with an infinite potential. The parameters used in the calculations are as follows [26, 28]: $\tau = 10^{-12}s; R = 80\mathring{A}; \phi = 10^{14}Wm^{-2}; \rho = 2{\times}10^{13}kgm^{-3}; v_s = 5370ms^{-1}; E = 10^6Vm^{-1}; \omega_{\vec{q}} = 10^{10}s^{-1}; m = 0.067m_e$, m_e being the mass of free electron.

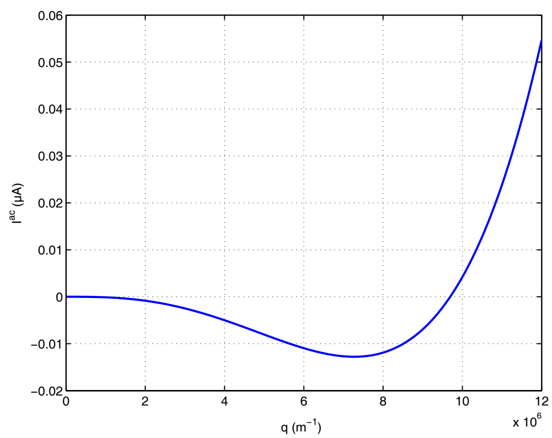

Fig. *The dependence of the acoustoelectric current I^{ac} on the acoustic wave numbers q.*

Figure shows the dependence of the acoustoelectric current on the acoustic wave number q when the relaxation time of momentum τ is constant approximation and degenerate electron gas. The curve of the acoustoelectric current I^{ac} decreases when the small value range of the acoustic wave number q and strongly increases when the large value range of the acoustic wave number q.

IV. CONCLUSION

In this paper, we have analytically investigated the possibility of the acoustoelectric effect in the cylindrical quantum wire with an infinite potential. We have obtained analytically expressions for the acoustoelectric effect in the cylindrical quantum wire with an infinite potential for the case degenerate electron gas. The dependences of the expression for the acoustoelectric current I^{ac} on the frequency $\omega_{\vec{q}}$ of the acoustic wave, the acoustic wave numbers q and the radius R of the quantum wire are obtained. The result is different compared to those obtained in the normal bulk semiconductors [5], according to [5] in the case $\tau = constant$ the effect only exists if the electron gas is non-degenerate, if the electron gas is degenerate, the effect is not appear, however, our result indicates that in the cylindrical quantum wire with an infinite potential the acoustoelectric effect exists both non-degenerate and degenerate electron gas when $\tau = constant$. Unlike the normal bulk semiconductors, in the cylindrical quantum wire with an infinite potential the acoustoelectric current I^{ac} is nonlinear with the acoustic wave number q.

We have numerically calculated and graphed expressing the dependence of the acoustoelectric current I^{ac} on the acoustic wave number q are performed for $AlGaAs/GaAs$ cylindrical quantum wire with an infinite potential. The result shows that, the acoustoelectric effect exists when the acoustic wave vector $\vec{q}$ complies with specific conditions in Eq.(14) which condition dependences on the frequency $\omega_{\vec{q}}$ of the acoustic wave, Fermi energy, the mass of electron and the radius R of the quantum wire. That is mean to have acoustoelectric current I^{ac}, the acoustic phonons energy is high enough and satisfied in the some interval to impact much momentum to the conduction electrons. The curve of the acoustoelectric current I^{ac} strongly decreases when the small value range of the acoustic wave number q and strongly increases when the large value range of the acoustic wave number q.

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