Proc. Natl. Conf. Theor. Phys. **35** (2010), pp. 124-134

PARAMETRIC TRANSFORMATION AND PARAMETRIC RESONANCE OF CONFINED ACOUSTIC PHONONS AND CONFINED OPTICAL PHONONS IN QUANTUM WELLS

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Abstract. *The parametric transformation and parametric resonance of confined acoustic phonons and confined optical phonons in quantum wells in the presence of an external electromagnetic field are theoretically studied by using a set of quantum kinetic equations for phonons. The analytic expression of the parametric transformation coefficient (K_1) and the threshold amplitude (E_{th}) of the field in quantum wells are obtained. Unlike the case of unconfined phonons, the formula of K_1 and contains a quantum number m characterizing confined phonons. Their dependence on the temperature T of the system and the frequency Ω of the electromagnetic field is studied. Numerical computations have been performed for GaAs/AlAsAl quantum wells. The result have been compared with the case of unconfined phonons which show that confined phonons cause some unusual effects.*
Keyword: Parametric transformation and parametric resonance, quantum well

I. INTRODUCTION

It is well known that in the presence of an external electromagnetic field (EEF), an electron gas becomes non-stationary. When the conditions of parametric resonance (PR) are satisfied, parametric interactions and transformations (PIT) of same kinds of excitations, such as phonon - phonon, plasmon - plasmon, or of different kinds of excitations, such as plasmon - phonon, will arise; i.e., energy exchange processes between these excitations will occur [1, 2]. The PIT of acoustic and optical phonons has been considered in bulk semiconductors [3 - 5]. The physical picture can be described as follows: due to the electron - phonon interaction, propagation of an acoustic phonon with a frequency $\omega_{\vec{q}}$ is accompanied by a density wave with the same frequency. When an EEF with frequency Ω is presented, a charge density waves (CDW) with a combination frequency $\omega_{\vec{q}} \pm N\Omega$ (N = 1, 2,) will appear. If among the CDW there exists a certain wave having a frequency which coincides, or approximately coincides, with the frequency of the optical phonon, $\nu_{\vec{q}}$, optical phonons will appear. These optical phonons cause a CDW with a combination frequency of $\nu_{\vec{q}} \pm N\Omega$, and when $\nu_{\vec{q}} \pm N\Omega \cong \omega_{\vec{q}}$, a certain CDW causes the acoustic phonons mentioned above. The PIT can speed up the damping process for one excitation and the amplification process for another excitation. There have been a lot of works on the PIT for low dimensional semiconductors in the case of unconfined phonons [6 - 8]. However, parametric transformation and parametric resonance of acoustic and optical phonons in quantum wells in the case of confined phonons have not been studied yet. Therefore, in this paper, we have studied parametric transformation and parametric resonance of acoustic and optical phonons in quantum wells in the case of confined phonons. The comparison of the result of confined phonons to one of unconfined phonons shows that confined phonons causes some unusual effects. To this clarify, we estimate numerical values for a GaAs/AlAsAl quantum well, and we discuss the conditions under which the parametric resonance occurs.

II. THE PARAMETRIC RESONANCE OF CONFINED ACOUSTIC PHONONS AND CONFINED OPTICAL PHONONS IN QUANTUM WELLS.

It is well known that the motion of an electron and phonon in a quantum wells is confined and that its energy spectrum is quantized into discrete levels. In this paper, we assume that the quantization direction is the z direction. The Hamiltonian of the electron - confined acoustic (confined optical) phonon system in a quantum well in the second quantization representation can be written as:

$$H(t) = \sum_{n,\vec{k}_\perp} \varepsilon_n\left(\vec{k}_\perp - \frac{e}{\hbar c}\vec{A}(t)\right) a^+_{n,\vec{k}_\perp} a_{n,\vec{k}_\perp} + \sum_{m,\vec{q}_\perp} \hbar\omega_{m,\vec{q}_\perp} b^+_{m,\vec{q}_\perp} b_{m,\vec{q}_\perp}$$

$$+ \sum_{m,\vec{q}_\perp} \hbar\nu_{m,\vec{q}_\perp} c^+_{m,\vec{q}_\perp} c_{m,\vec{q}_\perp} + \sum_{m,\vec{q}_\perp} \sum_{n,n',\vec{k}_\perp} C^m_{\vec{q}_\perp} I^m_{n,n'} a^+_{n',\vec{k}_\perp+\vec{q}_\perp} a_{n,\vec{k}_\perp} \left(b_{m,\vec{q}_\perp} + b^+_{m,-\vec{q}_\perp}\right)$$

$$+ \sum_{m,\vec{q}_\perp} \sum_{n,n',\vec{k}_\perp} D^m_{\vec{q}_\perp} I^m_{n,n'} a^+_{n',\vec{k}_\perp+\vec{q}_\perp} a_{n,\vec{k}_\perp} \left(c_{m,\vec{q}_\perp} + c^+_{m,-\vec{q}_\perp}\right), \qquad (1)$$

here, n, n' are denotes the quantization of the energy spectrum in the z direction (n, n' = 1, 2, 3, ...), $\left(n, \vec{k}_\perp\right)$ and $\left(n', \vec{k}_\perp + \vec{q}_\perp\right)$ are electron states before and after scattering, respectively; $\left(\vec{k}_\perp, \vec{q}_\perp\right)$ is the in-plane (x, y) wave vector of the electron (phonon); $a^+_{n,\vec{k}_\perp} a_{n,\vec{k}_\perp}$, $(b^+_{m,\vec{q}_\perp}, b_{m,\vec{q}_\perp}; c^+_{m,\vec{q}_\perp}, c_{m,\vec{q}_\perp})$ are the creation and the annihilation operators of the electron (acoustic phonon; optical phonon), e is the charge of the electron, c is the of light velocity, $\vec{A}(t)$ is the vector potential of an EEF, respectively. $\vec{A}(t) = \frac{c}{\Omega}\vec{E}_0\cos(\Omega t)$ and $\hbar\omega_{m,\vec{q}_\perp}$ $\left(\hbar\nu_{m,\vec{q}_\perp}\right)$ is the energy of the confined acoustic (optical) phonon, $\omega_{m.\vec{q}}$ $(\nu_{m,\vec{q}_\perp})$ is frequency confined acoustic (optical) phonon; m is the quantum number characterizing confined phonons. $\varepsilon_n\left(\vec{k}_\perp\right)$ is the energy spectrum of the electron in quantum wells take the simple form [7]:

$$\varepsilon_n\left(\vec{k}_\perp\right) = \frac{\hbar^2 \vec{k}^2_\perp}{2m^*} + \frac{\hbar^2\pi^2 n^2}{2m^* L^2} \qquad (2)$$

where, L is the well width, m^* is the effective mass. $C^m_{\vec{q}_\perp}$ $\left(D^m_{\vec{q}_\perp}\right)$ is the electron - confined acoustic (electron - confined optical) phonon interaction constant take the form [9]

$$\left|C^m_{\vec{q}_\perp}\right|^2 = \frac{\xi^2\hbar}{\rho\upsilon_a V}\sqrt{\vec{q}^{\,2}_\perp + \left(\frac{m\pi}{L}\right)^2} \tag{3}$$

$$\left|D^m_{\vec{q}_\perp}\right|^2 = \frac{e^2\hbar\nu_{m,\vec{q}_\perp}}{V}\frac{1}{\varepsilon_0}\left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0}\right)\frac{1}{\vec{q}^{\,2}_\perp + \left(\frac{m\pi}{L}\right)^2} \tag{4}$$

Here, V, ρ, υ_a, and ξ are the volume, the crystal density, the acoustic wave velocity, and the deformation potential constant, respectively. ε_0 is the electronic constant; χ_0 and χ_∞ are the static and high - frequency dielectric constant, respectively. The electron form factor $I^m_{n,n'}(q_z)$, is written as [10]

$$I^m_{n,n'}(q_z) = \frac{2}{L}\int\limits_0^L N_m(z)\sin\left(\frac{n'\pi z}{L}\right)\sin\left(\frac{n\pi z}{L}\right)dz \tag{5}$$

$$N_m(z) = \eta(m)\cos\left(\frac{m\pi z}{L}\right) + \eta(m+1)\sin\left(\frac{m\pi z}{L}\right) \tag{6}$$

($\eta(m) = 0$ if m even and $\eta(m) = 1$ if m ext) In order to establish a set of quantum transport equations for confined acoustic and confined optical phonons in quantum wells, we use the general quantum distribution function [11] for the confined acoustic (confined optical) phonons,$\left\langle b_{m,\vec{q}_\perp}\right\rangle_t$ and $\left\langle c_{m,\vec{q}_\perp}\right\rangle_t$:

$$i\frac{\partial}{\partial t}\left\langle b_{m,\vec{q}_\perp}\right\rangle = \frac{1}{\hbar}\left\langle\left[b_{m,\vec{q}_\perp}, H(t)\right]\right\rangle_t \tag{7}$$

and

$$i\frac{\partial}{\partial t}\left\langle c_{m,\vec{q}_\perp}\right\rangle = \frac{1}{\hbar}\left\langle\left[c_{m,\vec{q}_\perp}, H(t)\right]\right\rangle_t \tag{8}$$

Where $\langle\psi\rangle_t$ denotes a staticical average value at the moment t and $\langle\psi\rangle_t = Tr(\hat{W}\hat{\psi})$($\hat{W}$ being the density matrix operator) Hamiltonian Eq. (1), (7) and (8) and using the commutative relations of the creation and the annihilation operators, we obtain the quantum kinetic equation for confined acoustic (confined optical) phonon in quantum wells:

$$\begin{aligned}
&\frac{\partial}{\partial t}\left\langle b_{m,\vec{q}_\perp}\right\rangle_t + i\omega_{m,\vec{q}_\perp}\left\langle b_{m,\vec{q}_\perp}\right\rangle_t = -\frac{1}{\hbar^2}\sum_{n,n',\vec{k}_\perp}\sum_{\nu,\mu=-\infty}^{\infty}\left|I^m_{n,n'}\right|^2 J_\nu\left(\frac{\lambda}{\Omega}\right)J_\mu\left(\frac{\lambda}{\Omega}\right)\\
&\times\left[f_{n'}\left(\vec{k}_\perp - \vec{q}_\perp\right) - f_n\left(\vec{k}_\perp\right)\right]\int\limits_{-\infty}^{t} dt_1 e^{\frac{i}{\hbar}\left[\varepsilon_n\left(\vec{k}_\perp\right) - \varepsilon_{n'}\left(\vec{k}_\perp - \vec{q}_\perp\right)\right](t_1 - t) - i\nu\Omega t_1 + i\mu\Omega t}\\
&\times\left\{\left|C_{m,\vec{q}_\perp}\right|^2\left(\left\langle b_{m,\vec{q}_\perp}\right\rangle_{t_1} + \left\langle b^+_{m,-\vec{q}_\perp}\right\rangle_{t_1}\right) + C^m_{-\vec{q}_\perp}D^m_{\vec{q}_\perp}\left(\left\langle c_{m,\vec{q}_\perp}\right\rangle_{t_1} + \left\langle c^+_{m,-\vec{q}_\perp}\right\rangle_{t_1}\right)\right\}
\end{aligned} \tag{9}$$

and

$$\frac{\partial}{\partial t}\left\langle c_{m,\vec{q}_\perp}\right\rangle_t + i\nu_{m,\vec{q}_\perp}\left\langle c_{m,\vec{q}_\perp}\right\rangle_t = -\frac{1}{\hbar^2}\sum_{n,n',\vec{k}_\perp}\sum_{\nu,\mu=-\infty}^{\infty}\left|I^m_{n,n'}\right|^2 J_\nu\left(\frac{\lambda}{\Omega}\right)J_\mu\left(\frac{\lambda}{\Omega}\right)$$

$$\times\left[f_{n'}\left(\vec{k}_\perp-\vec{q}_\perp\right)-f_n\left(\vec{k}_\perp\right)\right]\int_{-\infty}^{t}dt_1 e^{\frac{i}{\hbar}\left[\varepsilon_n\left(\vec{k}_\perp\right)-\varepsilon_{n'}\left(\vec{k}_\perp-\vec{q}_\perp\right)\right](t_1-t)-i\nu\Omega t_1+i\mu\Omega t}$$

$$\times\left\{D^m_{-\vec{q}_\perp}C^m_{\vec{q}_\perp}\left(\left\langle b_{m,\vec{q}_\perp}\right\rangle_{t_1}+\left\langle b^+_{m,-\vec{q}_\perp}\right\rangle_{t_1}\right)+\left|D_{m,\vec{q}_\perp}\right|^2\left(\left\langle c_{m,\vec{q}_\perp}\right\rangle_{t_1}+\left\langle c^+_{m,-\vec{q}_\perp}\right\rangle_{t_1}\right)\right\} \tag{10}$$

Where, $f_n\left(\vec{k}_\perp\right)$ is the distribution function of the electron in the state $\left|n,\vec{k}_\perp\right\rangle$; $J_\nu\left(\frac{\lambda}{\Omega}\right)$ is the Bessel function; $\lambda=\frac{e\hbar\vec{E}_0\vec{q}_\perp}{m\Omega}$. One finds that the final result consists of an infinite set of coupled equations for the Fourier transformations $B_{m,\vec{q}_\perp}(\omega)$ and $C_{m,\vec{q}_\perp}(\omega)$ of $\left\langle b_{m,\vec{q}_\perp}\right\rangle_t$ and $\left\langle c_{m,\vec{q}_\perp}\right\rangle_t$, respectively. For instance, the equations for $B_{m,\vec{q}_\perp}(\omega)$ and $C_{m,\vec{q}_\perp}(\omega)$, can be written as:

$$\left(\omega-\omega_{m,\vec{q}_\perp}\right)B_{m,\vec{q}_\perp}(\omega)=\frac{2}{\hbar}\sum_{n,n'}\left|I^m_{n,n'}\right|^2\left|C^m_{\vec{q}_\perp}\right|^2\omega_{m,\vec{q}_\perp}\frac{\Pi_0\left(m,\vec{q}_\perp,\omega\right)}{\omega+\omega_{m,\vec{q}_\perp}}B_{m,\vec{q}_\perp}(\omega)$$

$$+\frac{2}{\hbar}\sum_{n,n'}\left|I^m_{n,n'}\right|^2 C^m_{-\vec{q}_\perp}D^m_{\vec{q}_\perp}\sum_{s=-\infty}^{\infty}\nu_{m,\vec{q}_\perp}\frac{\Pi_s\left(m,\vec{q}_\perp,\omega\right)}{\omega-s\Omega+\nu_{m,\vec{q}_\perp}}C_{m,\vec{q}_\perp}(\omega-s\Omega) \tag{11}$$

and

$$\left(\omega-\nu_{m,\vec{q}_\perp}\right)C_{m,\vec{q}_\perp}(\omega)=\frac{2}{\hbar}\sum_{n,n'}\left|I^m_{n,n'}\right|^2\left|D^m_{\vec{q}_\perp}\right|^2\nu_{m,\vec{q}_\perp}\frac{\Pi_0\left(m,\vec{q}_\perp,\omega\right)}{\omega+\nu_{m,\vec{q}_\perp}}C_{m,\vec{q}_\perp}(\omega)$$

$$+\frac{2}{\hbar}\sum_{n,n'}\left|I^m_{n,n'}\right|^2 C^m_{\vec{q}_\perp}D^m_{-\vec{q}_\perp}\sum_{s=-\infty}^{\infty}\omega_{m,\vec{q}_\perp}\frac{\Pi_s\left(m,\vec{q}_\perp,\omega\right)}{\omega-s\Omega+\omega_{m,\vec{q}_\perp}}B_{m,\vec{q}_\perp}(\omega-s\Omega) \tag{12}$$

From Eq. (11), we have:

$$\left(\omega^2-\omega^2{}_{m,\vec{q}_\perp}-\frac{2}{\hbar}\sum_{n,n'}\left|I^m_{n,n'}\right|^2\left|C^m_{\vec{q}_\perp}\right|^2\omega_{m,\vec{q}_\perp}\Pi_0\left(m,\vec{q}_\perp,\omega\right)\right)B_{m,\vec{q}_\perp}(\omega)$$

$$=\frac{2}{\hbar}\sum_{n,n'}\left|I^m_{n,n'}\right|^2 C^m_{-\vec{q}_\perp}D^m_{\vec{q}_\perp}\nu_{m,\vec{q}_\perp}\left(\omega+\omega_{m,\vec{q}_\perp}\right)\sum_{s=-\infty}^{\infty}\frac{\Pi_s\left(m,\vec{q}_\perp,\omega\right)}{\omega+s\Omega+\nu_{m,\vec{q}_\perp}}C_{m,\vec{q}_\perp}(\omega+s\Omega) \tag{13}$$

From Eq. (12), after some mathematical transformations, and for $\omega=\omega_{m,\vec{q}_\perp}$ and s = N, we find the expression

$$C_{m,\vec{q}_\perp}\left(\omega_{m,\vec{q}_\perp}+N\Omega\right)=\frac{\frac{2}{\hbar}\sum\limits_{n,n'}\left|I^m_{n,n'}\right|^2 C^m_{-\vec{q}_\perp}D^m_{\vec{q}_\perp}\omega_{m,\vec{q}_\perp}\Pi_N\left(m,\vec{q}_\perp,\omega+N\Omega\right)B_{m,\vec{q}_\perp}\left(\omega_{m,\vec{q}_\perp}\right)}{\left[\omega+N\Omega-\nu_{m,\vec{q}_\perp}-\frac{\frac{2}{\hbar}\sum\limits_{n,n'}\left|I^m_{n,n'}\right|\left|D^m_{\vec{q}_\perp}\right|^2\nu_{m,\vec{q}_\perp}\Pi_0\left(m,\vec{q}_\perp,\omega_{m,\vec{q}_\perp}+N\Omega\right)}{\omega_{m,\vec{q}_\perp}+N\Omega+\nu_{m,\vec{q}_\perp}}\right]2\omega_{m,\vec{q}_\perp}}$$

We obtain equations dispersion describe interaction between confined acoustic phonon and confined optical phonon in quantum wells:

$$\begin{gathered}\left(\omega^2-\omega^2{}_{m,\vec{q}_\perp}-\frac{2}{\hbar}\sum_{n,n'}\left|I^m_{n,n'}\right|^2\left|C^m_{\vec{q}_\perp}\right|^2\omega_{m,\vec{q}_\perp}\Pi_0\left(m,\vec{q}_\perp,\omega\right)\right)\\ \times\left[(\omega+N\Omega)^2-\nu^2{}_{m,\vec{q}_\perp}-\frac{2}{\hbar}\sum_{n,n'}\left|I^m_{n,n'}\right|^2\left|D^m_{\vec{q}_\perp}\right|^2\nu_{m,\vec{q}_\perp}\Pi_0\left(m,\vec{q}_\perp,\omega+N\Omega\right)\right]\\ =\frac{4}{\hbar^2}\sum_{n,n'}\left|I^m_{n,n'}\right|^4\left|C^m_{\vec{q}_\perp}\right|^2\left|D^m_{\vec{q}_\perp}\right|^2\omega_{m,\vec{q}_\perp}\nu_{m,\vec{q}_\perp}\sum_{s=-\infty}^{\infty}\Pi_N\left(m,\vec{q}_\perp,\omega\right)\Pi_N\left(m,\vec{q}_\perp,\omega+N\Omega\right)\end{gathered}\tag{14}$$

In Eq. (14), the first terms describe the interaction between phonons that belong to the same kind (acoustic - acoustic phonon or optical - optical phonon) while the second terms describe interaction between phonons that belong to different kinds (acoustic - optical phonon). We limit our calculation to the case of the first order resonance, $\omega_{m,\vec{q}}\pm\nu_{m,\vec{q}}=\Omega$. Because the solution to the general dispersion equation, Eq. (14), is complex. Here, we assume that the electron - phonon interactions satisfy the condition $\left|I^m_{n,n'}\right|^4\left|C^m_{\vec{q}_\perp}\right|^2$, $\left|D^m_{\vec{q}_\perp}\right|^2\ll 1$ approximation can be regarded as $\left|I^m_{n,n'}\right|^4\left|C^m_{\vec{q}_\perp}\right|^2\left|D^m_{\vec{q}_\perp}\right|^2$ zero, and the solution Eq. (14) by means of the disturbance, we obtain:

$$\left(\omega^2-\omega^2{}_{m,\vec{q}_\perp}-\frac{2}{\hbar}\sum_{n,n'}\left|I^m_{n,n'}\right|^2\left|C^m_{\vec{q}_\perp}\right|^2\omega_{m,\vec{q}_\perp}\Pi_0\left(m,\vec{q}_\perp,\omega\right)\right)=0\tag{15}$$

$$\left[(\omega+N\Omega)^2-\nu^2{}_{m,\vec{q}_\perp}-\frac{2}{\hbar}\sum_{n,n'}\left|I^m_{n,n'}\right|^2\left|D^m_{\vec{q}_\perp}\right|^2\nu_{m,\vec{q}_\perp}\Pi_0\left(m,\vec{q}_\perp,\omega+N\Omega\right)\right]=0\tag{16}$$

In these limitations, if we write the dispersion relations for confined acoustic and confined optical phonons as $\omega_{ac}\left(m,\vec{q}_\perp\right)=\omega_a+i\tau_a$ and $\nu_{oc}\left(m,\vec{q}_\perp\right)=\omega_0+i\tau_0$ with conditions $|\omega_a|\gg|\tau_a|$ and $|\omega_0|\gg|\tau_0|$, and consider the case of N = 1, we obtain:

$$\omega_a\approx\omega_{m,\vec{q}_\perp}+\frac{1}{\hbar}\sum_{n,n'}\left|I^m_{n,n'}\right|^2\left|C^m_{\vec{q}_\perp}\right|^2\mathrm{Re}\Gamma_{m,\vec{q}_\perp}\left(\omega_{m,\vec{q}_\perp}\right)\tag{17}$$

$$\omega_0\approx\nu_{m,\vec{q}_\perp}+\frac{1}{\hbar}\sum_{n,n'}\left|I^m_{n,n'}\right|^2\left|D^m_{\vec{q}_\perp}\right|^2\mathrm{Re}\Gamma_{m,\vec{q}_\perp}\left(\nu_{m,\vec{q}_\perp}\right)\tag{18}$$

$$\tau_a = -\frac{1}{\hbar}\sum_{n,n'} \left|I^m_{n,n'}\right|^2 \left|C^m_{\overrightarrow{q}_\perp}\right|^2 \frac{m^{3/2} f_0}{2\sqrt{2\pi\beta\hbar^2 \overrightarrow{q}_\perp}} \exp\left[-\frac{\beta \mathrm{m}^* \varepsilon_{n,n'}\left(\omega_{m,\overrightarrow{q}_\perp}\right)}{2\hbar \overrightarrow{q}^2_\perp}\right] e^{-\beta\varepsilon_n}\left[1 - e^{\beta\hbar\omega_{m,\overrightarrow{q}_\perp}}\right] \tag{19}$$

$$\tau_0 = -\frac{1}{\hbar}\sum_{n,n'} \left|I^m_{n,n'}\right|^2 \left|D^m_{\overrightarrow{q}_\perp}\right|^2 \frac{m^{3/2} f_0}{2\sqrt{2\pi\beta\hbar^2 \overrightarrow{q}_\perp}} \exp\left[-\frac{\beta \mathrm{m}^* \varepsilon_{n,n'}\left(\nu_{m,\overrightarrow{q}_\perp}\right)}{2\hbar \overrightarrow{q}^2_\perp}\right] e^{-\beta\varepsilon_n}\left[1 - e^{\beta\hbar\nu_{m,\overrightarrow{q}_\perp}}\right] \tag{20}$$

$$\Lambda = \frac{\lambda}{\hbar\Omega}\sum_{n,n'} \left|I^m_{n,n'}\right|^2 C^m_{\overrightarrow{q}_\perp} D^m_{\overrightarrow{q}_\perp} \mathrm{Re}\Gamma_{m,\overrightarrow{q}_\perp}\left(\omega_{m,\overrightarrow{q}_\perp}\right) \tag{21}$$

and

$$\mathrm{Re}\Gamma_{m,\overrightarrow{q}_\perp}\left(\omega_{m,\overrightarrow{q}_\perp}\right) = \frac{f_0 m^*}{2\pi\beta A\hbar^2}\left[\exp\left(-\beta\frac{\pi^2\hbar^2 n'^2}{2m^* L^2}\right) - \exp\left(-\beta\frac{\pi^2\hbar^2 n^2}{2m^* L^2}\right)\right] \tag{22}$$

$$A = \frac{\pi^2\hbar^2\left(n'^2 - n^2\right)}{2m^* L^2} + \frac{\hbar^2 \overrightarrow{q}^2_\perp}{2m^*} + \hbar\omega_{m,\overrightarrow{q}_\perp} \tag{23}$$

$$\mathrm{Re}\Gamma_{m,\overrightarrow{q}_\perp}\left(\nu_{m,\overrightarrow{q}_\perp}\right) = \frac{f_0 m^*}{2\pi\beta A_1\hbar^2}\left[\exp\left(-\beta\frac{\pi^2\hbar^2 n'^2}{2m^* L^2}\right) - \exp\left(-\beta\frac{\pi^2\hbar^2 n^2}{2m^* L^2}\right)\right] \tag{24}$$

$$A_1 = \frac{\pi^2\hbar^2\left(n'^2 - n^2\right)}{2m^* L^2} + \frac{\hbar^2 \overrightarrow{q}^2_\perp}{2m^*} + \hbar\nu_{m,\overrightarrow{q}_\perp} \tag{25}$$

$$\varepsilon_{n,n'}\left(\omega_{m,\overrightarrow{q}_\perp}\right) = \varepsilon_n - \varepsilon_{n'} - \frac{\hbar^2 \overrightarrow{q}^2_\perp}{2m^*} - \hbar\omega_{m,\overrightarrow{q}_\perp} \tag{26}$$

$$\varepsilon_{n,n'}\left(\nu_{m,\overrightarrow{q}_\perp}\right) = \varepsilon_n - \varepsilon_{n'} - \frac{\hbar^2 \overrightarrow{q}^2_\perp}{2m^*} - \hbar\nu_{m,\overrightarrow{q}_\perp} \tag{27}$$

We obtain the resonant acoustic phonon modes

$$\omega^\pm_\pm = \omega_a + \frac{1}{2}\left[\left(\upsilon_a \pm \upsilon_0\right)\Delta\left(q\right) - i\left(\tau_a + \tau_0\right) \pm \sqrt{\left[\left(\upsilon_a \mp \upsilon_0\right)\Delta\left(q\right) - i\left(\tau_a - \tau_0\right)\right]^2 \pm \Lambda^2}\right] \tag{28}$$

In Eq. (27) the signs ($\pm$) in the sub-script of $\omega^{(\pm)}_\pm$ correspond to the signs in front of the root and the sings ($\pm$) in the superscript of $\omega^{(\pm)}_\pm$ correspond to the other sign pairs. These signs depend on the resonance condition $\nu_{m,\overrightarrow{q}_\perp} \pm \omega_{m,\overrightarrow{q}_\perp} = N\Omega$. For instance, the existence of a positive imaginary part of $\omega^{(-)}_+$ implies a parametric amplification of the confined acoustic phonons. ω_a and ω_0 are the renormalization (by the electron - phonon interaction) frequency of the acoustic phonon and optical phonon; $\Delta\left(q\right) = q - q_0$, the distance to the intersection of dispersion curves, q_0 being the wave number for which the resonance is satisfied;υ_a (υ_0) is the group velocity of the acoustic (optical) phonon;τ_a τ_0, are electronic decrease constant of the acoustic and optical phonons, $\beta = 1/k_B T$, $k_B$is the Boltzmann constant, $f_0$is the density of electron. In such case that $\lambda \ll 1$, the maximal resonance, q = qx (qy = qz=0), we obtain:

$$F = \mathrm{Im}\omega^-_+ = \mathrm{Im}\left\{\omega_a + \frac{1}{2}\left[-i\left(\tau_a - \tau_0\right) + \sqrt{\left[-\left(\tau_a - \tau_0\right)\right]^2 - \Lambda^2}\right]\right\} \tag{29}$$

The condition for the resonant acoustic phonon modes to have a positive imaginary part $F > 0$ so $\frac{1}{2}\left\{-i\left(\tau_a-\tau_0\right)+\sqrt{\left(\tau_a-\tau_0\right)^2+|\Lambda|^2}\right\} > 0$ leads to $|\Lambda|^2 > 4\tau_a\tau_o$, Using this condition and Eqs. (19) - (21), yields the threshold amplitude for EEF:

$$E_{th}=\frac{2m^*\Omega^2}{e\hbar\overrightarrow{q}_{\perp}}\left[\frac{\mathrm{Im}\Gamma_{m,\overrightarrow{q}_{\perp}}\left(\omega_{m,\overrightarrow{q}_{\perp}}\right)\mathrm{Im}\Gamma_{m,\overrightarrow{q}_{\perp}}\left(\nu_{m,\overrightarrow{q}_{\perp}}\right)}{\left[\mathrm{Re}\Gamma_{m,\overrightarrow{q}_{\perp}}\left(\omega_{m,\overrightarrow{q}_{\perp}}\right)\right]^2}\right]^{\frac{1}{2}} \tag{30}$$

$$\begin{aligned}\mathrm{Im}\Gamma_{m,\overrightarrow{q}_{\perp}}\left(\omega_{m,\overrightarrow{q}_{\perp}}\right)&=-\frac{m^*f_0}{2\pi\hbar^3q_{\perp}}\sqrt{\frac{2m^*\pi}{\beta}}\exp\left(-\frac{\beta m^*A}{2\hbar^2(\overrightarrow{q}_{\perp})^2}\right)\\&\times\exp\left(-\beta\frac{\pi^2\hbar^2n^2}{2m^*L^2}\right)\exp\left(\beta\frac{\hbar\omega_{m,\overrightarrow{q}_{\perp}}}{2}\right)sh\left(\beta\frac{\hbar\omega_{m,\overrightarrow{q}_{\perp}}}{2}\right)\end{aligned} \tag{31}$$

$$\begin{aligned}\mathrm{Im}\Gamma_{m,\overrightarrow{q}_{\perp}}\left(\nu_{m,\overrightarrow{q}_{\perp}}\right)&=-\frac{m^*f_0}{2\pi\hbar^3q_{\perp}}\sqrt{\frac{2m^*\pi}{\beta}}\exp\left(-\frac{\beta m^*A_1^2}{2\hbar^2(\overrightarrow{q}_{\perp})^2}\right)\\&\times\exp\left(-\beta\frac{\pi^2\hbar^2n^2}{2m^*L^2}\right)\exp\left(\beta\frac{\hbar\nu_{m,\overrightarrow{q}_{\perp}}}{2}\right)sh\left(\beta\frac{\hbar\nu_{m,\overrightarrow{q}_{\perp}}}{2}\right)\end{aligned} \tag{32}$$

Equation (30) means that parametric amplification of the confined acoustic phonons is achieved when the amplitude of the EEF is higher than some threshold amplitude and easy to come back to the case of unconfined phonons [7] when $m \to 0$.

III. THE PARAMETRIC TRANSFORMATION OF CONFINED ACOUSTIC PHONONS AND CONFINED OPTICAL PHONONS IN QUANTUM WELLS

Parametric transformation of confined acoustic phonons and confined optical phonons in quantum well is determined by the formula:

$$K_N=\left|\frac{C_{m,\overrightarrow{q}_{\perp}}\left(\nu_{m,\overrightarrow{q}_{\perp}}\right)}{B_{m,\overrightarrow{q}_{\perp}}\left(\omega_{m,\overrightarrow{q}_{\perp}}\right)}\right|$$

$C_{m,\overrightarrow{q}_{\perp}}\left(\nu_{m,\overrightarrow{q}_{\perp}}\right)$ are determined from Eq. (13). Using the parametric resonant conditions $\omega_{m,\overrightarrow{q}_{\perp}}+N\Omega\approx\nu_{m,\overrightarrow{q}_{\perp}}$, the parametric transformation coefficient is obtained:

$$K_N=\left|\frac{\frac{1}{\hbar}\sum\limits_{n,n'}\left|I^m_{n,n'}\right|^2C^m_{\overrightarrow{q}_{\perp}}D^m_{\overrightarrow{q}_{\perp}}\Pi_N\left(m,\overrightarrow{q}_{\perp},\omega_{m,\overrightarrow{q}_{\perp}}\right)}{\delta+i\gamma_0}\right| \tag{33}$$

where, the quantity δ is infinitesimal. Consider the case of N = 1 and note $|\delta| \ll \gamma_0$, we get:

$$K_1 = \left|\frac{\Gamma}{2\gamma_0}\right| \tag{34}$$

with

$$\Gamma = \frac{\lambda}{\hbar\Omega}\sum_{n,n'}\left|I^m_{n,n'}\right|^2 C^m_{\vec{q}_\perp} D^m_{\vec{q}_\perp} \mathrm{Re}\Gamma_{m,\vec{q}_\perp}\left(\omega_{m,\vec{q}_\perp}\right) \tag{35}$$

$$\gamma_0 = -\frac{1}{\hbar}\sum_{n,n'}\left|I^m_{n,n'}\right|^2\left|D^m_{\vec{q}_\perp}\right|^2 \mathrm{Im}\Gamma_{m,\vec{q}_\perp}\left(\nu_{m,\vec{q}_\perp}\right) \tag{36}$$

Where, $\mathrm{Re}\Gamma_{m,\vec{q}_\perp}\left(\omega_{m,\vec{q}_\perp}\right)$ and $\mathrm{Im}\Gamma_{m,\vec{q}_\perp}\left(\nu_{m,\vec{q}_\perp}\right)$ are determined by the formula (22), and (32). γ_0 is the decline electronic constant of the optical phonon. Equation (34) means that parametric transformation coefficient of confined acoustic phonons and confined optical phonons in quantum well is achieved when the amplitude of the EEF is higher. When $m \to 0$, easy to come back to the case of unconfined phonons, is determined by the formula:

$$K = \left|\frac{\Gamma^*}{2\gamma^*}\right| \tag{37}$$

Where,

$$\Gamma^* = \frac{\lambda}{\hbar\Omega}\sum_{n,n'} D\left(\vec{q}_\perp\right) C\left(\vec{q}_\perp\right)\mathrm{Re}\Gamma_{\vec{q}_\perp}\left(\omega_{\vec{q}_\perp}\right) \tag{38}$$

$$\gamma^* = -\frac{1}{\hbar}\sum_{n,n'}\left|D\left(\vec{q}_\perp\right)\right|^2 \mathrm{Im}\Gamma_{\vec{q}_\perp}\left(\nu_{\vec{q}_\perp}\right) \tag{39}$$

$$\mathrm{Re}\Gamma_{\vec{q}_\perp}\left(\omega_{\vec{q}_\perp}\right) = \frac{f_0 m^*}{2\pi\beta A_2\hbar^2}\left[\exp\left(-\beta\frac{\pi^2\hbar^2 n'^2}{2m^*L^2}\right) - \exp\left(-\beta\frac{\pi^2\hbar^2 n^2}{2m^*L^2}\right)\right] \tag{40}$$

$$\mathrm{Im}\Gamma_{\vec{q}_\perp}\left(\omega_{\vec{q}_\perp}\right) = -\frac{{m^*}^{3/2} f_0}{2\sqrt{2\pi\beta}\hbar^2 q_\perp}\exp\left(-\frac{\beta m^* A_2^2}{2\hbar^2\vec{q}_\perp^{\,2}}\right)\exp\left(-\frac{\beta\hbar^2\pi^2 A_2^2}{2m^*L^2}\right)\left(1-\exp\left(\beta\hbar\omega_{\vec{q}_\perp}\right)\right) \tag{41}$$

$$A_2 = \frac{\pi^2\hbar^2\left(n'^2 - n^2\right)}{2m^*L^2} + \frac{\hbar^2\vec{q}_\perp^{\,2}}{2m^*} + \hbar\omega_{\vec{q}_\perp} \tag{42}$$

IV. NUMERICAL RESULTS AND DISCUSSIONS

IV.1. In the case parametric resonance

In order to clarify the mechanism for parametric resonance of acoustic and optical phonons in the case of confined phonons, we consider a AlAs/GaAsAl quantum well. The parameters used in this calculation are as follows [12]: $\chi_\infty = 10.9$, $\chi_0 = 12.9$, $L = 100A^0, m = 0.067m_0$, (m_0 being the mass of free electron), $\hbar\nu_0 = 36.25mev$, $\Omega = 2 \times 10^{14}Hz$, $\xi = 13.5ev$ $\rho = 5.32g.cm^{-3}$ $\upsilon_s = 5370m.s^{-1}$, $E_0 = 10^6 v/m$, $e = 1.60219\times10^{-19}C$, $\hbar = 1.05459 \times 10^{-34}J.s$

Figure 1 show the dependence of the threshold amplitude E_{th} on the magnitude of wave vector $\vec{q}$ at temperature T = 72K. As shown in, the threshold amplitude reaches

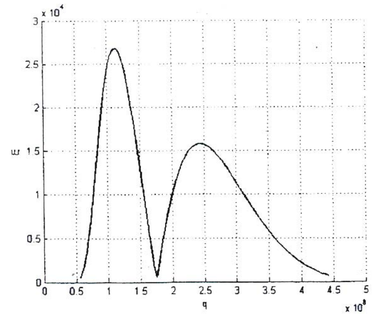

Fig. 1. The dependence of $E_{th}\left(v.cm^{-1}\right)$ on the $q\left(m^{-1}\right)$ with T = 72K

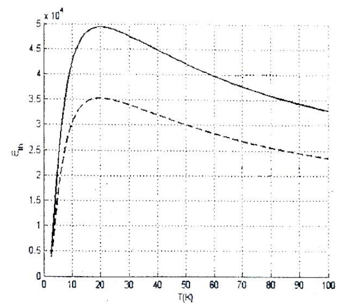

Fig. 2. The dependence of $E_{th}\left(v.cm^{-1}\right)$ on the T with $q = 2.8 \times 10^8\left(m^{-1}\right)$

the maximum value when $q = 1.2 \times 10^8\left(m^{-1}\right)$. Other cases of unconfined phonon, the curve has a sub-maximal when $q = 2.5 \times 10^8\left(m^{-1}\right)$. The cause of this difference is due to the wave vector of phonon quantum chemical confined phonon. Because the wave vector of phonon is quantized of the energy in the confined phonon direction.

Figure 2 (solid line - confined and dot line - unconfined) show the dependence of the threshold amplitude on the temperature T for both the confined phonon and unconfined phonon. From the graph shows, at the same temperature, the confined phonons makes the threshold amplitude increases.

IV.2. In the case parametric transformation

In order to clarify the mechanism for the parametric transformation of acoustic and optical phonons in the case of confined phonons, in this section, we will consider quantum wells. The parameters used in this calculation are as follow [12]:$\chi_\infty = 10.9$, $\chi_0 = 12.9$, $L = 100A^0, m = 0.067m_0$, (m_0 being the mass of free electron), $\hbar\nu_0 = 36.25mev$, $\Omega = 2 \times 10^{14}Hz$, $\xi = 13.5ev$ $\rho = 5.32g.cm^{-3}$ $\upsilon_s = 5370m.s^{-1}$, $E_0 = 10^6v/m$, $e = 1.60219 \times 10^{-19}C$, $\hbar = 1.05459 \times 10^{-34}J.s$

Figure 3, and Figure 4 shows the influence of confined phonon on the changing phenomenon of the parameter between the acoustic phonon and optical phonon. Concretely, the confinement of phonon makes an increase of the coefficient-changing parameter between the acoustic phonon and optical phonon in quantum well. In a same range of temperature T, (in the case confined phonon) the coefficient oscillation around the unit, with the case unconfined phonon the threshold amplitude is very small . Because, when phonon is confined, the energy bands of phonon are divided into mini-bands like electrons in potential well. Therefore, the probability of occurrence greater resonance conditions . In other words, the chance of changing acoustic phonon into optical phonon and vice versa becomes bigger. In short, the coefficient of parametric transformation between acoustic phonon and optical when phonon is confined is more stronger than unconfined phonon. From all figures above, we can see clearly the effect of confined phonons on the parameter transformation coefficient. Namely, the confined phonons increase the phonon transformation coefficient in quantum wells.

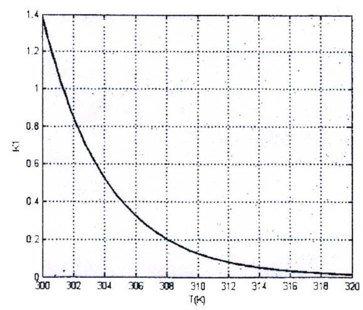

Fig. 3. The dependence of K_1 on the T (In the case confined phonon)

V. CONCLUSIONS

In this paper, we analytically investigated the possibility of parametric transformation and parametric resonance of confined acoustic phonons and confined optical phonons. We obtained a general dispersion equation for parametric amplification and transformation

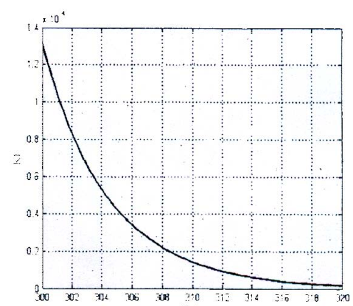

Fig. 4. The dependence of K_1 on the T (In the case unconfined phonon)

of phonons. However, an analytical solution to the equation can only be obtained within some limitations. Using these limitations for simplicity, we obtained dispersions of the resonant confined acoustic phonon and confined optical phonon modes and the threshold amplitude of the field for acoustic phonon parametric amplification and optical phonon parametric amplification. Similarly to the mechanism pointed out by several authors for bulk semiconductors, parametric amplification for acoustic phonons in a quantum well can occur under the condition that the amplitude of the external electromagnetic field is higher than some threshold amplitude. We have numerically calculated and graphed the threshold amplitude and the parametric coefficient for AlAs/GaAsAl quantum well clearly show the predicted mechanism. Parametric amplification for acoustic phonons and optical phonon and the threshold amplitude depend on the physical parameters of the system and are sensitive to the temperature. Calculated result shows that the confinement of phonon makes an increase of the coefficient changing parameter between acoustic and optical phonon. Based on this idea, we can put forward a capability about changing the functions of low - semiconductors. It plays important sense in application especially in material science, electronics. In addition, we can manufacture super mini (based nanostructures) and multi - functions (based on devices properties which could be controlled from outside.

ACKNOWLEDGMENT

This research is completed with financial support from the Vietnam - NAFOSTED (N0 103.01.18.09)

REFERENCES

[1] P. Silin, *Parametric Action of the High - Power Radiation on Plasma*, 1973 National Press on Physics Theory, Literature, Moscow.
[2] G. M. Shmelev, N. Q. Bau, *Physical phenomena in semiconductors*, 1981 Kishinev.
[3] E. M. Epstein, *Sov. Phys. Semicond.* **10** (1976) 1164.
[4] M. V. Vyazovskii, V. A. Yakovlev, *Sov. Phys. Semicond.* **11** (1977) 809.
[5] S. M. Komirenko, K. W. Kim, A. A. Dimidenko, V. A. Kochelap, M. A. Stroscico, *Phys. Rev. B* **62** (2000) 7459; *J. Appl. Phys.* **90** (2001) 3934.
[6] K. Ploog, G. H. Doller, *Adv. Phys.* **32** (1983) 285.
[7] T. C. Phong, N. Q. Bau, *J. Korea. Phys. Soc.* **42** (2003) 647.
[8] T. C. Phong, L. V. Tung, N. Q. Bau, *J. Korea. Phys. Soc.* **53** (2008) 1971.
[9] N. Q. Bau, D. M. Hung, N. B. Ngoc, *J. Korea. Phys. Soc.* **54** (2009) 765.
[10] N. Q. Bau, D. M. Hung, L. T. Hung, *Journal of the USA Physics Progress In Electromagnetics Research Letters* (2010) 175.
[11] N. Q. Bau, D. M. Hung, *Journal of the USA Physics Progress In Electromagnetics Research B* (2010) 39.
[12] Y. He, Z. Yin, M. S. Zhang, T. Lu, Y. Zheng, *Mat. Sci. Eng. B* **75** (2000) 130.

Received 10-10-2010.