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FISSION BARRIERS OF SUPERHEAVY NUCLEI

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Abstract. *The aim of this paper is to give an overview of theoretical predictions on heights of fission barriers for superheavy nuclei. The macroscopic-microscopic model of the potential energy as a function of nuclear shape is briefly presented. Immersion method of searching for saddle points determining the barrier heights which has been recently adapted to nuclear physics is discussed. Predictions of fission barrier heights for a wide region of superheavy nuclei are given, as well as a comparison to results obtained by other approaches and to existing experimental data.*

I. INTRODUCTION

The first evidence of atomic nuclei have appeared almost a century ago (1911) and is connected with E. Rutherford who gave the theoretical interpretation of experiments performed by his students H. Geiger and E. Marsden. The great progress in the field of nuclear physics came later, when the construction of particle accelerators enabled for a qualitative change from passive observations to active experiments. The use of accelerators allowed for production, in physics laboratories, of new elements, heavier than uranium ($Z = 92$), the heaviest element found on the Earth. Generally, the heaviest nuclei become less and less stable when one considers proton numbers greater than 92. The mean lifetime of nuclei around $Z \approx 100$ decreases to days or minutes and for $Z \approx 110$ goes down to an order of miliseconds. Therefore the production of new elements through a synthesis of projectile nucleus bombarding a target nucleus became more and more difficult because the cross section for the creation of the compound nuclei decreases rapidly with $Z_p Z_t$, where $Z_{p/t}$ is the proton number of the projectile/target. Despite those obstacles the process of synthesis of new nuclides is still continuing [1, 2] and the liftimes of nuclides produced in these experiments are again of the order of miliseconds. For example, Fig. 1 shows only those nuclei that have been produced after the year 1994. Moreover, it is worth mentioning that even in this year the new element with atomic number $Z = 117$ [3] has been observed (element $Z = 118$ has been synthesized in 2006 [4]) and additionally in the coming years the synthesis of even heavier systems with $119, 120$ or even 122 protons is planned. Of course, the natural question appears: `whether or not there exists any limit for the process of synthesis of the heaviest nuclei?`

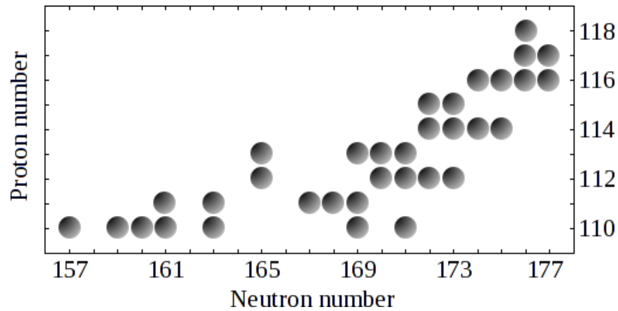

Fig. 1. Presently known isotopes of superheavy elements with proton number $110 \leq Z$, all discovered after the year 1994.

Partial answer to this question can be obtained by theoretical estimations of the cross section σ for the synthesis of those systems within existing nuclear models. The height of the static fission barrier B_f is an important quantity needed for the calculations of σ. This height is a decisive quantity in the competition between processes of neutron evaporation and fission of the compound nucleus in the process of its cooling to form a residual nucleus in its ground state. A large sensitivity of σ to B_f stresses a need for accurate predictions, because even small change of this quantity can make the difference between success and failure in experiments aimed at production of a new superheavy nuclei.

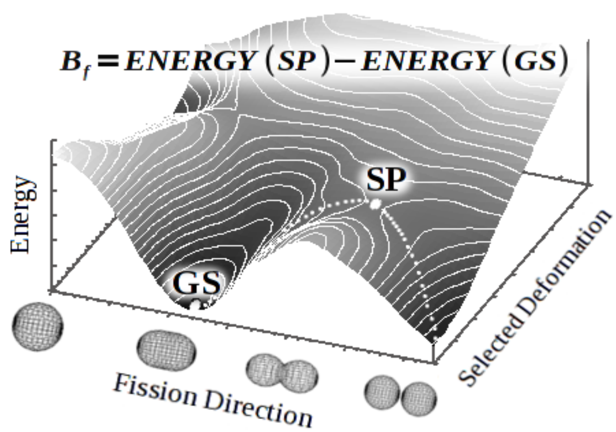

Fig. 2. Schematic picture of the potential energy surface of a heavy nucleus with marked location of the ground state point (GS) and saddle point (SP). As can be seen in this case nucleus in the ground state is deformed.

As shown in Fig. 2 quantity B_f is calculated as a difference between two values of stationary points: one representing the ground-state minimum (GS) and the other: saddle point (SP), that is the maximum of the potential energy on the path to fission. The aim of this paper is the presentation of recent results concerning predictions of fission barriers of even-even superheavy nuclei [5] and statistical comparison of those barriers with the predictions of other groups [6, 7], [8, 9] in cases when fission barriers estimated from experimantal data are known [10, 11].

II. METHOD OF THE ANALYSIS

All calculations were done within a macroscopic-microscopic approach in which the total energy as a function of shape is a sum of two parts. The main part comes from a macroscopic model a kind of liquid drop model which gives a good average description of many nuclear properties. The second, microscopic part takes into account quantum corrections to that overall description which arise from shell structure and residual short range interactions between nucleons. In our model we apply the Yukawa-plus-exponential model [12] as the macroscopic part of the energy, and the Strutinski shell correction plus pairing correction as the microscopic part. We use the deformed Woods-Saxon single-particle potential [13] for the nuclear mean field to calculate the Strutinski shell correction and BCS theory for pairing correction. It is worth mentioning that the total energy presented in the following maps was additionally renormalized by subtracting the macroscopic energy for the spherical shape. For all details of the approach we refer to [5] and references therein.

The shape of the nucleus and the nuclear potential has been parametrized by expansion of the nuclear radius $R(\vartheta,\varphi)$ in spherical harmonics:

$$R(\vartheta,\varphi) = R_0\,\{1 \;+\; \beta_2\left[\cos\gamma_2 \mathrm{Y}_{20} + \sin\gamma_2 \mathrm{Y}_{22}^{(+)}\right] + a_{40}\mathrm{Y}_{40} + a_{42}\mathrm{Y}_{42}^{(+)} + a_{44}\mathrm{Y}_{44}^{(+)} \\ +\; \beta_3\mathrm{Y}_{30} + \beta_5\mathrm{Y}_{50} + \beta_7\mathrm{Y}_{70} + \beta_6\mathrm{Y}_{60} + \beta_8\mathrm{Y}_{80}\} \qquad (1)$$

where the dependence of R_0 on the deformation parameters is determined by the volume-conservation condition. The real functions $\mathrm{Y}_{\lambda\mu}^{(+)}$ are defined as:

$$\mathrm{Y}_{\lambda\mu}^{(+)} = \frac{1}{\sqrt{2}}\left[\mathrm{Y}_{\lambda\mu} + (-1)^{\mu}\mathrm{Y}_{\lambda-\mu}\right], \qquad \text{for} \quad \mu > 0. \qquad (2)$$

A few examples of nuclear shapes which can by described by above parametrization are shown in Fig. 3 where for illustrative purposes the values of deformation parameters are exaggerated.

II.1. Method of the analysis: Immersion method

A useful and very effective method for searching saddle point is the immersion method described in detail in [11, 9]. In Fig. 4 we show a scheme of the algorithm determining the saddle point energy and position. For simplicity we explain the method on an example of three-dimensional potential energy map (shown at the top left panel) where parameters of deformation are $[\beta_2\cos(\gamma_2), \beta_2\sin(\gamma_2), \beta_4]$. In this figure energy dependence on β_4 is hidden because for each $[\beta_2\cos(\gamma_2), \beta_2\sin(\gamma_2)]$ the energy is minimized with respect to β_4.

The algorithm starts by setting two points: *GS* and *EX*, where *GS* corresponds to the position of the ground state (with the energy $E(GS)$) and *EX* denotes the position of the point outside the barrier, i.e. exit point (with the energy $E(EX)$). The whole energy landscape is stored in the matrix $E(i,j,k)$ which has three indices corresponding to the values of the three shape coordinates i,j,k defined on a grid. The values of elements

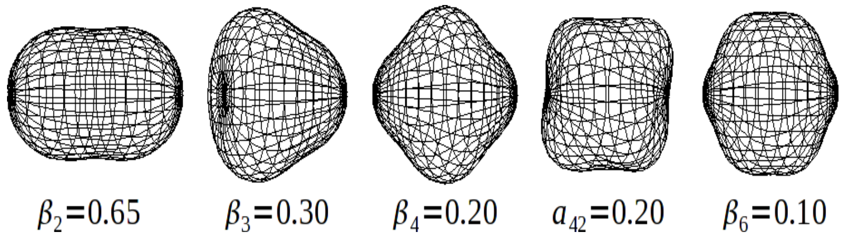

Fig. 3. Some examples of nuclear shapes which can be described by the expression (1). In each presented case only a single non-zero deformation parameter was used.

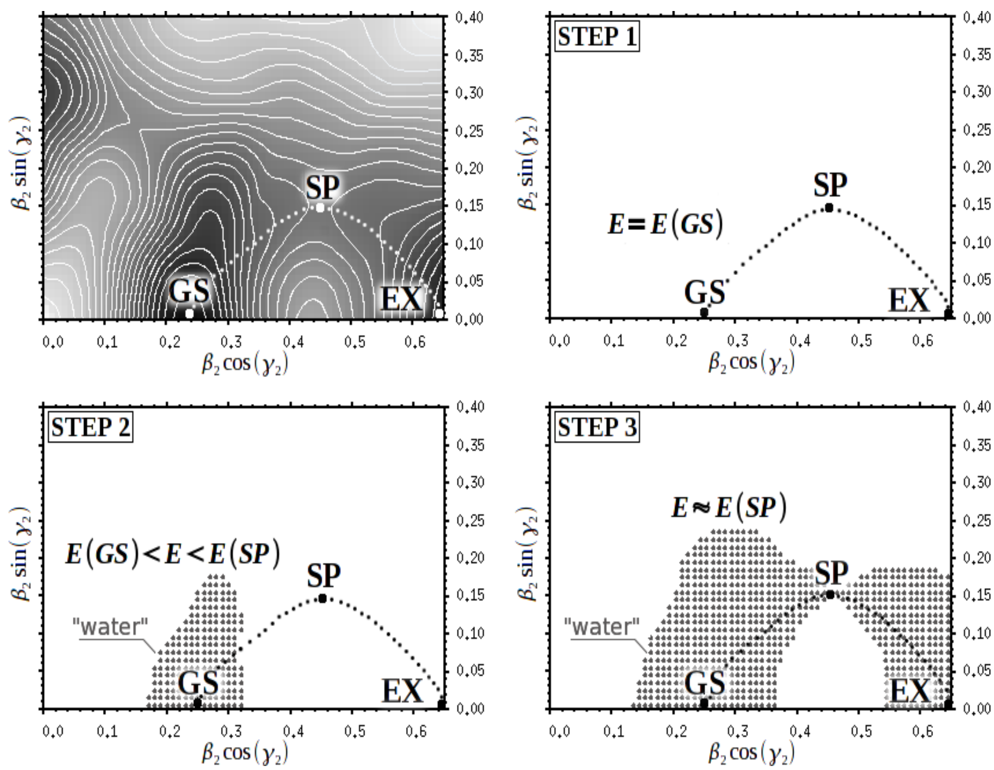

Fig. 4. A scheme of determination of the saddle point in the immersion method.

of another matrix $I(i,j,k)$ indicate if a given grid point is "wet" $I(i,j,k) = 1$ or "dry" $I(i,j,k) = 0$. In first step we set $I(GS) = 1$ making the ground state point "wet", and initialize all other elements of $I(i,j,k)$ to 0 (Fig. 4: top right). Next we fill the potential energy well around the ground state with "water" to the energy $E = E(GS) + \Delta E$ and replace the value of $I(i,j,k)$ to 1 for all grid points conneted to each other which energies fulfil the condition $E(GS) < E(i,j,k) \leq E(GS) + \Delta E$ (Fig. 4: bottom left). This step correspond to an immersion of the potential energy landscape around the ground state shape up to the energy $E(GS) + \Delta E$. If in this step the area of "wet" points does not contain the exit point we increase the considered energy up to $E(GS) + 2\Delta E$, $E(GS)+3\Delta E, \ldots$, until the exit point EX becomes "wet" (i.e. $I(EX) = 1$). This situation occurs only when the "water" has flooded the saddle SP between the entry GS and exit EX points (Fig. 4: bottom right). In this case we know that the saddle point energy lies between $E(GS) + n\Delta E$ and $E(GS) + (n + 1)\Delta E$. Then we can define a smaller energy increment ΔE_1 and by repeating the same steps in a new smaller range of the energy, starting from the points with the energy $E(GS) + n\Delta E$ we restrict our search area again and again. After several iterations our search area is narrowed to a very small region (in the limit to one point) which is a saddle point up to required precision. In the case of more dimensions all steps described below will be the same. This method is intuitively, fast in operation and easy to implement - its only limitation is the size of the grid which have to be stored in the computer memory.

III. RESULTS AND CONCLUSION

In order to check a predictive power of our approach we have first calculated the fission barrier heights B_f for actinide nuclei and compared our results with values extracted from experiments, available for the heaviest nuclei [10, 11], and with those obtained in other approaches.

Table 1. Statistical parameters of calculted inner fission barrier heights in various models. Except the number of nuclei N all quantities are in MeV.

Models:	LSD	FRLDM	HN
N	16	18	18
$<\mid B_f^{th} - B_f^{exp} \mid>$	0.9	1.0	0.4
Max $\mid B_f^{th} - B_f^{exp} \mid$	1.8	2.2	1.0
R.M.S.	1.0	1.1	0.5

In Table 1 we compare statistical properties of the calculated B_f values obtained in three theoretical models: LSD [6, 7], FRLDM [8, 9] and our HN [5]. All results are refered to experimental values. All three theoretical models are based on macroscopic-microscopic approach, however they differ from each other by using different versions of macroscopic part, different details of the mean field potential and different shape parametrization. Our method, particularly devoted for a description of many properties on Heavy Nuclei (HN) [14] exhibits much better reproduction of experimental values of B_f than the other models. The parameters compared are: the mean value of deviations $<\mid B_f^{th} - B_f^{exp} \mid>$, the maximal deviation Max $\mid B_f^{th} - B_f^{exp} \mid$ and root mean square of deviations R.M.S. All those quantities are in our model roughly two times smaller than those in other models. Therefore we hope that our predictions for yet non-observed nuclei are the most precise.

Fission barrier heights for a vast region of even-even nuclei with $Z \in [98, 126]$ and $N \in [134, 192]$ obtained by our group are presented in Fig. 5. We predict relatively high values of B_f (5-6 MeV) only for region of $Z \lesssim 122, 124$ and $170 \lesssim N \lesssim 184$. The numerical values for all considered nuclei are given in [5].

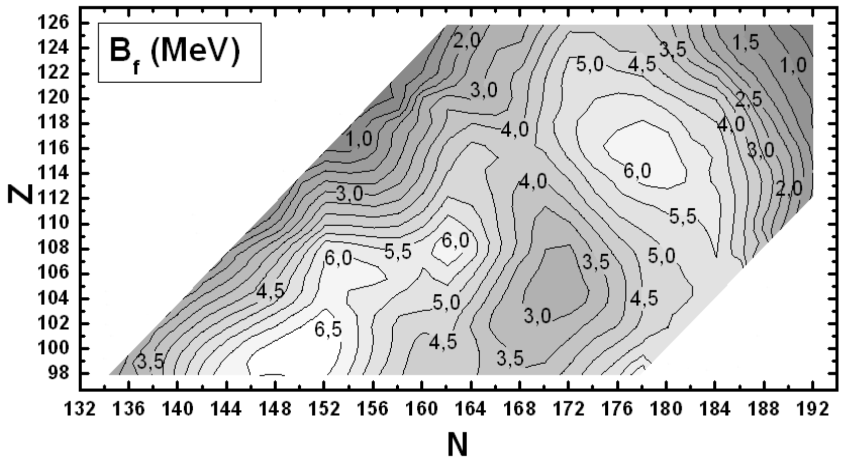

Fig. 5. Contour map of fission barrier heights B_f calculated for even-even superheavy nuclei.

It is worth to mention that the other approaches, like Hartree-Fock self-consistent theory, based on several versions of nucleon-nucleon interaction give in general much worse description of fission properties of heavy nuclei than macroscopic-microscopic approach.

There may be several reasons causing that the results of HN model are the closest to the experimental data. Firstly, the parameters of our calculations for the heavy nuclei have been several years ago chosen such that many properties of a large region of heavy nuclei were reproduces by the model calculations as well as possible. Next, our shape paramtrization is the richest one allowing to find a shape configuration of the lowest energy for the fission process.

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