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PHOTOELECTRON SPECTRA INDUCED BY BROAD-BAND CHAOTIC LIGHT FROM THE DOUBLE FANO CONTINUUM

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Abstract. *In this paper, we consider a model for laser-induced autoionisation introduced before in [1]. Following [2], we assume that a laser light is decomposed into two parts: a deterministic or coherent part and a randomly fluctuating chaotic component, which is a δ-correlated, Gaussian, Markov and stationary process (white noise). We solve a set of coupled stochastic integro-differential equations and describe a double Fano model for autoionisation. We determine the exact photoelectron spectrum and compare it with our results obtained before in [1] and [2].*

I. INTRODUCTION

Over the period of last three decades, we have noticed a particular interest in the research on the different ionisation processes of atoms in laser fields. One of the most interesting examples is the so-called laser-induced autoionisation (LIA). On that particular subject, several papers have been published already, including the study of detailed characteristics of electron and photon spectra associated with photoexcited autoionisation. The most commonly used atomic model is Fano model [3] of a number of discrete states and one continuum that can be diagonalized. The Fano diagonalisation, which is based on Coulomb-mixing of the ionising states with continuum, leads to the nontrivial structure in the continuum [2–4]. Systems comprising autoionising levels can behave in diverse and nontrivial way, which may lead to various interesting physical phenomena like quantum interferences discussed in [2–4] (and the references quoted therein), electromagnetically induced transparency of light slowdown modifications [5], or quantum anti-Zeno effect [6], for example.

In [2] we treated the laser field as a white noise in the Fano model for autoionisation. Then the set of coupled stochastic integro-differential equations were exactly solved. We also determined there the exact photoelectron spectrum. In our present work, we extend the formalism introduced there to the case of the double Fano System [1] in which instead of one autoionising state we have two discrete states embedded in one continuum. The model of the white noise for the field is interesting by itself because it describes the electric field amplitude of the multimode laser, operating without any correlation between the modes. We will determine the exact photoelectron spectrum and compare it with our results obtained before in [1] and [2].

Our paper is organized as follows: In the second section, some details of our model are described and the set of equations for atomic operators involved in the problem is derived. These equations are more exact than those introduced in [2]. In the third part, our results are presented and discussed. Instead of presenting a rather complicated formula, we have restricted ourselves to two interesting physical limits: infinite and finite assymetry parameter introduced by Fano [3] and generalized to the case of the double Fano system in [1]. The last section contains conclusions.

II. THE MODEL WITH THE DOUBLE FANO SYSTEM

We consider the model shown in Fig. 1, where besides the ground state $|0\rangle$, there are two discrete states $|1\rangle$ and $|2\rangle$ lying above the ionisation threshold and interacting with the continuum by means of configurational Coulomb interaction. The two levels, as well as the continuum, are coupled to the ground state by a strong laser beam of frequency ω_L.

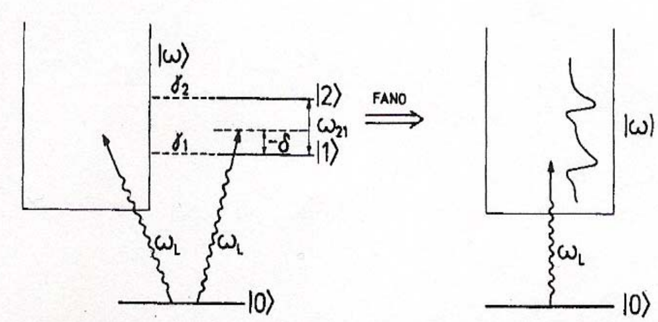

Fig. 1. Simplified atomic-level scheme. Configuration-interaction coupling of levels $|1\rangle$ and $|2\rangle$ to $|\omega\rangle$ leads to the double Fano continuum $|\omega\rangle$. This continuum is coupled to discrete state $|0\rangle$ by a laser of frequency ω_L.

Following the procedure used in [7] we can obtain the so called double Fano continuum. The state of this new structured continuum is denoted by round brackets $|\omega)$ (see the scheme on the right side of Fig. 1). The matrix element $\Omega(\omega)$ that describes the coupling of the ground state with this new state (the effective Rabi frequency) is given by

$$\Omega(\omega) = \frac{\Omega_0}{\sqrt{4\pi\Gamma}} \left(\frac{A_+}{\omega - \omega_+} + \frac{A_-}{\omega - \omega_-} + \frac{1}{Q + i} \right), \tag{1}$$

where $\Gamma = \Gamma_1 + \Gamma_2$ is the total autoionisation rate, the effective asymmetry parameter is expressed by $Q = (q_1\Gamma_1 + q_2\Gamma_2)/\Gamma$ and $\omega_\pm$ are the complex roots of the denominator of Eq. (1.1) given by [1]

$$\omega_\pm = \frac{\omega_1 + \omega_2 \pm \theta}{2} + i\frac{\Gamma \pm \phi}{2}, \tag{2}$$

with

$$\begin{aligned} \phi =& \frac{1}{\sqrt{2}}\left\{\left[\left(\omega_{21}^2-\Gamma^2\right)^2+4\omega_{21}^2\left(\Gamma_2-\Gamma_1\right)^2\right]^{1/2}-\omega_{21}^2+\Gamma^2\right\}^{1/2}, \\ \theta =& \frac{1}{\sqrt{2}}\left\{\left[\left(\omega_{21}^2-\Gamma^2\right)^2+4\omega_{21}^2\left(\Gamma_2-\Gamma_1\right)^2\right]^{1/2}+\omega_{21}^2-\Gamma^2\right\}^{1/2}, \end{aligned} \tag{3}$$

where $\omega_{21}=\omega_2-\omega_1$, ω_1 and ω_2 are the bare energies of the discrete atomic states.

The complex amplitudes $A_\pm$ are given by the following expressions:

$$A_\pm=\frac{\Gamma}{2}\left(1\pm\frac{\omega_{21}K+i\Gamma}{\theta+i\phi}\right), \tag{4}$$

where

$$K=\frac{q_2\Gamma_2-q_1\Gamma_1+i\left(\Gamma_2-\Gamma_1\right)}{\Gamma\left(Q+i\right)}. \tag{5}$$

The form (Eq. (1)) of the radiative matrix element $\Omega\left(\omega\right)$ is a generalization of the corresponding formula of Rzewski and Eberly [7] to the case of two autoionizing levels, both of which are radiatively coupled to the ground state. It is a superposition of two Lorentzians and a flat background. Thus in the case of the double Fano profile we have an additional Lorentzian, which is due to the presence of an additional autoionising state.

For the sake of convenience, we denote further on the new state $|\omega)$ again by the ket $|\omega\rangle$. As in [2], we start with the Hamiltonian, which describes a model with a bound state lying below the edge of the continuum, with the bound and continuum states coupled by the electromagnetic field

$$H=\hbar\omega_0P_0+\int\hbar\omega C_{\omega\omega}d\omega+\int\Omega\left(\omega\right)|0\rangle\langle\omega|\,d\omega+H.C \tag{6}$$

where P_0 and $C_{\omega\omega}$ are occupation-number operators for the ground state and for the continuum states respectively. The interaction here is described by the function $\Omega\left(\omega\right)$, which marks how strongly different points of the continuous spectrum are coupled to the bound state, and is given by the formula (1). By $|0\rangle$ we mean the bound state and $|\omega\rangle$ stands for the excited state in the dressed continuum. We define the following operators

$$\begin{aligned} B_\omega &= |0\rangle\langle\omega|\,, \\ C_{\omega\omega'} &= |\omega\rangle\langle\omega'|\,. \end{aligned} \tag{7}$$

Then the Heisenberg equations of motion for the atomic operators $P_0, B_\omega, B_\omega^+, C_{\omega\omega}$ form the complete set and can be easily found by simple commutations of that operators with Hamiltonian (6). For the sake of simplicity we assumed that$\omega_0=0$.

In the Heisenberg picture, one obtains the linear equations for the dynamical variables, so the equations for corresponding averaged quantities are easily found using different well-known results from the theory of multiplicative stochastic processes. We assume now that $\Omega\left(\omega\right)$ has the form:

$$\Omega\left(\omega\right)=f\left(\omega\right)\left(E_0+E\left(t\right)\right)e^{i\omega_Lt}, \tag{8}$$

where $E\left(t\right)$ is characterized by a Gaussian, Markov and stationary process (white noise),

$$\langle\langle E(t)\,E^*\left(t'\right)\rangle\rangle = a\delta\left(t-t'\right), \tag{9}$$

and E_0 is a deterministic coherent component of the laser field. The double brackets in (9) indicate an average over the ensemble of realisations of the process $E(t)$. We consider the following stochastic differential equation

$$\frac{dQ}{dt} = \{A + x(t)\,B + x^*(t)\,C\}\,Q, \tag{10}$$

where Q is a vector function of time and A, B and C are constant matrices. Then it is a known result in the theory of multiplicative stochastic process that $\langle\langle Q\rangle\rangle$exactly satisfies the nonstochastic equation:

$$\frac{d\langle\langle Q\rangle\rangle}{dt} = [A + a\{B, C\}/2]\,\langle\langle Q\rangle\rangle \tag{11}$$

where $\{B, C\}$ is the anticommutator of B and C. Before averaging however, we transform dynamical variables to the rotating frame:

$$\begin{aligned} B_\omega &= f^*(\omega)\,D_\omega e^{-i\omega_L t}\\ C_{\omega\omega'} &= f(\omega')\,f^*(\omega)\,E_{\omega\omega'}. \end{aligned} \tag{12}$$

The new quantities $D_\omega, E_{\omega\omega'}$ together with P_0 satisfy the following closed set of equations:

$$\begin{aligned} \frac{dP_0}{dt} =& -i\int d\omega\,|f(\omega)|^2\left[(E_0 + E(t))\,D_\omega - (E_0^* + E^*(t))\,D_\omega^+\right]\\ \frac{dD_\omega}{dt} =& i(\omega_L - \omega)\,D_\omega - i(E_0^* + E^*(t))\,P_0 + i\int d\omega'\left|f\left(\omega'\right)\right|^2(E_0^* + E^*(t))\,E_{\omega'\omega}\\ \frac{dE_{\omega\omega'}}{dt} =& i\left(\omega - \omega'\right)E_{\omega\omega'} + i(E_0 + E(t))\,D_{\omega'} - i(E_0^* + E^*(t))\,D_\omega^+. \end{aligned} \tag{13}$$

Next using Eq. (11) we obtain the system of equations for stochastic averages of the variables (double brackets have been dropped for convenience):

$$\begin{aligned} \frac{dP_0}{dt} =& -aFP_0 - ib\int d\omega'\,|f(\omega')|^2\left(D_{\omega'} - D_\omega^+\right) + a\int\int|f(\omega')|^2\,|f(\omega")|^2\,E_{\omega'\omega"}d\omega' d\omega" \quad (a),\\ \frac{dD_\omega}{dt} =& -ibP_0 + \left[i(\omega_L - \omega) - \frac{aF}{2}\right]D_\omega - \frac{a}{2}\int d\omega'\,|f(\omega')|^2\,D_{\omega'} + ib\int d\omega'\,|f(\omega')|^2\,E_{\omega'\omega} \quad (b),\\ \frac{dE_{\omega\omega'}}{dt} =& aP_0 + ib\left(D_{\omega'} - D_\omega^+\right) + i(\omega - \omega')\,E_{\omega\omega'} - \frac{a}{2}\int d\omega"\,|f(\omega")|^2\,(E_{\omega"\omega'} + E_{\omega\omega"}) \quad (c), \end{aligned} \tag{14}$$

where $F = \int|f(\omega)|^2, b = |E_0|$. The equation corresponding to the adjoint operator D_ω^+ is easily found from D_ω by complex conjugation. In comparison with the equations (2.9a) and (2.9b) in [2], we have here in (14a) and (14b) the additional terms containing the coherent part of the laser field b which are not correctly omitted there.

Then using the Laplace transform technique and assuming the separation property of $\tilde{E}_{\omega\omega'}$:

$$\left[z - i\left(\omega - \omega'\right)\right]\tilde{E}_{\omega\omega'} = \xi_\omega\left(z\right) + \eta_{\omega'}\left(z\right), \tag{15}$$

we can obtain the exact analytical expressions for the ξ_ω and η_ω. The calculations are very long and tedious but rather simple and will be published elsewhere [8]. In the next section we shall use the results obtained above to calculate the steady state spectrum of photoelectrons in a strong field.

III. PHOTOELECTRON SPECTRUM IN THE CASE OF TWO LORENTZIANS

As has been mentioned before, we discuss the two lorentzians case of the double Fano profile. Our model allows the spectrum, i.e.

$$W\left(\omega\right) = \lim_{t\to\infty} C_{\omega\omega}\left(t\right), \tag{16}$$

to be computed directly and completely analytically.

The spectral distribution of excited electrons is determined from $C_{\omega\omega}\left(t\right)$. In the steady state at $t \to \infty$, only the pole at $z = 0$ in the Laplace domain contributes. From the definition of $\tilde{E}_{\omega\omega'}$ and its separation property one obtains

$$\tilde{C}_{\omega\omega}\left(z\right) = \left|f\left(\omega\right)\right|^2 \left|\frac{\xi_\omega\left(z\right) + \eta_\omega\left(z\right)}{z}\right|. \tag{17}$$

Thus the spectrum $W\left(\omega\right)$ is given by

$$W\left(\omega\right) = \left|f\left(\omega\right)\right|^2 \left|2Re\xi_\omega\left(0\right)\right|. \tag{18}$$

Because analytical formula of $W\left(\omega\right)$ for our model is very long and much more complicated than for the single Fano profile, we don't give it here. We will consider photoelectron spectra in the two cases separately: infinite asymmetry parameter and finite asymmetry parameter. As in [1] all the frequencies (energies) are given in units of Γ.

III.1. Infinite asymmetry parameter ($q \to \infty$)

In the degenerate case $\omega_{21} = 0$ photoelectron spectrum is the same as the one described in [2] for the case, when the value of coherent component b is fixed, while the value of chaotic component a is changed (Fig. 2). But in the case when we keep the constant value of a and change b, our peaks are higher than in [2] because Eq. (14a) and Eq. (14c)) contain coherent component b which is not correctly omitted in the corresponding equations in [2] (see Fig. 3). When a is small, i.e. the coherent part of the light dominates over the fluctuations, photoelectron spectra exhibit characteristic Autler-Townes splitting.

The photoelectron spectra for the nondegenerate case ($\omega_{21} \neq 0$) are presented in a future publication [8].

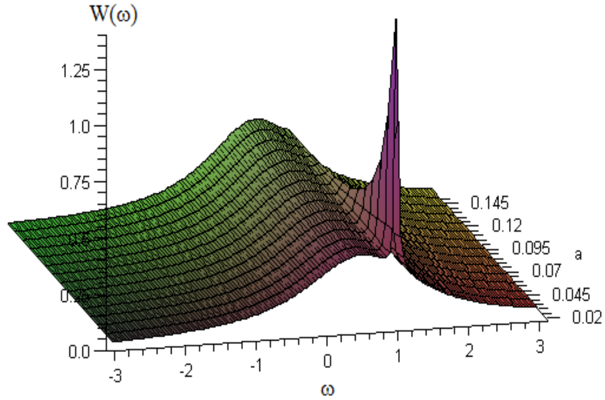

Fig. 2. Photoelectron spectrum in the case $\omega_1 = \omega_2 = 0.5$; with $\omega_L = 1.0$; autoionisation widths $\Gamma_1 = \Gamma_2 = 0.5$ and coherent component $b = 0.1$.

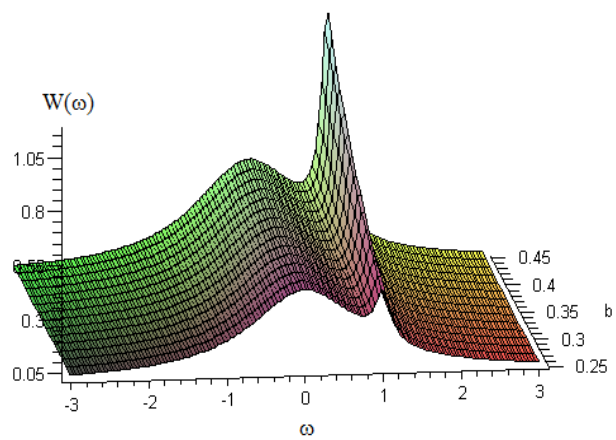

Fig. 3. Photoelectron spectrum in the case $\omega_1 = \omega_2 = 0.001$; with $\omega_L = 1.0$; autoionisation widths $\Gamma_1 = \Gamma_2 = 0.5$ and chaotic component $a = 0.12$.

III.2. Finite asymmetry parameter

When q_1 and q_2 are finite, in the degenerate case ($\omega_{21} = 0$), photoelectron spectrum is presented in the Fig. 4 and Fig. 5. In Fig. 4 photoelectron spectrum is presented when the chaotic component is absent ($a = 0$) and $q$takes the large values. For the weak field, the symmetric spectrum only has one peak. When the field grows, a sharp wedge in the spectrum appears. This spectrum exactly reproduces that obtained by W. Leouński et al [1]. However, when the chaotic component is present ($a \neq 0$) (Fig. 5) sharp wedge in the spectrum disappears, the symmetric spectrum only remains one peak, with intensity peak smaller than in the case $a = 0$.

The results for the nondegenerate case will be presented elsewhere [8].

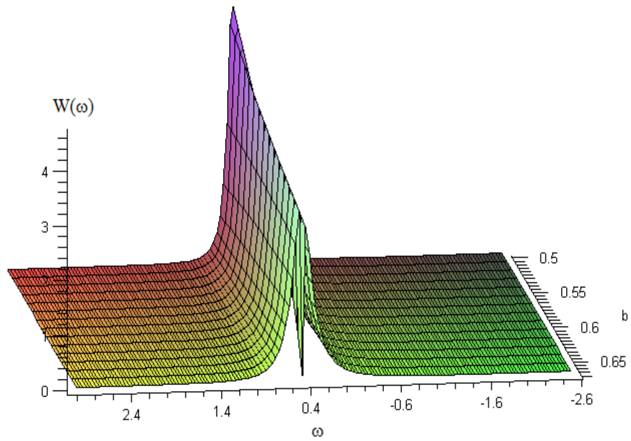

Fig. 4. Photoelectron spectrum in the degenerate case ($\omega_1 = \omega_2 = 0.5$) with chaotic component $a = 0.0$, $\omega_L = 0.5$, autoionisation widths $\Gamma_1 = \Gamma_2 = 0.5$ and asymmetry parameters $q_1 = 90$, $q_2 = 100$.

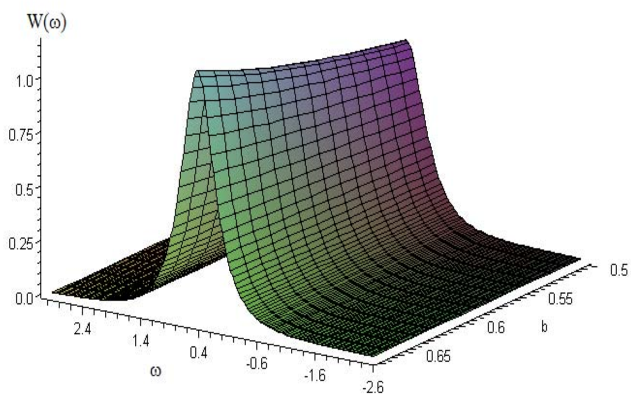

Fig. 5. Photoelectron spectrum in the degenerate case ($\omega_1 = \omega_2 = 0.5$) with chaotic component a= 0.5, ω_L= 0.5, autoionisation widths $\Gamma_1 = \Gamma_2 = 0.5$ and asymmetry parameters $q_1 = 90$, $q_2 = 100$.

IV. CONCLUSIONS

In this paper, we consider a model for laser-induced autoionisation introduced before in [1] in which instead of one autoionising state we have two discrete states embedded in one continuum, the so-called double Fano model. As in [2], we assume in this paper that the laser light is decomposed into two parts: the deterministic or coherent part and the one that, being a randomly fluctuating chaotic component, is called white noise. Then we introduce and solve exactly a set of coupled stochastic integro-differential equations and describe the double Fano model for autoionisation. These equations are more correct than those introduced in [2]. We determine the exact photoelectron spectrum and compare it with our results obtained before in [1] and [2]. We have considered photoelectron spectra in the two situations separately: infinite asymmetry parameter and finite asymmetry parameter. The spectra have been discussed only for the degenerate case $\omega_{12}= 0$. The results for the nondegenerate case ($\omega_{12} \neq 0$) will be published in a near future [8].

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