

RENORMALIZATION GROUP AND 3-3-1 MODEL WITH THE DISCRETE FLAVOUR SYMMETRIES

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Abstract. *Renormalization group equations of the 3-3-1 models with A_4 and S_4 flavor symmetries as the only intermediate gauge group between the standard model and the scale of unification of the three coupling constants are presented. We shall assume that there is no necessarily a group of grand unification at the scale of convergence of the couplings.*

I. INTRODUCTION

Since the birth of the Standard Model (SM) many attempts have been done to go beyond it, and solve some of the problems of the model such as the unification of coupling constants. In looking for unification of the coupling constant by passing through a 3-3-1 models [1, 2], we shall assume that 1) The 3-3-1 gauge group is the only extension of the SM before the unification of the running coupling constants. 2) The hypercharge associated with the 3-3-1 gauge group is adequately normalized such that the three gauge couplings unify at certain scale M_U . and 3) There is no necessarily a unified gauge group at the scale of convergence of the couplings M_U . In the absence of a grand unified group, there are no restriction on M_U coming from proton decay.

If the unification came from a grand unified symmetry group G , the normalization of the hypercharge Y would be determined by the group structure. However, under our assumptions, this normalization factor is free and the problem could be addressed the opposite way, since the values obtained for a could in turn suggest possible groups of grand unification in which the 3-3-1 group is embedded, we shall explore this possibility as well.

II. RGE ANALYSIS

Renormalization group equations are

$$\alpha_U^{-1} = \frac{1}{a^2 - \frac{4b^2}{3}} \left\{ \alpha_{EM}(M_Z)^{-1} - \frac{4b^2}{3} \alpha_{2L}(M_Z)^{-1} - \frac{b_Y - \frac{4b^2}{3} b_{2L}}{2\pi} \ln \left(\frac{M_X}{M_Z} \right) - \frac{b_X}{2\pi} \ln \left(\frac{M_U}{M_X} \right) \right\}, \quad (1)$$

$$\alpha_U^{-1} = \alpha_{2L}(M_Z)^{-1} - \frac{b_{2L}}{2\pi} \ln \left(\frac{M_X}{M_Z} \right) - \frac{b_{3L}}{2\pi} \ln \left(\frac{M_U}{M_X} \right), \quad (2)$$

$$\alpha_U^{-1} = \alpha_s(M_Z)^{-1} - \frac{b_s}{2\pi} \ln \left(\frac{M_X}{M_Z} \right) - \frac{b_{3C}}{2\pi} \ln \left(\frac{M_U}{M_X} \right). \quad (3)$$

The input parameters from precision measurements are [3]

$$\begin{aligned} \alpha_{EM}^{-1}(M_Z) &= 127.934 \pm 0.027, \\ \sin^2 \theta_w(M_Z) &= 0.23113 \pm 0.00015, \\ \alpha_s(M_Z) &= 0.1172 \pm 0.0020, \\ \alpha_{2L}^{-1}(M_Z) &= 29.56938 \pm 0.00068. \end{aligned} \quad (4)$$

The M_U scale, where all the well-normalized couplings have the same value, can be calculated from (2) and (3) as a function of the symmetry breaking scale M_X

$$M_U = M_X \left(\frac{M_X}{M_Z} \right)^{-\frac{b_s - b_{2L}}{b_{3C} - b_{3L}}} \exp \left\{ 2\pi \frac{\alpha_s(M_Z)^{-1} - \alpha_{2L}(M_Z)^{-1}}{b_{3C} - b_{3L}} \right\}. \quad (5)$$

The hierarchy condition $M_X \leq M_U \leq M_{Planck}$, must be satisfied. We shall however impose a stronger condition of $M_U \simeq 10^{17}$ GeV, in order to avoid gravitational effects. Hence, the hierarchy condition becomes

$$M_X \leq M_U \leq 10^{17} GeV \quad (6)$$

Such condition can establish an allowed range for the symmetry breaking scale M_X in order to obtain grand unification for a given normalizing parameter a .

With a similar procedure, the expression for a^2 is found, and is given by

$$a^2 = \frac{4}{3} b^2. \quad (7)$$

This convergence occurs at the scale

$$M_X = M_Z \exp \left[\frac{2\pi [\alpha_{2L}(M_Z)^{-1} - \alpha_s(M_Z)^{-1}]}{(b_{2L} - b_s)} \right] \quad (8)$$

It worths emphasizing that this scenario leads to a unique value of M_X and not to an allowed range. Finally, Eq. (7) for a^2 must also be recalculated to find

$$\begin{aligned}
a^2 &= \frac{F_1(M_X) - \frac{b_X}{2\pi} \ln\left(\frac{M_U}{M_X}\right)}{F_2(M_X) - \frac{b_3}{2\pi} \ln\left(\frac{M_U}{M_X}\right)} + \frac{4b^2}{3} \\
F_1(M_X) &= \alpha_{EM}(M_Z)^{-1} - \frac{4b^2}{3} \alpha_{2L}(M_Z)^{-1} - \frac{b_Y - \frac{4b^2}{3} b_{2L}}{2\pi} \\
&\quad \times \left\{ \frac{2\pi [\alpha_{2L}(M_Z)^{-1} - \alpha_s(M_Z)^{-1}]}{(b_{2L} - b_s)} \right\} \\
F_2(M_X) &= \alpha_{2L}(M_Z)^{-1} - \frac{b_{2L}}{2\pi} \left\{ \frac{2\pi [\alpha_{2L}(M_Z)^{-1} - \alpha_s(M_Z)^{-1}]}{(b_{2L} - b_s)} \right\} \quad (9)
\end{aligned}$$

The case $(b_{3C} - b_{3L}) = (b_s - b_{2L}) = 0$, does not lead to unification as can be seen by trying to equate Eqs. (2) and (3). Since the first scenario is the most common one, we shall only indicate when the other two scenarios appear.

III. RGE IN THE 3-3-1 MODEL WITH A_4 FLAVOUR SYMMETRY

III.1. Particle content

Let us briefly mention on the above mentioned model [4]

Let us summarize the Higgs content of the model:

$$\phi = \sim (3, 2/3, \underline{3}, -1/3), \quad (10)$$

$$\eta = \sim (3, -1/3, \underline{1}, -1/3), \quad (11)$$

$$\rho = \sim (3, 2/3, \underline{1}, -1/3), \quad (12)$$

$$\chi = \sim (3, -1/3, \underline{1}, 2/3), \quad (13)$$

$$\sigma = \left(\begin{array}{ccc} \sigma_{11}^0 & \sigma_{12}^+ & \sigma_{13}^0 \\ \sigma_{12}^+ & \sigma_{22}^{++} & \sigma_{23}^+ \\ \sigma_{13}^0 & \sigma_{23}^+ & \sigma_{33}^0 \end{array} \right) \sim (6^*, 2/3, \underline{1}, -4/3), \quad (14)$$

$$s = \sim (6^*, 2/3, \underline{3}, -4/3), \quad (15)$$

where the parentheses denote the quantum numbers based on $(SU(3)_L, U(1)_X, A_4, U(1)_\mathcal{L})$ symmetries, respectively. The subscripts to the component fields are indices of $SU(3)_L$. The $\underline{3}$ indices of A_4 for ϕ and s are discarded and understood.

III.2. Calculation of b_i

With the above particle content, we have

(1) For the group $SU(3)_C$:

$$b_C^{A_4} = 5 \quad (16)$$

where n_g is the number of families, and is taken to be 3.

- (2) For the group $SU(3)_L$: Note that, in the group $SU(3)$, sextet is symmetric 2nd-rank tensor with common property of $SU(N)$

$$T(\text{2nd rank}) = \frac{1}{2}(N+2) \quad \text{for symmetric 2nd-rank tensor.} \quad (17)$$

Hence

$$b_{3L}^{A4} = \frac{11}{3} \times 3 - \frac{2}{3} \times \frac{1}{2} \times (3+1) \times n_g - \frac{1}{6} \times n_T - \frac{5}{6} \times n_S = \frac{8}{3} \quad (18)$$

where n_T is number of Higgs triplets and n_S is number of scalar sextets. In the model under consideration, these numbers are equal to 6 and 4, respectively.

- (3) For the group $U(1)_X$:

$$\begin{aligned} C(\text{vector}) &= 0, \\ C(\text{Dirac fermion, scalar}) &= X^2 \end{aligned} \quad (19)$$

Therefore we have

- (a) Contribution of the leptons $\sum_X X^2$: one right-handed lepton, 3 left-handed one in triplet **3**:

$$(-1)^2 + 3 \times \frac{(-1)^2}{3^2} = \frac{4}{3} \quad (20)$$

Hence, for three generation, we get a total contribution from leptons $\frac{4}{3} \times n_g$.

- (b) Contribution of the colour quarks N_c : three in triplet with $X = \frac{1}{3}$, and right-handed quarks (u, T) with $X = \frac{2}{3}$ and (d, D_α) with $X = -\frac{1}{3}$ in singlet **1**:

$$\begin{aligned} N_c \left[3 \times \frac{1}{3^2} + \frac{(-1)}{3^2} + 2 \times \frac{2^2}{3^2} + 2 \times 2 \times \frac{(-1)^2}{3^2} + 2 \times \frac{2^2}{3^2} \right] \\ = 8 \end{aligned} \quad (21)$$

Therefore contributions of leptons and quarks are given by

$$-\frac{2}{3} \left(\frac{4}{3} \times n_g + 8 \right) = -8 \quad (22)$$

- (c) For scalar fields: 2 triplets with $X = -\frac{1}{3}$, 4 triplets with $X = \frac{2}{3}$ and 4 sextets with $X = \frac{2}{3}$. Thus

$$2 \times 3 \times \frac{(-1)^2}{3^2} + 4 \times 3 \times \frac{2^2}{3^2} + 4 \times 6 \times \frac{2^2}{3^2} = \frac{50}{3} \quad (23)$$

Therefore contribution from scalar fields is

$$-\frac{1}{3} \times \frac{50}{3} = -\frac{50}{9} \quad (24)$$

The sum of (22) and (24) gives

$$b_X^{A4} = -\frac{122}{9} \quad (25)$$

Therefore a set of three beta functions in the model under consideration is

$$(b_C^{A4}, b_{3L}^{A4}, b_X^{A4}) = \left(5, \frac{8}{3}, -\frac{122}{9}\right) \quad (26)$$

IV. RGE IN THE 3-3-1 MODEL WITH S_4 FLAVOUR SYMMETRY

IV.1. Particle content

The fermions in this model under $[SU(3)_L, U(1)_X, U(1)_L, \underline{S}_4]$ symmetries [5] transform as

$$\psi_L \equiv \psi_{1,2,3L} \sim [3, -1/3, 2/3, \underline{\mathfrak{3}}], \quad (27)$$

$$l_{1R} \sim [1, -1, 1, \underline{\mathfrak{1}}], \quad l_R \equiv l_{2,3R} \sim [1, -1, 1, \underline{\mathfrak{2}}], \quad (28)$$

$$Q_{3L} \sim [3, 1/3, -1/3, \underline{\mathfrak{1}}], \quad Q_L \equiv Q_{1,2L} \sim [3^*, 0, 1/3, \underline{\mathfrak{2}}], \quad (29)$$

$$u_R \equiv u_{1,2,3R} \sim [1, 2/3, 0, \underline{\mathfrak{3}}], \quad d_R \equiv d_{1,2,3R} \sim [1, -1/3, 0, \underline{\mathfrak{3}}], \quad (30)$$

$$U_R \sim [1, 2/3, -1, \underline{\mathfrak{1}}], \quad D_R \equiv D_{1,2R} \sim [1, -1/3, 1, \underline{\mathfrak{2}}], \quad (31)$$

where the subscript numbers on field indicate to respective families which also in order define components of their S_4 multiplet. This is under $U(1)_L$ symmetry to prevent unwanted interactions in order to perform the tribimaximal form as shown below. U and $D_{1,2}$ are exotic quarks carrying lepton numbers $L(U) = -1$, $L(D_{1,2}) = 1$, thus called leptoquarks.

To generate masses for the charged leptons, we need two scalar multiplets:

$$\phi \sim [3, 2/3, -1/3, \underline{\mathfrak{3}}], \quad \phi' \sim [3, 2/3, -1/3, \underline{\mathfrak{3}}'], \quad (32)$$

with the vacuum expectation values (VEVs) $\langle \phi \rangle = (v, v, v)$ and $\langle \phi' \rangle = (v', v', v')$ written as those of S_4 components respectively (these will be derived from the potential minimization conditions). Here and after, the number subscripts on the component scalar fields are indices of $SU(3)_L$. The S_4 indices are discarded and should be understood.

The antisextets in this model transform as

$$\sigma = \begin{pmatrix} \sigma_{11}^0 & \sigma_{12}^+ & \sigma_{13}^0 \\ \sigma_{12}^+ & \sigma_{22}^{++} & \sigma_{23}^+ \\ \sigma_{13}^0 & \sigma_{23}^+ & \sigma_{33}^0 \end{pmatrix} \sim [6^*, 2/3, -4/3, \underline{\mathfrak{1}}], \quad (33)$$

$$s \sim [6^*, 2/3, -4/3, \underline{\mathfrak{3}}]. \quad (34)$$

The VEV of s is set as $(\langle s_1 \rangle, 0, 0)$ under S_4 (which is also a natural minimization condition for the scalar potential), where

$$\langle s_1 \rangle = \begin{pmatrix} \lambda_s & 0 & v_s \\ 0 & 0 & 0 \\ v_s & 0 & \Lambda_s \end{pmatrix}. \quad (35)$$

To generate masses for quarks, we additionally acquire the following scalar multiplets:

$$\chi = \sim [3, -1/3, 2/3, \underline{\mathfrak{1}}], \quad (36)$$

$$\eta = \sim [3, -1/3, -1/3, \underline{\mathfrak{3}}], \quad \eta' \sim [3, -1/3, -1/3, \underline{\mathfrak{3}}']. \quad (37)$$

IV.2. Calculation of b_i with S_4 group

With the above particle content, we have

- (1) For the group $SU(3)_C$: The same as in the above mentioned model, i.e.,

$$b_C^{S_4} = 5 \quad (38)$$

- (2) For the group $SU(3)_L$: With just one change - 13 scalar triplets, we get

$$b_{3L}^{S_4} = \frac{11}{3} \times 3 - \frac{2}{3} \times \frac{1}{2} \times (3+1) \times n_g - \frac{1}{6} \times n_T - \frac{5}{6} \times n_S = \frac{2}{3} \quad (39)$$

- (3) For the group $U(1)_X$: As before, we have

(a) Contribution of the leptons is $\frac{4}{3} \times n_g$.

(b) Contribution of the colour quarks is 8.

(c) For scalar fields: 7 triplets with $X = -\frac{1}{3}$, 6 triplets with $X = \frac{2}{3}$ and 4 sextets with $X = \frac{2}{3}$. Thus

$$7 \times 3 \times \frac{(-1)^2}{3^2} + 6 \times 3 \times \frac{2^2}{3^2} + 4 \times 6 \times \frac{2^2}{3^2} = 21 \quad (40)$$

Therefore contribution from scalar fields is

$$-\frac{1}{3} \times 21 = -7 \quad (41)$$

Thus, the coefficient in this case is:

$$b_X^{S_4} = -8 - 7 = -15. \quad (42)$$

Therefore a set of three beta functions in the model under consideration is

$$(b_C^{S_4}, b_{3L}^{S_4}, b_X^{S_4}) = \left(5, \frac{2}{3}, -15\right) \quad (43)$$

V. CONCLUSION

From (26) and (43) we see that the mass of unification M_U in the 3-3-1 model with A_4 symmetry is larger than those in the model with S_4 symmetry. Numerical study on these equations will be presented elsewhere.

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