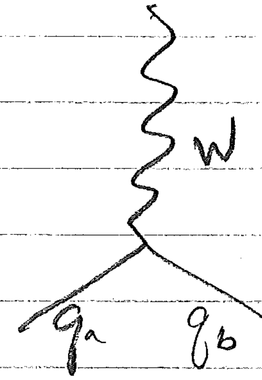
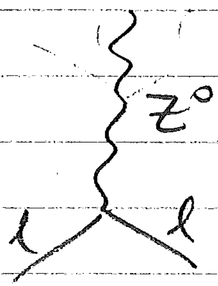
(3) charged hadronic vertex



$$\frac{-ig_W}{2\sqrt{2}}\gamma^{\mu}(1-\gamma^5)V^{ab}$$ ← CKM mixing element

(4) neutral leptonic vertex
↑
Z^0



$$\frac{-ig_Z}{2}\gamma^{\mu}(c_V^f - c_A^f\gamma^5)$$

(See Griffiths Table 9.1:)

f	c_V	c_A
ν	$\frac{1}{2}$	$\frac{1}{2}$
l^-	$-\frac{1}{2}+2\sin^2\theta_W$	$-\frac{1}{2}$
u,c,t	$\frac{1}{2}-\frac{4}{3}\sin^2\theta_W$	$\frac{1}{2}$
d,s,b	$-\frac{1}{2}+\frac{2}{3}\sin^2\theta_W$	$-\frac{1}{2}$

weak or Weinberg angle

$$g_W = \frac{g_e}{\sin\theta_W}$$

$$g_Z = \frac{g_e}{\sin\theta_W \cos\theta_W}$$

$$M_W = M_Z \cos\theta_W$$

$$\theta_W = 28.75^\circ \quad (\sin^2\theta_W = 0.2314)$$

Maybe the deepest truths about nature are NOT the physical laws, which have certain symmetries among their properties.

Maybe the deepest truths are the symmetries themselves, which, in order to be realized in our universe, generate the forms of the physical laws.

(We codify this assertion by trying to show how the relations among particles + interactions can be described by group theory.)

Noether's Theorem

For every continuous transformation under which a Lagrangian is invariant, there exists a conserved current.

$\searrow$

Every kind of transformation that leaves the laws of physics unchanged implies the existence of a conservation law.

$\swarrow$

Example to show that

spatial translation symmetry implies conservation of linear momentum:

intuitively We expect that space is homogeneous

$\downarrow$

The results of your experiment shouldn't depend on where you do it.

$\downarrow$

If something in your experiment has $\vec{p}$ in one location, + it experiences no force, then its $\vec{p}$ elsewhere should be identical.

now this mathematically

Recall Lagrange's Equation:

$$\frac{d}{dt}\left(\frac{d\mathcal{L}}{d\dot{x}}\right) - \frac{d\mathcal{L}}{dx} = 0$$

Consider a non-interacting (force-free) particle:

$$\mathcal{L} = \frac{p^2}{2m}$$

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Notice:

...hat we ...pect physically...

System is invariant w.r.t. a parameter (System = homogeneous space invariance = translation parameter = x	$\longleftrightarrow$	$\mathcal{L}$ does not contain the parameter

..., borne out ...mathematically...

$\frac{d\mathcal{L}}{dx} = 0$. Plug this into Lagrange's Eq:

$$\downarrow$$

$$\frac{d}{dt}\left(\frac{d\mathcal{L}}{d\dot{x}}\right) - \underbrace{\frac{d\mathcal{L}}{dx}}_{0} = 0$$

$$\downarrow$$

$$\frac{d}{dt}\left(\frac{d\mathcal{L}}{d\dot{x}}\right) = 0$$

$$\downarrow$$

$$\frac{d}{dt}(p) = 0$$

$$\downarrow$$

p = constant

Other examples:

<u>Symmetry</u>	<u>Conservation Law</u>
Rotation	Angular momentum
Translation in Time	Energy
<u>Gauge</u>	Charge

Focus on gauge invariance:

$\boxed{\text{Gauge}}$ just means $\boxed{\text{phase}}$. (We'll show why.)

1. Recall gauge invariance in classical E+M.
2. Recall the Hamiltonian for the system of a charged particle in an EM field.
3. Phase invariance in quantum mechanics.
4. The connection between gauges + phases
5. Maybe the requirement of local gauge invariance is a deep truth about nature that determines what the fundamental forces (strong, electroweak, gravity) can be like....

$\boxed{1.}$ Recall gauge invariance in classical E+M:

Combine:

Maxwell's Eq. $\vec{\nabla}\cdot\vec{B} = 0$

+

Vector calculus $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0$ (any $\vec{A}$)

$\longrightarrow \vec{B} = \vec{\nabla}\times\vec{A}$

$\vec{B} = \vec{\nabla}\times\vec{A}$

+

Vector calculus $\vec{\nabla}\times(\vec{\nabla}\Lambda) = 0$ (any Λ)

$\longrightarrow$ For a $\vec{B}$ and $\vec{A}_0$ if $\vec{B} = \vec{\nabla}\times\vec{A}_0$, then $\vec{B} = \vec{\nabla}\times(\vec{A}_0 + \vec{\nabla}\Lambda)$ <u>also</u> works, for <u>any</u> Λ.

So we can pick any Λ as long as it satisfies the rest of Maxwell's Equations. Consider those...

⋮

Combine:

$$\left.\begin{array}{l} \text{Maxwell's Eq. } \vec{\nabla}\times\vec{E} = -\dfrac{d\vec{B}}{dt} \\ \qquad + \\ \vec{B} = \vec{\nabla}\times\vec{A} \end{array}\right\} \rightarrow \begin{array}{l} \vec{\nabla}\times\vec{E} = -\dfrac{d}{dt}(\vec{\nabla}\times\vec{A}), \text{ or} \\ \\ \vec{\nabla}\times\left(\vec{E}+\dfrac{d\vec{A}}{dt}\right) = 0 \end{array}$$

$$\left.\begin{array}{l} \nabla\times\left(E+\dfrac{d\vec{A}}{dt}\right) = 0 \\ \qquad + \\ \text{Vector calculus } \vec{\nabla}\times\vec{\nabla}\Lambda = 0 \text{ (any } \Lambda) \end{array}\right\} \rightarrow \vec{E} + \frac{d\vec{A}}{dt} = -\vec{\nabla}\phi$$

call this $\Lambda = -\phi$

$$\left.\begin{array}{l} \vec{E} + \dfrac{d\vec{A}}{dt} = -\vec{\nabla}\phi \\ \qquad + \\ \vec{A} \rightarrow \vec{A}^{+} \equiv \vec{A} + \vec{\nabla}\Lambda \end{array}\right\} \rightarrow \begin{array}{l} \vec{E} + \dfrac{d}{dt}\left(\vec{A}^{+} - \vec{\nabla}\Lambda\right) = -\vec{\nabla}\phi, \text{ or} \\ \\ \vec{E} + \dfrac{d\vec{A}^{+}}{dt} = -\vec{\nabla}\underbrace{\left(\phi - \dfrac{d\Lambda}{dt}\right)}_{\text{call this } \phi^{+}} \end{array}$$

Conclusion: Maxwell's Laws are unchanged by the simultaneous transformations

$$\left\{\begin{array}{l} \vec{A} \rightarrow \vec{A} + \vec{\nabla}\Lambda \\ \phi \rightarrow \phi - \dfrac{d\Lambda}{dt} \end{array}\right\} \text{ for any scalar } \Lambda.$$

*This combined choice of A and ϕ is the choice of gauge.

2. Recall that

the Hamiltonian for a charged particle in an EM field is

$$\boxed{H = \frac{1}{2m}\left(\vec{p} - e\vec{A}\right)^2 - e\varphi}$$

Compare that to:

The Hamiltonian for a free particle:

$$\boxed{H = \frac{p^2}{2m}}$$

So adding the EM field $(\varphi, \vec{A})$ is the same as

- offsetting the particle's energy by $e\varphi$

AND

- changing its momentum $\vec{p} \to \vec{p} - e\vec{A}$.

$\boxed{3.}$ Phase invariance in Quantum Mechanics:

Recall: any QM prediction about a physical process requires a $\psi^*\psi$ combination (e.g., expectation values, probability densities).

$\Downarrow$

So the transformation $\psi \to \psi \cdot e^{i\theta}$ has no effect on predictions.

If θ is NOT a function of coordinates, we say

"ψ has global phase invariance."

Consider the implications if the phase θ DOES depend on coordinates...

So $\theta = \theta(x)$ "local phase"

... the phase ~~of~~ ψ can be different at every point in space.

It may seem unlikely that any physical law could be invariant with respect to local phase...

Surprisingly, if we require that QM be invariant with respect to local phase, we find that

the Schrödinger/ Dirac Equations, with their d^2/dx^2

The electromagnetic force is required to exist!

Furthermore,

Global phase invariance causes charge conservation.

-AND-

Local phase invariance forces the photon to be massless.

$\boxed{\text{Show this:}}$

Suppose Nature insists that QM be invariant with respect to local phase of ψ.

Start with Dirac Eq $(i\gamma^\mu \partial_\mu - m)\psi = 0$

What we mean by "invariance of the Dirac Eq." is preservation of the equation's form.

So suppose that Nature insists:

"if $(i\gamma^\mu \partial_\mu - m)\psi(x) = 0$,

then $(i\gamma^\mu \partial_\mu - m)\, e^{i\theta(x)}\psi(x) = 0$ must be true too."

Can $(i\gamma^\mu \partial_\mu - m) e^{i\theta(x)} \psi(x) = 0$ be true as it stands?

Consider the term $\partial_\mu e^{i\theta(x)} \psi(x)$:

By the chain rule of calculus, this is

$$\partial_\mu e^{i\theta(x)} \psi(x) = e^{i\theta(x)} \left[\partial_\mu \psi(x) + i(\partial_\mu \theta(x)) \psi(x) \right]$$

which will NOT reproduce the form of the Dirac Eq.

So $\psi \to e^{i\theta(x)} \psi$ alone does not work.

Nature has to try a little harder to make this local phase invariance thing work...

Try the combination:

$$\left\{ \begin{array}{l} \psi \to e^{i\theta(x)} \psi \\ \partial_\mu \to \partial_\mu - ieA_\mu \\ A_\mu \to A_\mu + \frac{1}{e} \partial_\mu \theta(x) \end{array} \right\}$$

A_μ is for now an arbitrary 4-dimensional vector

Now try

$$(i\gamma^\mu \partial_\mu - m)\psi(x) \Longrightarrow \underbrace{\left[i\gamma^\mu\left\{\partial_\mu - ie\left(A_\mu + \frac{1}{e}\partial_\mu\theta(x)\right)\right\} - m\right] e^{i\theta(x)}\psi(x)}$$

$\psi(x) \to e^{i\theta(x)}\psi(x)$

$\partial_\mu \to \partial_\mu - ieA_\mu$

$A_\mu \to A_\mu + \frac{1}{e}\partial_\mu\theta(x)$

$$e^{i\theta(x)}\left[i\gamma^\mu\underbrace{(\partial_\mu - ieA_\mu)} - m\right]\psi(x)$$

If we call this "D_μ", the "gauge-covariant derivative," then the form of the Dirac Equation is preserved as

$$e^{i\theta(x)}\left[i\gamma^\mu D_\mu - m\right]\psi(x) = 0$$

So it takes simultaneous changes of ψ and ∂_μ to make local phase invariance work.

We're doing gauge theory! Is it worth it? Why bother?...

Why bother...

Imposing the requirement of local phase invariance upon QM (the Dirac Eq.) <u>mandates</u> that

$$\partial_\mu \rightarrow D_\mu = \partial_\mu - ieA_\mu$$

What is this A_μ?

Recall QM identifies the derivative operator with momentum:

$$P_{op} = -i\vec{\nabla} \qquad (\hbar = 1)$$

So relativistically,

$$p_\mu = -i\partial_\mu$$

(E, p_x, p_y, p_z)

So another way to write $\partial_\mu \rightarrow D_\mu \equiv \partial_\mu - ieA_\mu$ is

$$ip_\mu \longrightarrow ip_\mu - ieA_\mu$$

$$\downarrow$$

$$p_\mu \longrightarrow p_\mu - eA_\mu$$

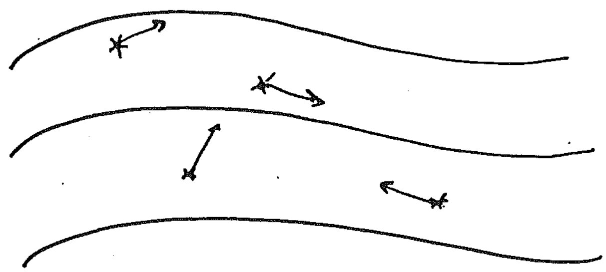
But this is exactly the way the momentum operator transforms in the presence of an EM field $(\phi, \vec{A})$!

So the A_μ in $D_\mu \equiv \partial_\mu - ieA_\mu$ is <u>not</u> arbitrary, but is the usual pair of potentials $(\phi, \vec{A})$ of the EM field.

Requiring local gauge symmetry calls into existence the electromagnetic interaction, A_μ

What this means physically:

Consider a group of electric charges moving in an electric field:



Global phase change $\Leftrightarrow$ uniform change in ϕ everywhere

does not affect charges' motions

Local phase change $\Leftrightarrow$ $\Delta\phi$ depends on x.

$\vec{E}$ by itself is not invariant to this

but the charges generate $\vec{B}$'s (with associated $\vec{A}$'s) that exactly compensate the $\Delta\phi$'s.

so the <u>total</u> field is invariant.

Because the requirement of local gauge symmetry makes the force (e.g., electromagnetism) exist, we call the propagators of the force (e.g., photons) gauge bosons.

$\boxed{4.}$ The connection between gauges and phases.

Begin with Schrödinger Equation:

$$H\psi = i\hbar \frac{d\psi}{dt}$$

Plug in the EM Hamiltonian $H = \frac{1}{2m}\left(-i\hbar\nabla + eA\right)^2 - e\phi$:

$$\left[\frac{1}{2m}\left(-i\hbar\nabla + eA\right)^2 - e\phi\right]\psi = i\hbar\frac{d\psi}{dt}$$

Now do either of 2 things:

Make the gauge transform:

$$\begin{Bmatrix} A \to A + \nabla\Lambda \\ \phi \to \phi - \frac{d\Lambda}{dt} \end{Bmatrix}$$

Make the phase transform:

$$\psi \to e^{+ie\Lambda/\hbar}\psi$$

Plug either into here.

They produce identical equations.

So "gauge = phase"