Introduction to Particle Physics
Homework 4

1. (a) Show that $\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = I$ (where the σ_i are the Pauli matrices and I is the 2x2 identity matrix.

(b) Show that $\sigma_i \sigma_j = \delta_{ij} + i\varepsilon_{ijk}\sigma_k$ (where δ_{ij} is the Kronecker delta and ε_{ijk} is the Levi-Civita symbol).

(c) Use the above information to show that $[\sigma_i, \sigma_j] = 2i\varepsilon_{ijk}\sigma_k$.

(d) Also show the anticommutation relation, $\{\sigma_i, \sigma_j\} = 2\delta_{ij}$.

(e) Show that for any two vectors $\vec{a}$ and $\vec{b}$, $(\vec{\sigma} \cdot \vec{a})(\vec{\sigma} \cdot \vec{b}) = \vec{a} \cdot \vec{b} + i\vec{\sigma} \cdot (\vec{a} \times \vec{b})$.

(f) Apply the result in (e) to the special case where $\vec{a} = \vec{b} = \vec{p}$, momentum.

2. Define the helicity operator $H = \dfrac{\vec{\sigma} \cdot \vec{p}}{|\vec{p}|}$. Apply this operator to u_A, u_B, v_A, and v_B. and explicitly show the helicity eigenvalues that result. You may use information derived in Problem (1) above.

3. The Dirac Equation was derived in class for the case of a free particle. Consider the case in which an electromagnetic potential A_μ is present. Modify the Dirac Equation by replacing $-i\hbar\partial_\mu$ with $-i\hbar\partial_\mu - eA_\mu / c$, then show that the Continuity Equation still holds.

4. Griffiths 7.48a.