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3-3-1-1 model for dark matter

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We show that the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (3-3-1) model of strong and electroweak interactions can naturally accommodate an extra $U(1)_N$ symmetry behaving as a gauge symmetry. The resulting theory based on $SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes U(1)_N$ (3-3-1-1) gauge symmetry realizes $B - L = -(2/\sqrt{3})T_8 + N$ as a charge of $SU(3)_L \otimes U(1)_N$. Consequently, a residual symmetry, W parity, resulting from broken $B - L$, similarly to R-parity in supersymmetry, is always conserved and may be unbroken. There is a specific fermion content recently studied in which all new particles that have wrong lepton numbers are odd under W parity, while the standard model particles are even. Therefore, the lightest wrong-lepton particle (LWP) responsible for dark matter is naturally stabilized. We explicitly show that the non-Hermitian neutral gauge boson (X^0) as the LWP cannot be dark matter. However, the LWP as a new neutral fermion (N_R) can be dark matter if its mass is in the range 1.9 TeV $\leq m_{N_R} \leq$ 2.5 TeV, provided that the new neutral gauge boson (Z') mass satisfies 2.2 TeV $\leq m_{Z'} \leq$ 2.5 TeV. Moreover, the scalar dark matter candidate ($H' \simeq \eta_3$) that has traditionally been studied is only stabilized by W parity. All the unwanted interactions and vacuums that are often encountered in the 3-3-1 model are naturally suppressed. And, the standing issues on tree-level flavor changing neutral currents and CPT violation also disappear.

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I. INTRODUCTION

The obvious experimental evidence that proves we must go beyond the standard model of fundamental particles and interactions comes from neutrino oscillations [1], which means that the neutrinos have hierarchically small masses and mixing. Among the extensions known, the seesaw mechanism [2] is perhaps the most natural one for explaining the above problem with the introduction of heavy right-handed neutrinos (ν_R) or some kind of new neutral fermion (N_R). However, while these assumed particles have not been observed, it is useful to ask what their natural origin is. They may arise as fundamental constituents in left-right models [3] or $SO(10)$ unification [4]. The presence of these particles might also lead to interesting consequences such as baryon asymmetry via leptogenesis [5]. In this work, we will show that they can also exist in a gauge model which implies to a class of new particles. These particles are odd under a parity symmetry responsible for dark matter.

The approach is based on $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ gauge symmetry (thus named 3-3-1) in which the last two groups are extended from the electroweak symmetry of the standard model, while the QCD symmetry is retained. The right-handed neutrinos or new neutral fermions may be used in fundamental lepton triplets or antitriplets of $SU(3)_L$ to complete the representations,

$$\begin{pmatrix} \nu_L \\ e_L \\ \nu_R^c \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} \nu_L \\ e_L \\ N_R^c \end{pmatrix},$$

the so-called 3-3-1 model with right-handed neutrinos [6] or the 3-3-1 model with neutral fermions [7], respectively (see also [8] for a variant). In addition, this approach has intriguing features. The number of fermion families must be an integral multiple of fundamental color number, which is 3, in order to cancel the $SU(3)_L$ anomaly [9]. There are nine flavors of quarks due to the enlarged electroweak gauge symmetry, so the family number must also be smaller than or equal to 5 to ensure QCD asymptotic freedom. The result is an exact family number of 3, coinciding with the observation [1]. Since the third family of quarks transforms under $SU(3)_L$ differently from the first two, this can explain why the top quark is so heavy [10]. The extension can also provide some insight into the electric charge quantization observed in nature [11].

The extended sectors from the standard model in the 3-3-1 models, such as scalar, fermion and gauge, might by themselves provide dark matter candidates. This is strongly proven by the gauge interactions, the minimal Yukawa Lagrangian, and the minimal scalar potential that normally put the new particles (similar to the so-called wrong-lepton particles as defined in the following section) in pairs, similarly to the case of superparticles in supersymmetry.

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This automatically results from the specific structure of 3-3-1 gauge symmetry by itself [6,8], which is unlike the conclusion in [12,13]. The first attempts to identify the dark matter candidates of 3-3-1 models can be found in [14,15]; however, a strict treatment on their stability issue and relic abundance has not been given. The stabilization of dark matter in the 3-3-1 models due to extra symmetries was first discussed in [12,13]. To this aim, the lepton number symmetry has been imposed in [12] so that the lightest bilepton particle could be stabilized. It is noteworthy that all the unwanted interactions of the Yukawa Lagrangian and scalar potential (other than the minimal interactions mentioned) explicitly violate the lepton number [16], which is naturally suppressed due to this symmetry (except the coupling of two lepton and one scalar triplets that violates only flavor lepton numbers, but leading to an unrealistic neutrino mass spectrum; however, in our model discussed below this coupling explicitly disappears due to the total lepton number violation). The Z_2 as introduced therein is, in fact, not needed. An alternative problem encountered is that the lepton number should also be violated due to five-dimensional effective interactions responsible for the neutrino masses, as cited therein.

In [13], the bilepton character of new particles has been arranged to disappear, and the lepton number symmetry no longer prevents those unwanted interactions from turning on. So, a Z_2 symmetry has been included by hand with appropriate Z_2 representation assignments in order to eliminate those unwanted interactions, and this symmetry has been regarded as the one for stabilizing dark matter [13]. However, since the Z_2 that acts on the model multiplets must be spontaneously broken by the Higgs vacuum, there is no reason why the scalar dark matter that carries no lepton number cannot develop a vacuum expectation value (VEV) and then decay. Also in [13], a continuous symmetry called $U(1)_G$ that acts on component particles, not commuting with the gauge symmetry like the lepton charge before, has been introduced to be used instead of the Z_2 in interpreting dark matter, in part because of the gauge interactions. Let us remind the reader that these interactions of gauge bosons with fermions, scalars or gauge self-interactions are automatic consequences of and already restricted by the gauge symmetry itself, as already mentioned. They are always present, not excluded or added by other interactions, whether the $U(1)_G$ or even lepton charge is imposed or not. For the purpose of suppressing those unwanted interactions and vacuums, obviously there are many other symmetries behaving like $U(1)_G$ or lepton charge found as respective solutions of the gauge interactions' conservation. However, all these continuous symmetries can face problems.

The continuous symmetries above are assumed to be exact symmetries responsible for the dark matter stability. Therefore, they can naturally be regarded as respective residual symmetries of higher symmetries that span the 3-3-1 group (since they do not commute with the gauge symmetry, as mentioned) acting at the Lagrangian level, under which the unwanted interactions are explicitly suppressed. In other words, the minimal Lagrangian of the theory actually contains larger symmetries spanning the gauge symmetry that shall be spontaneously broken down to those residual symmetries, respectively. As a specific property of the 3-3-1 models, the lepton charge [or even any kind of $U(1)_G$ if included independently, neglecting the lepton charge symmetry] should work as the residual gauge symmetry of some higher symmetry (as shown in the next section) and must be spontaneously broken so that the resulting gauge boson gets a large enough mass to make the theory consistent. On the other hand, this lepton charge symmetry or even a general continuous symmetry is also known to actually be violated due to its anomalies. Therefore, such symmetries [lepton number or $U(1)_G$] would no longer be able to protect the dark matter stability from decays. For the stability issue of dark matter, similarly to the R parity in supersymmetry, it is more natural to search for an exact and unbroken residual discrete symmetry of some anomaly-free continuous symmetry spanning either the lepton number and other necessary symmetries, such as baryon charge, or $U(1)_G$. Let us remark that, among the continuous symmetries analyzed, the one concerning lepton charge is perhaps the most motivated and natural because of the following: (i) all the unwanted interactions in ordinary 3-3-1 models that should be prevented in fact violate the lepton numbers [16]; (ii) while the discrete symmetry is responsible for dark matter stability, the lepton or baryon numbers could be broken in several ways, necessarily to account for the observed neutrino masses and baryon-number asymmetry. We will see that this is similar to enlarging the $SU(5)$ theory to $SO(10)$, in which the $B-L$ charge is naturally gauged. In this work the lepton charge will be taken into account, which is different from the $U(1)_G$ symmetry *ad hoc* input.

By investigating nontrivial lepton number behavior and a resulting W parity (similar to the R parity in supersymmetry) in a specific 3-3-1 model [7], we show that the theory can contain natural dark matter candidates. For details, we consider the 3-3-1 model with neutral fermions (N_R), which is different from the model of [13]. These neutral fermions possess no lepton number, as already studied before in a TeV seesaw extension of the standard model [17] and in the 3-3-1 model with flavor symmetries [7]. We investigate lepton number symmetry, its dynamics and other symmetries which result in a new 3-3-1-1 gauge model. We show that there is an unbroken residual symmetry of such (anomaly-free) 3-3-1-1 theory behaving like the R parity in supersymmetry under which most of the new particles given are odd. It is interesting that the model can contain several kinds of dark matter, such as singlet scalar, fermion and gauge boson, as often presented in other extensions of the standard model and similar to the conclusion in [13]. However, these candidates may be as heavy as some TeV, which is unlike light candidates in the familiar standard model extensions. Before [13] and our work, the previous considerations of the 3-3-1 model recognized only the scalar singlet [15] and lightest supersymmetric particles of respective supersymmetric 3-3-1 versions. The reason behind this may be that the mentioned symmetries such as 3-3-1-1 and W parity, under which dark matter is dynamically stabilized, have not been explored yet. The dark matter phenomenologies in our models will be different from the other extensions. The model can work better under the experimental constraints than the ordinary 3-3-1 model with right-handed neutrinos due to the W parity mentioned. The above procedure will fail when applied to the other 3-3-1 models, such as the model of [13], the 3-3-1 model with right-handed neutrinos [6] and the minimal 3-3-1 model [8]. In fact, all the particles, including the new ones in those models, would transform trivially under the W parity. Therefore, this parity may only be realized in the class of 3-3-1 models with flavor symmetries [7].

The rest of this article is organized as follows. In Sec. II we give a review of the 3-3-1 model with neutral fermions by stressing baryon and lepton numbers as well as proposing wrong-lepton particles. We next construct the 3-3-1-1 gauge symmetry and W parity and show possible dark matter candidates and direct consequences. We also identify physical scalar particles and give a discussion of all the masses. In Sec. III we provide detailed calculations of relic densities of possible dark matter and show constraints. Finally, we summarize our results and give an outlook in the last section—Sec. IV.

II. PROPOSAL OF 3-3-1-1 MODEL

A. The 3-3-1 model with neutral fermions and wrong-lepton particles

The gauge symmetry of the 3-3-1 model under consideration is given by $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$, where the first factor is the usual QCD symmetry while the last two $SU(3)_L \otimes U(1)_X$ are extended from the electroweak symmetry of the standard model. The electric charge operator is the only unbroken residual charge of the gauge symmetry, and it is defined by $Q = T_3 - (1/\sqrt{3})T_8 + X$, where T_i ($i = 1, 2, 3, \dots, 8$) are the $SU(3)_L$ charges while X is that of $U(1)_X$. The weak hypercharge of the standard model is therefore identified as $Y = -(1/\sqrt{3})T_8 + X$.

The fermion content which is anomaly-free is assigned by

$$\psi_{aL} = \begin{pmatrix} \nu_{aL} \\ e_{aL} \\ (N_{aR})^c \end{pmatrix} \sim (1, 3, -1/3), \quad e_{aR} \sim (1, 1, -1), \tag{1}$$

$$Q_{\alpha L} = \begin{pmatrix} d_{\alpha L} \\ -u_{\alpha L} \\ D_{\alpha L} \end{pmatrix} \sim (3, 3^*, 0), \quad Q_{3L} = \begin{pmatrix} u_{3L} \\ d_{3L} \\ U_L \end{pmatrix} \sim (3, 3, 1/3), \tag{2}$$

$$u_{aR} \sim (3, 1, 2/3), \qquad d_{aR} \sim (3, 1, -1/3), \tag{3}$$

$$U_R \sim (3, 1, 2/3), \qquad D_{\alpha R} \sim (3, 1, -1/3), \tag{4}$$

where $a = 1, 2, 3$ and $\alpha = 1, 2$ are family indices. The values defined in parentheses are quantum numbers based on $(SU(3)_C, SU(3)_L, U(1)_X)$ gauge symmetries, respectively. The N_{aR} and U, D_α are the new neutral fermions (which are singlets under the standard model symmetry such as the right-handed neutrinos often considered) and exotic quarks, respectively. The electric charges of exotic quarks $Q(U) = 2/3$ and $Q(D_\alpha) = -1/3$ are the same as ordinary quarks. As mentioned, the lepton number of N_{aR} will be taken to be zero: $L(N_{aR}) = 0$. This is due to the fact that the conventional seesaw mechanism with right-handed neutrinos, including that of the 3-3-1 model, can require a very high seesaw scale of $M_R \sim 10^{10}$–10^{14} GeV [2,18]. As already shown in [7,17], if we suppose such an N_R, one can have a natural TeV seesaw scale matching the 3-3-1 breaking scale. And, the lepton mixing matrix under flavor symmetries can be naturally explained in those models [7,19]. The presence of N_R also implies a kind of new particle that is odd under a parity symmetry, which is responsible for dark matter candidates as shown below.

To break the gauge symmetry and generate the masses, this kind of 3-3-1 model actually requires three scalar triplets [6],

$$\rho = \begin{pmatrix} \rho_1^+ \\ \rho_2^0 \\ \rho_3^+ \end{pmatrix} \sim (1, 3, 2/3), \tag{5}$$

$$\eta = \begin{pmatrix} \eta_1^0 \\ \eta_2^- \\ \eta_3^0 \end{pmatrix} \sim (1, 3, -1/3), \tag{6}$$

$$\chi = \begin{pmatrix} \chi_1^0 \\ \chi_2^- \\ \chi_3^0 \end{pmatrix} \sim (1, 3, -1/3). \tag{7}$$

Here, the gauge symmetry is broken in two stages: The first stage $SU(3)_L \otimes U(1)_X$ is broken down to that of the standard model generating the masses of new particles such as exotic quarks U, D_α, as well as the new gauge bosons—one neutral Z' coupled to the generator that is orthogonal to the weak hypercharge and two charged $X^{0/0*}$, $Y^{\mp}$

corresponding to $T_4 \pm iT_5$ and $T_6 \pm iT_7$ raising or lowering operators. In the second stage, the standard model electroweak symmetry is broken down to $U(1)_Q$, which is responsible for the masses of ordinary particles such as $W^{\pm}$, Z, e_a, u_a, and d_a.

The lepton number (L) of lepton triplet components is given by $(+1, +1, 0)$, which does not commute with the $SU(3)_L$ gauge symmetry unlike the standard model case:

$$[L, T_4 \pm iT_5] = \pm(T_4 \pm iT_5) \neq 0,\quad [L, T_6 \pm iT_7] = \pm(T_6 \pm iT_7) \neq 0, \tag{8}$$

which means that X and Y bosons carry lepton numbers with one unit. This also happens for the 3-3-1 model with right-handed neutrinos and the minimal 3-3-1 model. It is a characteristic property of this kind of model. Hence, if the lepton number is a symmetry of the theory, it can be regarded as a residual charge of conserved symmetries,

$$\mathcal{G} \equiv SU(3)_L \otimes U(1)_{\mathcal{L}}, \tag{9}$$

where the second factor is a new symmetry that is supposed because the lepton number and the gauge symmetry generators do not form a closed algebra. [This is because, in order for L to be some generator of $SU(3)_L$, the L charges of a complete multiplet must add up to zero, which is not correct. Also, the $\mathcal{L}$ and X charges must be distinct because, for the $SU(3)_L$ fermion singlets, the lepton number and electrical charge generally do not coincide.] The lepton number that is a combination of $SU(3)_L \otimes U(1)_{\mathcal{L}}$ diagonal generators (due to the conservation of lepton number as strictly required by the experiments, similar to the case of the electric charge operator) can be easily obtained by using a lepton triplet given by

$$L = \frac{2}{\sqrt{3}} T_8 + \mathcal{L}, \tag{10}$$

where T_8 is the charge of $SU(3)_L$ while $\mathcal{L}$ is that of $U(1)_{\mathcal{L}}$ [16]. On the grounds of known lepton numbers, the $\mathcal{L}$ charges of all the model multiplets can be easily obtained, as given in Table I (notice that the lepton numbers of χ_3^0, ρ_2^0 and η_1^0 must be zero since these fields are normally required to develop VEVs that are responsible for the gauge symmetry breaking and mass generation). Moreover, we point out that all the ordinary interactions of the theory (i.e., the minimal interactions as mentioned) conserve $\mathcal{L}$ [16]. For convenience, we also give the lepton numbers of model particles L in Table II. We see that the standard model particles have lepton numbers, as usual. However, most of the new particles, such as N_R, U, D, X, Y, ρ_3, η_3, and $\chi_{1,2}$, possess abnormal lepton numbers, in comparison to those of the standard model. For example, $L(U, D) = 0$ like ordinary quarks instead of ± 1. This kind of particle is called a "wrong-lepton particle" or sometimes "W particle" for short.

In this work, we suppose that the $\mathcal{G}$ symmetry, and thus $U(1)_{\mathcal{L}}$ which is responsible for the lepton number, is an exact symmetry. However, since the scalar triplets as given are all charged under $\mathcal{G}$ and will get VEVs, the $\mathcal{G}$ symmetry must be broken spontaneously (in accordance with the gauge symmetry breaking). It is also easily shown that the scalar potential can be stabilized by the following solution of the potential minimization conditions:

$$\langle \chi_1^0 \rangle = \langle \eta_3^0 \rangle = 0, \tag{11}$$

$$\langle \rho_2^0 \rangle \neq 0, \qquad \langle \eta_1^0 \rangle \neq 0, \qquad \langle \chi_3^0 \rangle \neq 0. \tag{12}$$

In fact, this solution of the potential minimization has been formally used in the literature as a standard vacuum structure [6]. Also, with the VEVs given in (11) and (12), i.e.,

$$\langle \rho \rangle = \frac{1}{\sqrt{2}}(0, v, 0)^T, \qquad \langle \eta \rangle = \frac{1}{\sqrt{2}}(u, 0, 0)^T, \qquad \langle \chi \rangle = \frac{1}{\sqrt{2}}(0, 0, \omega)^T, \tag{13}$$

all the particles in this model (except for ν_L and N_R) will get consistent masses at tree level, similarly to those of the ordinary 3-3-1 model with right-handed neutrinos [6]. Let us note that ω is responsible for the first stage of electroweak symmetry breaking $SU(3)_L \otimes U(1)_X \to SU(2)_L \otimes U(1)_Y$ providing the masses for new particles, whereas u, v act on the second stage $SU(2)_L \otimes U(1)_Y \to U(1)_Q$ giving the masses for ordinary particles. For consistency with the standard model, we should suppose

$$u^2, v^2 \ll \omega^2. \tag{14}$$

For the $\mathcal{G}$ symmetry, although both the $\mathcal{L}$ and T_8 (and all other generators) are broken, the combination of lepton number L in this case is conserved by the VEVs, which can

TABLE I. The $\mathcal{L}$ charge of model multiplets.

Multiplet	ψ_{aL}	e_{aR}	Q_{3L}	$Q_{\alpha L}$	u_{aR}	d_{aR}	U_R	$D_{\alpha R}$	ρ	η	χ
$\mathcal{L}$	$\frac{2}{3}$	1	$-\frac{1}{3}$	$\frac{1}{3}$	0	0	-1	1	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$

TABLE II. The lepton number of model particles.

Particle	ν_{aL}	e_a	N_{aR}	u_a	d_a	U	D_α	ρ_1^+	ρ_2^0	ρ_3^+	η_1^0	η_2^-	η_3^0	χ_1^0	χ_2^-	χ_3^0	γ	W	Z	Z'	X^0	Y^-
L	1	1	0	0	0	-1	1	0	0	-1	0	0	-1	1	1	0	0	0	0	0	1	1

be verified directly from Table II. The breakdown of $\mathcal{G}$ symmetry into the lepton number,

$$\mathcal{G} = SU(3)_L \otimes U(1)_{\mathcal{L}} \to U(1)_L, \quad (15)$$

implies the existence of eight Goldstone bosons contained in the scalar sector ρ, η, χ. However, these Goldstone bosons are just those associated with the gauge symmetry breaking $SU(3)_L \otimes U(1)_X \to U(1)_Q$ that will be subsequently gauged away (they are unphysical because they are already the Goldstone bosons of the gauge symmetries, as stated).

Moreover, with a similar ingredient we may find that the baryon number (B) does not commute with gauge symmetry and that it results from some broken exact symmetries,

$$\mathcal{G}' \equiv SU(3)_L \otimes U(1)_{\mathcal{B}} \to U(1)_B, \quad (16)$$

because the baryon numbers of U, D_α are unknown and, in principle, could be arbitrary (in this case the unwanted interactions also violate the baryon number and are suppressed due to the $\mathcal{B}$ charge conservation). The χ_1^0 and η_3^0 will carry the baryon number in this case, which can only be conserved by the above potential minimization condition. For simplicity, in this work let us take $B(U) = B(D_\alpha) = 1/3 = B(u_a) = B(d_a)$, which vanishes for other fields, as in the literature for this kind of model [16], so that

$$B = \mathcal{B} \quad (17)$$

commutes with the gauge symmetry and is always conserved at the renormalizable level, as in the standard model (i.e., the baryon number of our general theory is always an exact and unbroken symmetry since the unwanted interactions conserve B while the χ_1^0 and η_3^0 are neutral under this charge). The $\mathcal{B}$ charges of model multiplets are given in Table III. [Let us remark on alternative cases: (i) If $\langle\chi_1^0\rangle \neq 0$ or $\langle\eta_3^0\rangle \neq 0$, the L would be broken too, along with T_8 and $\mathcal{L}$. (ii) The conservation of L in this model is not an automatic consequence of the theory like in the standard model. This is because if the $U(1)_{\mathcal{L}}$ symmetry is not imposed there would be unwanted interactions explicitly violating L, as actually seen in the Yukawa sector and/or scalar sector [16,18]. (iii) If the baryon numbers of U, D_α were alternatively chosen, points (i) and (ii) would also apply for the baryon number.]

The above ingredients of lepton and baryon numbers have been presented only for the 3-3-1 model with neutral fermions. In general, they can also be applied to the minimal 3-3-1 model [8] and 3-3-1 model with right-handed neutrinos [6]. Here the crucial discrimination is that in these models wrong-lepton particles differ from the ordinary ones by 2 units in lepton charge and have also been called bilepton particles, whereas in the present model they differ by only 1 unit due to the possible nature of the neutral fermions N_R. However, they are completely distinguished when applied to the dark matter problems, as shown below.

B. 3-3-1-1 gauge symmetry and *W* parity

Let us recall that the lepton numbers L and $\mathcal{L}$ which satisfy (10) were primarily introduced in other 3-3-1 models [16]; however, their dynamical nature has been completely unrealized and not yet examined. In the second article of [7], we have given the first notes on this lepton dynamics. And, in the current work it is to be analyzed in more detail. Since T_8 is a gauged charge of the $SU(3)_L$ symmetry, the L, and thus the $\mathcal{L}$, and vice versa, must be subsequently gauged. This is because of a contrasting assumption that since both the L and $\mathcal{L}$ are global generators, the $T_8 \sim L - \mathcal{L}$ is also global, which is incorrect. In this case, the anomalies coupled to L, and thus to $\mathcal{L}$, are obviously unable to be suppressed, which spoils the model's consistency. To regard the lepton number as such a local symmetry for this kind of model (leptonic dynamics can be considered), we must deal with the leptonic anomaly cancellation issue. All of this also applies for the baryon number if it satisfies point (iii); however, it does not happen by our choice.

A solution to canceling the leptonic anomalies is that we can consider the model with gauged symmetry $N \equiv \mathcal{B} - \mathcal{L}$ (where $\mathcal{B}$ is the baryon number as given above) as well as introduce three new right-handed neutrinos transforming as singlets under the 3-3-1 symmetry,

$$\nu_{aR} \sim (1, 1, 0). \quad (18)$$

Here, these particles that have lepton and baryon numbers as usual, $\mathcal{L}(\nu_{aR}) = L(\nu_{aR}) = 1$ and $\mathcal{B}(\nu_{aR}) = B(\nu_{aR}) = 0$, are necessarily included in order to cancel the gravitational anomaly $[\text{Gravity}]^2 U(1)_N$ (since the anomaly of ν_L and N_R is not canceled out). It is explicitly checked that the resulting theory is free from all the anomalies, as presented in Appendix A. Hence, the new theory, as an important investigation of this article, is obtained by the following gauge symmetry:

$$SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes U(1)_N, \quad (19)$$

the so-called 3-3-1-1 model. And, the multiplets of the 3-3-1-1 model, as well as their N charges, can be easily counted and are given in Table IV. Here, the complex scalar 3-3-1 singlet,

TABLE III. The $\mathcal{B}$ charge of model multiplets.

Multiplet	ψ_{aL}	e_{aR}	Q_{3L}	$Q_{\alpha L}$	u_{aR}	d_{aR}	U_R	$D_{\alpha R}$	ρ	η	χ
$\mathcal{B}$	0	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0	0	0

TABLE IV. The 3-3-1-1 model multiplets and respective N charges.

Multiplet	ψ_{aL}	e_{aR}	ν_{aR}	Q_{3L}	$Q_{\alpha L}$	u_{aR}	d_{aR}	U_R	$D_{\alpha R}$	ρ	η	χ	ϕ
$N = \mathcal{B} - \mathcal{L}$	$-\frac{2}{3}$	-1	-1	$\frac{2}{3}$	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{4}{3}$	$-\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$	2

$$\phi \sim (1, 1, 0), \tag{20}$$

with $\mathcal{B}(\phi) = B(\phi) = 0$, $\mathcal{L}(\phi) = L(\phi) = -2$, has been included, along with η, ρ, χ, for breaking the 3-3-1-1 symmetry and generating the masses in a correct way. Let us remind the reader that the $B - L$ gauge charge, which can be directly derived from (10) and (17) as follows,

$$B - L = -\frac{2}{\sqrt{3}}T_8 + N, \tag{21}$$

is a residual symmetry of $SU(3)_L \otimes U(1)_N$ that does not commute with the 3-3-1 gauge symmetry, similarly to the lepton charge L. The extension from the 3-3-1 gauge symmetry to the new 3-3-1-1 gauge symmetry, which must also apply for the ordinary 3-3-1 models that respect the lepton number symmetry by this view, is very intriguing and quite similar to enlarging the $SU(5)$ theory to $SO(10)$, in which the $B - L$ charge is naturally gauged.

While this possibility of a phenomenological 3-3-1-1 model is interesting and worth exploring elsewhere [20], in this work we will focus on only its consequence of a discrete residual symmetry responsible for the dark matter stabilization, as shown below. Therefore, the leptonic and baryonic dynamics, as associated with the new gauge charge N, will be neglected. The lepton number will be understood as a consequence of the charge conservation associated with $\mathcal{G} = SU(3)_L \otimes U(1)_\mathcal{L}$ symmetry, in which the first factor is a global version of the gauge symmetry. [Namely, in calculating lepton numbers all $SU(3)_L$ global quantum numbers for model multiplets are the same gauged ones. And, both T_8 and $\mathcal{L}$, when responsible for the lepton number, will be taken as global charges and not gauged, which should not be confused with the similar ones of the $SU(3)_L \otimes U(1)_X$ gauge symmetry.] Similarly, the baryon number B will be regarded as an ordinary global charge. Since the general theory is always conserved by the baryon number, $\mathcal{L}$ or L is equivalent to the N charge, which should be understood in the following discussions. In other words, the 3-3-1 model with neutral fermions and $\mathcal{L}$ charge (plus the new right-handed neutrinos and scalar singlet) is also understood as the 3-3-1-1 model in which the gauge interactions or dynamics, as associated with N charge (thus B and L), are omitted in this work.

Although the $U(1)_\mathcal{L}$ and $U(1)_\mathcal{B}$ symmetries have been imposed and L, B are conserved by the VEVs of η, ρ, χ as given, it is evident that B, L should be broken in some way in order to account for the matter-antimatter asymmetry of the universe and even neutrino masses included later. On the other hand, as stated, the nature of L in this model is a gauged charge since it is a result of T_8. The theory with L gauged simply takes $N = \mathcal{B} - \mathcal{L}$ into account since this new charge is necessarily independent of the anomalies, and the complete breakdown of the N charge must be achieved so that its gauge boson Z_N gets a large enough mass to escape from the current detectors. This can all be achieved by the scalar singlet ϕ when it develops a VEV,

$$\langle\phi\rangle = \frac{1}{\sqrt{2}}\Lambda. \tag{22}$$

Therefore, we will assume that the matter parity (quite similar to the minimal supersymmetric standard model (MSSM) case), a residual discrete symmetry of broken $B - L = -(2/\sqrt{3})T_8 + N$ [or $SU(3)_L \otimes U(1)_N$], is an exact and unbroken symmetry of the 3-3-1-1 theory. Thus R parity when including spin is given by

$$P = (-1)^{3(B-L)+2s} = (-1)^{-2\sqrt{3}T_8+3N+2s}, \tag{23}$$

which still conserves all the vacuum structures above. (For a detailed proof, see Appendix B.) The R parity of model particles is given in Table V. We see that all the particles having unusual (abnormal) characteristic lepton-number differing from the ordinary one by 1 unit, e.g., $L(N_R) = 0$, $L(U) = -1$, $L(X) = 1$, $L(\rho_3) = -1$, already called wrong-lepton particles, are odd. Otherwise, the ordinary particles such as the standard model particles or new particles that remain would-be-ordinary properties of the lepton number (or differing from the ordinary ones by an even lepton number as ϕ due to the parity symmetry) are even. It is remarkable that the R parity (P) which originates in the 3-3-1-1 gauge symmetry is a natural symmetry of wrong-lepton particles in this model. The lightest wrong-lepton particle (LWP) within the odd ones is stable and able to contribute to dark matter since this parity is exact, not broken by the VEVs. Simultaneously, as mentioned, we

TABLE V. The R parity of 3-3-1-1 model particles that separates wrong-lepton particles from ordinary particles. Here the family indices for fermions have been suppressed and should be understood.

$+1$ (ordinary or bilepton particle)	ν_L e u d γ W Z $\rho_{1,2}$ $\eta_{1,2}$ χ_3 ϕ ν_R Z' Z_N
-1 (wrong-lepton particle)	N_R U D ρ_3 η_3 $\chi_{1,2}$ X Y

can have several violations of L or B (one example is that $L = \pm 2$ is broken by Λ) in order to make the model phenomenologically viable while still protecting the parity. From this point of view, it is noteworthy that there can be R-parity odd particles of wrong-lepton particles, even in nonsupersymmetric theories like ours. This is due to a possible property of neutral fermions $L(N_R) = 0$, as also implemented by a class of 3-3-1 models with flavor symmetries [7]. By contrast, every particle in the 3-3-1 model, with right-handed neutrinos $L(\nu_R) = 1$, and the minimal 3-3-1 model is even under the parity.

Also, it can be explicitly pointed out that in the interactions of the theory all the odd particles are only coupled in pairs, and thus linked to ordinary particles of the standard model due to the 3-3-1 gauge symmetry, the $U(1)_\mathcal{L}$ symmetry and the vacuum respecting R parity (i.e., the 3-3-1-1 symmetry with conserved R parity). Let us show now examples and consequences (the scalar potential also possessing these properties will be expressed later).

(1) Yukawa sector:

$$\begin{aligned}\mathcal{L}_Y = {} & h^e_{ab}\bar{\psi}_{aL}\rho e_{bR} + h^\nu_{ab}\bar{\psi}_{aL}\eta\nu_{bR} \\ & + h'^\nu_{ab}\bar{\nu}^c_{aR}\nu_{bR}\phi + h^U\bar{Q}_{3L}\chi U_R \\ & + h^D_{\alpha\beta}\bar{Q}_{\alpha L}\chi^* D_{\beta R} + h^u_a\bar{Q}_{3L}\eta u_{aR} \\ & + h^d_a\bar{Q}_{3L}\rho d_{aR} + h^d_{\alpha a}\bar{Q}_{\alpha L}\eta^* d_{aR} \\ & + h^u_{\alpha a}\bar{Q}_{\alpha L}\rho^* u_{aR} + \text{H.c.}\end{aligned} \tag{24}$$

We see that the odd scalars ρ_3, η_3 and $\chi_{1,2}$ do not interact with ordinary fermions which only couple to eN, νN, uU, dU, dD, uD with an even-odd particle pair due to the 3-3-1 symmetry. There are not similar $\mathcal{L}$-charge violating interactions (which lead to violations of R parity) such as $\bar{\psi}_{aL}\chi\nu_{bR}$, $\bar{\psi}^c_{aL}\psi_{bL}\rho$, $\bar{Q}_{3L}\chi u_{aR}$, $\bar{Q}_{\alpha L}\chi^* d_{aR}$, $\bar{Q}_{3L}\eta U_R$ and so forth. In addition, since R parity is conserved the VEVs of η^0_3 and χ^0_1 vanish. Because of these conditions, the ordinary quarks and exotic quarks do not mix, which means that the flavor changing neutral currents at tree level disappear. The 3-3-1 model with right-handed neutrinos, as often considered [6], is strictly improved by this parity. It is also noted that N_R does not mix with ν_L and ν_R because of the parity symmetry.

(2) Gauge boson sector: The odd gauge bosons X, Y do not couple to the standard model gauge boson pairs either, except for a similar type as mentioned such as WX, WY or other types such as W-W-odd-odd, etc., due to the 3-3-1 symmetry. This can be verified directly from [21]. Because of the R-parity symmetry, the neutral gauge boson X cannot mix with Z and Z' bosons. The CPT violation at tree level, as stated in [22], is suppressed. Again, constraints on the 3-3-1 model with right-handed neutrinos [6] are improved by the parity.

We notice that in the MSSM, the spins or angular-momenta of component particles within a supermultiplet do not commute with supersymmetry (comparing to our case in an alternative scenario, the lepton number does not commute with the gauge symmetry). However, the residual R parity of spin, lepton and baryon numbers (which must also not commute with the supersymmetry) is conserved and unbroken, even though the conservation of its global symmetry (that spans such spin, lepton and baryon numbers, known as R-symmetry) is actually broken along with the supersymmetry breaking. It is also emphasized that the lepton and baryon number conservation of the MSSM superpotential is not an automatic consequence of the theory at the renormalizable level, unlike the standard model, which must be an assumption similar to our case with the $U(1)_\mathcal{L}$ symmetry associated with the lepton number. Our R parity obviously has a different origin of the 3-3-1-1 gauge symmetry, as mentioned. This is due to the nature of the lepton charge that nontrivially resides in $SU(3)_L \otimes U(1)_\mathcal{L}$, baryon charge $U(1)_\mathcal{B}$ and spin, uniform for both kinds of respective particles (ordinary and W particles), instead of those in the MSSM. Particularly, the $\mathcal{G}$ symmetry or $U(1)_\mathcal{L}$ of the lepton number is broken due to the gauge symmetry breaking. The L-symmetry conservation for this model can also be protected by the R parity instead. The breaking of $N = \mathcal{B} - \mathcal{L}$ gauge symmetry can occur just above the TeV scale or at a very high scale. Therefore, we call the R parity in this model W parity, where W means the "wrong lepton," as already pointed out.

Depending on the parameter space of the model, the LWP may belong to a vector particle (X^0), scalar (χ^0_1 or η^0_3), or fermion (N^0_R), which must be considered here to be electrically neutral if they are expected to contribute dark matter. Before considering those cases in the next section, let us make some comments on the scalar particle identifications and the masses of particles, with emphasis on those of the wrong-lepton particles.

C. Scalar potential and masses

If the scalar singlet ϕ which is responsible for completely breaking $U(1)_N$, as mentioned, is in the same scale of 3-3-1 breaking ($\Lambda \sim \omega$), it will couple to ordinary scalars η, ρ, χ via the scalar potential. And, the phenomenologies associated with broken $B - L$ symmetry via Z_N happen simultaneously with the new physics of the 3-3-1 model, just in the TeV scale. This possibility is very interesting [20]. Otherwise, the ϕ that can be integrated out from the low-energy effective potential of η, ρ, χ should be very massive. Also, the Z_N will be decoupled from the gauge boson spectrum. This is the case considered in this work. It is to be noted that the behavior of W parity in both cases is unchanged. For the ϕ, we can expand

$$\phi = \frac{1}{\sqrt{2}}(\Lambda + R + iI). \tag{25}$$

The imaginary part (I) of ϕ is just the Goldstone boson of Z_N, while the real part (R) is a new physical Higgs boson carrying a lepton number of 2 units and being W-parity even. The mass of Z_N is proportional to the VEV Λ of the ϕ scalar.

At low energy, the scalar potential of η, ρ, χ, after integrating out the ϕ that must conserve the 3-3-1 symmetry and W parity, is given by

$$\begin{aligned} V &= \mu_1^2\rho^\dagger\rho + \mu_2^2\chi^\dagger\chi + \mu_3^2\eta^\dagger\eta + \lambda_1(\rho^\dagger\rho)^2 \\ &+ \lambda_2(\chi^\dagger\chi)^2 + \lambda_3(\eta^\dagger\eta)^2 + \lambda_4(\rho^\dagger\rho)(\chi^\dagger\chi) \\ &+ \lambda_5(\rho^\dagger\rho)(\eta^\dagger\eta) + \lambda_6(\chi^\dagger\chi)(\eta^\dagger\eta) \\ &+ \lambda_7(\rho^\dagger\chi)(\chi^\dagger\rho) + \lambda_8(\rho^\dagger\eta)(\eta^\dagger\rho) \\ &+ \lambda_9(\chi^\dagger\eta)(\eta^\dagger\chi) + (f\epsilon^{mnp}\eta_m\rho_n\chi_p + \mathrm{H.c.}), \end{aligned} \quad (26)$$

where $\mu_{1,2,3}$ and f have mass dimension whereas $\lambda_{1,2,3,\ldots,9}$ are dimensionless. The unwanted terms, such as $\eta^\dagger\chi$, $(\eta^\dagger\chi)^2$, $(\rho^\dagger\rho)(\eta^\dagger\chi)$ and so on, which violate L (or $\mathcal{L}$), are prevented due to the parity symmetry. Let us note that the f coupling conserves all the natural symmetries of the theory as imposed, and there is no reason why it is not presented. In the literature, it is ordinarily excluded [23] (see also the first entry of [15]). Therefore, we need to clarify that its presence makes all the extra Higgs bosons massive, reasonably leading to a phenomenologically consistent scalar spectrum, as shown below. For partial solutions of the potential minimization and scalar spectrum, see [12,13].

To identify the scalar particles, let us expand the neutral fields (and the conservation of W parity must be maintained, as determined above):

$$\rho = \begin{pmatrix} \rho_1^+ \\ \frac{1}{\sqrt{2}}(v + S_2 + iA_2) \\ \rho_3^+ \end{pmatrix}, \quad \eta = \begin{pmatrix} \frac{1}{\sqrt{2}}(u + S_1 + iA_1) \\ \eta_2^- \\ \frac{1}{\sqrt{2}}(S_3' + iA_3') \end{pmatrix}, \quad \chi = \begin{pmatrix} \frac{1}{\sqrt{2}}(S_1' + iA_1') \\ \chi_2^- \\ \frac{1}{\sqrt{2}}(\omega + S_3 + iA_3) \end{pmatrix}, \quad (27)$$

where $S'_{1,3}$ and $A'_{1,3}$ are W-parity odd while $S_{1,2,3}$ and $A_{1,2,3}$ are even. Two kinds of these particles do not mix. Similarly, for the charged scalars, ρ_1 and η_2 do not mix with ρ_3 and χ_2. All of this can be seen in the results given below. The potential minimization conditions are obtained by

$$v\mu_1^2 + v^3\lambda_1 + \frac{1}{2}v\omega^2\lambda_4 + \frac{1}{2}vu^2\lambda_5 + \frac{1}{\sqrt{2}}fu\omega = 0, \quad (28)$$

$$u\mu_3^2 + u^3\lambda_3 + \frac{1}{2}uv^2\lambda_5 + \frac{1}{2}u\omega^2\lambda_6 + \frac{1}{\sqrt{2}}fv\omega = 0, \quad (29)$$

$$\omega\mu_2^2 + \omega^3\lambda_2 + \frac{1}{2}\omega v^2\lambda_4 + \frac{1}{2}\omega u^2\lambda_6 + \frac{1}{\sqrt{2}}fuv = 0. \quad (30)$$

The pseudoscalars A_1, A_2 and A_3 mix because $f \neq 0$. One combination of these fields is a physical pseudoscalar (A) with mass,

$$A = \frac{u^{-1}A_1 + v^{-1}A_2 + \omega^{-1}A_3}{\sqrt{u^{-2} + v^{-2} + \omega^{-2}}}, \quad m_A^2 = -\frac{f}{\sqrt{2}}\frac{u^2v^2 + u^2\omega^2 + v^2\omega^2}{uv\omega}. \quad (31)$$

We see that $f < 0$ if u, v, $\omega > 0$. The two other fields are massless, orthogonal to A, and identified as the Goldstone bosons of Z and Z', as given by

$$G_{Z'} = \frac{-\omega^{-1}(u^{-1}A_1 + v^{-1}A_2) + (u^{-2} + v^{-2})A_3}{\sqrt{(u^{-2} + v^{-2} + \omega^{-2})(u^{-2} + v^{-2})}}, \quad G_Z = \frac{-uA_1 + vA_2}{\sqrt{u^2 + v^2}}. \quad (32)$$

The A mass is proportional to ω if we suppose that f is (here and below) in the ω scale ($f \sim -\omega$). At the leading order, $\omega \gg u, v$, we have $G_{Z'} \simeq A_3$ and $A \simeq (vA_1 + uA_2)/\sqrt{u^2 + v^2}$.

The scalars S_1, S_2 and S_3 mix via the mass Lagrangian:

$$(S_1 \quad S_2 \quad S_3) \times \begin{pmatrix} \lambda_3u^2 - \frac{fv\omega}{2\sqrt{2}u} & \frac{1}{2}\lambda_5uv + \frac{f\omega}{2\sqrt{2}} & \frac{1}{2}\lambda_6u\omega + \frac{fv}{2\sqrt{2}} \\ \frac{1}{2}\lambda_5uv + \frac{f\omega}{2\sqrt{2}} & \lambda_1v^2 - \frac{fu\omega}{2\sqrt{2}v} & \frac{1}{2}\lambda_4v\omega + \frac{fu}{2\sqrt{2}} \\ \frac{1}{2}\lambda_6u\omega + \frac{fv}{2\sqrt{2}} & \frac{1}{2}\lambda_4v\omega + \frac{fu}{2\sqrt{2}} & \lambda_2\omega^2 - \frac{fuv}{2\sqrt{2}\omega} \end{pmatrix} \times \begin{pmatrix} S_1 \\ S_2 \\ S_3 \end{pmatrix}. \quad (33)$$

This mass matrix always gives a physical light state to be identified as the standard model Higgs boson (H). Since $f \sim -\omega$, the two other physical states ($H_{1,2}$) are heavy in ω scale. At the leading order ($-f$, $\omega \gg u$, v), those physical fields with respective masses can be obtained by

$$H_1 \simeq \frac{-vS_1 + uS_2}{\sqrt{u^2 + v^2}}, \quad m_{H_1}^2 \simeq -\frac{f\omega}{\sqrt{2}}\left(\frac{u}{v} + \frac{v}{u}\right), \quad H_2 \simeq S_3, \quad m_{H_2}^2 \simeq 2\lambda_2\omega^2, \quad (34)$$

$$H \simeq \frac{uS_1 + vS_2}{\sqrt{u^2+v^2}}, \qquad m_H^2 \simeq \frac{4\lambda_2(\lambda_5 u^2 v^2 + \lambda_3 u^4 + \lambda_1 v^4) - (\sqrt{2}uv(f/\omega) + \lambda_6 u^2 + \lambda_4 v^2)^2}{2\lambda_2(u^2+v^2)}. \tag{35}$$

One combination of S'_1 and S'_3 is a physical field $S' = (\omega S'_3 + uS'_1)/\sqrt{u^2+\omega^2}$ with mass $m^2_{S'} = (\frac{1}{2}\lambda_9 - \frac{fv}{\sqrt{2}u\omega}) \times (u^2+\omega^2)$. The orthogonal state $G'_S = (-uS'_3 + \omega S'_1)/\sqrt{u^2+\omega^2}$ is a massless Goldstone field. Similarly, one combination of A'_1 and A'_3 is a physical field $A' = (\omega A'_3 - uA'_1)/\sqrt{u^2+\omega^2}$ with mass $m^2_{A'} = (\frac{1}{2}\lambda_9 - \frac{fv}{\sqrt{2}u\omega}) \times (u^2+\omega^2)$. The orthogonal state $G'_A = (uA'_3 + \omega A'_1)/\sqrt{u^2+\omega^2}$ is a massless Goldstone field. It is easy to realize that the G'_S and G'_A are Goldstone bosons of ReX and ImX gauge fields, respectively. Therefore, their combination can be identified as the Goldstone boson of the X gauge boson:

$$G_X = \frac{1}{\sqrt{2}}(G'_S + iG'_A) = \frac{\omega\chi_1 - u\eta_3^*}{\sqrt{u^2+\omega^2}}. \tag{36}$$

Simultaneously, we also have a physical neutral complex field as a combination of S' and A' (and obviously orthogonal to G_X) with the mass given as

$$H' = \frac{1}{\sqrt{2}}(S' + iA') = \frac{u\chi_1^* + \omega\eta_3}{\sqrt{u^2+\omega^2}}, \quad m^2_{H'} = \left(\frac{1}{2}\lambda_9 - \frac{fv}{\sqrt{2}u\omega}\right)(u^2+\omega^2). \tag{37}$$

H' is the only physical neutral scalar field which is odd under W parity and responsible for dark matter, as shown below. It is to be noted that the H' mass is always in the ω scale. At the leading order ($\omega \gg u, v$) we have $H' \simeq \eta_3$ (which is a scalar singlet of the standard model) and $G_X \simeq \chi_1$.

There are two physical charged scalars, one W-parity odd (H_3) and another even (H_4):

$$H_3^- = \frac{v\chi_2^- + \omega\rho_3^-}{\sqrt{v^2+\omega^2}}, \qquad H_4^- = \frac{v\eta_2^- + u\rho_1^-}{\sqrt{u^2+v^2}}, \tag{38}$$

with respective masses

$$m^2_{H_3} = \left(\frac{1}{2}\lambda_7 - \frac{fu}{\sqrt{2}v\omega}\right)(v^2+\omega^2), \quad m^2_{H_4} = \left(\frac{1}{2}\lambda_8 - \frac{f\omega}{\sqrt{2}uv}\right)(u^2+v^2). \tag{39}$$

The states orthogonal to these scalars are Goldstone bosons of the Y and W bosons, respectively:

$$G_Y^- = \frac{\omega\chi_2^- - v\rho_3^-}{\sqrt{v^2+\omega^2}}, \qquad G_W^- = \frac{u\eta_2^- - v\rho_1^-}{\sqrt{u^2+v^2}}. \tag{40}$$

The masses of H_3 and H_4 are in the ω scale. At the leading order, we have $H_3 \simeq \rho_3$ and $G_Y \simeq \chi_2$.

We conclude that only the standard-model-like Higgs boson H is light in the u, v scale. All the other physical scalars such as A, $H_{1,2,3,4}$ and H' are heavy in the ω scale, while R is in the Λ scale. The number of Goldstone bosons matches those of the massive gauge bosons [when the $U(1)_N$ gauge symmetry is turned on, the extra scalar ϕ, as required, will completely break this charge as well as provide the Goldstone boson I for it]. On the other hand, if the f coupling is suppressed as in the literature, the field $A \sim vA_1 + uA_2$ becomes a physical massless Goldstone field living in the doublets of the standard model, which is very unrealistic. In addition, the identification in [14] of another 3-3-1 version of Imχ_3^0 (similar to A_3 in this model) as dark matter is incorrect since it is already the Goldstone boson of the Z' gauge boson ($G_{Z'}$).

For the gauge boson sector, after integrating out Z_N, the masses of the remaining gauge bosons are given as usual:

$$c_W^2 m_Z^2 \simeq m_W^2 = \frac{g^2}{4}(u^2+v^2), \qquad m_Y^2 = \frac{g^2}{4}(v^2+\omega^2), \quad m_X^2 = \frac{g^2}{4}(u^2+\omega^2), \tag{41}$$

and Z' obtains a mass in the ω scale as X and Y (to be specified in the next section), where g is the $SU(3)_L$ gauge coupling constant. We therefore identify $u^2+v^2 = (246\ \text{GeV})^2$. It is noted that W and Y do not mix. Similarly, Z, Z' and Z_N do not mix with X. All this is due to the W-parity symmetry that forces the VEVs of χ_1^0 and η_3^0 to vanish. Moreover, the mass spectrum of the neutral gauge boson sector will be changed if Z_N gets a mass in the 3-3-1 scale ($\Lambda \sim \omega$). By our convention, as given, this should be skipped in the present work.

For the fermion sector, we first note that the Dirac masses that appear below will be written in the Lagrangian form $-\bar{f}_L m_f f_R$ + H.c. The masses of exotic quarks are given by

$$m_U = -\frac{1}{\sqrt{2}}h^U\omega, \qquad [m_D]_{\alpha\beta} = -\frac{1}{\sqrt{2}}h^D_{\alpha\beta}\omega, \tag{42}$$

which are all in the ω scale. The ordinary quarks and charged leptons have consistent masses at tree level, as in the 3-3-1 model with right-handed neutrinos:

$$[m_u]_{\alpha a} = \frac{1}{\sqrt{2}}h^u_{\alpha a}v, \qquad [m_u]_{3a} = -\frac{1}{\sqrt{2}}h^u_a u, \tag{43}$$

for up quarks,

$$[m_d]_{\alpha a} = -\frac{1}{\sqrt{2}}h^d_{\alpha a}u, \qquad [m_d]_{3a} = -\frac{1}{\sqrt{2}}h^d_a v, \tag{44}$$

for down quarks, and

$$[m_e]_{ab} = -\frac{1}{\sqrt{2}} h^e_{ab} v, \tag{45}$$

for charged leptons. Here, we see that the up quarks do not mix with U, and the down quarks do not mix with D_α either, as already mentioned.

The ν_L and ν_R coupled to η_1^0 will have Dirac masses:

$$[m^D_\nu]_{ab} = -\frac{1}{\sqrt{2}} h^\nu_{ab} u. \tag{46}$$

However, the right-handed neutrinos (ν_R) by themselves coupled via ϕ will get large Majorana masses (in the Λ scale) in the form $-\frac{1}{2}\bar{\nu}^c_R m^M_\nu \nu_R + \mathrm{H.c.}$, where

$$[m^M_\nu]_{ab} = -\sqrt{2} h'^\nu_{ab} \Lambda. \tag{47}$$

Consequently, the (observed) active neutrinos ($\sim \nu_L$) naturally get small masses via a type I seesaw mechanism, as given by

$$m^{\mathrm{eff}}_\nu = -m^D_\nu (m^M_\nu)^{-1} (m^D_\nu)^T \sim \frac{(h^\nu)^2}{h'^\nu} \frac{u^2}{\Lambda}. \tag{48}$$

If the Λ is proportional to ω acting on the TeV scale, the Dirac mass parameters (m^D_ν) should get values in the electron mass range in order for m^{eff}_ν to be in sub eV. In any case, the masses of physical sterile neutrinos ($\sim \nu_R$) are in the Λ scale responsible for the $U(1)_N$ breaking.

Unlike the previous model [6], the N_R have vanishing masses at the renormalizable level because ρ does not couple to $\psi_L \psi_L$ (in addition, χ is also not coupled to $\psi_L \nu_R$) because of the $\mathcal{L}$ charge or $U(1)_N$ symmetry. (In the 3-3-1 model with right-handed neutrinos [6], the status is not better. Although the $\psi_L \psi_L \rho$ coupling is allowed, the tree-level neutrinos have only three Dirac masses, not Majorana type, in which one mass is zero and two others are degenerate and are unrealistic in the data [1]). Fortunately, the masses of N_R can be generated by the scalar content by itself via an effective operator invariant under the 3-3-1-1 symmetry and W parity:

$$\frac{\lambda_{ab}}{M} \bar{\psi}^c_{aL} \psi_{bL} (\chi\chi)^* + \mathrm{H.c.} \tag{49}$$

There are no other operators of types $\psi\psi\eta\eta$ and $\psi\psi\eta\chi$ as often considered because of the 3-3-1-1 symmetry. Consequently, only the neutral fermions get masses via this kind of interaction:

$$[m_{N_R}]_{ab} = -\lambda_{ab} \frac{\omega^2}{M}. \tag{50}$$

The mass scale of N_R is unknown; however, it can be taken in the TeV order (i.e., M is not so high compared to ω) because of the following facts: (i) In the 3-3-1 model with neutral fermions, m_{N_R} were proved to be naturally in the ω scale (but the W parity should be violated) [7], and (ii) we can introduce a new scalar sextet coupled to $\psi_L \psi_L$ conserving the 3-3-1-1 symmetry and W parity. The 33 component of the sextet provides the masses for N_R. However, it is also responsible for the 3-3-1 symmetry breaking, which should be in the same ω scale. On the other hand, this sextet, if included, can also be reserved for totally breaking the N-charge and obtaining W parity because of the VEV of the 11-component carrying a lepton number of 2 units [7,18]. The ν_L neutrinos also get small masses via a type II seesaw in this case. However, let us neglect the scalar sextet in this work because the scalar singlet ϕ as included is just enough for all purposes. Finally, it is noted that, because of W parity, $\nu_{L,R}$ do not mix with N_R. Also, the mass sources of N_R and ν_R might come from different kinds of 3-3-1-1 breaking.

III. DARK MATTER ABUNDANCE AND DIRECT DETECTION

Let us note that all the W particles, including the dark matter candidates X^0, N^0_R and H'^0, are heavy particles with masses proportional to ω. Among the W particles, supposing X^0 as a LWP ($m_X < m_{N_R}, m_U, m_{D_\alpha}, m_{H_3}, m_{H'}$), it will be stabilized and responsible for dark matter. Notice that X cannot decay into Y and vice versa. This is the first case discussed below. For the second case, N_R will be assumed as a LWP ($m_{N_R} < m_X, m_Y, m_U, m_{D_\alpha}, m_{H_3}, m_{H'}$) for dark matter. The work in [13] has presented numerical calculations for relic densities using the MicrOMEGAs package. In the following, we will give an analytic evaluation. For the constraints from the direct and indirect detection experiments, see [13]. Below, we provide only one of these kinds for analytic calculation so that our conclusions are viable.

The scalar dark matter candidate H', which behaves as a LWP ($m_{H'} < m_X, m_Y, m_U, m_{D_\alpha}, m_{H_3}, m_{N_R}$), has been traditionally studied in the 3-3-1 model [15]. In the following consideration, this candidate will be neglected. For detailed calculations and experimental constraints, see Refs. [12,13,24]. However, let us make some remarks on this particle: (i) The previous studies [12,13,15] that identify the massive scalar $H' \simeq \eta_3$ as dark matter are unnatural since the symmetries protecting it from decay are either neglected or included in terms of lepton charge, G charge, or even Z_2, which must be broken because of the problems shown above. In this case, it will develop a VEV, allowing decay channels into the standard model particles such as $H' \to HH$ since this field should be naturally heavy (the fine-tuning in mass was needed in [15] and is very awkward). In our model, by investigating W parity, the stability issue of H' has been solved, similar to the standard model extension with a Z_2-odd scalar singlet. (ii) $H' \simeq \eta_3$ is a singlet under the standard model symmetry, and it annihilates into the standard model particles via the scalar portal, exotic quarks, and new gauge bosons.

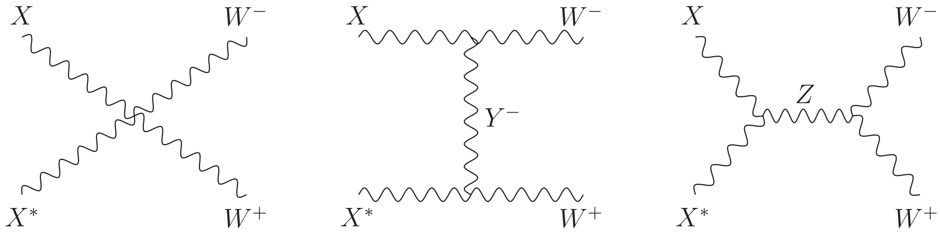

FIG. 1. Dominant contributions to annihilation of X^0 into W^+W^-.

A. Relic density of X^0 gauge boson

The annihilation of X into the standard model particles is dominated by the following channels,

$$XX^* \to W^+W^-, ZZ, HH, \nu\nu^c, l^+l^-, qq^c, \qquad (51)$$

where $\nu = \nu_e, \nu_\mu, \nu_\tau$, $l = e, \mu, \tau$, and $q = u, d, c, s, t, b$. Let us consider the channel $XX^* \to W^+W^-$ among them. This process is contributed by the diagrams in Fig. 1. The Feynman rules can be found in [21]. To the leading order, the thermal average of the cross-section times relative velocity [25] is given as follows:

$$\langle \sigma v_{\mathrm{rel}} \rangle_{v\to 0} \simeq \frac{5\alpha^2 m_X^2}{8 s_W^4 m_W^4}. \qquad (52)$$

Because $m_X^2 \gg m_W^2$, this result is too large so X can meet the criteria of dark matter. In fact, the relic density of the candidate [25] is bounded by

$$\Omega_X h^2 \simeq \frac{0.1\ \mathrm{pb}}{\langle \sigma_{\mathrm{tot}} v_{\mathrm{rel}} \rangle} < \frac{0.1\ \mathrm{pb}}{\langle \sigma v_{\mathrm{rel}} \rangle} \simeq 0.0024 \times \left(\frac{m_W}{m_X}\right)^2 < 0.00008, \qquad (53)$$

if we take a previous limit on the mass of X: $m_X > 440$ GeV [26]. The upper bound of the relic density is too small to compare to the WMAP data $\Omega_{\mathrm{DM}} h^2 \simeq 0.11$ [1]. The X cannot be dark matter. This conclusion coincides with a mere note in [13].

B. Relic density of neutral fermion N_R

Among the three neutral fermions, N_{aR}, the lightest particle is denoted as N_R. In addition, suppose ν and l are the left-handed neutrino and charged lepton that directly couple to N_R via the new gauge bosons X and Y, respectively. There are two other neutrinos and two other charged leptons to be denoted by ν_α and l_α, respectively. The annihilation of N_R into the standard model particles is dominated by the following channels:

$$NN^c \to \nu\nu^c, l^-l^+, \nu_\alpha\nu_\alpha^c, l_\alpha^- l_\alpha^+, qq^c, ZH, \qquad (54)$$

which are given in terms of Feynman diagrams in Fig. 2. Let us remind the reader that there are no channels into HH, W^-W^+ and ZZ bosons because ν_L and N_R do not mix because of W parity. On the other hand, there may also be some contributions coming from mediated scalars instead of the new gauge bosons, but they are all small and thus neglected.

The Feynman rules for the above processes can be found in [6,27] (see also [28]). An evaluation of the thermal average cross-section times relative velocity is given by

$$\begin{aligned}\langle \sigma v_{\mathrm{rel}} \rangle \simeq{}& \frac{g^4 m_{N_R}^2}{32\pi}\left(1+\frac{8}{x_F}\right)\left[\frac{1}{m_X^4}+\frac{1}{m_Y^4}-\frac{2c_{2W}}{(3-4s_W^2)m_{Z'}^2}\left(\frac{1}{m_X^2}+\frac{1}{m_Y^2}\right)+\frac{60-104s_W^2+196s_W^4}{3(3-4s_W^2)^2 m_{Z'}^4}\right]\\ &+\frac{g^4 m_{N_R}^2}{32\pi}\sqrt{1-\frac{m_t^2}{m_{N_R}^2}}\left\{\left[1+\frac{1}{x_F}\left(8+\frac{m_t^2}{m_{N_R}^2}\frac{m_{N_R}^2+2m_t^2}{m_{N_R}^2-m_t^2}\right)\right]\frac{9-12s_W^2+20s_W^4}{3(3-4s_W^2)^2 m_{Z'}^4}+\left[1+\frac{6}{x_F}\left(1+\frac{1}{2}\frac{m_t^2}{m_{N_R}^2-m_t^2}\right)\right]\right.\\ &\left.\times\frac{4s_W^2(3-2s_W^2)}{3(3-4s_W^2)^2 m_{Z'}^4}\frac{m_t^2}{m_{N_R}^2}\right\}+\frac{g^4 m_{N_R}^2}{32\pi}\sqrt{1-\frac{m_H^2+m_Z^2}{2m_{N_R}^2}}\frac{c_W^2 c_{2W}^2}{(3-4s_W^2)^2 m_{Z'}^4}\left[1+\frac{2m_Z^2-m_H^2}{2m_{N_R}^2}+\frac{1}{x_F}\left(4+\frac{5m_H^2+11m_Z^2}{2m_{N_R}^2}\right)\right],\end{aligned} \qquad (55)$$

where we have used the facts that $m_\nu, m_l, m_q (q \neq t) \ll m_t, m_Z, m_H < m_{N_R} < m_X, m_Y, m_{Z'}$. In addition, the above cross section has been expanded in the nonrelativistic limit of N_R as usual up to the squared velocity, in which $\langle v^2 \rangle = 6/x_F$ and $x_F \equiv m_{N_R}/T_F \sim 20$ at the freeze-out temperature [25].

To have a numerical value for the WMAP data, let us use the condition $\omega^2 \gg u^2, v^2$, which follows the tree-level relation (the mass of Z' can be found in [29])

$$m_X^2 \simeq m_Y^2 \simeq \frac{3-t_W^2}{4} m_{Z'}^2. \qquad (56)$$

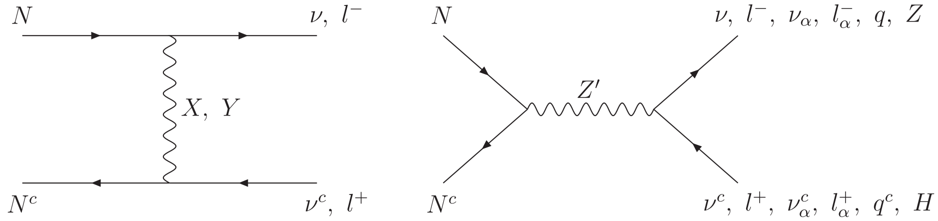

FIG. 2. Dominant contributions to annihilation of N_R.

Also, let the neutral fermion mass be large enough, $m^2_{N_R} \gg m^2_{t,W,H}$, so that the ratios $m^2_{t,H,W}/m^2_{N_R}$ can be neglected and the new physics is safe. We have

$$\langle \sigma v_{\rm rel}\rangle \simeq \frac{\alpha^2}{(150\ \text{GeV})^2}\frac{(2557.5\ \text{GeV})^2 m^2_{N_R}}{m^4_{Z'}}, \tag{57}$$

where we have used $s^2_W = 0.231$ and $x_F = 20$. Because $\alpha^2/(150\ \text{GeV})^2 \simeq 1$ pb, the WMAP data on the dark matter relic density for N_R ($\Omega_N h^2 \simeq 0.1\ \text{pb}/\langle\sigma v_{\rm rel}\rangle \simeq 0.11$) imply

$$m_{N_R} \simeq \frac{m^2_{Z'}}{2557.5\ \text{GeV}}. \tag{58}$$

Since $m_{N_R} < m_{Z'}$ we derive $m_{Z'} \lesssim 2.5$ TeV. This upper limit of the Z' mass is needed in order to make the dark matter candidate N_R stable. Several lower limits on the Z' mass have been given in the literature at some TeV scale [1,30], so let us take the strong one recently studied in the second article of [30], $m_{Z'} \gtrsim 2.2$ TeV. Consequently, the mass of N_R is limited by $m_{N_R} \gtrsim 1.9$ TeV. In summary, the N_R is dark matter if it has a mass in the range

$$1.9\ \text{TeV} \lesssim m_{N_R} \lesssim 2.5\ \text{TeV}. \tag{59}$$

The mass of N_R is completely fixed via $m_{Z'}$ or the VEV ω as a single-valued function because of the relic density as given, which is unlike that in [13] numerically calculated with the MicrOMEGAs package. Our result above is in agreement with the large range, among others, as dedicated in [13].

C. Direct detection of dark matter N_R

The direct detection experiments measure the recoil energy deposited by the scattering of dark matter with the nuclei in a large detector. At the fundamental level, the scattering is due to the interactions of dark matter with quarks, as confined in the nucleons. In this model, the leading contribution to the N_R-quark scattering amplitude comes from the t-channel exchange of the Z' boson (there may be another contribution from the Z boson; however, it is very suppressed because of the constrained small mixing of $Z - Z'$). Therefore, the effective Lagrangian is given by

$$\mathcal{L}^{\rm eff}_{N_R\text{-quark}} = \bar{N}_R\gamma^\mu N_R[\bar{q}\gamma_\mu(\alpha_q P_L + \beta_q P_R)q], \tag{60}$$

where the relevant couplings are evaluated as [6,27,28]

$$\alpha_{u,d,c,s} = -\frac{g^2}{6m^2_{Z'}}, \qquad \beta_{u,c} = \frac{2g^2 s^2_W}{3(4c^2_W - 1)m^2_{Z'}},$$
$$\beta_{d,s} = -\frac{g^2 s^2_W}{3(4c^2_W - 1)m^2_{Z'}}. \tag{61}$$

In the nonrelativistic limit, there are only two operators in the effective Lagrangian that survive (the other operators vanish), as given by [31]

$$\mathcal{L}^{\rm eff}_{N_R\text{-quark}} = \lambda_{q,o}\bar{N}\gamma^\mu N\bar{q}\gamma_\mu q + \lambda_{q,e}\bar{N}\gamma^\mu\gamma_5 N\bar{q}\gamma_\mu\gamma_5 q, \tag{62}$$

where $\lambda_{q,o} \equiv (\beta_q + \alpha_q)/4$ is for the odd operator while $\lambda_{q,e} \equiv (\beta_q - \alpha_q)/4$ is for the even operator.

The N_R-nucleon amplitudes can be directly converted from the amplitudes above via the nucleon form factors. as obtained by [31]

$$\mathcal{L}^{\rm eff}_{N_R\text{-nucleon}} = \lambda_{\psi,o}\bar{N}\gamma^\mu N\bar{\psi}\gamma_\mu\psi + \lambda_{\psi,e}\bar{N}\gamma^\mu\gamma_5 N\bar{\psi}\gamma_\mu\gamma_5\psi, \tag{63}$$

where ψ is the nucleon, $\psi \equiv (p, n)$, and $\lambda_{\psi,e} = \sum_{q=u,d,s}\Delta^\psi_q\lambda_{q,e}$ with the Δ^ψ_q values provided in [31], while $\lambda_{\psi,o}$ is given by

$$\lambda_{\psi,o} = \sum_{q=u,d} f^\psi_{Vq}\lambda_{q,o}, \qquad f^p_{Vu} = 2,$$
$$f^p_{Vd} = 1, \qquad f^n_{Vu} = 1, \qquad f^n_{Vd} = 2. \tag{64}$$

The $\lambda_{\psi,o}$ and $\lambda_{\psi,e}$ are spin-independent (SI) and spin-dependent interactions, respectively.

For the large nuclei, the N_R-nucleus scattering cross section is strongly enhanced because of the SI interaction, while there is no strong enhancement from the spin-dependent interaction [31]. Therefore, the dominant contribution to the cross section comes from the SI interaction given by

$$\sigma^{\rm SI}_{N_R\text{-nucleus}} = \frac{4\mu^2_A}{\pi}(\lambda_p Z + \lambda_n(A - Z))^2, \tag{65}$$

where Z is the nucleus charge, A the total number of nucleons, and

$$\mu_A = \frac{m_{N_R} m_A}{m_{N_R} + m_A}, \qquad \lambda_p = \frac{\lambda_{p,o}}{2} = \frac{3(2s_W^2 - 1)g^2}{16(3 - 4s_W^2)m_{Z'}^2},$$
$$\lambda_n = \frac{\lambda_{n,o}}{2} = -\frac{g^2}{16m_{Z'}^2}. \quad (66)$$

The experimental data of N_R-nucleon cross section are defined by the above result averaged over a nucleon,

$$\sigma^{\mathrm{SI}}_{N_R\text{-nucleon}} = \frac{4\mu^2_{\mathrm{nucleon}}}{\pi}\left(\lambda_p \frac{Z}{A} + \lambda_n \frac{A-Z}{A}\right)^2,$$
$$\mu_{\mathrm{nucleon}} = \frac{m_{N_R} m_{p,n}}{m_{N_R} + m_{p,n}} \simeq m_{\mathrm{nucleon}}. \quad (67)$$

The strongest limit on the N_R-nucleon cross section presently comes from the XENON100 experiment. Taking the Xe nucleon with $Z = 54$, $A = 131$, and $m_{\mathrm{nucleon}} \simeq 1$ GeV, $g^2 = 4\pi\alpha/s_W^2$ with $\alpha = 1/128$ and $s_W^2 = 0.231$, we have

$$\sigma^{\mathrm{SI}}_{N_R\text{-nucleon}} \simeq 2.9 \times 10^{-43}\left(\frac{1\ \mathrm{TeV}}{m_{Z'}}\right)^4\ \mathrm{cm}^2. \quad (68)$$

With the Z' mass limit given above, $m_{Z'} \geq 2.2$ TeV, this leads to

$$\sigma^{\mathrm{SI}}_{N_R\text{-nucleon}} \leq 1.2 \times 10^{-44}\ \mathrm{cm}^2. \quad (69)$$

This limit is in good agreement with the constraint from the XENON100 experiment [32] since our dark matter is heavy, with mass in the TeV range.

IV. CONCLUSION AND OUTLOOK

We have given a detailed analysis of lepton and baryon numbers in the 3-3-1 model with neutral fermions. If they are (residual) symmetries of the theory, which are strictly respected by the gauge interactions, the minimal Yukawa Lagrangian and the minimal scalar potential, they behave as local symmetries. We have also given a classification of the wrong-lepton particles that have anomalous lepton (even baryon) numbers, whereas the ordinary particles, including the standard model ones, do not have this property. Moreover, all the unwanted interactions, which lead to the tree-level flavor changing neutral currents, inconsistent neutrino masses, instability of the lightest wrong-lepton particle, and, further, tree-level *CPT* violation, are naturally suppressed because of one of those symmetries. The above ingredients are also generally applied to the 3-3-1 model with right-handed neutrinos and the minimal 3-3-1 model if these theories conserve the lepton and baryon number symmetries.

The lepton ($\mathcal{L}$) and baryon ($\mathcal{B}$) numbers can be unified in a single charge ($N = \mathcal{B} - \mathcal{L}$) of natural gauge symmetry $SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes U(1)_N$, independent of all the anomalies, such as leptonic and baryonic, recognizing $B - L = -(2/\sqrt{3})T_8 + N$ as a charge of $SU(3)_L \otimes U(1)_N$ (see also [33] for another 3-3-1-1 symmetry derived from grand unifications). All the unwanted interactions are not invariant under this gauge symmetry, which is prevented, while the minimal Lagrangian of the 3-3-1 theory respects the symmetry. A direct consequence of the 3-3-1-1 model is that it contains, by itself, a residual discrete symmetry after breaking, W parity: $P_W = (-1)^{-2\sqrt{3}T_8+3N+2s}$, as a natural symmetry of the wrong-lepton particles. The unwanted vacuums, which also lead to the problems mentioned above, are naturally discarded due to this parity. The lightest wrong-lepton particle is truly stabilized because of the 3-3-1-1 gauge symmetry with unbroken W parity which is responsible for the dark matter. With the aid of W parity, it is completely understood why the wrong-lepton particles are only coupled in pairs in the minimal Lagrangian with the standard vacuum structure, because of the specific 3-3-1 gauge symmetry (which has been explicitly shown in the text too). This W parity has the natural origin of 3-3-1-1 gauge symmetry by itself, which is discriminated from those in the MSSM and others with gauged $B - L$ [3,4].

We have also provided a general analysis of the scalar sector, identifying physical scalars and Goldstone bosons. The trilinear coupling of scalars should be present in order to make all extra Higgs bosons massive, keeping the model obviously consistent with the low energy theory. The pseudoscalar part of χ_3 should be the Goldstone boson of the Z' gauge boson, which is unlike the conclusion in [34]. Therefore, it should not be dark matter [14]. Finally, this sector will be charged if the $U(1)_N$ gauge symmetry is turned on.

We have explicitly shown that the non-Hermitian neutral gauge boson (X) cannot be dark matter. However, the neutral fermion (N_R) can contribute dark matter if its mass is given in the range 1.9 TeV $\leq m_{N_R} \leq$ 2.5 TeV, provided that the mass of the Z' gauge boson satisfies 2.2 TeV $\leq m_{Z'} \leq$ 2.5 TeV. In these calculations, we have neglected the contributions of the new gauge boson Z_N (it should be assumed to be massive or weakly interacting). If its mass and coupling are comparable to those of X, Y, Z', our results may change. Also, phenomenologies in the 3-3-1-1 model, such as the baryon number asymmetry, neutrino masses, and new physics associated with the Z_N gauge boson, will be very interesting. All of these and the one above are devoted to further studies to be published elsewhere [20].

The W parity transforms trivially in the 3-3-1 model with right-handed neutrinos; i.e., all the particles in the model are even under this parity. While the model might predict potential dark matter candidates, let us ask what the mechanism is other than the useless W parity for stabilizing dark matter. In the past [12], the first works were focused on the conservation of lepton number and Z_2. However, this Z_2 is really broken by the VEVs of scalars, while the lepton number should also be violated by five-dimensional effective interactions or broken by the vacuum responsible for the neutrino masses. On the other hand, the lepton number respected as a symmetry of the theory is also broken as a gauge symmetry and it is anomalous. In [13], the lepton content was changed and the $U(1)_G$ was included, perhaps to avoid some of those problems. However, this $U(1)_G$ has the same status as the lepton number that acts as a gauge symmetry and is also broken. Fortunately, for the new lepton content this charge is anomaly-free like $N = \mathcal{B} - \mathcal{L}$. So, we propose one solution to the stability of dark matter in their model by imposing G parity ($P_G = (-1)^G$), which is odd for the G particles and even for ordinary particles. In this case, the gauge symmetry should be $SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes U(1)_{\mathcal{G}}$, where $G = \frac{2}{\sqrt{3}} T_8 + \mathcal{G}$. Now let us turn to the 3-3-1 model with right-handed neutrinos. The $U(1)_G$ is also useless as the lepton number or W parity since it also yields anomalies. To overcome all these difficulties, in the following, we suppose a new mechanism based on the idea of a potential "inert" scalar triplet naturally realized by a Z_2 symmetry with the economical 3-3-1 model as the base [28]. W parity is explicitly violated in this model, and the lepton number is no longer regarded as a gauge symmetry since it is only an approximate symmetry, explicitly violated by the Yukawa interactions.

We know that the 3-3-1 model with right-handed neutrinos can work with three scalar triplets (ρ, η, χ) two of which (η, χ) have the same gauge symmetry quantum numbers [6]. If we exclude one of these two triplets (we assume η) the result is a new, consistent, predictive model, named the economical 3-3-1 model, working with only the two scalar triplets (ρ, χ) as recently investigated in a series of articles [28,35–38] (note that in those works ρ is called ϕ instead). Alternatively to that proposal, we can retain η, but introduce a Z_2 symmetry so that the η is odd, while the χ, ρ and all other fields are even. The resulting model will be very rich in phenomenology, other than the economical 3-3-1 model because of the contribution of η. In fact, the vacuum can also be conserved by Z_2 as a partial solution of the potential minimization, $\langle\eta\rangle = 0$, $\langle\rho\rangle = \frac{1}{\sqrt{2}}(0, v, 0)$, and $\langle\chi\rangle = \frac{1}{\sqrt{2}}(u, 0, \omega)$. We thus have a new economical 3-3-1 model with an inert scalar triplet η that is odd under the exact and unbroken Z_2 symmetry. The scalar triplets χ and ρ can break the gauge symmetry and generate the masses for the particles in the right way, like the economical 3-3-1 model [28]. The inert scalar triplet η can provide some dark matter candidates; however, they may belong to a scalar singlet or a scalar doublet under the standard model symmetry. In this sense, the model proposed is quite similar to the two Higgs doublet model in which one doublet is inert, well known as the inert doublet model [39]. However, this theory provides only doublet dark matter. The proposal is to be published elsewhere [40].

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APPENDIX A: CHECKING THE $U(1)_N$ ANOMALIES

The nontrivial anomalies associated with $U(1)_N$ that are potentially troublesome can be listed as follows: $[SU(3)_C]^2U(1)_N$, $[SU(3)_L]^2U(1)_N$, $[U(1)_X]^2U(1)_N$, $U(1)_X[U(1)_N]^2$, $[U(1)_N]^3$, and $[\text{Gravity}]^2U(1)_N$. The other anomalies associated with the usual 3-3-1 symmetry obviously vanish [9] and will not be considered in this appendix.

With the fermion content and N charges of the 3-3-1-1 model as given in Table IV, the mentioned anomalies can be calculated. The first one is proportional to

$$\begin{aligned}[SU(3)_C]^2U(1)_N &\sim \sum_{\text{all quarks}} (N_{q_L} - N_{q_R}) \\ &= 3N_{Q_3} + 2\times 3N_{Q_\alpha} - 3N_{u_a} - 3N_{d_a} - N_U - 2N_{D_\alpha} \\ &= 3(2/3) + 6(0) - 3(1/3) - 3(1/3) - (4/3) - 2(-2/3) = 0, \qquad \text{(A1)}\end{aligned}$$

which vanishes. The second anomaly also vanishes,

$$[SU(3)_L]^2U(1)_N \sim \sum_{\text{all (anti)triplets}} N_{F_L} = 3N_{\psi_a} + 3N_{Q_3} + 2\times 3N_{Q_\alpha} = 3(-2/3) + 3(2/3) + 6(0) = 0. \qquad \text{(A2)}$$

Here the number of fundamental colors (the 3's in the second and last terms) must be taken into account. In the following, the appearance of color numbers should be understood. Notice also that the relation $\text{Tr}[(-T_i^*)(-T_j^*)N] = \text{Tr}[T_iT_jN]$ has been used.

The third anomaly is given by

$$\begin{aligned}[U(1)_X]^2U(1)_N &= \sum_{\text{all fermions}}(X_{f_L}^2 N_{f_L} - X_{f_R}^2 N_{f_R}) \\ &= 3\times 3X_{\psi_a}^2 N_{\psi_a} + 3\times 3X_{Q_3}^2 N_{Q_3} + 2\times 3\times 3X_{Q_\alpha}^2 N_{Q_\alpha} - 3\times 3X_{u_a}^2 N_{u_a} - 3\times 3X_{d_a}^2 N_{d_a} - 3X_U^2 N_U \\ &\quad - 2\times 3X_{D_\alpha}^2 N_{D_\alpha} - 3X_{e_a}^2 N_{e_a} - 3X_{\nu_a}^2 N_{\nu_a} \\ &= 3\times 3(-1/3)^2(-2/3) + 3\times 3(1/3)^2(2/3) + 2\times 3\times 3(0)^2(0) - 3\times 3(2/3)^2(1/3) \\ &\quad - 3\times 3(-1/3)^2(1/3) - 3(2/3)^2(4/3) - 2\times 3(-1/3)^2(-2/3) - 3(-1)^2(-1) - 3(0)^2(-1)^2 \\ &= 0. \end{aligned} \tag{A3}$$

The fourth anomaly is similarly calculated,

$$\begin{aligned}U(1)_X[U(1)_N]^2 &= \sum_{\text{all fermions}}(X_{f_L} N_{f_L}^2 - X_{f_R} N_{f_R}^2) \\ &= 3\times 3X_{\psi_a} N_{\psi_a}^2 + 3\times 3X_{Q_3} N_{Q_3}^2 + 2\times 3\times 3X_{Q_\alpha} N_{Q_\alpha}^2 - 3\times 3X_{u_a} N_{u_a}^2 - 3\times 3X_{d_a} N_{d_a}^2 \\ &\quad - 3X_U N_U^2 - 2\times 3X_{D_\alpha} N_{D_\alpha}^2 - 3X_{e_a} N_{e_a}^2 - 3X_{\nu_a} N_{\nu_a}^2 \\ &= 3\times 3(-1/3)(-2/3)^2 + 3\times 3(1/3)(2/3)^2 + 2\times 3\times 3(0)(0)^2 - 3\times 3(2/3)(1/3)^2 \\ &\quad - 3\times 3(-1/3)(1/3)^2 - 3(2/3)(4/3)^2 - 2\times 3(-1/3)(-2/3)^2 - 3(-1)(-1)^2 - 3(0)(-1)^2 \\ &= 0. \end{aligned} \tag{A4}$$

The $U(1)_N$ self-anomaly is

$$\begin{aligned}[U(1)_N]^3 &= \sum_{\text{all fermions}}(N_{f_L}^3 - N_{f_R}^3) \\ &= 3\times 3N_{\psi_a}^3 + 3\times 3N_{Q_3}^3 + 2\times 3\times 3N_{Q_\alpha}^3 - 3\times 3N_{u_a}^3 - 3\times 3N_{d_a}^3 - 3N_U^3 - 2\times 3N_{D_\alpha}^3 - 3N_{e_a}^3 - 3N_{\nu_a}^3 \\ &= 3\times 3(-2/3)^3 + 3\times 3(2/3)^3 + 2\times 3\times 3(0)^3 - 3\times 3(1/3)^3 - 3\times 3(1/3)^3 \\ &\quad - 3(4/3)^3 - 2\times 3(-2/3)^3 - 3(-1)^3 - 3(-1)^3 = 0. \end{aligned} \tag{A5}$$

The last anomaly is given by

$$\begin{aligned}[\text{Gravity}]^2U(1)_N &\sim \sum_{\text{all fermions}}(N_{f_L} - N_{f_R}) \\ &= 3\times 3N_{\psi_a} + 3\times 3N_{Q_3} + 2\times 3\times 3N_{Q_\alpha} - 3\times 3N_{u_a} - 3\times 3N_{d_a} - 3N_U - 2\times 3N_{D_\alpha} - 3N_{e_a} - 3N_{\nu_a} \\ &= 3\times 3(-2/3) + 3\times 3(2/3) + 2\times 3\times 3(0) - 3\times 3(1/3) - 3\times 3(1/3) - 3(4/3) \\ &\quad - 2\times 3(-2/3) - 3(-1) - 3(-1) = 0. \end{aligned} \tag{A6}$$

These anomalies are only canceled when the right-handed neutrinos are included, similar to the standard model extensions with gauged $B-L$. Indeed, since $B-L=-(2/\sqrt{3})T_8+N$ with the T_8 obviously independent of anomalies, the cancellation of N anomalies is equivalent to that of $B-L$. It is noted that the 3-3-1 model with right-handed neutrinos is always free from the $U(1)_N$ anomalies because of its fermion content, while the minimal 3-3-1 model in our case is not. Also, if $U(1)_N$ is imposed in the model of [13], it is also not free from the gravitational anomaly.

APPENDIX B: DERIVATION OF *W* PARITY

The $SU(3)_L\otimes U(1)_N$ symmetry is broken down to $U(1)_{B-L}$ by the VEVs of η, ρ and χ because the charge $B-L=-(2/\sqrt{3})T_8+N$ annihilates these vacuums:

$$(B-L)\langle\eta\rangle=0, \qquad (B-L)\langle\rho\rangle=0, \qquad (B-L)\langle\chi\rangle=0. \tag{B1}$$

This is the first stage of symmetry breaking. In the second stage the $B-L$ will be broken. And, this is achieved by the VEV of ϕ since

$$(B-L)\langle\phi\rangle\neq 0. \tag{B2}$$

It is to be noted that the ϕ VEV also breaks $U(1)_N$ in the first stage. Therefore, the ϕ vacuum totally breaks the N charge.

Now, let us find an unbroken residual symmetry as a discrete subgroup of $U(1)_{B-L}$ [exactly of $SU(3)_L\otimes U(1)_N$]. It must satisfy the following condition:

$$e^{i\alpha(B-L)}\langle\phi\rangle = \langle\phi\rangle, \tag{B3}$$

where α is a parameter of the $U(1)_{B-L}$ Lie group. Because $B(\phi) = 0$, $L(\phi) = -2$, and $\langle\phi\rangle = (1/\sqrt{2})\Lambda \neq 0$, we have

$$e^{i2\alpha} = 1 = e^{i2k\pi} \Leftrightarrow \alpha = k\pi, \qquad k = 0, \pm1, \pm2, \ldots \tag{B4}$$

The subgroup that is conserved by the ϕ vacuum is obtained by the elements

$$e^{i\alpha(B-L)} = e^{ik\pi(B-L)} = (-1)^{k(B-L)} = \{1, (-1)^{3(B-L)}\}, \tag{B5}$$

which is exactly a Z_2 symmetry. When including the spin symmetry $(-1)^{2s}$, we have

$$P = (-1)^{3(B-L)+2s} \tag{B6}$$

as an exact, unbroken parity symmetry responsible for W-particles in the 3-3-1-1 model since

$$\begin{aligned} P|\text{wrong-lepton particle}\rangle &= -|\text{wrong-lepton particle}\rangle, \\ P|\text{ordinary or bilepton particle}\rangle &= +|\text{ordinary or bilepton particle}\rangle. \end{aligned} \tag{B7}$$

It is noted that, although $B-L$ is generally broken by the vacuum, it leaves a residual Z_2 symmetry that is invariant, as realized by the scalar singlet ϕ. Let us remark that if one includes a scalar sextet, as mentioned in the text, this also yields the W parity, as expected.

Finally, let us remind the reader that all the fields which develop VEVs given are even under W parity: ϕ^0, η_1^0, ρ_2^0, $\chi_3^0 \to \phi^0$, η_1^0, ρ_2^0, χ_3^0, respectively. However, the W-fields η_3^0 and χ_1^0 are odd, $\eta_3^0 \to -\eta_3^0$ and $\chi_1^0 \to -\chi_1^0$, which lead to the vanishing VEVs. The stability of a LWP is a consequence of W-parity conservation.

In the minimal 3-3-1 model and the 3-3-1 model with right-handed neutrinos (even the model of [13]), all the new particles are either ordinary or bilepton. Therefore, even if W parity is derived in those models, there is no particles that are odd under this parity.

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