Science & Technology Development Journal, 24(3):2084-2090

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Research Article

Decay of SM-like higgs boson $h \to Z\gamma$ in the 3-3-1 model with inverse seesaw neutrino masses

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ABSTRACT

The decays of the SM-like Higgs boson $h \to Z\gamma$ in the 3-3-1 model with inverse seesaw neutrino masses (331ISS) proposed in T. Phong Nguyen, et al., Phys. Rev. D 97, 0973003 (2018), are discussed. They were using well-known general results in L.T. Hue, et al., Eur. Phys. J. C 78, (2018) 885, analytic formulas for one-loop contributions are constructed. We will show that new particles predicted by the model under consideration may have significant effects on the above-mentioned decay channel of the SM-like Higgs boson. From numerical investigation, some details and properties of the decay are presented. They may be useful for comparing with the experimental results that could be detected in the future.

Key words: Extensions of electroweak Higgs sector, Electroweak radiative corrections, Neutrino mass, and mixing

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History

- Received: 2021-04-20
- Accepted: 2021-07-28
- Published: 2021-08-23

DOI : 10.32508/stdj.v24i3.2548

INTRODUCTION

After the discovery of the standard model-like (SM-like) Higgs boson particle at LHC in 2012[1], the standard model (SM) has been again confirmed its validity, although many problems are still unsolved in the SM framework. Hence, models beyond the SM have been introduced to explain them. The 3-3-1 models predict new particles, including new gauge and Higgs bosons; therefore the branching ratio (Br) of the SM-like Higgs boson decay into $Z\gamma$ is strongly affected by the above-mentioned particles. According to data in Refs.[2,3], within a mass of the Higgs boson m_h = 125.09 GeV, the branching ratio predicted by the SM is $Br(h \to Z\gamma) = 1.54 \times 10^{-3}$ ($\pm 5.7\%$). Recent experimental data indicate that the SM-like Higgs boson decay $Br(h \to \gamma\gamma)$ is well consistent with the SM's prediction, hence contributions from new physics to the above-mentioned decay must be small. In this work, we will discuss on $Br(h \to Z\gamma)$ in the 3-3-1 model with inverse seesaw neutrino masses introduced recently in Ref.[4]. Because the model under consideration contains many new particles, which contribute to the decay amplitude $h \to \gamma\gamma$ may be destructive, but all of the contributions to the amplitude decay $h \to Z\gamma$ are constructive. The branching ratio can be large in the limit of parameters in this model. Hence, these particles may affect significantly on the second channel but still, satisfy the experimental bound on the first one. In this work, the numerical results of the second decay channel are presented.

The paper is organized as follows. In the section brief review of the model, we present the overview of the 3-3-1 model with inverse seesaw neutrino masses. We also present the vertexes and their couplings which are relevant to the decay $h \to Z\gamma$. Numerical results are discussed in the next section. Finally, the summary is then given in the conclusions section.

BRIEF REVIEW OF THE MODEL

The model based on the gauge group $SU(3)_C \times SU(3)_L \times U(1)_X$ with inverse seesaw was introduced in Ref.[4]. The electric charge operator of the model is $Q = T_3 - \frac{1}{\sqrt{3}}T_s + X$, where $T_{3,8}$ are diagonal $SU(3)_L$ generators and X is the new charge of the group $U(1)_X$. Fermions, including leptons and quarks, are assigned as follows: Firstly, leptons are under the $SU(3)_L$ triplet $\psi_{aL} = (\nu_a, e_a, N_a)_L^T \sim \left(3, -\frac{1}{3}\right)$ and a right-handed charged lepton $e_{aR} \sim (1, -1)$ with a = 1,2,3. Each left-handed neutrino $N_{aL} = (N_{aR})^c$ implies a new right-handed neutrino beyond the SM. Secondly, the first two generations of quarks are under antitriplet, while the third one is under the triplet

$$\begin{array}{l} Q_{\alpha L} = (d_{\alpha L}, -u_{\alpha L}, D_{\alpha L})_L^T \sim (3^*, 0)\,,\ \alpha = 1,2, \\ Q_{3L} = (u_{3L}, d_{3L}, T_L)_L^T \sim \left(3, \frac{1}{3}\right) \\ D_{\beta r}, d_{\beta R} \sim \left(1, -\frac{1}{3}\right), u_{\alpha R}, T_R \sim \left(1, \frac{2}{3}\right), \beta = 1,2,3. \end{array} \quad (1)$$

There are totally nine electroweak (EW) gauge bosons, included in the following covariant derivative

$$D_\mu = \partial_\mu - igW_\mu^a T^a - ig_X T^9 X X_\mu,\ a = 1,2,...,8, \quad (2)$$

Cite this article : Hong T T, Nguyen T P, Hoang N L. **Decay of SM-like higgs boson $h \to Z\gamma$ in the 3-3-1 model with inverse seesaw neutrino masses**. *Sci. Tech. Dev. J.;* 24(3):2084-2090.

where W_μ^a and X_μ are the gauge bosons of $SU(3)_L$ and $U(1)_X$, respectively, $T_a = \frac{\lambda_a}{2}$ are the Gell-Mann matrices, $T^9 \equiv \frac{I}{6}$ and $\frac{1}{\sqrt{6}}$ for triplets and singlets[5].

The symmetry breaking happens in two steps: $SU(3)_L \otimes U(1)_X \xrightarrow{\omega} SU(2)_L \otimes U(1)_Y \xrightarrow{v_1, v_2} U(1)_Q$, leading to the limit $v_{1,2} \ll \omega$. To generate masses for gauge bosons and fermions, three scalar triplets are introduced as $\rho = \left(\rho_1^+, \rho^0, \rho_2^+\right)^T \sim \left(3, \frac{2}{3}\right)$, $\eta = \left(\eta_1^+, \eta^-, \eta_2^0\right) \sim \left(3, -\frac{1}{3}\right)$, and $\chi = \left(\chi_1^0, \chi^-, \chi_2^0\right) \sim \left(3, -\frac{1}{3}\right)$. The necessary vacuum expectation values for generating all tree-level quark masses are

$$\langle \rho \rangle = \left(0, \frac{v_1}{\sqrt{2}}, 0\right)^T, \ \langle \eta \rangle = \left(\frac{v_2}{\sqrt{2}}, 0, 0\right), \ \langle \eta \rangle = \left(0, 0, \frac{\omega}{\sqrt{2}}\right)^T \text{ and}$$

$$\eta_1^0 = \frac{v_2 + R_1 + iI_2}{\sqrt{2}}, \ \rho^0 = \frac{v_1 + R_2 + iI_2}{\sqrt{2}}, \quad \chi_2^0 = \frac{\omega + R_3 + iI_3}{\sqrt{2}}, \ \eta_2^0 = \frac{R_4 + iI_4}{\sqrt{2}}, \quad \chi_1^0 = \frac{R_5 + iI_5}{\sqrt{2}}. \tag{3}$$

It can be identified that

$$g = e s_W, \ \frac{g_X}{g} = \frac{3\sqrt{2} s_W}{\sqrt{3 - 4 s_W^2}}, \tag{4}$$

where e and s_W are the electric charge and sine of the Weinberg angle, $s_W^2 = 0.231$, respectively. The model includes two pairs of singly charged gauge bosons, denoted as $W^\pm$ and $Y^\pm$, defined as

$$W_\mu^\pm = \frac{W_\mu^1 \mp i W_\mu^2}{\sqrt{2}}, \ m_W^2 = \frac{g^2}{4}\left(v_1^2 + v_2^2\right), \quad Y_\mu^\pm = \frac{W_\mu^6 \pm i W_\mu^7}{\sqrt{2}}, \ m_Y^2 = \frac{g^2}{4}\left(v_1^2 + \omega^2\right). \tag{5}$$

The bosons $W^\pm$ are identified with the SM ones, leading to $v_1^2 + v_2^2 \equiv v^2 = (246 GeV)^2$. In the remainder of the text, we will consider the simple case $v_1 = v_2 = \frac{v}{\sqrt{2}} = \frac{\sqrt{2} m_W}{g}$ given in Refs.[6,7].

The most general Yukawa Lagrangian is given as follows[8]

$$\begin{aligned}
L_Y^\chi &= h_1 \bar{Q}_{3L} T_R \chi + h_{2\alpha\beta} \bar{Q}_{\alpha L} D_{\beta R} \chi^* + h.c \\
&= h_1 \left(\bar{u}_{3L} \chi_1^0 + \bar{d}_{3L} \chi_2^- + \bar{T}_L \chi_3^0\right) T_R \\
&+ h_{2\alpha\beta} \left(\bar{d}_{\alpha L} \chi_1^{0*} - \bar{u}_{\alpha L} \chi_2^+ + \bar{D}_{\alpha L} \chi_3^{0*}\right) D_{\beta R} \chi^* \\
&+ h.c. \\
L_Y^\eta &= h_{3a} \bar{Q}_{3L} u_{aR} \eta + h_{4\alpha a} \bar{Q}_{\alpha L} d_{aR} \mu^* + h.c. \\
&= h_{3a} \left(\bar{u}_{3L} \eta_1^0 + \bar{d}_{3L} \eta_2^- + \bar{T}_L \eta_3^0\right) u_{aR} \\
&+ h_{4\alpha a} \left(\bar{d}_{\alpha L} \eta_1^{0*} - \bar{u}_{\alpha L} \eta_2^+ + \bar{D}_{\alpha L} \eta_3^{0*}\right) d_{aR} \\
&+ h.c, \\
L_Y^\rho &= h_{5a} \bar{Q}_{3L} d_{aR} \rho + h_{6\alpha a} \bar{Q}_{\alpha L} u_{aR} \rho^* \\
&+ G_{ab} \bar{f}_L^a l_R^b \rho \\
&+ F_{ab} \varepsilon_{ijk} \left(\bar{f}_L\right)^{ai} \left(\bar{f}_L^C\right)^{bj} (\rho^*)^k + h.c \\
&= h_{5a} \left(\bar{u}_{3L} \rho_1^+ + \bar{d}_{3L} \rho_2^0 + \bar{T}_L \rho_3^+\right) d_{aR} \\
&+ h_{6\alpha a} \left(\bar{d}_{\alpha L} \rho_1^- - \bar{u}_{\alpha L} \rho_2^{0*} + \bar{D}_{\alpha L} \rho_3^-\right) u_{aR} \\
&+ G_{ab} \left(\bar{\nu}_L^\alpha \rho_1^+ + \bar{l}_L^\alpha \rho_2^0 + \bar{N}_L^a \rho_3^+\right) l_R^b \\
&+ F_{ab} \{ \bar{\nu}_L^\alpha \left(\left(l_C^L\right)^b \rho_3^- - \left(N_L^C\right)^b \rho_2^0\right) \\
&+ \bar{l}_L^\alpha \left(\left(N_L^C\right)^b \rho_1^- - \left(\nu_L^C\right)^b \rho_3^-\right) \\
&+ \bar{N}_L^a \left(\left(\nu_L^C\right)^b \rho_2^0 - \left(l_L^C\right)^b \rho_1^-\right) \} + h.c.
\end{aligned} \tag{6}$$

All tree-level lepton mass terms come from the following Yukawa part[8]

$$\begin{aligned}
L_l^Y &= -h_{ab}^e \bar{\Psi}_{aL} \rho e_{bR} \\
&+ h_{ab}^\nu e^{ijk} (\bar{\Psi}_{aL})_i (\psi_{bL})_j^c \rho_k^* - Y_{ab} \bar{\Psi}_{aL} \chi X_{bR} \\
&- \frac{1}{2} (\mu_X)_{ba}^* \left(\bar{X}_{aR}\right)^c X_{bR} + h.c.,
\end{aligned} \tag{7}$$

In the basis $\nu_L' = (\nu_L, N_L, (X_R)^c)^T$ and $\left(\nu_L'\right)^c = ((\nu_L)^c, (N_L)^c, X_R)^T$, Lagrangian (7) gives a neutrino mass term corresponding to a block form of the mass matrix[4], namely

$$L_{mass}^\nu = -\frac{1}{2} \bar{\nu}_L' M^{\nu+} \left(\nu_L'\right)^c + h.c., \ where \quad M^{\nu+} = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M_R \\ 0 & M_R^T & \mu_X^+ \end{pmatrix}, \tag{8}$$

where M_R is a 3×3 matrix $(M_R)_{ab} = Y_{ab} \frac{\omega}{\sqrt{2}}$, $(m_D)_{ab} \equiv \sqrt{2} h_{ab}^\nu v_1$ with $a,b = 1,2,3$. Neutrino sub-bases are denoted as $\nu_R = ((\nu_{1L})^c, (\nu_{2L})^c, (\nu_{3L})^c)^T$, $N_R = ((N_{1L})^c, (N_{2L})^c, (N_{3L})^c)^T$, and $X_L = ((X_{1R})^c, (X_{2R})^c, (X_{3R})^c)^T$. In the model under consideration, the Dirac neutrino mass matrix m_D must be antisymmetric. The matrix μ_X defined in Eq. (7) is symmetric, and it can be diagonalized by a transformation U_X:

$$U_X^T \mu_X U_X = diag\left(\mu_{X_1}, \mu_{X_2}, \mu_{X_3}\right). \tag{9}$$

The matrix U_X will be absorbed in the redefinitions the neutral fermion states X_a; hence we will consider μ_X as a diagonal matrix given in Eq. (9).
The mass matrix M^ν is diagonalized by a 9×9 unitary matrix U^ν,

$$\begin{aligned} &U^{\nu T} M^\nu U^\nu = \widehat{M}^\nu \\ &= diag\left(m_{n_1}, m_{n_2}, ..., m_{n_9}\right) \\ &= diag\left(\widehat{m}_\nu, \widehat{M}_N\right), \end{aligned} \tag{10}$$

where $m_{n_1}\,(i=1,2,...,9)$ are masses of the nine physical neutrinos states n_{iL}, namely $\widehat{m}_\nu = diag\left(m_{n_1}, m_{n_2}, m_{n_3}\right)$ corresponding to the three active neutrinos $n_{aL}\,(a=1,2,3)$ and $\widehat{M}_N = diag\left(m_{n_4}, m_{n_5}, ...m_{n_9}\right)$ corresponding the six extra neutrinos $n_{IL}\,(I=4,5,...,9)$. The ISS mechanism leads to the following approximate solution of U^ν,

$$\begin{aligned} &U^\nu = \Omega \begin{pmatrix} U_{PMNS} & O \\ O & V \end{pmatrix}, \\ &\Omega = exp\begin{pmatrix} O & R \\ -R^\dagger & V \end{pmatrix} \\ &= \begin{pmatrix} 1-\frac{1}{2}RR^\dagger & R \\ -R^\dagger & 1-\frac{1}{2}RR^\dagger \end{pmatrix} + \mathrm{O}\left(R^3\right), \end{aligned} \tag{11}$$

where

$$\begin{aligned} &R^* \simeq \left(-m_D^* M^{-1}, m_D^*\left(M_R^\dagger\right)\right)^{-1}, \\ &M \equiv M_R^* \mu_X^{-1} M_R^\dagger, \end{aligned} \tag{12}$$

$$-m_D^* M^{-1} m_D^\dagger \simeq m_\nu \equiv U_{PMNS}^* \widehat{m}_\nu U_{PMNS}^\dagger, \tag{13}$$

$$\begin{aligned} &V^* \widehat{M}_N V^\dagger \simeq \\ &M_N + \frac{1}{2} R^T R^* M_N + \frac{1}{2} M_N R^\dagger R, \\ &M_N \equiv \begin{pmatrix} 0 & M_R^* \\ M_R^\dagger & \mu_X \end{pmatrix}. \end{aligned} \tag{14}$$

The relations between the flavor and mass eigenstates are

$$\nu_L^{'} = U^\nu n_L,\ and\ \nu_L^{'} = U^\nu n_L, \tag{15}$$

where $n_L \equiv \left(n_{1L}, n_{2L}, ...n_{9L}\right)^T$ and $\left(n_L\right)^c = \left(\left(n_{1L}\right)^c, \left(n_{2L}\right)^c, ..., \left(n_{9L}\right)^c\right)^T$. The detailed calculation has been shown in Ref.[4].
The model predicts three neutral gauge bosons: the massless photon, Z boson, and Z′ boson, where Z is the SM's Z boson which was found experimentally. The relations between the original weak and physical states of the neutral gauge boson are given in Ref.[9]. In this case $s_{331} = 1,\ c_\theta = 1$, we can write

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}. \tag{16}$$

The general Higgs potential has been discussed in Ref.[10], and the simple Higgs potential of it has been studied in Ref.[11], namely

$$\begin{aligned} &V_{Higgs} = \mu_1^2\left(\eta^+\eta + \rho^+\rho\right) + \mu_2^2 \chi^+\chi \\ &+\lambda_1\left(\eta^+\eta + \rho^+\rho\right)^2 + \lambda_2\left(\chi^+\chi\right)^2 \\ &+\lambda_{12}\left(\eta^+\eta + \rho^+\rho\right)\left(\chi^+\chi\right) \\ &-\sqrt{2} f\left(\varepsilon_{ijk}\eta_i\rho_j\chi_k + h.c.\right), \end{aligned} \tag{17}$$

where f is a mass parameter and is assumed to be real. This simple potential has been used in our research because it helps to reduce independent parameters in the Higgs potential. Relations between mass eigenstates and the original states of the charged Higgs bosons are

$$\begin{aligned} &\begin{pmatrix} \rho_1^\pm \\ \eta_1^\pm \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} G_W^\pm \\ H_2^\pm \end{pmatrix}, \\ &\begin{pmatrix} \rho_2^\pm \\ \chi^\pm \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} -s_\theta & c_\theta \\ c_\theta & s_\theta \end{pmatrix} \begin{pmatrix} G_Y^\pm \\ H_1^\pm \end{pmatrix}, \\ &\begin{pmatrix} h_1^0 \\ h_2^0 \\ h_3^0 \end{pmatrix} = \begin{pmatrix} -\frac{c_\alpha}{\sqrt{2}} & -\frac{c_\alpha}{\sqrt{2}} & s_\alpha \\ \frac{s_\alpha}{\sqrt{2}} & \frac{s_\alpha}{\sqrt{2}} & c_\alpha \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix} \begin{pmatrix} R_1 \\ R_2 \\ R_3 \end{pmatrix}, \end{aligned} \tag{18}$$

where $s_\theta = \frac{v_2}{\omega}$, $G_V^\pm$ and $G_Y^\pm$ are the Goldstone bosons eaten by gauge bosons $W, Y^\pm$, respectively, $h_1^0 \equiv h$ is identified with the SM-like Higgs boson found at the LHC. The mixing parameters $s_\alpha = \sin_\alpha$, $c_\alpha = \cos_\alpha$ depend on many free parameters, but $s_\alpha \to 0$ and $c_\alpha \to 1$, when SM-like Higgs couplings are identified with those in the SM. The relevant couplings in the first term of the Lagrangian (7) are

$$-h_{ab}^e \overline{\psi_{aL}} \rho E_{bR} + h.c. \supset \frac{m_a g c_\alpha}{2 m_W} h \bar{e}_a e_a. \tag{19}$$

The couplings of Higgs and gauge bosons are contained in the kinetic terms of the Higgs bosons

$$\begin{aligned} &L_{kin}^H = \left(D_\mu \chi\right)^\dagger \left(D_\mu \chi\right) \\ &+ \left(D_\mu \rho\right)^\dagger \left(D_\mu \rho\right) + \left(D_\mu \eta\right)^\dagger \left(D_\mu \eta\right) \\ &\to g_{hvv} g_{\mu\nu} h v^{-Q\mu} v^{Q\nu}, \\ &-ig^*_{hsv} v^{-Q\mu}\left(s^{+Q}\partial_\mu h - h\partial_\mu s^{+Q}\right), \\ &ig_{hsv} v^{Q\mu}\left(s^{-Q}\partial_\mu h - h\partial_\mu s^{-Q}\right), \\ &ig_{Zss} Z^\mu\left(s^{-Q}\partial_\mu s^Q - s^Q\partial_\mu s^{-Q}\right), \\ &ig_{Zvs} Z^\mu v^{Q\nu} s^{-Q} g_{\mu\nu}, \\ &ieQA^\mu\left(s^{-Q}\partial_\mu s^Q - s^Q\partial_\mu s^{-Q}\right), \end{aligned} \tag{20}$$

where $s = H_1^{\pm}, H_2^{\pm}$. From the second line in (20) we get the relevant terms contributing to the decays $h \to Z\gamma$. Similarly, for other vertices: the couplings of three gauge bosons arise from the kinetic term in Lagrangian of the non-Abelian gauge bosons, the Higgs self-couplings of the SM-like Higgs boson arise from the Lagrangian $L_{hHH} = -V_h$. All these couplings and vertices are shown in Table 1. Here we have used a notation $\Gamma_{\mu\nu\lambda}\left(p^0, p^+, p^-\right) = \left(p^0 - p^+\right)_\lambda g_{\mu\nu} + \left(p^+ - p^-\right)_\mu g_{\nu\lambda} + \left(p^- - p^0\right)_\nu g_{\lambda\mu}$ where all momenta are incoming and $p^{0,\pm}$ are respective momenta of h and charged gauge and charged Higgs bosons.

Based on the Yukawa Lagrangians (6) and (7), the couplings of the SM-like Higgs boson with SM fermions can be determined, see also in Table 2. The notation of the Feynman rule is $-i\left(Y_{h\bar{f}f_L}P_L + Y_{h\bar{f}f_R}P_R\right)$ for each vertex $h\bar{f}f$. We note that the couplings of $u_3 d_3$ the same as the couplings of u_α and d_α, respectively.

NUMERICAL DISCUSSIONS

In the unitary gauge, one-loop diagrams contributing to the decay $h \to Z\gamma$ are shown in **Figure 1**.

The branching ratio of the decay $h \to Z\gamma$ is determined by

$$\begin{aligned} &Br^{331ISS}(h \to Z\gamma) = \frac{\Gamma^{331ISS}(h\to Z\gamma)}{\Gamma_h^{331ISS}}, \\ &\Gamma^{331ISS}(h \to Z\gamma) = \frac{m_h^3}{32\pi}\left(1 - \frac{m_Z^2}{m_h^2}\right)\left|F_{21}^{331ISS}\right|^2, \end{aligned} \quad (21)$$

where Γ_h^{331ISS} is the total decay width of the SM-like Higgs boson h and Γ_h^{331ISS} is the partial decay width predict by the 331ISS model. The form factor F_{21}^{331ISS} is written as

$$\begin{aligned} F_{21}^{331ISS} = &F_{21,f_{ijj}}^{331ISS} + F_{21,V_{ijj}}^{331ISS} \\ &+ F_{21,S_{ijj}}^{331ISS} + F_{21,VS_{jj}}^{331ISS} + F_{21,SV_{jj}}^{331ISS}, \end{aligned} \quad (22)$$

where particular contributions are derived based on the general formulas in Ref.[12], namely

$$\begin{aligned} &F_{21,f}^{331ISS} = -\frac{eQ_f N_c K_{LL,RR}^{f+}}{16\pi^2} \times \\ &\left[16\left(C_{12} + C_{22} + C2\right) + 4C_0\right],\ F_{5f}^{331ISS} = 0, \\ &F_{21,H_{1,2}^{\pm}}^{331ISS} = K_{H_1 S} \times \left[4\left(C_{12} + C_{22} + C_2\right)\right], \\ &S = H_1^{\pm}, H_2^{\pm}, \\ &F_{21,G}^{331ISS} = K_G \times [(C_{12} + C_{22} + C_2) \times \\ &\frac{2\left(4m_G^2 - m_Z^2\right)C_0}{m_G^2} + \left(8 + \frac{\left(2m_G^2 + m_h^2\right)\left(2m_G^2 - m_Z^2\right)}{m_G^4}\right)], \\ &F_{21,VSS}^{331ISS} = K_{GSS} \times [2\left(1 + \frac{-m_S^2 + m_h^2}{m_V^2}\right) \\ &\times \left(C_{12} + C_{22} + C_2\right) + 4\left(C_1 + C_2 + C_2\right)], \\ &F_{21,SVV}^{331ISS} = K_{SGG} \times [2\left(1 + \frac{-m_S^2 + m_h^2}{m_V^2}\right) \\ &\times \left(C_{12} + C_{22} + C_2\right) - 4\left(C_1 + C_2\right), \end{aligned} \quad (23)$$

where $G = W, Y$ are gauge bosons; $K_{LL,RR}^{f+} = m_f\left(Y_{hfL} \times g_{ZfL}^* + Y_{hfR} \times g_{ZfR}^*\right)$. Here we use the following notations: m_f is mass of fermion, while m_F is mass of the exotic quark. Other factors are

$$\begin{aligned} &K_{hH_{1,2}^{\pm}} = \frac{\lambda_{hH_{1,2}^{\pm}} \times g_{ZS_{ij}}}{16\pi^2}, \\ &K_G = \frac{2eg_{hG_{ij}} \times g_{ZG_{ij}}}{16\pi^2}, \\ &K_{SGG} = K_{GSS} = \frac{eg_{hG_iS_j} \times g_{ZG_iS_j}}{16\pi^2}. \end{aligned} \quad (24)$$

Here $C_{0,i,ij}$ with $i,\ j = 1,2$ are the Passarino-Veltman functions (for analytic formulas, see Ref.[12]). The signal strength of the decay when $v^2 \ll \omega^2$, that means $O\left(\frac{v^2}{\omega^2}\right)$; 0 is defined as below

$$\begin{aligned} &\mu_{Z\gamma}^{331ISS} = \\ &\frac{\sigma^{331ISS}(pp \to h)}{\sigma^{SM}(pp \to h)} \times \frac{Br^{331ISS}(h \to Z\gamma)}{Br^{SM}(h \to Z\gamma)} \\ &= \left|\frac{F_{21}^{331ISS}}{F_{21}^{SM}}\right|^2. \end{aligned} \quad (25)$$

Remind that in the SM, $Br^{SM}(h \to Z\gamma) = 1.57 \times 10^{-3}$ and $\Gamma_h^{SM} = 4.07 \times 10^{-3}$ GeV with $m_h = 125.1$ GeV.

In order to numerically investigate the decay of the SM-like Higgs boson $h \to Z\gamma$, we will use the following well-known experimental parameters as in Ref.[13]: the mass of the W boson $m_W = 80.385$ GeV, the charged lepton masses $m_e = 5 \times 10^{-4}$ GeV, $m_\mu = 0.105$ GeV, and $m_\tau = 1.776$ GeV, the SM-like Higgs mass $m_h = 125.1$ GeV, and the gauge coupling of the $SU(2)_L$ symmetry $g = 0.651$.

Combined with the discussion in Ref.[4], the independent parameters are the exotic quark $m_D = m_T$, the heavy gauge boson mass m_Y, the charged Higgs boson mass $m_{H_2^{\pm}}$, and the two Higgs selfcouplings $\lambda_{1,12}$. Other parameters can be calculated in terms of the above free ones, namely,

$$\begin{aligned} &v_1 = v_2 = \frac{\sqrt{2}m_W}{g};\ s_\theta = \frac{m_W}{m_Y\sqrt{2}};\ \omega = \frac{2m_Y}{gc_\theta} \\ &f = \frac{gc_\theta m_{H_2^{\pm}}^2}{4m_Y},\ m_{H_1^{\pm}}^2 = \frac{m_{H_2^{\pm}}^2}{2}\left(t_\theta^2 + 1\right) \end{aligned} \quad (26)$$

The Higgs self coupling λ_2 is determined as in Refs.[7,11]

$$\lambda_2 = \frac{t_\theta^2}{2}\left(\frac{m_h^2}{v_1} - \frac{m_{H_2^{\pm}}^2}{2\omega^2}\right) + \frac{\left(\lambda_{12} - \frac{m_{H_2^{\pm}}^2}{2\omega^2}\right)^2}{4\lambda_1 - \frac{m_h^2}{v_1}}. \quad (27)$$

Combining with the discussion in Ref.[4], we will be investigated in the range of $5 \times 10^4 \geq m_{H_2^{\pm}} \geq 300$ GeV,

Table 1: Couplings related to the SM-like Higgs boson decay $h \to Z\gamma$ in the 331ISS model. All momenta in the Feynman rule corresponding to these vertices are incoming

Vertex	Coupling
$\bar{h}\bar{e}_a e_a$	$\frac{igm_a}{2m_w} c_\alpha$
$H_1^+ h Y_\mu^-$	$\frac{ig}{2\sqrt{2}}\left(c_\alpha c_\theta + \sqrt{2} s_\alpha s_\theta\right)\left(p_{H_1^-} - p_h\right)^\mu$
$hW_\mu^+ W_\nu^-$	$-igm_W c_\alpha g^{\mu\nu}$
$hY_\mu^+ Y_\nu^-$	$\frac{igm_Y}{\sqrt{2}}\left(\sqrt{2} s_\alpha c_\theta - c_\alpha s_\theta\right) g^{\mu\nu}$
$hH_1^+ H_1^-$	$-i\omega\left\{s_\alpha c_\theta^2 \lambda_{12} + 2 s_\alpha s_\theta^2 \lambda_2 - \sqrt{2}\left(2c_\alpha c_\theta^2 \lambda_1 + c_\alpha c_\theta^2 \lambda_{12}\right) t_\theta + \frac{\sqrt{2} f c_\alpha c_\theta s_\theta}{\omega}\right\}$
$hH_2^+ H_2^-$	$-iv_1\left(2\sqrt{2} c_\alpha \lambda_1 + \frac{s_\alpha \omega \lambda_{12} + f s_\alpha}{v_1}\right)$
$Z_\mu H_1^+ Y_\nu^-$	$\frac{1}{4} i g^2 v_1 c_\theta \left(c_W + \frac{S_W^2}{\sqrt{3\left(3-4s_W^2\right)}}\right) g_{\mu\nu}$
$Z_\mu H_1^+ H_1^-$	$ig\left(p_{H_1^+} - p_{H_1^-}\right)_\mu \left(\frac{4\sqrt{3} c_\theta^2 s_W^2 + \left(2\sqrt{3} s_W^2 - 3c_W \sqrt{3-4s_W^2}\right) s_\theta^2}{6\sqrt{3-4s_W^2}}\right)$
$Z_\mu W^{+\nu} W^{-\lambda}$	$igc_W \Gamma_{\mu\nu\lambda}\left(p_0, p_+, p_-\right)$
$Z_\mu Y^{+\nu} Y^{-\lambda}$	$\frac{ig}{2c_W} \Gamma_{\mu\nu\lambda}\left(p_0, p_+, p_-\right)$

Table 2: The couplings of the Z boson to fermion and antifermion.

Vertex	g_L	g_R
$Z_\mu \bar{d}_\alpha d_\alpha$	$\frac{1}{2} - \frac{s_W^2 c_W}{\sqrt{3\left(3-4s_W^2\right)}}$	$-\frac{s_W^2 c_W}{\sqrt{3\left(3-4s_W^2\right)}}$
$Z_\mu \bar{\mu}_\alpha \mu_\alpha$	$-\frac{1}{2} - \frac{s_W^2 c_W}{\sqrt{3\left(3-4s_W^2\right)}}$	$-\frac{2s_W^2 c_W}{\sqrt{3\left(3-4s_W^2\right)}}$
$Z_\mu \bar{D}_\alpha D_\alpha$	$\frac{s_W^2 c_W}{\sqrt{3\left(3-4s_W^2\right)}}$	$\frac{s_W^2 c_W}{\sqrt{3\left(3-4s_W^2\right)}}$
$Z_\mu \bar{e}_a e_a$	$-\frac{1}{2} + \frac{s_W^2 c_W}{\sqrt{3\left(3-4s_W^2\right)}}$	$\frac{3s_W^2 c_W}{\sqrt{3\left(3-4s_W^2\right)}}$
$Z_\mu \bar{T}_3 T_3$	$-\frac{2s_W^2 c_W}{\sqrt{3\left(3-4s_W^2\right)}}$	$-\frac{2s_W^2 c_W}{\sqrt{3\left(3-4s_W^2\right)}}$

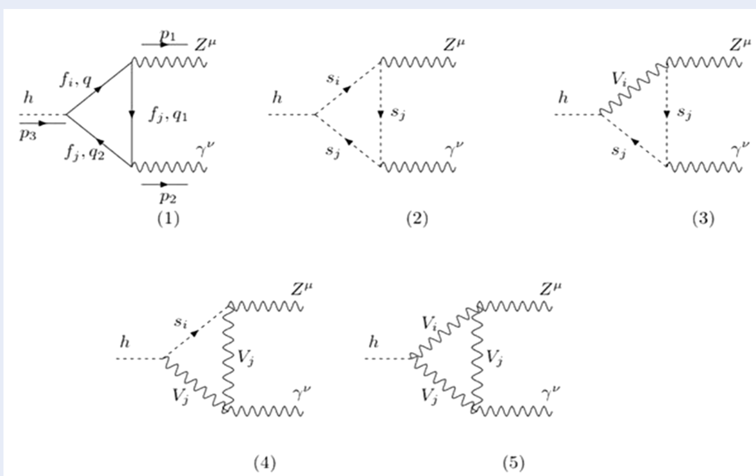

Figure 1: One-loop diagrams contributing to the decay $h \to Z\gamma$ in the unitary gauge. Here f_{ij}, s_{ij} and $V_{i,j}$ are fermions, Higgs bosons and gauge bosons, respectively.

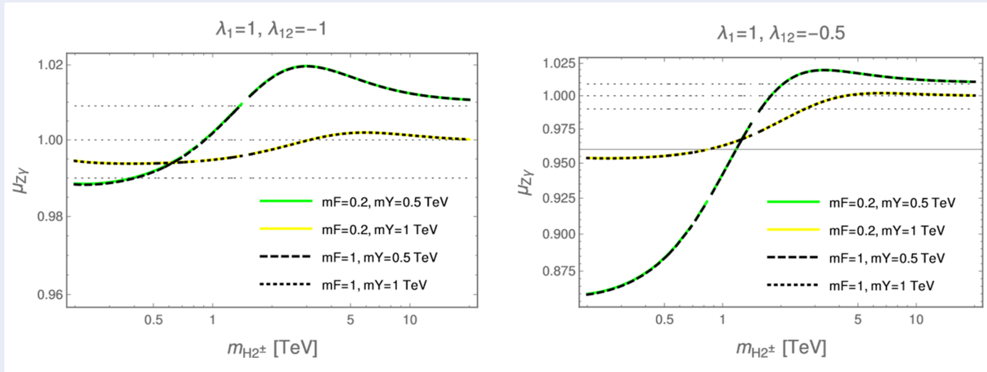

Figure 2: The signal strength $\mu_{Z\gamma}^{331ISS}$ in the 331ISS as a function of $m_{H_2^\pm}$

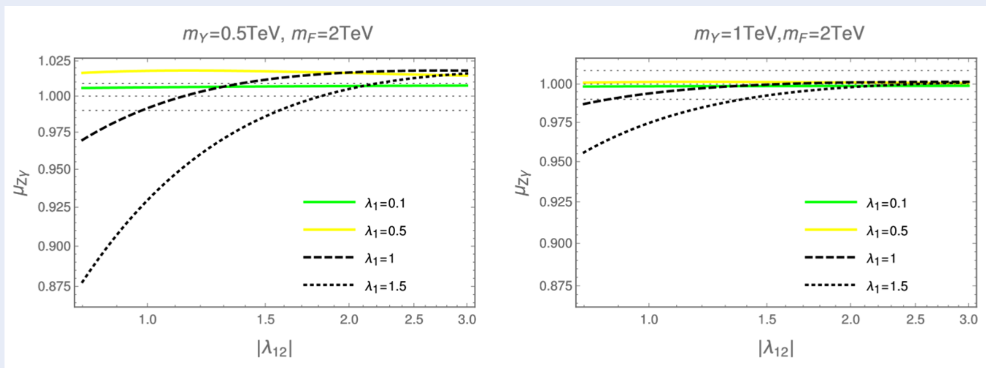

Figure 3: The signal strength $\mu_{Z\gamma}^{331ISS}$ in the 331ISS as a function of λ_{12}

$4.5 \geq m_Y \geq 0.5$ TeV, $\lambda_1 > 0, \lambda_{12} < 0$. We will investigate $\mu_{Z\gamma}^{331ISS}$ case $\lambda_1 = 1, \lambda_{12} = -1$ and $\lambda_1 = 1, \lambda_{12} = -0.5$. The signal strength $\mu_{Z\gamma}^{331ISS}$ as a function of $m_{H_2^\pm}$, the results are plotted in **Figure 2**.

From the numerical analysis, we see that the $Br(h \to Z\gamma)$ in the 331ISS is the same as the SM prediction at large $m_{H_2^+}$. On the other hand, small $m_{H_2^+}$ predicts $\mu_{Z\gamma}^{331ISS} < 1$, namely the largest deviation, is about 0.126. In the SM, the branching ratio is $Br^{SM}(h \to Z\gamma) = 1.54 \times 10^{-3} \pm 5.7\% | m_h = 125.09$ GeV [2,3], corresponding to the deviation from the SM $\mu_{Z\gamma}^{SM} = 1 \pm 0.01$ or $1.01 \geq \mu_{Z\gamma}^{SM} \geq 0.99$. The latest constraints of the signal strength for the decay $h \to Z\gamma$ is $\mu_{Z\gamma} = 1.00 \pm 0.23$ in Ref. [13]. Besides, the ATLAS expected significance to the SM-like Higgs boson channel $h \to Z\gamma$ is hoped to be 4.9σ with 300fb^{-1}. The 331ISS (BSM) predicts the values $\mu_{Z\gamma}^{331ISS}$ outside the range of SM, implying that there is a contribution of new particles. Namely, one-loop contribution from the new gauge and Higgs bosons to the decay amplitude $h \to Z\gamma$, but this value is still far from the expected sensitivity $\delta_{\mu_{Z\gamma}} = \pm 0.23$ in the HL-LHC project. If $\delta_{\mu_{Z\gamma}}$ can be detected at current colliders, they require the charged Higgs mass $m_{H_2^\pm}$ to be smaller than a TeV. Besides, $\delta_{\mu_{Z\gamma}}$ will only deviate much from SM's prediction if m_F is small enough and depends on the relationship of the Higgs self-coupling $\lambda_{1,12}$.

Let us consider the effect of self-couplings $\lambda_{1,12}$ on the signal strength $\mu_{Z\gamma}^{331ISS}$ in the 331ISS. The signal strength $\mu_{Z\gamma}^{331ISS}$ as a function of $\lambda_{1,12}$ is depicted in **Figure 3**, where m_Y was chosen to be from 1 TeV to 2 TeV and $m_{H_2^\pm} = 0.5$ TeV. The figure shows that there are two important features of $\mu_{Z\gamma}^{331ISS}$. Firstly, $\mu_{Z\gamma}^{331ISS}$ always returns the constant value when $|\lambda_{12}|$ is large enough. And secondly, $\mu_{Z\gamma}^{331ISS}$ depends very weakly on the small values of λ_1 and with large value of λ_1 will allow $\mu_{Z\gamma}^{331ISS} < 1$ and the big difference compared to that of the SM's value.

CONCLUSIONS

The signal strength of the decay $h \to Z\gamma$ has been investigated for the charged Higgs mass $m_{H_2^\pm}$ varied in the range from 100 GeV to O (10) TeV. The $Br(h \to Z\gamma)$ is the same as the SM prediction at large $m_{H_2^\pm}$. On the other hand, small $m_{H_2^\pm}$ predicts $\mu_{Z\gamma}^{331ISS} < 1$, implying that there is a new contribution of gauge bosons and charged Higgs bosons in the decay amplitude $h \to Z\gamma$. This shows that there exists an extension of the standard group for $\mu_{Z\gamma}^{331ISS}$ which takes the large values and still satisfies the experimental limit of $\mu_{\gamma\gamma}$.

ACKNOWLEDGMENT

This research is funded by the Vietnam National Foundation for Science and Technology Development (NAFOSTED) under grant number No.103.01-2019.387.

CONFLICT OF INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

AUTHORS CONTRIBUTIONS

T. T. Hong: Write the contents of the brief review of the model and the numerical discussions.

T. Phong Nguyen: Write the contents of the research literature review and the conclusion.

N. L. Hoang: Write the content of the article summary and the introduction section.

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