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Two-Way Remote Preparations of Inequivalent Quantum States Under a Common Control

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Abstract

In this paper we design a deterministic controllable quantum communication protocol for remotely preparing three-qubit GHZ-type and four-qubit W-type entangled states at two different locations at the same time. Two parties (the preparers) along with a third party (the controller) are connected through a proper multipartite entangled resource. The states to be prepared are not physically available, only the classical information of each state is known by a party who intends to remotely prepare it at the end of the other party under the same control of a third party. Concretely, it is a multi-tasking protocol in which remote preparations of the two inequivalent states are accomplished not only simultaneously but also controllably through execution of a single protocol by utilizing an eleven-qubit entangled state as shared quantum resource. We also propose a generation process of the multipartite quantum resource used.

Keywords Multiqubit quantum entanglement · Two-way remote state preparation · Common control · GHZ-type state · W-type state · Quantum measurement · Unitary operator

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1 Introduction

Quantum communication science had its origination in the work of Bennett et al. [1] in the year 1993 when a protocol for transferring a single-qubit state from one to another remote location using quantum nonlocal means assisted with local operations and classical communication was presented. It is known as quantum teleportation protocol. Subsequently the same idea was utilized for transfer of several kinds of quantum states. Today this class of protocols has a large literature developed since the publication of the pioneer work by Bennett et al. and still remains an active branch of research.

One feature of quantum teleportation is that the state to be transferred is arbitrarily unknown at least from a class of states. There is another class of protocols somewhat similar to quantum teleportation in the sense that its objective is also to produce a quantum state at a distant location but the state which is intended to be remotely created is known for the sender. These protocols are known as remote state preparation (RSP) protocols which in some cases can save the classical communication cost and reduce the complexity in measurements [2–19].

Subsequently, RSP has been made to be controllable, i.e., its execution can be supervised by parties other than the preparer and the receiver. They are referred to as controlled remote state preparation (CRSP) protocols [20–22].

All the above-mentioned protocols are designed in a one-way scenario allowing only one party, say Alice, to remotely help the other party, say Bob, to prepare a known quantum state, but not vice versa, i.e., Bob is unable to prepare his state for Alice. As a fact in every-day practice, mutual exchange of information between distant parties at equal right and under control is highly demanded. Of course, such a demand can be achieved trivially if two independent CRSP protocols are performed in parallel. However, this is naively awkward. Therefore bidirectional controlled remote state preparation (BCRSP) protocols have appeared. Here “bidirectional” implies that Alice and Bob are equally able to prepare their states for each other at the same time within a single protocol, while “controlled” means that the mutual remote state preparations are put under a common control, i.e., the same control managed by a third party, say, by name Candy. Here, for our taste, we call them two-way remote state preparation under a common control. As in a controlled protocol, Candy needs not to know any information of the states to be prepared but participates quantumly in the sense that he holds and measures his qubit which is entangled with those of the two preparers and only when he discloses his measurement outcome can the preparers fulfill their tasks. In other words, neither Alice nor Bob can reconstruct the desired state at their locations until Candy permits them to do so.

The first protocol for two-way remote state preparations under a common control [23] uses a five-qubit nonlocal resource for the simplest situation when the states of both Alice and Bob are single-qubit ones. The same situation is studied in [24, 25] via six-qubit entangled quantum channels, while an entangled state of seven qubits is utilized in [26]. Reference [27] considers two-way remote preparation under a common control of three-qubit states. The effect of noisy environment and the possibilities of experimental realizations are addressed in [28].

Of further concerns is exploration of the possible asymmetry feature in the protocol. A two-way remote state preparation protocol is regarded as asymmetric when Alice’s state and Bob’s state belong to different classes. For example, Alice may have a general single-qubit state, while Bob an arbitrary two-qubit one [29]. Or, a four-qubit cluster-type state is to be prepared by Alice for Bob, while a single-qubit one should be created for Alice by

Bob [30]. Asymmetric controlled preparation of two- and three-qubit equatorial states has also recently been reported with noise and fidelity effects taken into account [31]. Two-way protocols for hybrid tasks are of interest as well. These include situations in which in a single protocol in one direction state teleportation is carried out by Alice, while in the opposite direction Bob performs remote state preparation [32–35].

In this paper we propose a quantum protocol for three parties (two preparers Alice and Bob plus one controller Candy) who may be arbitrarily far apart from each other. Alice has a three-qubit GHZ-type state which she wishes to prepare for Bob at his location. At the same time Bob has a four-qubit W-type state which he wishes to prepare for Alice at her location. Also, such a two-way state preparation should be controllable by Candy. As Alice, Bob and Candy are distant with only local operations and classical communication allowed, the task should only be done via a suitable entanglement shared among the three parties. In the next section, Section 2, we explicitly specify the states to be prepared and the working quantum channel. In Section 3 a scheme for construction of the working quantum channel is put forward. Section 4 describes in detail the steps of our two-way asymmetric state preparation under a common control. We conclude in Section 5.

2 The States to be Prepared and the Working Quantum Channel

Let Alice's state and Bob's state be respectively of the forms

$$|\chi\rangle = \alpha_0|000\rangle + \alpha_1|111\rangle \tag{1}$$

and

$$|\tau\rangle = \beta_0|0001\rangle + \beta_1|0010\rangle + \beta_2|0100\rangle + \beta_3|1000\rangle, \tag{2}$$

where, for simplicity, all the coefficients α_0, α_1, β_0, β_1, β_2 and β_3 are assumed to be real and to satisfy the normalization conditions $\sum_{i=0}^{1} \alpha_i^2 = \sum_{i=0}^{3} \beta_i^2 = 1$. Clearly, $|\chi\rangle$ belongs to the GHZ-type class, while $|\tau\rangle$ to the W-type one (for the genuine GHZ and W states see [36]). The states (1) and (2) not only differ in the number of qubits but also are inequivalent in the sense that they cannot be converted to each other by any unitarities even stochastically or/and with ancillas [36]. Each of (1) and (2) has it own domain of significant applications in the field of quantum informatic tasks and quantum computational algorithms. In the protocol under our consideration Alice has full information of $|\chi\rangle$, *i.e.*, she knows the exact values of both the α_0 and α_1, but has no ideas about β_0, β_1, β_2 and β_3. On the other hand, Bob has full information of $|\tau\rangle$, i.e., he knows the exact values of all β_0, β_1, β_2 and β_3, but has no ideas about α_0 nor α_1. Note that here the selections for the purposes of remote state preparations are due to their necessity for some specific quantum problems that Alice and Bob have to solve. For instance, GHZ-type states have been utilized in teleportation problems in [37–41] and W-type states have been useful in quantum communication problems in [42–48]. Broadly speaking, our choices of the states stem from the general interest in transferring and remotely creating entangled quantum states which are considered as valuable quantum resources by themselves. Also, needless to remind that, unlike in quantum teleportation, in remote state preparation neither Alice nor Bob possesses their state physically. As for Candy, he has no information at all about any of the two states $|\chi\rangle$ and $|\tau\rangle$. Nevertheless, Candy serves as the powerful controller in the entire protocol and his role is pivotal as he is the one who decides the execution of the entire two-way protocol.

As a rule for any global quantum protocols, *a priori* sharing of right entangled resources is necessary. For our task, we find out that a quantum entangled state consisting of eleven qubits in the form

$$
\begin{aligned}
|Q\rangle \equiv & |Q\rangle_{a_1a_2a_3a_4a_5b_1b_2b_3b_4b_5c} \\
= & \frac{1}{4}(|00010000000\rangle + |00011111000\rangle \\
& + |01000000100\rangle + |01001111100\rangle \\
& + |00100000010\rangle + |00101111010\rangle \\
& + |10000000110\rangle + |10001111110\rangle \\
& + |10000000001\rangle - |10001111001\rangle \\
& + |00100000101\rangle - |00101111101\rangle \\
& + |01000000011\rangle - |01001111011\rangle \\
& + |00010000111\rangle - |00011111111\rangle)_{a_1a_2a_3a_4a_5b_1b_2b_3b_4b_5c},
\end{aligned} \tag{3}
$$

with the qubits a_1, a_2, a_3, a_4, a_5 held by Alice, the qubits b_1, b_2, b_3, b_4, b_5 being in the possession of Bob and the single qubit c belonging to Candy, would be a suitable working channel.

Our proposal is invalid if the working multiqubit quantum channel (3) does not really exist. So, before presenting our protocol in detail, we construct in the next section a possible quantum circuit that produces the eleven-qubit entangled state given in (3).

3 Construction of the Working Quantum Channel

Perhaps there might exist many ways to create the eleven-qubit entangled state (3) which serves as the working quantum channel for our protocol. In this section we propose one of them by means of the quantum circuit, sketched in Fig. 1.

At the beginning all the eleven qubits are set in the 'zero' state, *i.e.*, the input state to the quantum circuit reads

$$|Q_0\rangle = |00000000000\rangle_{a_4a_3a_2a_1b_5b_4a_5b_3b_2b_1c}\,. \tag{4}$$

Then, after applications of a single-qubit bit-flip gate X,

$$X\,|k\rangle_{a_4} = |k \oplus 1\rangle_{a_4}\,, \tag{5}$$

and the four Hadamard gates H_{a_5}, H_{b_4}, H_{b_5} and H_c, where

$$H\,|k\rangle_a = \frac{1}{\sqrt{2}}\Big((-1)^k\,|k\rangle + |k \oplus 1\rangle\Big)_a\,, \tag{6}$$

with $k \in \{0, 1\}$ and $\oplus$ an addition modulus 2, $|Q_0\rangle$ changes to

$$|Q_1\rangle = |1000+++000+\rangle_{a_4a_3a_2a_1b_5b_4a_5b_3b_2b_1c}\,, \tag{7}$$

with

$$|+\rangle_a = H\,|0\rangle_a \equiv \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_a. \tag{8}$$

Next, the five two-qubit controlled-X (also called controlled-NOT) gates $CX_{b_4a_1}$, $CX_{b_5a_2}$, $CX_{a_5b_1}$, $CX_{a_5b_2}$and $CX_{a_5b_3}$, where

$$CX_{ab}\,|m, n\rangle_{ab} = |m, n \oplus m\rangle_{ab}\,, \tag{9}$$

with a the control qubit and b the target one, transform $|Q_1\rangle$ to

$$|Q_2\rangle = |10\rangle_{a_4a_3}\,|B\rangle_{a_2b_5}\,|B\rangle_{a_1b_4}\,|G\rangle_{a_5b_3b_2b_1}\,|+\rangle_c\,, \tag{10}$$

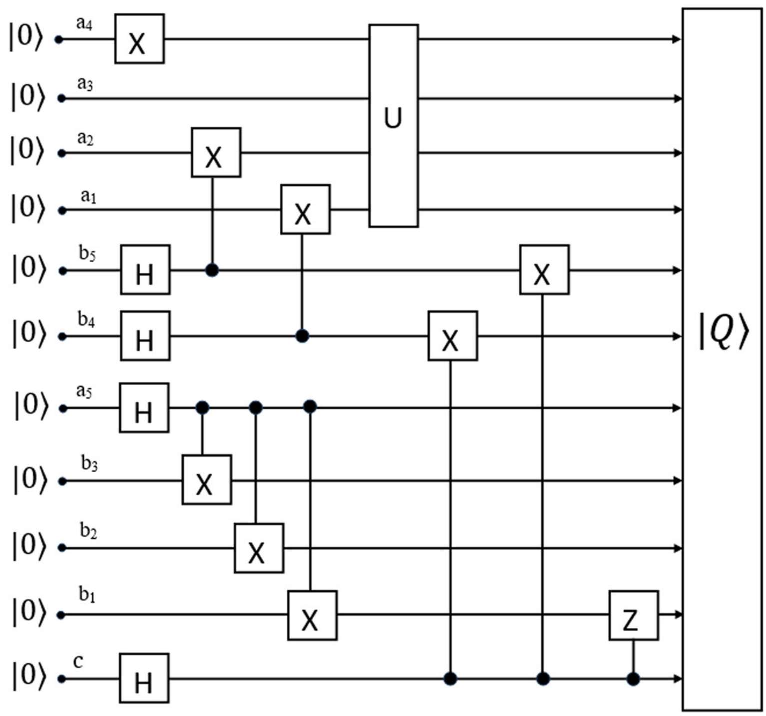

Fig. 1 Quantum circuit for production of the eleven-qubit entangled state $|Q\rangle$ given in (3). X, H and Z denote single-qubit bit-flip, Hadamard and phase-flip gates. U is the four-qubit gate defined in (23). The big black dot represents the control qubit

with

$$|B\rangle_{ab} \equiv \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)_{ab} \tag{11}$$

and

$$|G\rangle_{abcd} \equiv \frac{1}{\sqrt{2}}\left(|0000\rangle + |1111\rangle\right)_{abcd}. \tag{12}$$

Note that, for our convenience, we re-express $|Q_2\rangle$ in the form

$$\begin{aligned} |Q_2\rangle = & \left(|0001\rangle_{a_1a_2a_3a_4}|00\rangle_{b_4b_5} + |0101\rangle_{a_1a_2a_3a_4}|01\rangle_{b_4b_5}\right. \\ & \left. + |1001\rangle_{a_1a_2a_3a_4}|10\rangle_{b_4b_5} + |1101\rangle_{a_1a_2a_3a_4}|11\rangle_{b_4b_5}\right) \\ & |G\rangle_{a_5b_3b_2b_1}|+\rangle_c. \end{aligned} \tag{13}$$

The four-qubit U gate in Fig. 1 is determined by

$$U \equiv U_{a_1a_2a_3a_4} = CXXX_{a_3a_aa_2a_4}CCX_{a_1a_2a_3}CX_{a_2a_4}CX_{a_1a_4}, \tag{14}$$

where CCX_{abc} is a three-qubit controlled-controlled-X gate (also called Toffoli gate which can be composed from single-qubit gates and two-qubit CX gates [49]) with a, b the control qubits and c the target one,

$$CCX_{abc}|m, n, l\rangle_{abc} = |m, n, l \oplus mn\rangle_{abc} \tag{15}$$

while $CXXX_{abcd}$ is a four-qubit controlled-X$^{\otimes 3}$ gate with a the control qubit and b, c, d the target ones,

$$CXXX_{abcd} \, |m, n, l, h\rangle_{abcd} = |m, n \oplus m, l \oplus m, h \oplus m\rangle_{abcd} \,. \tag{16}$$

In fact, $CXXX_{abcd}$ can be regarded as a triple controlled-X gate, i.e.,

$$CXXX_{abcd} \equiv CX_{ab} CX_{ac} CX_{ad}. \tag{17}$$

In general, the action of the $U_{a_1a_2a_3a_4}$ gate on the state $|m, n, l, h\rangle_{a_1a_2a_3a_4}$, with $m, n, l, h \in \{0, 1\}$, reads

$$\begin{aligned} U_{a_1a_2a_3a_4} \, |m, n, l, h\rangle_{a_1a_2a_3a_4} &\equiv CXXX_{a_3a_aa_2a_4} CCX_{a_1a_2a_3} CX_{a_2a_4} \\ &\quad CX_{a_1a_4} \, |m, n, l, h\rangle_{a_1a_2a_3a_4} \\ &= CXXX_{a_3a_aa_2a_4} CCX_{a_1a_2a_3} \\ &\quad CX_{a_2a_4} \, |m, n, l, h \oplus m\rangle_{a_1a_2a_3a_4} \\ &= CXXX_{a_3a_aa_2a_4} CCX_{a_1a_2a_3} \\ &\quad |m, n, l, h \oplus m \oplus n\rangle_{a_1a_2a_3a_4} \\ &= CXXX_{a_3a_aa_2a_4} \\ &\quad |m, n, l \oplus mn, h \oplus m \oplus n\rangle_{a_1a_2a_3a_4} \\ &= |m \oplus l \oplus mn, n \oplus l \oplus mn, l \oplus mn, \\ &\quad h \oplus m \oplus n \oplus l \oplus mn\rangle_{a_1a_2a_3a_4}. \end{aligned} \tag{18}$$

In particular, by virtue of (18) we have explicitly

$$U_{a_1a_2a_3a_4} \, |0001\rangle_{a_1a_2a_3a_4} = |0001\rangle_{a_1a_2a_3a_4} \,, \tag{19}$$

$$U_{a_1a_2a_3a_4} \, |0101\rangle_{a_1a_2a_3a_4} = |0100\rangle_{a_1a_2a_3a_4} \,, \tag{20}$$

$$U_{a_1a_2a_3a_4} \, |1001\rangle_{a_1a_2a_3a_4} = |1000\rangle_{a_1a_2a_3a_4} \,, \tag{21}$$

$$U_{a_1a_2a_3a_4} \, |1101\rangle_{a_1a_2a_3a_4} = |0010\rangle_{a_1a_2a_3a_4} \,. \tag{22}$$

Effectively, the above transformations (19)–(22) implies

$$\begin{aligned} U_{a_1a_2a_3a_4} &= |0001\rangle_{a_1a_2a_3a_4} \langle 0001| + |0100\rangle_{a_1a_2a_3a_4} \langle 0101| \\ &\quad + |1000\rangle_{a_1a_2a_3a_4} \langle 1001| + |0010\rangle_{a_1a_2a_3a_4} \langle 1101| \,, \end{aligned} \tag{23}$$

which are represented in terms of controlled-X, controlled-controlled-X and controlled-X$^{\otimes 3}$ gates in Fig. 2. Applying $U_{a_1a_2a_3a_4}$ in (23) to $|Q_2\rangle$ we get

$$\begin{aligned} |Q_3\rangle &= (|0001\rangle_{a_1a_2a_3a_4} |00\rangle_{b_4b_5} + |0100\rangle_{a_1a_2a_3a_4} |01\rangle_{b_4b_5} \\ &\quad + |1000\rangle_{a_1a_2a_3a_4} |10\rangle_{b_4b_5} + |0010\rangle_{a_1a_2a_3a_4} |11\rangle_{b_4b_5}) \\ &\quad |G\rangle_{a_5b_3b_2b_1} |+\rangle_c \,. \end{aligned} \tag{24}$$

Finally, applications of the two controlled-X gates CX_{cb_4}, CX_{cb_5} and the two-qubit controlled-Z gate

$$CZ_{cb_1} \, |mn\rangle_{cb_1} = (-1)^{mn} \, |mn\rangle_{cb_1} \,, \tag{25}$$

on $|Q_3\rangle$ of (24) produces a output state of the quantum circuit in Fig. 1, which is nothing else but the desired eleven-qubit entangled state $|Q\rangle$ given in (3).

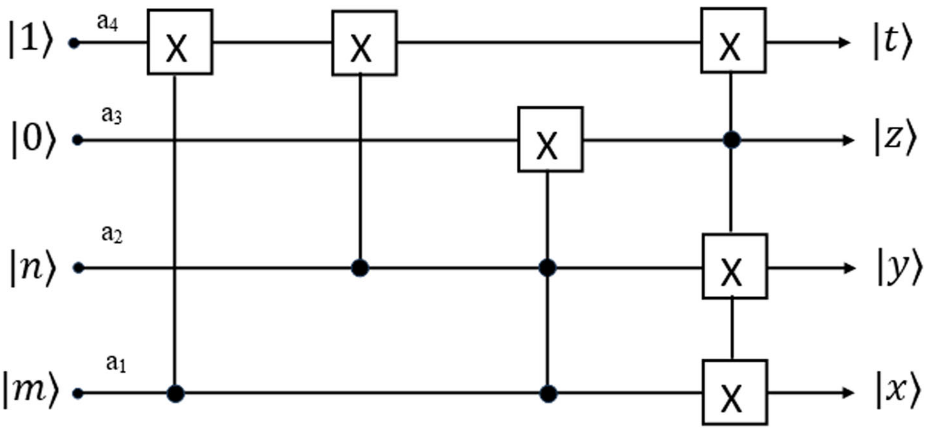

Fig. 2 Quantum circuit describing the explicit action of the four-qubit gate $U \equiv U_{a_1a_2a_3a_4}$ of (23) on a particular state $|mn01\rangle_{a_1a_2a_3a_4}$ with $m, n \in \{0, 1\}$, X the single-qubit bit-flip gate and $|x\rangle \equiv |m \oplus mn\rangle$, $|y\rangle \equiv |n \oplus mn\rangle$, $|z\rangle \equiv |mn\rangle$ and $|t\rangle \equiv |1 \oplus m \oplus n \oplus mn\rangle$. The big black dot represents the control qubit

4 Our Protocol

The production of the multiqubit entangled state $|Q\rangle$ as outlined in the previous section can be done locally in one place, no matter where. This may be at Alice's, Bob's or Candy's site or somewhere else. Yet, of importance is the right distribution of qubits among the authorized parties after the production. In order for $|Q\rangle$ to be a working quantum channel the qubits a_1, a_2, a_3, a_4, a_5 should be distributed to Alice, the qubits b_1, b_2, b_3, b_4, b_5 to Bob and the qubit c to Candy. Only after succeeding such a distribution of qubits the three parties become ready to start executing the protocol.

Thanks to the effect of 'spooky action at a distance' exhibited by the shared entanglement, Alice, Bob and Candy can independently manipulate their qubits, regardless of who does first. However, in what follows we assume that Candy will act after Alice and Bob to highlight his role as the deciding controller.

So, firstly, Alice performs a measurement on her qubit a_5 with the measuring basis $\{|\rho_i\rangle_{a_5}; i \in \{0, 1\}\}$,

$$\begin{pmatrix} |\rho_0\rangle_{a_5} \\ |\rho_1\rangle_{a_5} \end{pmatrix} = \begin{pmatrix} \alpha_0 & \alpha_1 \\ \alpha_1 & -\alpha_0 \end{pmatrix} \begin{pmatrix} |0\rangle_{a_5} \\ |1\rangle_{a_5} \end{pmatrix}, \tag{26}$$

while Bob measures his two qubits b_4, b_5 in the basis $\{|\sigma_{mn}\rangle_{b_4b_5}; m, n \in \{0, 1\}\}$,

$$\begin{pmatrix} |\sigma_{00}\rangle_{b_4b_5} \\ |\sigma_{01}\rangle_{b_4b_5} \\ |\sigma_{10}\rangle_{b_4b_5} \\ |\sigma_{11}\rangle_{b_4b_5} \end{pmatrix} = \begin{pmatrix} \beta_0 & \beta_1 & \beta_2 & \beta_3 \\ \beta_1 & -\beta_0 & \beta_3 & -\beta_2 \\ \beta_2 & -\beta_3 & -\beta_0 & \beta_1 \\ \beta_3 & \beta_2 & -\beta_1 & -\beta_0 \end{pmatrix} \begin{pmatrix} |00\rangle_{b_4b_5} \\ |01\rangle_{b_4b_5} \\ |10\rangle_{b_4b_5} \\ |11\rangle_{b_4b_5} \end{pmatrix}. \tag{27}$$

Such choices of measurement bases as in (26) and (27) are fully possible since Alice has full knowledge of α_0, α_1 and the values of $\beta_0, \beta_1, \beta_2, \beta_3$ are certainly known to Bob. Alice's finding $|\rho_i\rangle_{a_5}$ implies that her measurement outcome is $i \in \{0, 1\}$, while the outcome of Bob's measurement is denoted as $mn \in \{00, 01, 10, 11\}$, if he finds $|\sigma_{mn}\rangle_{b_4b_5}$. After their measurements Alice and Bob publicly announce the obtained outcomes imn via a reliable classical communication channel.

Since the quantum resource $|Q\rangle \equiv |Q\rangle_{a_1a_2a_3a_4a_5b_1b_2b_3b_4b_5c}$ in (3) can be represented as

$$|Q\rangle = \frac{1}{2\sqrt{2}} \sum_{i,m,n=0}^{1} |\rho_i\rangle_{a_5} |\sigma_{mn}\rangle_{b_4b_5} |\Delta_{imn}\rangle_{a_1a_2a_3a_4b_1b_2b_3c}, \tag{28}$$

obtaining the measurement outcomes imn projects the remaining (unmeasured) qubits $a_1, a_2, a_3, a_4, b_1, b_2, b_3, c$ onto the state $|\Delta_{imn}\rangle \equiv |\Delta_{imn}\rangle_{a_1a_2a_3a_4b_1b_2b_3c}$, with

$$\begin{aligned}|\Delta_{000}\rangle = \frac{1}{\sqrt{2}}\{&\alpha_0\beta_0(|00010000\rangle + |10000001\rangle) + \alpha_0\beta_1(|00100000\rangle \\ &+|01000001\rangle) + \alpha_0\beta_2(|01000000\rangle + |00100001\rangle) \\ &+\alpha_0\beta_3(|10000000\rangle + |00010001\rangle) + \alpha_1\beta_0(|00011110\rangle \\ &-|10001111\rangle) + \alpha_1\beta_1(|00101110\rangle - |01001111\rangle) \\ &+\alpha_1\beta_2(|01001110\rangle - |00101111\rangle) + \alpha_1\beta_3(|10001110\rangle \\ &-|00011111\rangle)\}_{a_1a_2a_3a_4b_1b_2b_3c},\end{aligned} \tag{29}$$

$$\begin{aligned}|\Delta_{001}\rangle = \frac{1}{\sqrt{2}}\{&\alpha_0\beta_1(|00010000\rangle + |10000001\rangle) - \alpha_0\beta_0(|00100000\rangle \\ &+|01000001\rangle) + \alpha_0\beta_3(|01000000\rangle + |00100001\rangle) \\ &-\alpha_0\beta_2(|10000000\rangle + |00010001\rangle) + \alpha_1\beta_1(|00011110\rangle \\ &-|10001111\rangle) - \alpha_1\beta_0(|00101110\rangle - |01001111\rangle) \\ &+\alpha_1\beta_3(|01001110\rangle - |00101111\rangle) - \alpha_1\beta_2(|10001110\rangle \\ &-|00011111\rangle)\}_{a_1a_2a_3a_4b_1b_2b_3c},\end{aligned} \tag{30}$$

$$\begin{aligned}|\Delta_{010}\rangle = \frac{1}{\sqrt{2}}\{&\alpha_0\beta_2(|00010000\rangle + |10000001\rangle) - \alpha_0\beta_3(|00100000\rangle \\ &+|01000001\rangle) - \alpha_0\beta_0(|01000000\rangle + |00100001\rangle) \\ &+\alpha_0\beta_1(|10000000\rangle + |00010001\rangle) + \alpha_1\beta_2(|00011110\rangle \\ &-|10001111\rangle) - \alpha_1\beta_3(|00101110\rangle - |01001111\rangle) \\ &-\alpha_1\beta_0(|01001110\rangle - |00101111\rangle) + \alpha_1\beta_1(|10001110\rangle \\ &-|00011111\rangle)\}_{a_1a_2a_3a_4b_1b_2b_3c},\end{aligned} \tag{31}$$

$$\begin{aligned}|\Delta_{011}\rangle = \frac{1}{\sqrt{2}}\{&\alpha_0\beta_3(|00010000\rangle + |10000001\rangle) + \alpha_0\beta_2(|00100000\rangle \\ &+|01000001\rangle) - \alpha_0\beta_1(|01000000\rangle + |00100001\rangle) \\ &-\alpha_0\beta_0(|10000000\rangle + |00010001\rangle) + \alpha_1\beta_3(|00011110\rangle \\ &-|10001111\rangle) + \alpha_1\beta_2(|00101110\rangle - |01001111\rangle) \\ &-\alpha_1\beta_1(|01001110\rangle - |00101111\rangle) - \alpha_1\beta_0(|10001110\rangle \\ &-|00011111\rangle)\}_{a_1a_2a_3a_4b_1b_2b_3c},\end{aligned} \tag{32}$$

$$
\begin{aligned}
|\Delta_{100}\rangle = & \frac{1}{\sqrt{2}}\{\alpha_1\beta_0(|00010000\rangle + |10000001\rangle) + \alpha_1\beta_1(|00100000\rangle \\
& +|01000001\rangle) + \alpha_1\beta_2(|01000000\rangle + |00100001\rangle) \\
& +\alpha_1\beta_3(|10000000\rangle + |00010001\rangle) - \alpha_0\beta_0(|00011110\rangle \\
& -|10001111\rangle) - \alpha_0\beta_1(|00101110\rangle - |01001111\rangle) \\
& -\alpha_0\beta_2(|01001110\rangle - |00101111\rangle) - \alpha_0\beta_3(|10001110\rangle \\
& -|00011111\rangle)\}_{a_1a_2a_3a_4b_1b_2b_3c},
\end{aligned} \tag{33}
$$

$$
\begin{aligned}
\Delta_{101}\rangle = & \frac{1}{\sqrt{2}}\{\alpha_1\beta_1(|00010000\rangle + |10000001\rangle) - \alpha_1\beta_0(|00100000\rangle \\
& +|01000001\rangle) + \alpha_1\beta_3(|01000000\rangle + |00100001\rangle) \\
& -\alpha_1\beta_2(|10000000\rangle + |00010001\rangle) - \alpha_0\beta_1(|00011110\rangle \\
& -|10001111\rangle) + \alpha_0\beta_0(|00101110\rangle - |01001111\rangle) \\
& -\alpha_0\beta_3(|01001110\rangle - |00101111\rangle) + \alpha_0\beta_2(|10001110\rangle \\
& -|00011111\rangle)\}_{a_1a_2a_3a_4b_1b_2b_3c},
\end{aligned} \tag{34}
$$

$$
\begin{aligned}
\Delta_{110}\rangle = & \frac{1}{\sqrt{2}}\{\alpha_1\beta_2(|00010000\rangle + |10000001\rangle) - \alpha_1\beta_3(|00100000\rangle \\
& +|01000001\rangle) - \alpha_1\beta_0(|01000000\rangle + |00100001\rangle) \\
& +\alpha_1\beta_1(|10000000\rangle + |00010001\rangle) - \alpha_0\beta_2(|00011110\rangle \\
& -|10001111\rangle) + \alpha_0\beta_3(|00101110\rangle - |01001111\rangle) \\
& +\alpha_0\beta_0(|01001110\rangle - |00101111\rangle) - \alpha_0\beta_1(|10001110\rangle \\
& -|00011111\rangle)\}_{a_1a_2a_3a_4b_1b_2b_3c},
\end{aligned} \tag{35}
$$

and

$$
\begin{aligned}
|\Delta_{111}\rangle = & \frac{1}{\sqrt{2}}\{\alpha_1\beta_3(|00010000\rangle + |10000001\rangle) + \alpha_1\beta_2(|00100000\rangle \\
& +|01000001\rangle) - \alpha_1\beta_1(|01000000\rangle + |00100001\rangle) \\
& -\alpha_1\beta_0(|10000000\rangle + |00010001\rangle) - \alpha_0\beta_3(|00011110\rangle \\
& -|10001111\rangle) - \alpha_0\beta_2(|00101110\rangle - |01001111\rangle) \\
& +\alpha_0\beta_1(|01001110\rangle - |00101111\rangle) + \alpha_0\beta_0(|10001110\rangle \\
& -|00011111\rangle)\}_{a_1a_2a_3a_4b_1b_2b_3c}.
\end{aligned} \tag{36}
$$

Note that even after the outcomes i and mn of Alice's and Bob's measurements are open, the two preparers remain unable to obtain the desired states because they are still in entanglement with Candy through the entangled state $|\Delta_{imn}\rangle_{a_1a_2a_3a_4b_1b_2b_3c}$. Completion of the protocol depends on Candy's decision. Before making a decision Candy carefully examines every concerned circumstances. If, even at the very last minute, Candy observes something wrong, he will stop the remote states' preparation, by doing nothing! Otherwise, he shows his powerful role as a controller by measuring the qubit c in the computational basis $\{|j\rangle_c; j = 0, 1\}$:

$$
\begin{pmatrix} |0\rangle_c \\ |1\rangle_c \end{pmatrix}. \tag{37}
$$

It can be verified that

$$|\Delta_{imn}\rangle_{a_1a_2a_3a_4b_1b_2b_3c} = \frac{1}{\sqrt{2}}\sum_{j=0}^{1}|j\rangle_c \left|\Lambda_{mnj}\right\rangle_{a_1a_2a_3a_4}\left|\Omega_{ij}\right\rangle_{b_1b_2b_3}, \tag{38}$$

where

$$|\Lambda_{000}\rangle_{a_1a_2a_3a_4} = (\beta_0|0001\rangle + \beta_1|0010\rangle + \beta_2|0100\rangle + \beta_3|1000\rangle)_{a_1a_2a_3a_4}, \tag{39}$$

$$|\Lambda_{001}\rangle_{a_1a_2a_3a_4} = (\beta_0|1000\rangle + \beta_1|0100\rangle + \beta_2|0010\rangle + \beta_3|0001\rangle)_{a_1a_2a_3a_4}, \tag{40}$$

$$|\Lambda_{010}\rangle_{a_1a_2a_3a_4} = (-\beta_0|0010\rangle + \beta_1|0001\rangle - \beta_2|1000\rangle + \beta_3|0100\rangle)_{a_1a_2a_3a_4}, \tag{41}$$

$$|\Lambda_{011}\rangle_{a_1a_2a_3a_4} = (-\beta_0|0100\rangle + \beta_1|1000\rangle - \beta_2|0001\rangle + \beta_3|0010\rangle)_{a_1a_2a_3a_4}, \tag{42}$$

$$|\Lambda_{100}\rangle_{a_1a_2a_3a_4} = (-\beta_0|0100\rangle + \beta_1|1000\rangle + \beta_2|0001\rangle - \beta_3|0010\rangle)_{a_1a_2a_3a_4}, \tag{43}$$

$$|\Lambda_{101}\rangle_{a_1a_2a_3a_4} = (-\beta_0|0010\rangle + \beta_1|0001\rangle + \beta_2|1000\rangle - \beta_3|0100\rangle)_{a_1a_2a_3a_4}, \tag{44}$$

$$|\Lambda_{110}\rangle_{a_1a_2a_3a_4} = (-\beta_0|1000\rangle - \beta_1|0100\rangle + \beta_2|0010\rangle + \beta_3|0001\rangle)_{a_1a_2a_3a_4}, \tag{45}$$

$$|\Lambda_{111}\rangle_{a_1a_2a_3a_4} = (-\beta_0|0001\rangle - \beta_1|0010\rangle + \beta_2|0100\rangle + \beta_3|1000\rangle)_{a_1a_2a_3a_4} \tag{46}$$

and

$$|\Omega_{00}\rangle_{b_1b_2b_3} = (\alpha_0|000\rangle + \alpha_1|111\rangle)_{b_1b_2b_3}, \tag{47}$$

$$|\Omega_{01}\rangle_{b_1b_2b_3} = (\alpha_0|000\rangle - \alpha_1|111\rangle)_{b_1b_2b_3}, \tag{48}$$

$$|\Omega_{10}\rangle_{b_1b_2b_3} = (-\alpha_0|111\rangle + \alpha_1|000\rangle)_{b_1b_2b_3}, \tag{49}$$

$$|\Omega_{11}\rangle_{b_1b_2b_3} = (\alpha_0|111\rangle + \alpha_1|000\rangle)_{b_1b_2b_3}. \tag{50}$$

Hence, if Candy's measurement outcome is j (i.e., $|j\rangle_c$ is found when c is measured), the state of Alice's qubits a_1, a_2, a_3, a_4 turns out to be $\left|\Lambda_{mnj}\right\rangle_{a_1a_2a_3a_4}$ and, at the same time, Bob's qubits b_1, b_2, b_3 are projected onto the state $\left|\Omega_{ij}\right\rangle_{b_1b_2b_3}$. If Candy publicly reveals his measurement outcome j, then, conditioned on $imnj$ (Alice's and Bob's measurement outcomes imn were already broadcasted before), Alice and Bob become able to recover their desired states $|\tau\rangle_{a_1a_2a_3a_4}$ and $|\chi\rangle_{b_1b_2b_3}$ from the collapsed ones $\left|\Lambda_{mnj}\right\rangle_{a_1a_2a_3a_4}$ and $\left|\Omega_{ij}\right\rangle_{b_1b_2b_3}$, respectively. This comes out from the observation that

$$|\tau\rangle_{a_1a_2a_3a_4} = A_{mnj}\left|\Lambda_{mnj}\right\rangle_{a_1a_2a_3a_4} \tag{51}$$

and

$$|\chi\rangle_{b_1b_2b_3} = B_{ij}\left|\Omega_{ij}\right\rangle_{b_1b_2b_3}, \tag{52}$$

where

$$A_{mnj} = U_{a_1a_2a_3a_4}X_{a_1}^{m\oplus j}Z_{a_1}^{m}X_{a_2}^{n\oplus j}Z_{a_2}^{m\oplus n}U_{a_1a_2a_3a_4}^{\dagger} \tag{53}$$

and

$$B_{ij} = X_{b_1}^{i}Z_{b_1}^{i\oplus j}X_{b_2}^{i}Z_{b_2}^{i\oplus j}X_{b_3}^{i}Z_{b_3}^{i\oplus j} \tag{54}$$

with $U_{a_1a_2a_3a_4}^{\dagger}$ the complex conjugate with $U_{a_1a_2a_3a_4}$ in (23) while X_a and Z_a the bit-flip and phase-flip operators acting on qubit a. Thus, for the correct recovery Alice needs just to apply the operator A_{mnj} on $|\Lambda_{mnj}\rangle_{a_1a_2a_3a_4}$, while Bob applies the operator B_{ij} on $\left|\Omega_{ij}\right\rangle_{b_1b_2b_3}$.

As an illustration, consider a particular case with $i=1, m=1, n=0$ and $j=1$. If so, the collapsed states at Alice's and Bob's locations are $|\Lambda_{101}\rangle_{a_1a_2a_3a_4}$ and $|\Omega_{11}\rangle_{b_1b_2b_3}$, whose explicit forms are given in (44) and (50), respectively. As for the corresponding recovery operators, we have

$$A_{101} = U_{a_1a_2a_3a_4} Z_{a_1} X_{a_2} Z_{a_2} U^{+}_{a_1a_2a_3a_4} \tag{55}$$

and

$$B_{11} = X_{b_1} X_{b_2} X_{b_3}. \tag{56}$$

Then, straightforward calculations yield

$$\begin{aligned}
& A_{101}\,|\Lambda_{101}\rangle_{a_1a_2a_3a_4} \\
&= U_{a_1a_2a_3a_4} Z_{a_1} X_{a_2} Z_{a_2} U^{+}_{a_1a_2a_3a_4} \\
&\quad (-\beta_0|0010\rangle + \beta_1|0001\rangle + \beta_2|1000\rangle - \beta_3|0100\rangle)_{a_1a_2a_3a_4} \\
&= U_{a_1a_2a_3a_4} Z_{a_1} X_{a_2} Z_{a_2} \\
&\quad (-\beta_0|0100\rangle + \beta_1|0000\rangle + \beta_2|1100\rangle - \beta_3|1000\rangle)_{a_1a_2a_3a_4} \\
&= U_{a_1a_2a_3a_4} (\beta_0|0000\rangle + \beta_1|0100\rangle + \beta_2|1000\rangle + \beta_3|1100\rangle)_{a_1a_2a_3a_4} \\
&= \beta_0|0001\rangle + \beta_1|0010\rangle + \beta_2|0100\rangle + \beta_3|1000\rangle)_{a_1a_2a_3a_4} \\
&= |\tau\rangle_{a_1a_2a_3a_4}
\end{aligned} \tag{57}$$

Table 1 The explicit operators A_{mnj} by (53) and B_{ij} by (54) for Alice and Bob to recover the desired W-type state $|\tau\rangle_{a_1a_2a_3a_4}$ of the form (2) and the GHZ-type state $|\chi\rangle_{b_1b_2b_3}$ of the form (1), conditioned on Alice's, Bob's and Candy's measurement outcomes i, mn and j, respectively

Case #	i, mn, j	A_{mnj}	B_{ij}
1	0,00,0	$U_{a_1a_2a_3a_4} I_{a_1} I_{a_2} U^{+}_{a_1a_2a_3a_4}$	$I_{b_1} I_{b_2} I_{b_3}$
2	0,00,1	$U_{a_1a_2a_3a_4} X_{a_1} X_{a_2} U^{+}_{a_1a_2a_3a_4}$	$Z_{b_1} Z_{b_2} Z_{b_3}$
3	0,01,0	$U_{a_1a_2a_3a_4} X_{a_2} Z_{a_2} U^{+}_{a_1a_2a_3a_4}$	$I_{b_1} I_{b_2} I_{b_3}$
4	0,01,1	$U_{a_1a_2a_3a_4} X_{a_1} Z_{a_2} U^{+}_{a_1a_2a_3a_4}$	$Z_{b_1} Z_{b_2} Z_{b_3}$
5	0,10,0	$U_{a_1a_2a_3a_4} X_{a_1} Z_{a_1} Z_{a_2} U^{+}_{a_1a_2a_3a_4}$	$I_{b_1} I_{b_2} I_{b_3}$
6	0,10,1	$U_{a_1a_2a_3a_4} Z_{a_1} X_{a_2} Z_{a_2} U^{+}_{a_1a_2a_3a_4}$	$Z_{b_1} Z_{b_2} Z_{b_3}$
7	0,11,0	$U_{a_1a_2a_3a_4} X_{a_1} Z_{a_1} X_{a_2} U^{+}_{a_1a_2a_3a_4}$	$I_{b_1} I_{b_2} I_{b_3}$
8	0,11,1	$U_{a_1a_2a_3a_4} Z_{a_1} U^{+}_{a_1a_2a_3a_4}$	$Z_{b_1} Z_{b_2} Z_{b_3}$
9	1,00,0	$U_{a_1a_2a_3a_4} I_{a_1} I_{a_2} U^{+}_{a_1a_2a_3a_4}$	$X_{b_1} Z_{b_1} X_{b_2} Z_{b_2} X_{b_3} Z_{b_3}$
10	1,00,1	$U_{a_1a_2a_3a_4} X_{a_1} X_{a_2} U^{+}_{a_1a_2a_3a_4}$	$X_{b_1} X_{b_2} X_{b_3}$
11	1,01,0	$U_{a_1a_2a_3a_4} X_{a_2} Z_{a_2} U^{+}_{a_1a_2a_3a_4}$	$X_{b_1} Z_{b_1} X_{b_2} Z_{b_2} X_{b_3} Z_{b_3}$
12	1,01,1	$U_{a_1a_2a_3a_4} X_{a_1} Z_{a_2} U^{+}_{a_1a_2a_3a_4}$	$X_{b_1} X_{b_2} X_{b_3}$
13	1,10,0	$U_{a_1a_2a_3a_4} X_{a_1} Z_{a_1} Z_{a_2} U^{+}_{a_1a_2a_3a_4}$	$X_{b_1} Z_{b_1} X_{b_2} Z_{b_2} X_{b_3} Z_{b_3}$
14	1,10,1	$U_{a_1a_2a_3a_4} Z_{a_1} X_{a_2} Z_{a_2} U^{+}_{a_1a_2a_3a_4}$	$X_{b_1} X_{b_2} X_{b_3}$
15	1,11,0	$U_{a_1a_2a_3a_4} X_{a_1} Z_{a_1} X_{a_2} U^{+}_{a_1a_2a_3a_4}$	$X_{b_1} Z_{b_1} X_{b_2} Z_{b_2} X_{b_3} Z_{b_3}$
16	1,11,1	$U_{a_1a_2a_3a_4} Z_{a_1} U^{+}_{a_1a_2a_3a_4}$	$X_{b_1} X_{b_2} X_{b_3}$

I_{b_1}, I_{b_2}, I_{b_3} are the identity operators, while $U_{a_1a_2a_3a_4}$ is defined in (23) and

$$\begin{aligned} B_{11}\,|\Omega_{11}\rangle_{b_1b_2b_3} &= X_{b_1}X_{b_2}X_{b_3}(\alpha_0|111\rangle + \alpha_1|000\rangle)_{b_1b_2b_3} \\ &= (\alpha_0|000\rangle + \alpha_1|111\rangle)_{b_1b_2b_3} \\ &= |\chi\rangle_{b_1b_2b_3}\,, \end{aligned} \tag{58}$$

as should be. As an explicit summary of the obtained results, we provide in Table 1 the required recovery operators A_{mnj} and B_{ij} of Alice and Bob conditioned on the measurement outcomes i, mn and j of Alice, Bob and Candy, respectively. The steps of our protocol are displayed in Fig. 3.

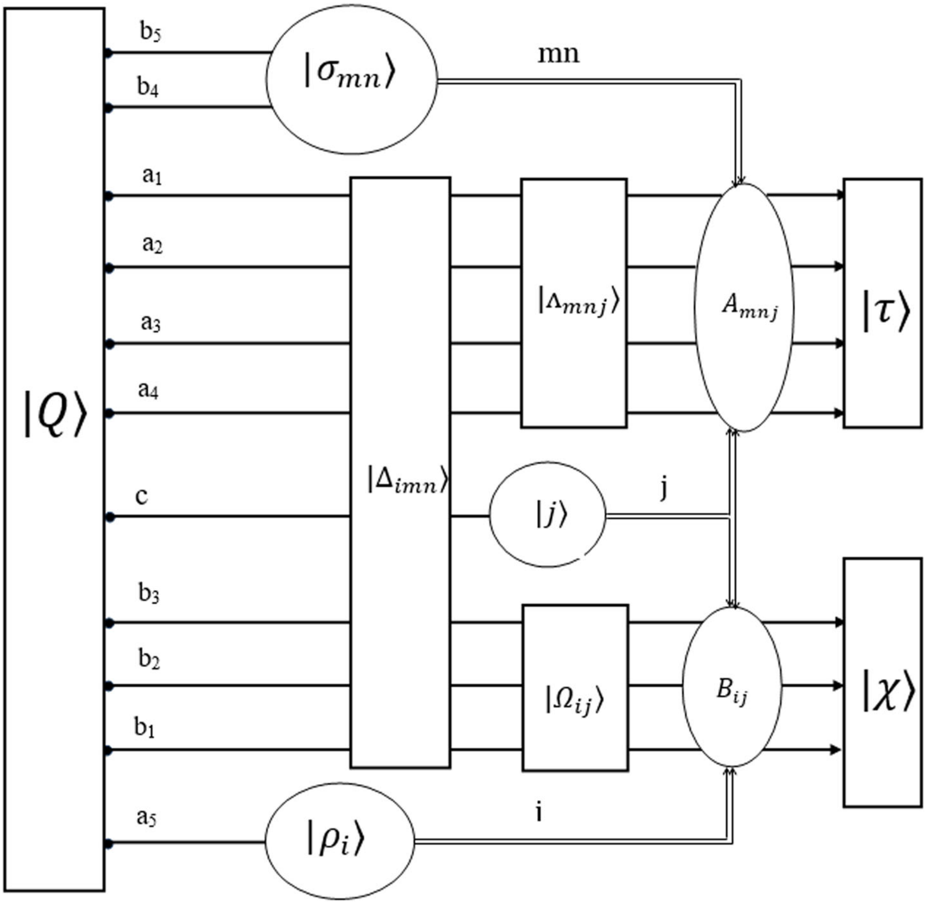

Fig. 3 Schematic diagram for executing our protocol of two-way remote preparations of two inequivalent states, the GHZ-type and the W-type ones, under a common quantum control via the eleven-qubit quantum channel $|Q\rangle$ in (3). The qubits a_1, a_2, a_3, a_4, a_5 and b_1, b_2, b_3, b_4, b_5 are respectively held by the preparers Alice and Bob, while the qubit c is in the possession of the controller Candy. The states $|\chi\rangle$, $|\tau\rangle$, $|\rho_i\rangle$, $|\sigma_{mn}\rangle$ and $|j\rangle$ are defined by (1), (2), (26), (27) and (37), respectively. The measurement-outcome-dependent recovery operators A_{mnj} and B_{ij} are determined by (53) and (54). The double lines represent reliable classical communications with i, mn and j being classical bits specifying the measurement outcomes of Alice, Bob and Candy, respectively

5 Conclusion

We simultaneously perform two tasks of quantum state preparation at two different locations where the two states are of different nature: one belongs to the three-qubit GHZ-type and the other to the four-qubit W-type. There can be a comparison with the asymmetric bidirectional teleportation schemes like those due to Nie et al. [50] and Choudhury et al. [51]. In those works, like in the present one, multi-particle entanglement resources are used for multi-tasking purposes. One practical difficulty for such kinds of schemes is that the shared quantum resources, that is, the suitable multi-particle entanglements are not easy to produce in general. However, in the present paper we succeed in proposing the generation process of the quantum resource we need for our purpose. Although measurement outcomes occur probabilistically our protocol is deterministic because for every set of possible outcomes there exist recovery operators which we are able to determine in terms of elementary quantum gates through explicit analytical formulae given by , (53) and (54).

The difference of the approaches adopted in the present work with those of [50, 51] is that in both the above mentioned works there have been exchanges of states which were physically in possessions of the two parties. But here the desired states are created at two distant ends not by exchange since none of the parties, neither Alice nor Bob, are in physical possessions of the states initially. They only have access to classical information about the respective states which they finally want to produce at the end of the other party by the execution of one and the same protocol controllable by the common controller Candy. The role of the controller Candy is indispensable. Even if the outcomes of Alice's and Bob's measurements are publicly revealed, the two parties remain unable to obtain the desired states because they are still entangled with Candy. The fulfillment of the intended tasks is at the discretion of Candy who can, if he wants, withhold the completion of the protocol, by not acting. Only when Candy cooperates (by measuring his qubit and disclosing his measurement outcome) can Alice and Bob complete their tasks. Remarkably the controller has no prior information of the states to be prepared.

The main cost to pay for our protocol is the ability to produce the eleven-qubit entanglment (3). The cost goes to the number and kind of gates to be used which, as seen from Fig. 1, include one X-gate, one Z -gate, three H-gate, seven $CNOTs$ and one U-gate (23), which is a particular case of the general U-gate defined by (14). Here the X-, Z- and H-gate are single-qubit which are simple to implement and relatively cheap. As for the general four-qubit U-gate (14), it is composed of two CX-gate (9), one CCX-gate (15) and one $CXXX$-gate (16). While the CX-gate is nothing else but the controlled-NOT ($CNOT$), the CXX-gate is the so-called Toffoli gate constructed by combiming single-qubit gates with $CNOTs$ and the $CXXX$-gate is in fact just a product of three $CNOTs$ (see (17)). Since general single-qubit gates plus $CNOTs$ constitute a minumum set of gates for any unitary operations and such set is regarded as off-the-shelf toolkits within the state-of-the-art technology, implementation of quantum circuit in Fig. 1 for production of the desired entangled state (3) is, though expensive, experimentally feasible.

Concerning the classical communication cost (CCC), Alice needs three bits (two from Bob and one from Candy) to reconstruct the four-qubit W-type state $|\tau\rangle_{a_1a_2a_3a_4}$, while Bob needs two bits (one from Alice and one from Candy) to reconstruct the three-qubit GHZ-type state$|\chi\rangle_{b_1b_2b_3}$. Because the bit from Candy is co-used by both Alice and Bob, the total CCC in our protocol is 4 bits.

Like in other quantum communication protocols, in ours the relevant entanglement (3) is assumed having been successfully shared in advance among Alice, Bob and Candy. Therefore, the security is unconditional because only the authorized parties who hold the right qubits are able to obtain the to-be-remotely-prepared states but any outsider cannot just by listening to the publicly published bits which are random quantum measurement outcomes containing no information at all for the outsider. Several security aspects pertaining to quantum communication protocols can be found, e.g., in works [28, 52, 53].

Lastly, the efficiency of a protocol is given by the formula $\eta = \frac{q_s}{q_u+b_t}$, where q_s is the number of qubits that consist of the quantum information shared, q_u is the number of qubits that is used in the quantum channel (except for those chosen for security checking) and b_t is the number of classical bits transmitted. According to the above formula [54, 55], the efficiency of our protocol is $\eta = \frac{(3+4)}{(11+4)} = \frac{7}{15} \approx 0.467$.

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