



$U(1)_{B-L}$ extension of the standard model with S_3 symmetry

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Abstract We propose a renormalizable $B - L$ Standard Model (SM) extension based on S_3 symmetry which successfully accommodates the observed fermion mass spectra and flavor mixing patterns as well as the CP violating phases. The small masses for the light active neutrinos are generated through a type I seesaw mechanism. The obtained physical parameters in the lepton sector are well consistent with the global fit of neutrino oscillations (Esteban et al. in J High Energy Phys 01:106, 2019) for both normal and inverted neutrino mass orderings. The model also predicts effective neutrino mass parameters of $\langle m_{ee} \rangle = 1.02 \times 10^{-2}$ eV, $m_\beta = 1.25 \times 10^{-2}$ eV for normal hierarchy (NH) and $\langle m_{ee} \rangle = 5.03 \times 10^{-2}$ eV, $m_\beta = 5.05 \times 10^{-2}$ eV for inverted hierarchy (IH) which are all well consistent with the future large and ultra-low background liquid scintillator detectors which has been discussed in Ref. (Zhao et al. in Chin Phys C 41(5):053001, 2017) or the limit of the effective neutrino mass can be reached by the planning of future experiments. The model results are consistent with and successfully accommodate the recent experimental values of the physical observables of the quark sector, including the six quark masses, the quark mixing angles and the CP violating phase in the quark sector.

1 Introduction

Although highly successful in describing most of the elementary particle phenomena at low energy scale, the SM leaves many unresolved issues such as the large hierarchy of charged fermion masses, the fermion mixings, and the

CP-violating phases, etc. Therefore, these issues can be considered as important evidences of physics beyond the SM.

Among the possible extensions of the SM, the one with an extra $U(1)_{B-L}$ gauge symmetry is one of promising extensions which has been considered in Refs. [1–8]. In this type of model, the presence of three right-handed neutrinos is essential to cancel the gauge and the mixed gauge-gravitational anomalies. However, it does not provide a natural explanation for fermion mixings.

Experimentally, the best-fit values for neutrino squared mass splittings, leptonic mixing angles and the Dirac CP violating phase, for both normal and inverted mass hierarchies with Super-Kamiokande atmospheric (wSK-atm) neutrino data, are given in Ref. [9]:

$$\begin{aligned} \Delta m_{21}^2 &= 7.39 \times 10^{-5} \text{eV}^2, \quad \sin^2 \theta_{12} = 0.310, \\ \sin^2 \theta_{23} &= 0.582, \\ \Delta m_{31}^2 &= 2.525 \times 10^{-3} \text{eV}^2 \text{ (NH)}, \\ \sin^2 \theta_{13} &= 0.02240 \text{ (NH)}, \quad \delta = 217^\circ \text{ (NH)}, \\ \Delta m_{32}^2 &= -2.512 \times 10^{-3} \text{eV}^2 \text{ (IH)}, \\ \sin^2 \theta_{13} &= 0.02263 \text{ (IH)}, \quad \delta = 280^\circ \text{ (IH)}. \end{aligned} \quad (1)$$

The discrete symmetry has revealed many outstanding advantages in explaining the observed pattern of SM fermionic masses and mixing angles given in Eq. (1). There have been many models based on discrete symmetries, see for instance, Refs. [10–51]. However, in most of these papers, the fermion masses and mixings are generated from non-renormalizable interactions or at loop levels. The renormalizable $B - L$ model with S_3 symmetry was first proposed by Gómez-Izquierdo and Mondragón [52], however, there are substantial differences between the work in Ref. [52] and our current work as follows¹: (a) Ref. [52] contains two other sym-

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¹ Another difference is that our current work is done with the complex representation of S_3 while in Ref. [52] the real representation is used.

Table 1 Fermion and scalar contents of the model and the charge assignment ($\alpha = 2, 3$)

Fields	ψ_{1L}	$\psi_{\alpha L}$	l_{1R}	$l_{\alpha R}$	$\nu_{\alpha R}$	ν_{1R}	Q_{1L}	$Q_{\alpha L}$	u_{1R}	$u_{\alpha R}$	d_{1R}	$d_{\alpha R}$	H	H'	H''	ϕ
$SU(2)_L$	2	2	1	1	1	1	2	2	1	1	1	1	2	2	2	1
$U(1)_{B-L}$	-1	-1	-1	-1	-1	-1	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0	0	0	2
S_3	<u>1</u>	<u>2</u>	<u>1</u>	<u>2</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>	<u>1</u>	<u>2</u>	<u>1</u>	<u>2</u>	<u>1</u>	<u>1'</u>	<u>2</u>	<u>2</u>
Z_4	i	i	i	i	i	i	i	$-i$	i	$-i$	i	$-i$	1	1	-1	-1

metries Z_2 and Z_2^c compared to S_3 symmetry where the couplings $\bar{L}H e_R$ and $\bar{L}\tilde{H}N$ are prevented by Z_2^c to obtain diagonal charged and neutrinos Dirac mass matrices but it does not modify the Majorana mass matrix form; (b) in Ref. [52], the Yukawa coupling constant y_2^q is assumed equal to zero ($a_q = 0$) which corresponds to an extra discrete symmetry (differs from the existing symmetries Z_2 , Z_2^c and S_3) is included to prohibit the Yukawa interaction term $\bar{Q}_{1L}H_3d_{1R} + \bar{Q}_{2L}H_3d_{2R}$; (c) in Ref. [52] the inverted and degenerate neutrino mass hierarchies were considered, however, the recent experimental data [9,53] favors the inverted or normal hierarchy; (d) The Dirac CP-violating phase was also not predicted by [52]; (e) The quark mass matrices $\mathbb{M}_{u,d}$ in our model seem simpler in the sense that two entries “23” and “32” are vanished.

The aim of this paper is to construct the renormalizable $B - L$ extension of the SM. In this work, the first generation of quarks and leptons are put in 1 while the two others are put in 2 under S_3 symmetry. This paper is organized as follows. In Sect. 2 we present a simple SM extension by adding $U(1)_{B-L}$ and S_3 symmetries. In Sect. 3 we present the lepton sector of the model and introduce necessary Higgs fields responsible for lepton masses and mixings. Section 4 deals with quark mass and mixing. The implications of our model in the Higgs diphoton decay rate are analyzed in Sect. 5. We conclude in Sect. 6. Appendix A provides a brief description of the Clebsch–Gordan coefficients for the S_3 group.

2 The model

The electroweak sector of the SM is supplemented by the $S_3 \otimes Z_4$ flavor symmetry and a gauge symmetry $U(1)_{B-L}$. To the particle content, we add three right-handed neutrinos (ν_{iR}), two $SU(2)_L$ scalar doublets H', H'' with $B - L = 0$ put in 1' and 2 under S_3 , respectively, and one flavon ϕ with $B - L = 2$ put in 2 under S_3 . The particle content of the model is then given in Table 1.

Since there are many Yukawa couplings in Higgs potential, it is easy to arrange a suitable Higgs potential. In order to generate the remarkable fermion mixing pattern, from the potential minimization conditions of the Higgs potential as presented in Appendix B, we choose the VEVs of scalar fields as follows

$$\langle H \rangle = (0 v_H)^T, \langle H' \rangle = (0 v_{H'})^T, \langle H'' \rangle = (\langle H''_1 \rangle, \langle H''_2 \rangle), \langle H''_1 \rangle = (0 v_{H''})^T, \langle \phi \rangle = (\langle \phi_1 \rangle, \langle \phi_2 \rangle), \langle \phi_1 \rangle = v_\phi. \quad (2)$$

In order to show that the alignment in Eq. (2) is a automatical solution from the minimization conditions of V_{total} , in the system of minimization equations, let us put $v_H^* = v_H$, $v_{H'}^* = v_{H'}$, $v_{H''}^* = v_{H''}$ and $v_\phi^* = v_\phi$, which reduces to

$$\mu_H^2 + 2\lambda_H v_H^2 + (\lambda_1^{HH'} + \lambda_2^{HH'})v_{H'}^2 + 2(\lambda_1^{HH''} + \lambda_2^{HH''})v_{H''}^2 + 2(\lambda_1^{H\phi} + \lambda_2^{H\phi})v_\phi^2 = 0, \quad (3)$$

$$\mu_{H'}^2 + (\lambda_1^{HH'} + \lambda_2^{HH'})v_H^2 + 2[\lambda_1^{H'H''} + \lambda_2^{H'H''})v_{H''}^2 + (\lambda_1^{H'\phi} + \lambda_2^{H'\phi})v_\phi^2] = 0, \quad (4)$$

$$\mu_{H''}^2 + (\lambda_1^{HH''} + \lambda_2^{HH''})v_H^2 + (\lambda_1^{H'H''} + \lambda_2^{H'H''})v_{H'}^2 + 2(2\lambda_1^{H''} + \lambda_3^{H''})v_{H''}^2 + (2\lambda_1^{H''\phi} + \lambda_3^{H''\phi} + 2\lambda_4^{H''\phi} + \lambda_6^{H''\phi})v_\phi^2 - \mu_{S_B}^2 = 0, \quad (5)$$

$$\mu_\phi^2 + (\lambda_1^{H\phi} + \lambda_2^{H\phi})v_H^2 + (\lambda_1^{H'\phi} + \lambda_2^{H'\phi})v_{H'}^2 + (2\lambda_+^{H''\phi}\lambda_3^{H''\phi} + 2\lambda_4^{H''\phi} + \lambda_6^{H''\phi})v_{H''}^2 + 2(2\lambda_1^\phi + \lambda_3^\phi)v_\phi^2 = 0. \quad (6)$$

The system of Eqs. (3)–(6) always give the solution $(v_H, v_{H'}, v_{H''}, v_\phi)$ as chosen in Eq. (2). It is noted that this aligned is only one solution to have the desirable results.

Note for the low energy observables, but lowering that scale, for example at the TeV scale, will imply lower values for the neutrino Yukawa couplings needed to reproduce the tiny values of the light active neutrino masses. The scale of breaking of the $B - L$ symmetry does not have implications in the quark sector given that the only scalar field charged under this symmetry is a gauge singlet that only appear in the Yukawa terms for the right handed Majorana neutrinos. Independently on the magnitude of the $B - L$ breaking scale, the vacuum expectation values of the Higgs doublets should satisfy the relation $v_H^2 + v_{H'}^2 + 2v_{H''}^2 = v^2$, with $v = 246$ GeV, which is needed to correctly reproduce the measured values of the W and Z gauge boson masses. If the $B - L/S_3$ breaking scale is very high, some collider signatures that can be tested in the model are: the production of neutral scalar via gluon fusion mechanism, the associated production of a neutral scalar with a SM gauge boson (via Vector Boson Fusion or Drell–Yan mechanism), the associated production of a charged scalar with a SM gauge boson

(via Vector Boson Fusion or Drell–Yan mechanism). Furthermore, flavor violating decay like $t \rightarrow hc$ (h being the 126 GeV Higgs), $h \rightarrow \mu\tau$, $\mu \rightarrow e\gamma$ can be used to constrain the model parameter space. Some low energy observables that can be used to constrain the model are the muon and electron anomalous magnetic moment, the oblique T and S parameters. Rather than avoiding, to make such process below the current experimental limits, some restrictions on the model parameter space need to be imposed, which could be constraints on the Yukawa couplings, on the mixing angles in the scalar sector and large masses for non SM scalars. Besides that, notice that our model has many free parameters, which allows us freedom to assume that the remaining scalars are sufficiently heavy to fulfill the current experimental bounds. Moreover, the loop effects of the heavy scalars contributing to precision observables can be suppressed by making an appropriate choice of the free parameters in the scalar potential. These adjustments do not affect the physical observables in the quark and lepton sectors, which are determined mainly by the Yukawa couplings. A detailed study of the phenomenology of the model is beyond the scope of this paper and is deferred for a future work.

3 Lepton masses and mixings

With the fermion content in Table 1 and the tensor products of S_3 group in Appendix A, the charged lepton masses can arise from the couplings of $\bar{\psi}_{(\alpha,1)L}l_{(\alpha,1)R}$ to scalars, where under $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L} \times S_3 \times Z_4$ symmetry, $\bar{\psi}_{1L}l_{1R}$ transforms as $(\mathbf{1}, \mathbf{2}, -1/2, 0, \underline{1}, 1)$. Similarly, we have $\bar{\psi}_{\alpha L}l_{1R} \sim (\mathbf{1}, \mathbf{2}, 1/2, 0, \underline{2}, 1)$, $\bar{\psi}_{1L}l_{\alpha R} \sim (\mathbf{1}, \mathbf{2}, 1/2, 0, \underline{2}, 1)$ and $\bar{\psi}_{\alpha L}l_{\alpha R} \sim (\mathbf{1}, \mathbf{2}, 1/2, 0, \underline{1} + \underline{1}' + \underline{2}, 1)$. Therefore, to generate masses for the charged leptons in the diagonal basis, we need two $SU(2)_L$ scalar fields H and H' respectively put in $\underline{1}$ and $1'$ under S_3 as given in Table 1.

The Yukawa interactions in charged lepton sector are:

$$-\mathcal{L}_Y^{lep} = h_1 \bar{\psi}_{1L} H l_{1R} + h_2 (\bar{\psi}_{\alpha L} l_{\alpha R})_{\underline{1}} H + h_3 (\bar{\psi}_{\alpha L} l_{\alpha R})_{1'} H' + \text{H.c.} \tag{7}$$

With the help of Eqs. (7) and (2), the Lagrangian mass term of the charged leptons can be written in the form:

$$-\mathcal{L}_l^{\text{mass}} = (\bar{l}_{1L}, \bar{l}_{2L}, \bar{l}_{3L}) M_l (l_{1R}, l_{2R}, l_{3R})^T + \text{H.c.}, \tag{8}$$

where

$$M_l = \text{diag}(h_1 v_H, h_2 v_H - h_3 v_{H'}, h_2 v_H + h_3 v_{H'}) \equiv \text{diag}(m_e, m_\mu, m_\tau), \tag{9}$$

which has the diagonal form. Thus, the charged lepton diagonalization matrices are $U_{lL} = U_{lR} = 1$ and the lepton mixing matrix depends only on that of the neutrino.

By comparing Eq. (9) with the experimental values for masses of the charged leptons given in Ref. [53], $m_e \simeq 0.51099 \text{ MeV}$, $m_\mu \simeq 105.65837 \text{ MeV}$, $m_\tau \simeq 1776.86 \text{ MeV}$, we get² $h_1 \sim 10^{-5}$, $h_2 \sim 10^{-2}$, $h_3 \sim 10^{-2}$, i.e. $h_1 \ll h_2 \simeq h_3$.

The neutrino masses arise from the couplings of $\bar{\psi}_{iL} \nu_{jR}$ and $\bar{\nu}_{iR}^c \nu_{jR}$ ($i, j = 1, 2, 3$) to scalars, where $\bar{\psi}_{\alpha L} \nu_{\alpha R}$ ($\bar{\nu}_{\alpha R}^c \nu_{\alpha R}$), $\bar{\psi}_{\alpha L} \nu_{1R}$ ($\bar{\nu}_{\alpha R}^c \nu_{1R}$) and $\bar{\psi}_{1L} \nu_{1R}$ ($\bar{\nu}_{1R}^c \nu_{1R}$) transforms as $SU(2)_L$ doublets (singlets) and $\underline{1} + \underline{1}' + \underline{2}, \underline{2}$ and $\underline{1}$ under S_3 , respectively. For the known $SU(2)_L$ scalar doublets, available interactions are $\bar{\psi}_{1L} \tilde{H} \nu_{1R}$, $(\bar{\psi}_{\alpha L} \nu_{\alpha R})_{\underline{1}} \tilde{H}$ and $(\bar{\psi}_{\alpha L} \nu_{\alpha R})_{1'} \tilde{H}'$ but they only generate Dirac mass terms. To generate Majorana mass terms for neutrinos we will therefore introduce one new $SU(2)_L$ singlets ϕ put in $\underline{2}$ under S_3 coupling to $\bar{\psi}_{1L} \nu_{1L}$ and $\bar{\psi}_{\alpha L} \nu_{\alpha L}$.

The Yukawa interactions, which are invariant under all the symmetries of the model, in neutrino sector are:

$$-\mathcal{L}_Y^\nu = \frac{x_1}{2} \bar{\psi}_{1L} \tilde{H} \nu_{1R} + \frac{x_2}{2} (\bar{\psi}_{\alpha L} \nu_{\alpha R})_{\underline{1}} \tilde{H} + \frac{x_3}{2} (\bar{\psi}_{\alpha L} \nu_{\alpha R})_{1'} \tilde{H}' + \frac{y_1}{2} (\bar{\nu}_{\alpha R}^c \nu_{\alpha R})_{\underline{2}} \phi + \frac{y_2}{2} (\bar{\nu}_{\alpha R}^c \nu_{1R} + \bar{\nu}_{1R}^c \nu_{\alpha R})_{\underline{2}} \phi + \text{H.c.} \tag{10}$$

With the VEVs given in Eq. (2), we get the Dirac neutrino mass matrix (M_D) and the right-handed Majorana neutrino mass matrix (M_R) as follows

$$M_D = \text{diag}(a_D, b_D - c_D, b_D + c_D), M_R = \begin{pmatrix} 0 & b_R & b_R \\ b_R & a_R & 0 \\ b_R & 0 & a_R \end{pmatrix}, \tag{11}$$

where

$$a_D = x_1 v_H, b_D = x_2 v_H, c_D = x_3 v_{H'}, a_R = y_1 v_\phi, b_R = y_2 v_\phi. \tag{12}$$

The effective neutrino mass matrix, obtained through the type-I seesaw mechanism, reads:

$$M_{\text{eff}} = -M_D^T M_R^{-1} M_D = \begin{pmatrix} A_0 & -B_0 & -B_0 \\ -B_0 & -C_0 & C_0 \\ -B_0 & C_0 & -C_0 \end{pmatrix} + \begin{pmatrix} 0 & a_1 & -a_1 \\ a_1 & -b_1 + a_2 & -b_1 \\ -a_1 & -b_1 & -b_1 - a_2 \end{pmatrix} \equiv M_0 + \delta M, \tag{13}$$

where

$$A_0 = \frac{a_D^2 a_R}{2b_R^2}, B_0 = \frac{a_D b_D}{2b_R}, C_0 = \frac{b_D^2}{2a_R}, \tag{14}$$

² We use $v_H \sim v_{H'} \sim 100 \text{ GeV}$ for their scale.

$$a_1 = \frac{a_D c_D}{2b_R}, \quad a_2 = \frac{b_D c_D}{a_R}, \quad b_1 = \frac{c_D^2}{2a_R}. \quad (15)$$

It is noted that \mathcal{A}_0 , \mathcal{B}_0 and \mathcal{C}_0 given in Eq. (14) accommodated in the first matrix of Eq. (13) due to the contribution of H and ϕ only while the last term in Eq. (13) is deviation from the contribution of H' only. If there is no contribution of H' , the deviations $a_{1,2}$ and b_1 will vanish and the matrix M_{eff} in (13) reduces to its first term which generates a $\mu - \tau$ mixing form. Thus, second term in (13) will take the role for a small deviation of θ_{13} and being responsible for the CP violating phase in the lepton sector.

The first matrix in Eq. (13) has three eigenvalues,

$$m_{1,2}^0 = \frac{1}{2} \left(\mathcal{A}_0 \pm \sqrt{\mathcal{A}_0^2 + 8\mathcal{B}_0^2} \right), \quad m_3^0 = -2\mathcal{C}_0, \quad (16)$$

and the corresponding lepton mixing matrix takes the form:

$$\mathbb{U}_0 = \begin{pmatrix} \cos \theta & \sin \theta & 0 \\ -\frac{\sin \theta}{\sqrt{2}} & \frac{\cos \theta}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{\sin \theta}{\sqrt{2}} & \frac{\cos \theta}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}, \quad (17)$$

where

$$\theta = \arcsin \left(\frac{\mathcal{K}}{\sqrt{\mathcal{K}^2 + 2}} \right), \quad \mathcal{K} = \frac{m_1^0 - \mathcal{A}_0}{\mathcal{B}_0} = -\frac{m_2^0}{\mathcal{B}_0}, \quad (18)$$

with \mathcal{A}_0 , \mathcal{B}_0 and $m_{1,2}^0$ are defined in Eqs. (14) and (16), respectively.

The matrix \mathbb{U}_0 in Eq. (17) implies $\theta_{13} = 0$, $\theta_{23} = \pi/4$ and $\theta_{12} = \theta$ which was rule out by the recent data.³ However, the contribution of the second term in Eq. (13) will improve this. Indeed, at the first order of perturbation theory, the second matrix in Eq. (13) contributes to both eigenvalues and eigenvectors. In this case, the neutrino masses are given as:

$$m_1 = m_1^0 - 2b_1 \sin^2 \theta, \quad m_2 = m_2^0 - 2b_1 \cos^2 \theta, \\ m_3 = m_3^0, \quad (19)$$

with b_1 , $m_{1,2}^0$ and θ are given in Eqs. (15), (16) and (18), respectively. The corresponding lepton mixing matrix becomes:

$$\mathbb{U} = \mathbb{U}_0 + \delta\mathbb{U}, \quad (20)$$

where \mathbb{U}_0 is defined by (17) and $\delta\mathbb{U}$ has the following entries:

$$(\delta\mathbb{U})_{11} = \frac{2b_1 \sin^2 \theta \cos \theta}{m_1^0 - m_2^0}, \quad (\delta\mathbb{U})_{12} = \frac{2b_1 \sin \theta \cos^2 \theta}{m_2^0 - m_1^0}, \\ (\delta\mathbb{U})_{13} = \frac{(a_2 \cos \theta + \sqrt{2}a_1 \sin \theta) \sin \theta}{m_3^0 - m_2^0} \\ + \frac{(a_2 \sin \theta - \sqrt{2}a_1 \cos \theta) \cos \theta}{m_1^0 - m_3^0},$$

³ In the case $\cos \theta = \sqrt{2/3}$, \mathbb{U}_0 becomes the exact tribimaximal form [54–57].

$$(\delta\mathbb{U})_{21} = \frac{1}{\sqrt{2}} \left(\frac{2b_1 \cos^2 \theta \sin \theta}{m_1^0 - m_2^0} + \frac{\sqrt{2}a_1 \cos \theta - a_2 \sin \theta}{m_1^0 - m_3^0} \right), \\ (\delta\mathbb{U})_{22} = \frac{1}{\sqrt{2}} \left(\frac{2b_1 \cos \theta \sin^2 \theta}{m_1^0 - m_2^0} + \frac{\sqrt{2}a_1 \sin \theta + a_2 \cos \theta}{m_2^0 - m_3^0} \right), \\ (\delta\mathbb{U})_{23} = \frac{(m_2^0 - m_1^0)a_1 \sin \theta \cos \theta}{(m_1^0 - m_3^0)(m_2^0 - m_3^0)} \\ + \frac{a_2}{\sqrt{2}} \left(\frac{\cos^2 \theta}{m_3^0 - m_2^0} + \frac{\sin^2 \theta}{m_3^0 - m_1^0} \right), \\ (\delta\mathbb{U})_{31} = \frac{1}{\sqrt{2}} \left(\frac{2b_1 \cos^2 \theta \sin \theta}{m_1^0 - m_2^0} + \frac{a_2 \sin \theta - \sqrt{2}a_1 \cos \theta}{m_1^0 - m_3^0} \right), \\ (\delta\mathbb{U})_{32} = \frac{1}{\sqrt{2}} \left(\frac{2b_1 \cos \theta \sin^2 \theta}{m_1^0 - m_2^0} - \frac{a_2 \cos \theta + \sqrt{2}a_1 \sin \theta}{m_2^0 - m_3^0} \right), \\ (\delta\mathbb{U})_{33} = (\delta\mathbb{U})_{23}. \quad (21)$$

In the three-neutrino framework, lepton mixing angles can be defined via the elements of the neutrino mixing matrix:

$$t_{12} = \frac{|\mathbb{U}_{12}|}{|\mathbb{U}_{11}|}, \quad t_{23} = \frac{|\mathbb{U}_{23}|}{|\mathbb{U}_{33}|}, \quad s_{13} = \mathbb{U}_{13} e^{i\delta}, \quad (22)$$

where $t_{12} = s_{12}/c_{12}$, $t_{23} = s_{23}/c_{23}$, $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$ with θ_{12} , θ_{23} and θ_{13} , respectively, being the solar angle, atmospheric angle and the reactor angle and δ is the Dirac CP violating phase.

From Eqs. (17), (20)–(22), we found that, in both normal and inverted hierarchies, \mathbb{U}_{11} and \mathbb{U}_{12} depend only on two parameters θ , θ_{12} :

$$\mathbb{U}_{11} = (\cos \theta + t_{12} \sin \theta)^{-1}, \\ \mathbb{U}_{12} = t_{12} (\cos \theta + t_{12} \sin \theta)^{-1}. \quad (23)$$

As pointed out in Ref. [9], at 3σ confidence level, $\mathbb{U}_{11} \in (0.797, 0.842)$. Thus, we get a value $\cos \theta = 0.831$ (i.e., $t_{12} = 0.670$, $\theta = 33.83^\circ$) and $\mathbb{U}_{12} = 0.557$, which is consistent with the best-fit value from the global analysis⁴ given in Ref. [9].

Furthermore, Eqs. (17), (20)–(22) provide a solution:

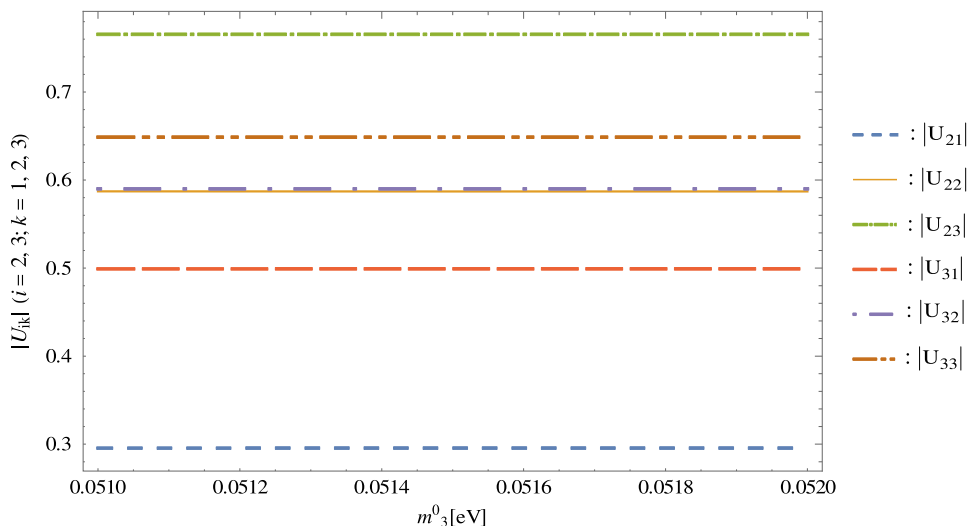
$$a_1 = -\frac{1}{\sqrt{2}} \frac{(m_1^0 - m_3^0)(m_2^0 - m_3^0)s_{13}e^{-i\delta} + a_2(m_1^0 - m_2^0) \sin \theta \cos \theta}{(m_1^0 - m_3^0) \sin^2 \theta + (m_2^0 - m_3^0) \cos^2 \theta}, \\ b_1 = \frac{(m_1^0 - m_2^0)(\sin \theta - t_{12} \cos \theta)}{\sin(2\theta)(\cos \theta + t_{12} \sin \theta)}, \\ a_2 = \begin{cases} \frac{\alpha - \sqrt{\beta}}{t_{23}^2 - 1} & \text{for NH,} \\ \frac{\alpha + \sqrt{\beta}}{t_{23}^2 - 1} & \text{for IH,} \end{cases} \quad (24)$$

where

$$\alpha = (t_{23}^2 - 1)(m_1^0 - m_2^0) \sin \theta \cos \theta s_{13} \cos \delta \\ - (t_{23}^2 + 1)(m_1^0 \sin^2 \theta + m_2^0 \cos^2 \theta - m_3^0),$$

⁴ The global analysis in Ref. [9] provides $t_{12} = 0.670$.

Fig. 1 $|\mathbb{U}_{ki}|$ ($k = 2, 3; j = 1, 2, 3$) as functions of m_3^0 with $m_3^0 \in (0.051, 0.052)$ eV in the NH



$$\beta = 4t_{23}^2(m_1^0 \sin^2 \theta + m_2^0 \cos^2 \theta - m_3^0)^2 - (t_{23}^2 - 1)^2(m_1^0 - m_2^0)^2 \sin^2 \theta \cos^2 \theta s_{13}^2 \sin^2 \delta. \quad (25)$$

Although the global analysis in Ref. [58] shows a hint in favor of the NH over the inverted one at more than 3σ and the global analysis in Ref. [9] obtain a preference for NH at about 2σ , but, it is the fact that the neutrino mass spectrum is currently unknown and it can be NH or IH depending on the sign of Δm_{31}^2 [53]. In the model under consideration, both NH and IH can be found in which the model parameters are in good agreement with the global analysis in Ref. [9].

3.1 Normal spectrum

By taking the best-fit values for neutrino squared mass splittings, leptonic mixing angles and the Dirac CP violating phase for NH with Super-Kamiokande atmospheric neutrino data as given in Eq. (1), we get a solution⁵:

$$m_1^0 = 1.56\sqrt{\alpha_1},$$

$$m_2^0 = \frac{[7.85 \times 10^{-9} - 3.14 \times 10^{-6}(m_3^0)^2 - 1.78\sqrt{\beta_1}]\sqrt{\alpha_1}}{3.39 \times 10^{-3} - 1.34(m_3^0)^2}, \quad (26)$$

where

$$\alpha_1 = -1.04 \times 10^{-3} + 0.411(m_3^0)^2 - 3.26 \times 10^{-7}\beta_1,$$

$$\beta_1 = 8.52 \times 10^{-6} - 6.85 \times 10^{-3}(m_3^0)^2 + 1.38(m_3^0)^4. \quad (27)$$

Now, \mathbb{U}_{2i} and \mathbb{U}_{3i} ($i = 1, 2, 3$) depend only on m_3^0 . In the NH, the estimated value of m_3 is [53] $m_3 \simeq 0.0506$ eV thus we find an allowed region of m_3^0 that can reach the constraint on the absolute values of the entries of the lepton mixing matrix given in Ref. [9] which has been depicted in Fig. 1.

In the NH, $m_1 < m_2 < m_3$ thus $m_1 \equiv m_{light}^N$ is the lightest neutrino mass. The effective neutrino masses governing the beta decay and neutrinoless double beta decay [59–63] $m_\beta = (\sum_{i=1}^3 |U_{ei}|^2 m_i^2)^{1/2}$, $\langle m_{ee} \rangle = |\sum_{i=1}^3 U_{ei}^2 m_i|$ and m_{light}^N as functions of m_3^0 has been plotted in Fig. 2.

In the case $m_3^0 = 0.051$ eV the other parameters are found in Table 2.

The absolute neutrino mass is found to be $\sum_{i=1}^3 m_i = 7.2 \times 10^{-2}$ eV which is well consistent with the strongest bound from cosmology, $\sum m_\nu < 0.078$ eV [64]. Furthermore, the effective neutrino masses $m_\beta = 1.25 \times 10^{-2}$ eV and $\langle m_{ee} \rangle = 1.02 \times 10^{-2}$ eV which are well consistent with the results in Ref. [65]. The magnitude of the elements of the leptonic mixing matrix in Eq. (20) then takes the form:

$$|\mathbb{U}^N| = \begin{pmatrix} 0.831 & 0.557 & 0.15 \\ 0.296 & 0.587 & 0.765 \\ 0.499 & 0.590 & 0.649 \end{pmatrix}, \quad (28)$$

which is consistent with the constraint on the absolute values of the entries of the lepton mixing matrix given in Ref. [9]. The Jarlskog invariant is $J_{CP}^N = \text{Im}(\mathbb{U}_{23}\mathbb{U}_{13}^*\mathbb{U}_{12}\mathbb{U}_{22}^*) = -0.0245$. The resulting effective neutrino mass parameter $\langle m_{ee} \rangle$ for normal hierarchy is below the upper bound arising from present $0\nu\beta\beta$ decay experiments. However, it is very well consistent with the future large and ultra-low background liquid scintillator detectors which has been discussed in Ref. [66] or the meV limit of the effective neutrino mass can be reached by the planning of future experiments [67–74].

3.2 Inverted spectrum

Similar to the normal spectrum, taking the best-fit values of neutrino oscillation parameters for IH with Super-Kamiokande atmospheric neutrino data as given in Eq. (1),

⁵ There are four solutions for $m_{1,2}$, but they give the same value of Δm_{ij}^2 and the same absolute values of $m_{1,2,3}$ thus we only consider in detail the solution in Eqs. (26) and (27).

Fig. 2 $\langle m_{ee} \rangle$, m_β and m_{light}^N as functions of m_3^0 with $m_3^0 \in (0.051, 0.052)$ eV in the NH

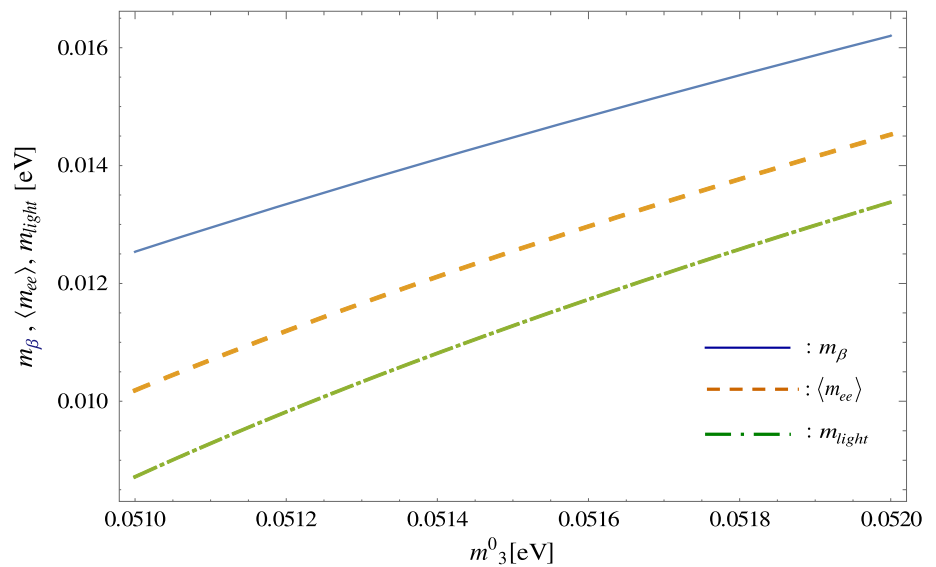


Table 2 The model parameters in the case $m_3^0 = 0.051$ eV in NH

Parameters	The derived values
a_1	$(-3.58 + 2.62i)10^{-3}$ eV
a_2	3.48×10^{-3} eV
b_1	-2.64×10^{-9} eV
m_1^0	8.72×10^{-3} eV
m_2^0	1.22×10^{-2} eV
m_1	8.72×10^{-3} eV
m_2	1.22×10^{-2} eV
m_3	5.10×10^{-2} eV

we get a solution⁶:

$$m_1^0 = -1.5 \times 10^{-6} m_2^0 + 1.74 \sqrt{\alpha_2}, m_3^0 = 0.356 \sqrt{\beta_2}, \quad (29)$$

with

$$\begin{aligned} \alpha_2 &= 0.331(m_2^0)^2 - 2.45 \times 10^{-5}, \\ \beta_2 &= -1.98 \times 10^{-2} + 7.88(m_2^0)^2 - 2.83 \times 10^{-5} m_2^0 \sqrt{\alpha_2}. \end{aligned} \quad (30)$$

The elements of the lepton mixing matrix \mathbb{U}_{2i}^I and \mathbb{U}_{3i}^I ($i = 1, 2, 3$) depend only on m_2^0 . In the IH, the estimated value [53] of m_2 is 0.0504 eV thus we find an allowed region of m_2^0 that can reach the constraint on the absolute values of the entries of the lepton mixing matrix given in Ref. [9] which has been depicted in Fig. 3.

In the IH, $m_3 < m_1 < m_2$ thus $m_3 \equiv m_{light}^I$ is the lightest neutrino mass. The effective neutrino masses governing the beta decay and neutrinoless double beta decay and m_{light}^I as functions of m_2^0 has been plotted in Fig. 4.

⁶ As before, there are four solutions for $m_{1,3}$ however they give the same value for Δm_{ij}^2 and the same absolute values of $m_{1,2,3}$ thus we only consider in detail the solution in Eqs. (29) and (30).

In the case $m_2^0 = 0.051$ eV the other parameters are found in Table 3.

The magnitude of the leptonic mixing matrix in Eq. (20) then takes the form:

$$|\mathbb{U}^I| = \begin{pmatrix} 0.831 & 0.557 & 0.15 \\ 0.387 & 0.532 & 0.765 \\ 0.42 & 0.649 & 0.649 \end{pmatrix}, \quad (31)$$

which is consistent with the constraint on the absolute values of the entries of the lepton mixing matrix given in Ref. [9]. The resulting effective neutrino mass parameter for inverted hierarchy is very well consistent with the future large and ultra-low background liquid scintillator detectors which has been discussed in Ref. [66] or the meV limit of the effective neutrino mass can be reached by the above mentioned future experiments [67–74].

4 Quark mass and mixing

The Yukawa interactions in quark sector are:

$$\begin{aligned} -\mathcal{L}_q &= h_{1u}(\bar{Q}_{1L}\tilde{H})_1 u_{1R} + h_{2u}(\bar{Q}_{\alpha L}\tilde{H})_2 u_{\alpha R} \\ &\quad + h_{3u}(\bar{Q}_{\alpha L}\tilde{H}')_2 u_{\alpha R} \\ &\quad + h_{4u}(\bar{Q}_{1L}\tilde{H}'')_2 u_{\alpha R} + h_{5u}(\bar{Q}_{\alpha L}\tilde{H}'')_1 u_{1R} \\ &\quad + h_{1d}(\bar{Q}_{1L}H)_1 d_{1R} + h_{2d}(\bar{Q}_{\alpha L}H)_2 d_{\alpha R} \\ &\quad + h_{3d}(\bar{Q}_{\alpha L}H')_2 d_{\alpha R} \\ &\quad + h_{4d}(\bar{Q}_{1L}H'')_2 d_{\alpha R} + h_{5d}(\bar{Q}_{\alpha L}H'')_2 d_{1R} + \text{H.c.} \end{aligned} \quad (32)$$

With the VEV alignments of H , H' and H'' in Eq. (2), the mass Lagrangian of quarks reads

Fig. 3

$|U_{ki}|$ ($k = 2, 3; j = 1, 2, 3$) as functions of m_2^0 with $m_2^0 \in (0.051, 0.052)$ eV in the IH

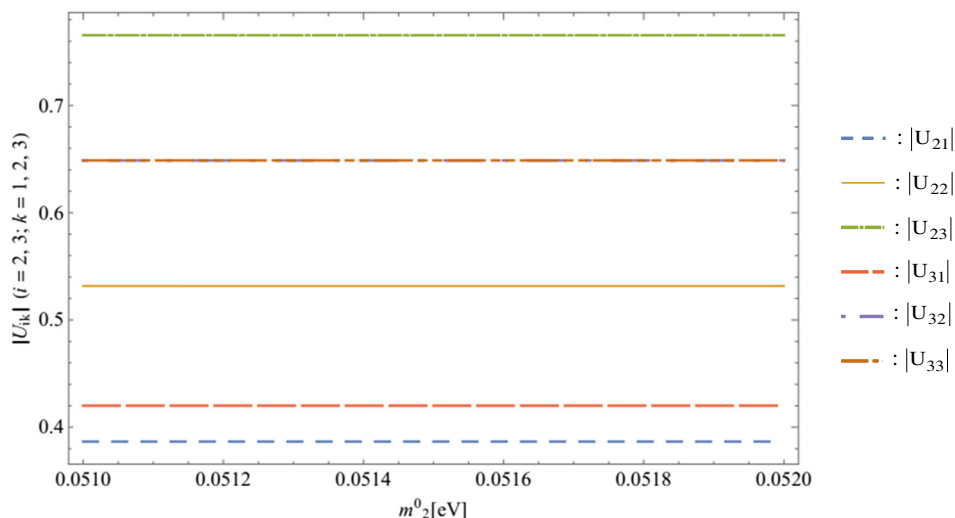


Fig. 4 $\langle m_{ee} \rangle, m_\beta$ and m_{light}^I as functions of m_2^0 with $m_2^0 \in (0.051, 0.052)$ eV in the IH

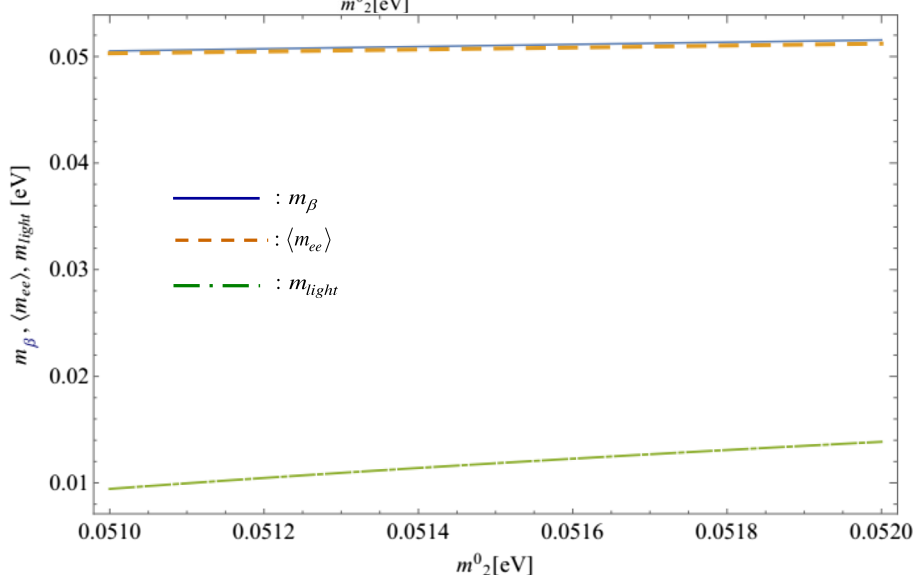


Table 3 Some model parameters in the case $m_2^0 = 0.051$ eV in IH

Parameters	The derived values
a_1	$(-0.778 - 4.3i)10^{-3}$ eV
a_2	-3.42×10^{-3} eV
b_1	-5.47×10^{-10} eV
m_1^0	5.03×10^{-2} eV
m_3^0	9.43×10^{-3} eV
m_1	5.03×10^{-2} eV
m_2	5.10×10^{-2} eV
m_3	9.43×10^{-3} eV
$\sum_{i=1}^3 m_i^I$	0.111 eV
m_β	5.05×10^{-2} eV
$\langle m_{ee} \rangle$	5.03×10^{-2} eV
J_{CP}^I	-3.27×10^{-2} eV

$$\begin{aligned}
 &+h_{4u}v_{H''}^*(\bar{u}_{1L}u_{3R} + \bar{u}_{1L}u_{2R}) \\
 &+h_{5u}v_{H''}^*(\bar{u}_{3L}u_{1R} + \bar{u}_{2L}u_{1R}) \\
 &+h_{1d}v_H\bar{d}_{1L}d_{1R} + (h_{2d}v_H + h_{3d}v_{H'})\bar{d}_{2L}d_{2R} \\
 &+(h_{3d}v_{H'} - h_{2d}v_H)\bar{d}_{3L}d_{3R} \\
 &+h_{4d}v_{H''}(\bar{u}_{1L}d_{3R} + \bar{u}_{1L}d_{2R}) \\
 &+h_{5d}v_{H''}(\bar{u}_{3L}d_{1R} + \bar{u}_{2L}d_{1R}) + \text{H.c.} \\
 &\equiv (\bar{u}_{1L}, \bar{u}_{2L}, \bar{u}_{3L})\mathbb{M}_u(u_{1R}, u_{2R}, u_{3R})^T \\
 &+(\bar{d}_{1L}, \bar{d}_{2L}, \bar{d}_{3L})\mathbb{M}_d(d_{1R}, d_{2R}, d_{3R})^T + \text{H.c.},
 \end{aligned} \tag{33}$$

where \mathbb{M}_u and \mathbb{M}_d take the form:

$$\begin{aligned}
 \mathbb{M}_u &= \begin{pmatrix} a_{1u} & a_{4u} & a_{4u} \\ a_{5u} & a_{2u} + a_{3u} & 0 \\ a_{5u} & 0 & -a_{2u} + a_{3u} \end{pmatrix}, \\
 \mathbb{M}_d &= \begin{pmatrix} a_{1d} & a_{4d} & a_{4d} \\ a_{5d} & a_{2d} + a_{3d} & 0 \\ a_{5d} & 0 & -a_{2d} + a_{3d} \end{pmatrix},
 \end{aligned} \tag{34}$$

$$\begin{aligned}
 -\mathcal{L}_q^{\text{mass}} &= h_{1u}v_H^*\bar{u}_{1L}u_{1R} + (h_{2u}v_H^* + h_{3u}v_{H'}^*)\bar{u}_{2L}u_{2R} \\
 &+(h_{3u}v_{H'}^* - h_{2u}v_H^*)\bar{u}_{3L}u_{3R}
 \end{aligned}$$

with

$$\begin{aligned}
 a_{1u} &= h_{1u}v_H^*, a_{2u} = h_{2u}v_H^*, a_{3u} = h_{3u}v_{H'}^*, \\
 a_{4u} &= h_{4u}v_{H''}^*, a_{5u} = h_{5u}v_{H''}^*, \\
 a_{1d} &= h_{1d}v_H, a_{2d} = h_{2d}v_H, a_{3d} = h_{3d}v_{H'}, \\
 a_{4d} &= h_{4d}v_{H''}, a_{5d} = h_{5d}v_{H''}.
 \end{aligned}
 \tag{35}$$

It is noted that without contribution of H'' , the matrices $\mathbb{M}_u, \mathbb{M}_d$ in Eq. (34) become diagonal ones, i.e, the quark mixing matrices are identity matrices, however, the quark masses can well fit to the experimental data given in Ref. [53].

The quark mass matrices satisfy the following relation:

$$M_q M_q^\dagger = \begin{pmatrix} X_q & e^{i\theta_q} Y_q & e^{i\psi_q} Z_q \\ e^{-i\theta_q} Y_q & R_q & W_q \\ e^{i\psi_q} Z_q & W_q & S_q \end{pmatrix}, \tag{36}$$

where

$$\begin{aligned}
 X_q &= x_{1q}^2 + 2x_{4q}^2, \quad W_q = x_{5q}^2, \\
 e^{i\theta_q} Y_q &= e^{-i(\kappa_{2q}-\kappa_{4q})} x_{2q} x_{4q} \\
 &\quad + e^{-i(\kappa_{3q}-\kappa_{4q})} x_{3q} x_{4q} + e^{i(\kappa_{1q}-\kappa_{5q})} x_{1q} x_{5q}, \\
 e^{i\psi_q} Z_q &= -e^{-i(\kappa_{2q}-\kappa_{4q})} x_{2q} x_{4q} + e^{-i(\kappa_{3q}-\kappa_{4q})} x_{3q} x_{4q} \\
 &\quad + e^{i(\kappa_{1q}-\kappa_{5q})} x_{1q} x_{5q}, \\
 R_q &= x_{2q}^2 + 2 \cos(\kappa_{2q} - \kappa_{3q}) x_{3q} x_{2q} + x_{3q}^2 + x_{5q}^2, \\
 S_q &= x_{2q}^2 - 2 \cos(\kappa_{2q} - \kappa_{3q}) x_{3q} x_{2q} + x_{3q}^2 + x_{5q}^2.
 \end{aligned}
 \tag{37}$$

Here $q = u, d$, $x_{iq} = |a_{iq}|$, $\kappa_{iq} = \arg(a_{iq})$ ($i = 1, 2, \dots, 5$) and X_q, Y_q, Z_q, W_q, R_q and S_q are real parameters.

In order to fit the measured values of the SM quark masses and CKM parameters given in Refs. [53, 75] as shown in Tables 4 and 5,⁷ we proceed by solving the eigenvalue problem for the SM quark masses. The following solution has been found:

$$\begin{aligned}
 M_u M_u^\dagger &= \begin{pmatrix} 29751. & -1920. + 761.398i & 1.84494 - 2.91915i \\ -1920. - 761.398i & 145.001 & -0.149911 \\ 1.84494 + 2.91915i & -0.149911 & 0.0140043 \end{pmatrix} \text{GeV}^2, \\
 M_d M_d^\dagger &= \begin{pmatrix} 17.2562 & -1.79614 + 0.7086i & 0.0134866 + 0.00200077i \\ -1.79614 - 0.7086i & 0.224489 & 0.000115589 \\ 0.0134866 - 0.00200077i & 0.000115589 & 0.000346993 \end{pmatrix} \text{GeV}^2.
 \end{aligned}
 \tag{38}$$

This show that our model is consistent with and successfully accommodate the experimental values of the physical observables of the quark sector: the six quark masses, the quark mixing angles and the CP violating phase in the quark sector.

⁷ To obtain the best-fit values of $(U_{CKM})_{ij}^{bf}$ in Tables 4 and 5 we use the Wolfenstein parameters given by Ref. [75]: $\lambda_C = 0.2250$, $\bar{\rho} = 0.148$, $\bar{\eta} = 0.348$ and $A = 0.826$.

Table 4 The best-fit results for quark masses and mixings taken from [76] and [53, 75], respectively. Note that the experimental values of the quark masses are given at the M_Z scale

Observable	Experimental best-fit value
m_u (MeV)	1.24 ± 0.22
m_c (GeV)	0.63 ± 0.02
m_t (GeV)	172.9 ± 0.4
m_d (MeV)	2.69 ± 0.19
m_s (MeV)	53.5 ± 4.6
m_b (GeV)	2.86 ± 0.03
$\sin \theta_{12}^{(q)}$	0.2245 ± 0.00044
$\sin \theta_{23}^{(q)}$	0.0421 ± 0.00076
$\sin \theta_{13}^{(q)}$	0.00365 ± 0.00012
$\delta^{(q)}$	$73.5_{-5.1}^{+4.2}$

Table 5 The best-fit results for CKM parameters taken from Refs. [53, 75]

Observable	Experimental best-fit value
$(V_{CKM}^{exp})_{11}$	0.974352
$(V_{CKM}^{exp})_{12}$	0.224998
$(V_{CKM}^{exp})_{13}$	$0.00138 - 0.003341i$
$(V_{CKM}^{exp})_{21}$	$-0.224858 - 0.00014i$
$(V_{CKM}^{exp})_{22}$	$0.973486 - 0.00003i$
$(V_{CKM}^{exp})_{23}$	0.0420
$(V_{CKM}^{exp})_{31}$	$0.00811 - 0.00332i$
$(V_{CKM}^{exp})_{32}^{bf}$	$-0.04123 - 0.00076i$
$(V_{CKM}^{exp})_{33}$	0.999111

5 Higgs diphoton decay rate

The decay rate for the $h \rightarrow \gamma\gamma$ process takes the form:

$$\begin{aligned}
 \Gamma(h \rightarrow \gamma\gamma) &= \frac{\alpha_{em}^2 m_h^3}{256\pi^3 v^2} \left| \sum_f a_{hff} N_C Q_f^2 F_{1/2}(\rho_f) + a_{hWW} F_1(\rho_W) \right. \\
 &\quad \left. + \sum_{k=1,2,3} \frac{C_{hH_k^\pm H_k^\mp} v}{2m_{H_k^\pm}^2} F_0(\rho_{H_k^\pm}) \right|^2,
 \end{aligned}
 \tag{39}$$

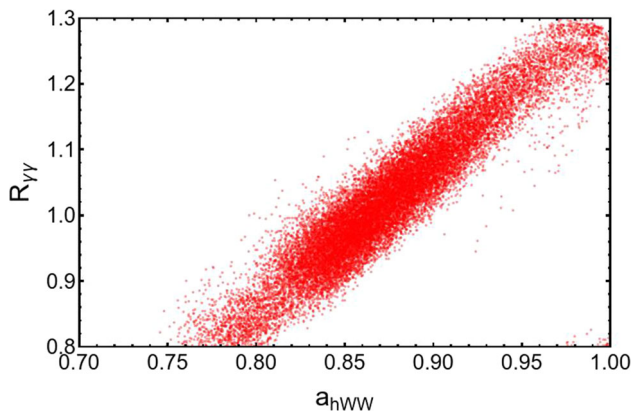


Fig. 5 Correlation of the Higgs diphoton signal strength with the a_{hWW} deviation factor from the SM Higgs-W gauge boson coupling

where ρ_i are the mass ratios $\rho_i = \frac{m_i^2}{4M_i^2}$ with $M_i = m_f, M_W$; α_{em} is the fine structure constant; N_C is the color factor ($N_C = 1$ for leptons and $N_C = 3$ for quarks) and Q_f is the electric charge of the fermion in the loop. From the fermion-loop contributions we only consider the dominant top quark term. Furthermore, $C_{hH_k^\pm H_k^\mp}$ is the trilinear coupling between the SM-like Higgs and a pair of charged Higgses, whereas a_{htt}

and a_{hWW} are the deviation factors from the SM Higgs-top quark coupling and the SM Higgs-W gauge boson coupling, respectively (in the SM these factors are unity). Such deviation factors are close to unity in our model, which is a consequence of the numerical analysis of its scalar, Yukawa and gauge sectors.

Furthermore, $F_{1/2}(z)$ and $F_1(z)$ are the dimensionless loop factors for spin-1/2 and spin-1 particles running in the internal lines of the loops. They are given by:

$$F_{1/2}(z) = 2(z + (z - 1)f(z))z^{-2}, \tag{40}$$

$$F_1(z) = -2(2z^2 + 3z + 3(2z - 1)f(z))z^{-2}, \tag{41}$$

$$F_0(z) = -(z - f(z))z^{-2}, \tag{42}$$

with

$$f(z) = \begin{cases} \arcsin^2 \sqrt{z} & \text{for } z \leq 1 \\ -\frac{1}{4} \left(\ln \left(\frac{1 + \sqrt{1 - z^{-1}}}{1 - \sqrt{1 - z^{-1}} - i\pi} \right) \right)^2 & \text{for } z > 1 \end{cases} \tag{43}$$

In order to study the implications of our model in the decay of the 126 GeV Higgs into a photon pair, one introduces the Higgs diphoton signal strength $R_{\gamma\gamma}$, which is defined as:

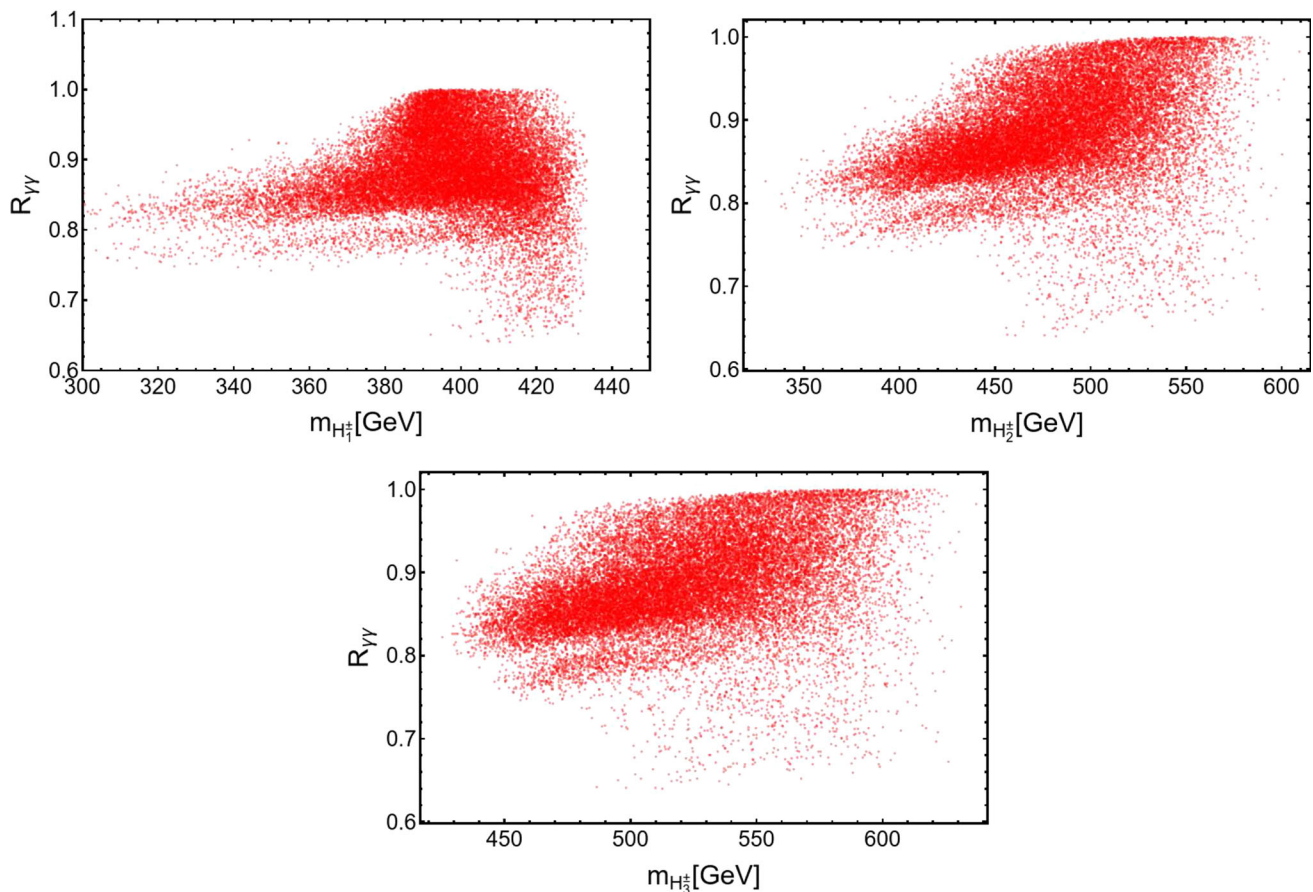


Fig. 6 Correlation of the Higgs diphoton signal strength with the charged scalar masses

$$R_{\gamma\gamma} = \frac{\sigma(pp \rightarrow h)\Gamma(h \rightarrow \gamma\gamma)}{\sigma(pp \rightarrow h)_{SM}\Gamma(h \rightarrow \gamma\gamma)_{SM}} \simeq a_{Htt}^2 \frac{\Gamma(h \rightarrow \gamma\gamma)}{\Gamma(h \rightarrow \gamma\gamma)_{SM}}. \quad (44)$$

That Higgs diphoton signal strength, normalizes the $\gamma\gamma$ signal predicted by our model in relation to the one given by the SM. Here we have used the fact that in our model, single Higgs production is also dominated by gluon fusion as in the Standard Model.

The ratio $R_{\gamma\gamma}$ has been measured by CMS and ATLAS collaborations with the best fit signals [77,78]:

$$R_{\gamma\gamma}^{CMS} = 1.18_{-0.14}^{+0.17} \quad \text{and} \quad R_{\gamma\gamma}^{ATLAS} = 0.96 \pm 0.14. \quad (45)$$

The correlations of the Higgs diphoton signal strength with the a_{hWW} deviation factor from the SM Higgs-W gauge boson coupling and with charged scalar masses are shown in Figs. 5 and 6, respectively. Such correlations indicate that our model successfully accommodates the current Higgs diphoton decay rate constraints.

6 Conclusions

We have proposed a renormalizable $B - L$ standard model extension based on the S_3 symmetry which successfully accommodates the current data on SM fermion masses, fermionic mixing angles and CP violating phases of both quark and lepton sectors. The tiny values of the light active neutrino masses are generated through a type I seesaw mechanism. The obtained physical observables of the lepton sector are well consistent with the global fit of neutrino oscillation experiments [9] for both normal and inverted neutrino mass hierarchies. The model also predicts effective neutrino mass parameters of $\langle m_{ee} \rangle = 1.02 \times 10^{-2}$ eV, $m_\beta = 1.25 \times 10^{-2}$ eV for NH and $\langle m_{ee} \rangle = 5.03 \times 10^{-2}$ eV, $m_\beta = 5.05 \times 10^{-2}$ eV for IH which are all well consistent with the future large and ultra-low background liquid scintillator detectors which has been discussed in Ref. [66] or the limit of the effective neutrino mass can be reached by the planning of future experiments. The model results are consistent with and successfully accommodate the recent experimental values of the physical observables of the quark sector, including the six quark masses, the quark mixing angles and the CP violating phase in the quark sector. Finally, we have also shown that our model successfully accommodates the current Higgs diphoton decay rate constraints.

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Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: The only experimental data used in our work are the ones reported in Table. Table IV, V, Eq. (45) as well as the experimental values of the charged lepton masses and the best-fit values of neutrino oscillation parameters given in Eq. (1) that we use to compare the predictions of our model in SM fermion masses and mixings with the experimental results.]

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Appendix A: The Clebsch–Gordan coefficients of S_3 group

S_3 is the permutation group of three objects, which is also the symmetry group of an equilateral triangle [51]. It has three irreducible representations denoted as $\underline{1}$, $\underline{1}'$ and $\underline{2}$. We will work in the complex representation in which the conjugation rules are given by $\underline{1}^*(1^*) = \underline{1}(1^*)$, $\underline{1}'^*(1^*) = \underline{1}'(1^*)$, $\underline{2}^*(1^*, 2^*) = \underline{2}(2^*, 1^*)$, and the decomposition rules are

$$\begin{aligned} \underline{1} \otimes \underline{1} &= \underline{1}(11), \quad \underline{1}' \otimes \underline{1}' = \underline{1}(11), \quad \underline{1} \otimes \underline{1}' = \underline{1}'(11), \\ \underline{1} \otimes \underline{2} &= \underline{2}(11, 12), \quad \underline{1}' \otimes \underline{2} = \underline{2}(11, -12), \\ \underline{2} \otimes \underline{2} &= \underline{1}(12 + 21) \oplus \underline{1}'(12 - 21) \oplus \underline{2}(22, 11), \end{aligned} \quad (A1)$$

where the first and second factors in parentheses respectively indicate to the multiplet components of the first and second representations, respectively.

Appendix B: Higgs potential

The renormalizable potential invariant under all symmetries $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_{B-L} \otimes S_3 \otimes Z_4$ is given by⁸:

$$\begin{aligned} V_{\text{total}} &= V(H) + V(H') + V(H'') + V(\phi) + V(H, H') \\ &\quad + V_{SB}(H'') + V(H, H'') + V(H, \phi) + V(H', H'') \\ &\quad + V(H', \phi) + V(H'', \phi), \end{aligned} \quad (B1)$$

⁸ Here, we denote $V(X \rightarrow X_1, Y \rightarrow Y_1, \dots) \equiv V(X, Y, \dots)_{\{X=X_1, Y=Y_1, \dots\}}$.

where

$$V(H) = \mu_H^2 H^\dagger H + \lambda^H (H^\dagger H)_\perp (H^\dagger H)_\perp, \\ V(H') = V(H \rightarrow H'), \quad (\text{B2})$$

$$V(H'') = \mu_{H''}^2 (H''^\dagger H'')_\perp + \lambda_1^{H''} (H''^\dagger H'')_\perp (H''^\dagger H'')_\perp \\ + \lambda_2^{H''} (H''^\dagger H'')_{1'} (H''^\dagger H'')_{1'} \\ + \lambda_3^{H''} (H''^\dagger H'')_{2'} (H''^\dagger H'')_{2'},$$

$$V(\phi) = V(H'' \rightarrow \phi), \quad (\text{B3})$$

$$V(H, H') = \lambda_1^{HH'} (H^\dagger H)_\perp (H'^\dagger H')_\perp \\ + \lambda_2^{HH'} (H^\dagger H')_{1'} (H'^\dagger H)_{1'}, \quad (\text{B4})$$

$$V(H, H'') = \lambda_1^{HH''} (H^\dagger H)_\perp (H''^\dagger H'')_\perp \\ + \lambda_2^{HH''} (H^\dagger H'')_{2'} (H''^\dagger H)_{2'}, \quad (\text{B5})$$

$$V(H, \phi) = V(H, H'' \rightarrow \phi),$$

$$V(H', H'') = V(H \rightarrow H', H''),$$

$$V(H', \phi) = V(H \rightarrow H', \phi), \quad (\text{B6})$$

$$V(H'', \phi) = \lambda_1^{H''\phi} (H''^\dagger H'')_\perp (\phi^\dagger \phi)_\perp \\ + \lambda_2^{H''\phi} (H''^\dagger H'')_{1'} (\phi^\dagger \phi)_{1'} \\ + \lambda_3^{H''\phi} (H''^\dagger H'')_{2'} (\phi^\dagger \phi)_{2'} \\ + \lambda_4^{H''\phi} (H''^\dagger \phi)_\perp (\phi^\dagger H'')_\perp \\ + \lambda_5^{H''\phi} (H''^\dagger \phi)_{1'} (\phi^\dagger H'')_{1'} \\ + \lambda_6^{H''\phi} (H''^\dagger \phi)_{2'} (\phi^\dagger H'')_{2'}, \quad (\text{B7})$$

$$V(H'')_{SB} = -\mu_{SB}^2 (H_1'' H_2''^\dagger + H_2'' H_1''^\dagger), \quad (\text{B8})$$

where the S_3 soft-breaking mass term $V(H'')_{SB}$ given by Eq. (B8) has been included to generate a non vanishing determinant for the squared mass matrix for the CP-even scalar fields. The scalars fields H , H' , H'' and ϕ with their VEVs aligned in Eq. (2) is a solution from the minimization condition of V_{total} as shown in Sect. 2.

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