

## QUASI THREE-PARAMETRIC R-MATRIX AND QUANTUM SUPERGROUPS $GL_{p,q}(1/1)$ AND $U_{p,q}[gl(1/1)]$

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**Abstract.** *An over-parametrized (three-parametric) R-matrix satisfying a graded Yang-Baxter equation is introduced. It turns out that such an over-parametrization is very helpful. Indeed, this R-matrix with one of the parameters being auxiliary, thus, reducible to a two-parametric R-matrix, allows the construction of quantum supergroups  $GL_{p,q}(1/1)$  and  $U_{p,q}[gl(1/1)]$  which, respectively, are two-parametric deformations of the supergroup  $GL(1/1)$  and the universal enveloping algebra  $U[gl(1/1)]$ . These two-parametric quantum deformations  $GL_{p,q}(1/1)$  and  $U_{p,q}[gl(1/1)]$ , to our knowledge, are constructed for the first time via the present approach. The quantum deformation  $U_{p,q}[gl(1/1)]$  obtained here is a true two-parametric deformation of Drinfel'd-Jimbo's type, unlike some other one obtained previously elsewhere.*

**Keywords:** quantum supergroup, R-matrix; Drinfel'd-Jimbo deformation; multi-parametric quantum deformation.

**Classification numbers:** 02.20.-a; 03.65.Fd.

## I. INTRODUCTION

The discovery of the Higgs boson by the LHC collaborations ATLAS and CMS [1,2] shows once again the might of the symmetry principle in physics (see, for instance, [3] and references therein for a review on the Higgs boson's search and discovery). In particular, the standard model (SM) based on the gauge symmetry  $SU(3) \otimes SU(2) \otimes U(1)$  (see, for example, [4,5]), has been verified by the experiment, specially, after the discovery of the Higgs model, as an excellent model of elementary particles and their interactions [6]. There are, however, a number of problems, which cannot be explained or described by the existing symmetry, for example, within the SM the problems like CP-violation (matter-antimatter asymmetry), neutrino masses and mixing, dark matter, dark energy, etc. cannot be solved. Such problems may require an extension by size, or even, a generalization by concept, of an underlying symmetry<sup>1</sup> adopted to a physics system. One of such generalizations is the concept of quantum deformed symmetry. Mathematically, if ordinary (or classical) symmetry is described by classical groups such as the above-mentioned group  $SU(3) \otimes SU(2) \otimes U(1)$ , the quantum deformed symmetry is described by the so-called quantum groups [7–12] (see, for example, [13, 14] for some physics applications of quantum groups).

Using the  $R$ -matrix formalism [7] is one of the approaches to quantum groups which can be interpreted as a kind of (quantum) deformations of ordinary (classical) groups or algebras. It has proved to be a powerful method in investigating quantum groups and related topics such as noncommutative geometry [8, 11, 12, 15], integrable systems [7, 13, 14], etc. A physical meaning of this approach is the so-called (universal)  $R$ -matrix associated to a quantum group satisfies the famous Yang-Baxter equation (YBE) representing an integrability condition of a physical system [7, 13, 14]. A mathematical advantage of this approach is both the algebraic and co-algebraic structure of the corresponding quantum group can be expressed in a few compact (matrix) relations. Quantum groups as symmetry groups of quantum spaces [7, 8, 15] or as deformations of universal enveloping algebras, called also Drinfel'd-Jimbo (DJ) deformation [9, 10], can be also derived in an elegant way in the framework of the  $R$ -matrix formalism. The DJ deformation, which is originally one-parametric, has an advantage that it has a clear algebraic structure (as a deformation from the classical algebraic structure) and it is convenient for a representation construction and a multi-parametric generalization. Combined with the supersymmetry idea [16–18] (see, for example, [19, 20], among a vast literature, for a more detailed introduction), the quantum deformations lead to the concept of quantum supergroups [21–24]. In this case, an  $R$ -matrix becomes graded and satisfies a graded YBE [24].

By original construction, a quantum (super) group depends on a, complex in general, parameter, but the concept of one-parametric quantum (super) groups can be generalized to that of multi-parametric quantum (super) groups. For about three decades quantum groups have been investigated in great details in many aspects. These investigations were carried out first and mainly on the one-parametric case and they were extended later to on the multi-parametric deformations [8, 25]. Having in principle richer structures, multi-parametric quantum groups are also a subject of interest of a number of authors (see Refs. [26–36] and references therein) and have been applied to considering some physics models (see in this context, for example, Refs. [36–39]) but in

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<sup>1</sup>Here we do not discuss the global- and the local symmetry separately.

comparison with the one-parametric quantum groups, they are considerably less understood (even, in some cases they can be proved to be equivalent to one-parametric deformations). Moreover, most of the multi-parametric deformations considered so far are two-parametric ones including those of supergroups [27–35] (it is clear that two-parametric deformations of supergroups cannot be always reduced to one-parametric ones [27–29, 31]). Here, we continue to deal with the case of two-parametric deformations, in particular, a two-parametric deformation of the supergroup  $GL(1/1)$ , which was also considered in [27]. The two-parametric deformation obtained there, however, does not lead to a "standard" DJ form of a two-parametric deformation of  $U[gl(1/1)]$  obtaining which is the purpose of the present work. It will be shown that such a two-parametric deformation of DJ type can be found via a quasi three-parametric deformation of  $Gl(1/1)$ .

**II. DRINFEL'D-JIMBO QUANTUM SUPERGROUPS AND THEIR TWO-PARAMETRIC GENERALIZATION**

A quantum group as a DJ deformation [9, 10] of an universal enveloping algebra of a (semi-) simple superalgebra  $g$  of rank  $r$  can be defined via a set of  $3r$  Cartan-Chevalley generators  $h_i, e_i, f_i, i = 1, 2, \dots, r$ , subject to the following defining relations (see, for example, [40, 41] and references therein):

a) the quantum Cartan-Kac supercommutation relations,

$$\begin{aligned} [h_i, h_j] &= 0, [h_i, e_j] = a_{ij}e_j, \\ [h_i, f_j] &= -a_{ij}f_j, [e_i, f_j] = \delta_{ij}[h_i]_{q_i}, \end{aligned} \tag{1}$$

b) the quantum Serre relations,

$$(ad_q E_i)^{1-\tilde{a}_{ij}} \mathcal{E}_j = 0, (ad_q \mathcal{F}_i)^{1-\tilde{a}_{ij}} \mathcal{F}_j = 0, \tag{2}$$

with  $\mathcal{E}_i = e_i q_i^{-h_i}$ ,  $\mathcal{F}_i = f_i q_i^{-h_i}$ , and

c) the quantum extra-Serre relations which for  $g$  being  $sl(m/n)$  or  $osp(m/n)$  have the form,

$$\begin{aligned} \{[e_{m-1}, e_m]_q, [e_m, e_{m+1}]_q\} &= 0, \\ \{[f_{m-1}, f_m]_q, [f_m, f_{m+1}]_q\} &= 0, \end{aligned} \tag{3}$$

where

$$[X]_q = \frac{q^X - q^{-X}}{q - q^{-1}}, \tag{4}$$

denotes a (one-parametric) quantum deformation of a number or operator  $X$ , and  $(\tilde{a}_{ij})$  is a matrix obtained from the non-symmetric Cartan matrix  $(a_{ij})$  of  $g$  by replacing the strictly positive elements in the rows with 0 on the diagonal entry by  $-1$ , while  $ad_q$  is the  $q$ -deformed adjoint operator given by the formula (2.8) in [40]. Here  $q_i = q^{d_i}$  where  $d_i$  are rational numbers symmetrizing the Cartan matrix,  $d_i a_{ij} = d_j a_{ji}, 1 \leq i, j \leq r$ . They take, for example, in case  $g = sl(m/n)$ , the values

$$d_i = \begin{cases} 1 & \text{for } 1 \leq i \leq m, \\ -1 & \text{for } m+1 \leq i \leq r = m+n-1. \end{cases} \tag{5}$$

Now let us define a two-parametric DJ deformation as a direct generalization of the above-defined one-parametric deformation (1)–(3) by extending (4) to

$$[X]_{p,q} = \frac{q^X - p^{-X}}{q - p^{-1}}, \tag{6}$$

where  $p$  and  $q$  are, in general, independent complex parameters. Thus  $[h]_{q_i}$  in (1) becomes

$$[h_i]_{p_i, q_i} \equiv \frac{q_i^{h_i} - p_i^{-h_i}}{q_i - p_i^{-1}}, \quad (7)$$

with  $q_i$  defined above and  $p_i = p^{d_i}$ . This kind two-parametric generalization of the DJ deformation was considered earlier in, for example, [29, 31, 32]. Next, according to this definition, we will derive via the  $R$ -matrix formalism a two-parametric DJ deformation of  $U[gl(1/1)]$  which, to our knowledge, has not yet been constructed. Since  $gl(1/1)$  is a rank-1 ( $r = 1$ ) superalgebra, the index  $i$  will be omitted.

As discussed in the previous section, one of the two-parametric quantum deformations of  $GL(1/1)$  was obtained elsewhere [27], however, the corresponding two-parametric deformation of the universal enveloping algebra  $U[gl(1/1)]$  has no DJ form. In fact, the two-parametric deformation of  $U[gl(1/1)]$  in [27] can be transformed to an one-parametric DJ deformation by re-scaling its generators appropriately. Indeed, starting from the defining relations of the deformation of  $U[gl(1/1)]$  given in [27],

$$\begin{aligned} [K, H] &= 0, [K, \chi_{\pm}] = 0, [H, \chi_{\pm}] = \pm 2\chi_{\pm}, \\ \{\chi_+, \chi_-\}_{q/p} &= \left(\frac{q}{p}\right)^{H/2} [K]_{\sqrt{qp}}, \end{aligned}$$

where

$$\begin{aligned} \{\chi_+, \chi_-\}_{q/p} &\equiv \left(\frac{q}{p}\right)^{1/2} \chi_+ \chi_- + \left(\frac{q}{p}\right)^{-1/2} \chi_- \chi_+, \\ [K]_{\sqrt{qp}} &= \frac{(qp)^{K/2} - (qp)^{-K/2}}{(qp)^{1/2} - (qp)^{-1/2}} \end{aligned}$$

and making re-scaling  $\chi_{\pm} \rightarrow \chi'_{\pm} = \left(\frac{q}{p}\right)^{-H/4} \chi_{\pm}$ , we get

$$\begin{aligned} [K, H] &= 0, [K, \chi'_{\pm}] = 0, \\ [H, \chi'_{\pm}] &= \pm 2\chi'_{\pm}, \{\chi'_+, \chi'_-\} = [K]_{\sqrt{qp}}. \end{aligned}$$

The latter relations are (conventional) defining relations of an one-parametric DJ deformation of  $U[gl(1/1)]$  with parameter  $\sqrt{qp}$  (cf. (1)–(4)). To obtain a true two-parametric deformation of both  $GL(1/1)$  and  $U[gl(1/1)]$  we start from a three-parametric  $R$ -matrix satisfying a graded YBE. This  $R$ -matrix approach will allow us to construct a (quasi) three-parametric deformation of  $GL(1/1)$  which in fact is equivalent upto a rescaling to a true two-parametric deformation of  $GL(1/1)$ . It also leads to a true two-parametric DJ deformation of  $U[gl(1/1)]$ .

### III. A QUASI THREE-PARAMETRIC DEFORMATION OF $GL(1/1)$

As the maximal number of parameters of a quantum deformation of  $GL(1/1)$  is two, the below-obtained deformation of  $GL(1/1)$  is in fact quasi-three parametric (so is the corresponding  $R$ -matrix). We will see below that such an over-parametrization is very convenient.

Let us start with the operator

$$R = q(e_1^1 \otimes e_1^1) + r(e_1^1 \otimes e_2^2) + s(e_2^2 \otimes e_1^1) + \lambda(e_2^1 \otimes e_1^1) + p(e_2^2 \otimes e_2^2), \quad (8)$$

where  $p, q, r, s$  and  $\lambda$  are complex deformation parameters, while  $e_j^i, i, j = 1, 2$ , are Weyl generators of  $GL(1|1)$  with a  $Z_2$ -grading given as follows:

$$[e_j^i] = [i] + [j] \pmod{2}, [i] = \delta_{i2}. \quad (9)$$

We call the latter operator an  $R$ -matrix although it has a (finite) matrix form only in a finite-dimensional representation. In the fundamental representation  $e_j^i$  are super-Weyl matrices,  $(e_j^i)_k^h = \delta_k^i \delta_j^h$ , and  $R$  is a  $4 \times 4$  matrix. Three of the five parameters, say,  $p, q$  and  $r$ , can be chosen to be independent, while the remaining parameters,  $s$  and  $\lambda$ , are subject to the constraints

$$rs = pq, \lambda = q - p. \quad (10)$$

By this choice of the parameters, the  $R$ -matrix (1) satisfies the graded YBE

$$R_{12}R_{13}R_{23} = R_{23}R_{13}R_{12}, \quad (11)$$

with

$$\begin{aligned} R_{12} &= R \otimes I \equiv R \otimes e_i^i, \quad i = 1, 2, \\ R_{13} &= q(e_1^1 \otimes e_i^i \otimes e_1^1) + r(e_1^1 \otimes e_i^i \otimes e_2^2) + s(e_2^2 \otimes e_i^i \otimes e_1^1) \\ &\quad + (-1)^{[i]} \lambda(e_2^1 \otimes e_i^i \otimes e_1^1) + p(e_2^2 \otimes e_i^i \otimes e_2^2), \\ R_{23} &= I \otimes R \equiv e_i^i \otimes R, \end{aligned} \quad (12)$$

where repeated indices are summation indices,  $I$  is the identity operator and the  $Z_2$ -grading is given in (9).

Now suppose the operator subject

$$T = a e_1^1 + \beta e_2^1 + \gamma e_1^2 + d e_2^2 \equiv t_i^j e_j^i \quad (13)$$

obeys the so-called  $RTT$  equation

$$RT_1T_2 = T_2T_1R, \quad (14)$$

where

$$\begin{aligned} T_1 &= T \otimes I \equiv (ae_1^1 + \beta e_2^1 + \gamma e_1^2 + de_2^2) \otimes e_j^i, \\ T_2 &= I \otimes T \\ &\equiv e_j^i \otimes [ae_1^1 + (-1)^{[i]} \beta e_2^1 + (-1)^{[i]} \gamma e_1^2 + de_2^2]. \end{aligned} \quad (15)$$

The Eq. (14) leads to the supercommutation relations between the elements of  $T$  :

$$\begin{aligned} a\beta &= \frac{r}{p}\beta a, \quad a\gamma = \frac{q}{r}\gamma a, \quad ad = da + \frac{\lambda}{r}\gamma\beta, \quad \beta^2 = 0 = \gamma^2, \\ \beta\gamma &= -\frac{s}{r}\gamma\beta \equiv -\frac{pq}{r^2}\gamma\beta, \quad \beta d = \frac{p}{r}d\beta, \quad \gamma d = \frac{r}{q}d\gamma. \end{aligned} \quad (16)$$

Let us denote  $G$  a set of all operators (13) satisfying (14) and let  $T$  and  $T'$  be two independent copies of (13) in the sense that all elements  $t_j^i$  of  $T$  commute with all those of  $T'$ . The fact that the

multiplication  $T.T'$  preserves the relation (14), that is, the relations (16), reflects the group nature of  $G$ . Next, since the quantity

$$\begin{aligned} D(T) &\equiv (a - \beta d^{-1} \gamma) d^{-1} = d^{-1} (a - \beta d^{-1} \gamma) \\ &= a (d - \gamma a^{-1} \beta)^{-1} \end{aligned} \tag{17}$$

commutes with  $T$  and has the "multiplicative" property  $D(T.T') = D(T).D(T')$  it can be identified with a representation of a quantum superdeterminant. Thus we can take  $G$  with  $D(T) \neq 0, \forall T \in G$ , as a quasi three-parametric deformation, denoted by  $GL_{p,q,r}(1/1)$ , of a representation of  $GL(1/1)$ . The latter deformation is equivalent upto a rescaling (e.g.,  $p/r \rightarrow p, q/r \rightarrow q$ ) to a two-parametric deformation, say  $GL_{p,q}(1/1)$ , but we keep the quasi three-parametric form until obtaining a true two-parametric deformation of  $U[gl(1/1)]$ . When we set  $D(T) = 1$  we get a quasi three-parametric deformation of  $SL(1/1)$ . We note that the form of the quantum superdeterminant  $D(T)$  is the same as that given in [27], that is, it remains non-deformed and belongs to the center of  $GL_{p,q,r}(1/1)$ . The Hopf structure is straightforward and given by the following maps:

- the co-product:

$$\Delta(T) = T \otimes T, \tag{18}$$

- the antipode:

$$S(T).T = I, \tag{19}$$

- the counit:

$$\varepsilon(T) = I. \tag{20}$$

In components they read

$$\Delta(t_j^i) = t_j^k \otimes t_k^i, \tag{21}$$

$$\begin{aligned} S(t_i^j e_j^i) &= S(t_i^j) e_j^i \\ &= a^{-1} (1 + \beta d^{-1} \gamma a^{-1}) e_1^1 - (a^{-1} \beta d^{-1}) e_2^1 \\ &\quad - (d^{-1} \gamma a^{-1}) e_1^2 + d^{-1} (1 - \beta a^{-1} \gamma d^{-1}) e_2^2, \end{aligned} \tag{22}$$

$$\varepsilon(t_j^i) = \delta_j^i. \tag{23}$$

A quantum superplane with symmetry (automorphism) group  $GL_{p,q,r}(1/1)$  is given by the coordinates

$$\begin{pmatrix} x \\ \theta \end{pmatrix} \text{ or } \begin{pmatrix} \eta \\ y \end{pmatrix} \tag{24}$$

subject to the commutation relations

$$x\theta = \frac{q}{r} \theta x \equiv \frac{s}{p} \theta x, \theta^2 = 0 \text{ or } \eta^2 = 0, \eta y = \frac{p}{r} y \eta, \tag{25}$$

respectively. Note that these quantum superplanes (which are "two-dimensional") are still two-parametric (of course, we cannot make relations between two coordinates to depend on more than two parameters). Finally, in order to complete our program we must construct a true two-parametric DJ deformation of the universal enveloping algebra  $U[gl(1/1)]$ . It can be obtained from a quasi-three parametric DJ deformation, denoted as  $U_{p,q,r}[gl(1/1)]$ , corresponding to the R-matrix (8).

#### IV. A TWO-PARAMETRIC DRINFEL'D–JIMBO DEFORMATION OF $U[\mathfrak{gl}(1/1)]$

First, following the technique of [7], we introduce two auxiliary operators

$$\begin{aligned} L^+ &= H_1^+ e_1^1 + H_2^+ e_2^2 + \lambda X^+ e_2^1, \\ L^- &= H_1^- e_1^1 + H_2^- e_2^2 + \lambda X^- e_1^2, \end{aligned} \quad (26)$$

with  $H_i^\pm$  and  $X^\pm$  belonging to  $U_{p,q,r}[\mathfrak{gl}(1/1)]$  to be constructed. Then, demanding

$$\begin{aligned} L_1^\pm &= L^\pm \otimes e_i^i, \\ L_2^+ &= e_i^i \otimes [H_1^+ e_1^1 + H_2^+ e_2^2 + (-1)^{|i|} \lambda X^+ e_2^1], \\ L_2^- &= e_i^i \otimes [H_1^- e_1^1 + H_2^- e_2^2 + (-1)^{|i|} \lambda X^- e_1^2] \end{aligned}$$

to obey the equations

$$RL_1^{\epsilon_1} L_2^{\epsilon_2} = L_2^{\epsilon_2} L_1^{\epsilon_1} R, \quad (27)$$

where  $(\epsilon_1, \epsilon_2) = (+, +), (-, -), (+, -)$ , we get the following commutation relations between  $H_i^\pm$  and  $X^\pm$ :

$$\begin{aligned} H_i^{\epsilon_1} H_j^{\epsilon_2} &= H_j^{\epsilon_2} H_i^{\epsilon_1}, \\ pH_i^+ X^+ &= rX^+ H_i^+, & qH_i^- X^+ &= rX^+ H_i^-, \\ rH_i^+ X^- &= pX^+ H_i^+, & rH_i^- X^- &= qX^- H_i^-, \\ rX^+ X^- + sX^- X^+ &= \lambda^{-1}(H_2^- H_1^+ - H_2^+ H_1^-), \end{aligned} \quad (28)$$

which are taken to be the defining relations of  $U_{p,q,r}[\mathfrak{gl}(1/1)]$ . Its Hopf structure is given by

$$\Delta(L^\pm) = L^\pm \otimes L^\pm, \quad (29)$$

$$S(L^\pm) = (L^\pm)^{-1}, \quad (30)$$

$$\varepsilon(L^\pm) = I, \quad (31)$$

or equivalently (no summation on  $i = 1, 2$ ),

$$\begin{aligned} \Delta(H_i^\pm) &= H_i^\pm \otimes H_i^\pm, \\ \Delta(X^+) &= H_1^+ \otimes X^+ + X^+ \otimes H_2^+, \\ \Delta(X^-) &= H_2^- \otimes X^- + X^- \otimes H_1^-, \end{aligned} \quad (32)$$

$$\begin{aligned} S(H_i^\pm) &= (H_i^\pm)^{-1}, \\ S(X^+) &= -(H_1^+)^{-1} X^+ (H_2^+)^{-1}, \\ S(X^-) &= -(H_2^-)^{-1} X^- (H_1^-)^{-1}, \end{aligned} \quad (33)$$

$$\varepsilon(H_i^\pm) = 1, \quad \varepsilon(X^\pm) = 0. \quad (34)$$

At first sight  $U_{p,q,r}[gl(1/1)]$  given in (28) is a three-parametric quantum supergroup depending on three parameters  $p$ ,  $q$  and  $r$  (or  $s$ ). However, making the substitution

$$\begin{aligned} H_1^+ &= \left(\frac{r}{p}\right)^{E_{11}}, & H_2^+ &= \left(\frac{p}{r}\right)^{E_{22}}, \\ H_1^- &= \left(\frac{r}{q}\right)^{E_{11}}, & H_2^- &= \left(\frac{q}{r}\right)^{E_{22}}, \\ E_{12} &= X^+ r^{E_{22}}, & E_{21} &= X^- s^{E_{11}}, \end{aligned} \quad (35)$$

in (28) and replacing  $p$  by  $p^{-1}$  (without loss of generality), we obtain a two-parametric deformation of  $U[gl(1/1)]$  generated by  $E_{ij}$ , which are two-parametric analogs of the Weyl generators, via the following relations

$$\begin{aligned} [E_{ii}, E_{jj}] &= 0, \\ [E_{ii}, E_{j,j\pm 1}] &= (\delta_{ij} - \delta_{i,j\pm 1})E_{j,j\pm 1}, \\ \{E_{12}, E_{21}\} &= [K]_{p,q}, \end{aligned} \quad (36)$$

where  $1 \leq i, j, j \pm 1 \leq 2$  and

$$[K]_{p,q} = \frac{q^K - p^{-K}}{q - p^{-1}}, \quad K = E_{11} + E_{22}. \quad (37)$$

The latter deformation denoted as  $U_{p,q}[gl(1/1)]$  is a true two-parametric DJ deformation of  $U[gl(1/1)]$ , which we are looking for, as it cannot be made to become one-parametric by a further rescaling of its generators. Of course, (35) is not the only realization of the generators of  $U_{p,q,r}[gl(1/1)]$  in terms of the deformed Weyl generators  $E_{ij}$ .

## V. CONCLUSION

We have suggested in the present paper an  $R$ -matrix satisfying a (quasi) three-parametric graded YBE. Using this overparametrized  $R$ -matrix we can obtain two-parametric deformations  $GL_{p,q}(1/1)$  and  $U_{p,q}[gl(1/1)]$ , respectively, of the supergroup  $GL(1/1)$  and the corresponding universal enveloping algebra  $U[gl(1/1)]$ , respectively. It is worth noting that the quantum superalgebra  $U_{p,q}[gl(1/1)]$  is a true two-parametric deformation of  $U[gl(1/1)]$  generalizing the Drinfel'd–Jimbo deformation  $U_q[gl(1/1)]$  which is one-parametric. That is  $U_{p,q}[gl(1/1)]$  cannot be reduced to any one-parametric deformation by any re-scaling or re-definition of generators. Physics interpretations and applications of these two-parametric deformations are a subject of our current interest.

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