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Deterministic joint remote preparation of an equatorial hybrid state via high-dimensional Einstein–Podolsky–Rosen pairs: active versus passive receiver

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Abstract Entanglement plays a vital and in many cases non-replaceable role in the quantum network communication. Here, we propose two new protocols to jointly and remotely prepare a special so-called bipartite equatorial state which is hybrid in the sense that it entangles two Hilbert spaces with arbitrary different dimensions D and N (i.e., a type of entanglement between a quDit and a quNit). The quantum channels required to do that are however not necessarily hybrid. In fact, we utilize four high-dimensional Einstein–Podolsky–Rosen pairs, two of which are quDit–quDit entanglements, while the other two are quNit–quNit ones. In the first protocol the receiver has to be involved actively in the process of remote state preparation, while in the second protocol the receiver is passive as he/she needs to participate only in the final step for reconstructing the target hybrid state. Each protocol meets a specific circumstance that may be encountered in practice and both can be performed with unit success probability. Moreover, the concerned equatorial hybrid entangled state can also be jointly prepared for two receivers at two separated locations by slightly modifying the initial particles' distribution, thereby establishing between them an entangled channel ready for a later use.

Keywords Deterministic joint remote state preparation · Equatorial state · Hybrid dimension · High-dimensional EPR pair · Active receiver · Passive receiver

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1 Introduction

Quantum entanglement, a kind of 'spooky action at a distance' [1], has been recognized as a useful resource for quantum information processing and quantum computing [2]. Physically, quantum entanglement is the peculiar nonlocal correlation possessed by distant, even space-like, quantum systems. The best-known entanglement-based applications are quantum cryptography [3], quantum dense coding [4], quantum teleportation (QT) [5] and exponentially speeding-up quantum algorithm [6]. As far as transferring quantum state between remote nodes by means of local operation and classical communication (LOCC) is concerned, QT is undoubtedly the most celebrated and powerful protocol since it works for any unknown state. However, QT requires physically supplying the state to be transferred that is not always at will (because of both inconvenience and secrecy matter). Avoiding such supplying of the state is possible if its full classical knowledge is known by the teleporter (or, more generally, the sender or the preparer). The protocol in this case is referred to as remote state preparation (RSP) [7,8] which has also obtained remarkable results in both theory and experiment by using different methods via different entangled channels. For examples, theoretical protocols such as low-entanglement RSP [9], higher-dimension RSP [10], optimal RSP [11] and so on have been proposed, while experimental RSP has been realized by using the technique of nuclear magnetic resonance [12], via dephasing entanglement by using spontaneous parametric down-conversion and linear-optics elements [13], etc. Although the demanded amount of ebit is the same as in QT, the local operation is simpler and the classical communication cost (CCC) is cheaper than those in QT, RSP does not always succeed in general, not already speaking about the fact that the whole secrecy encoded in the state is disclosed for free to the preparer. To circumvent such sensitive drawbacks RSP was extended to a new kind of protocol called joint RSP (JRSP) [14–16] (see also [17–35] for many abundant aspects of the problem). In JRSP, the number of preparers is more than one and the secrecy is intentionally shared among them so that no one has the full information about the state, thus providing the security level higher than that in RSP. More than that, by adopting wise strategies for secret sharing and for the preparers' measurements, success probability of JRSP can always be made 100%, a feature transparently superior to RSP. Yet, following the rule 'nothing is fully for gratis', JRSP needs a larger amount of ebit compared to that in QT and RSP.

Conventionally qubits, quantum states in 2-dimensional Hilbert space, are utilized to encode/manipulate information. Recently, it turns out that using quNits, quantum states in N-dimensional ($N > 2$) Hilbert space, in mutually unbiased bases [36,37] may be advantageous in many situations. For example, the security level is enhanced (e.g., it is safer against eavesdropping) and the noise effect is reduced in quNit-based quantum key distribution protocols [38–43] compared to that based on qubits, the local realism violation is stronger by quNit–quNit entanglements than by qubit–qubit ones [44], the efficiency of Bell state measurements in QT is increased if information is encoded in quNits [45], also the detection efficiency required for closing the detection loophole in Bell inequality tests can be significantly lowered in case of using dimensions larger than two [46], etc.

Entanglement can exist between systems of different natures as well. Preparation of such hybrid entanglement would allow mapping between parts and thus flexible working of a future quantum internet where information can be stored, transferred and processed by means of very distinct ways of encoding based on widely different physical platforms. A whole set of novel hybrid protocols involving discrete-continuous-variable techniques has been provided experimentally [47]. Theoretical ideas of applying micro–macro hybrid entangled states to perform QT [48,49] and test Bell–Clauser-Horne-Shimony-Holt inequality [50] have been considered. Furthermore, mechanisms and realization of such quantum-classical hybrid entanglement have been devised and experimented [51–53]. Of interest are also entanglements between Hilbert spaces with different dimensions [54,55].

Here, motivated by a very recent publication [56], which deals with JRSP of equatorial bi- and multipartite entangled states with hybrid dimensions, we consider the same task but by different methods suitable to different practical circumstances. Namely, while the authors in Ref. [56] used high-dimensional Greenberger–Horne–Zeilinger (GHZ) trios [57] as quantum channels, we in this work instead employ only high-dimensional Einstein–Podolsky–Rosen (EPR) [58] pairs which are not only easier producible but also make simpler the checking process during the entanglement distribution stage prior to actual execution of the task. Furthermore, of our concern is the technical ability of the receiver who may in practice be not so well or very well-equipped. Depending on the circumstances the receiver should participate in the JRSP protocol actively or passively, as will be detailed in our first and second protocol. It is worthy to note that though the methods in the two protocols differ, both achieve the same goal with 100% success probability. The first protocol with an active receiver will be presented in Sect. 2. Section 3 is devoted to the case of passive receiver. Some discussion will be given in Sect. 4, and Sect. 5 is the conclusion.

2 Protocol with an active receiver

Suppose that a boss, whose name is Bos, wants to securely and faithfully prepare for a remote Charlie an equatorial hybrid quDit–quNit state of the form

$$|\psi\rangle = \frac{1}{\sqrt{DN}} \sum_{d=0}^{D-1} \sum_{n=0}^{N-1} e^{ic_{dn}} |d,\ n\rangle\,, \tag{1}$$

which is characterized by a set of real numbers $\{c_{dn} \in \mathcal{R};\, d = 0, 1, \ldots, D-1;\ n = 0, 1, \ldots, N-1\}$ with D and N being any integers greater than 2.

In order to do that Bos secretly splits $\{c_{dn}\}$ into two subsets $S_A = \{a_{dn}\} \in R$ and $S_B = \{b_{dn}\} \in R$ such that $a_{dn} + b_{dn} = c_{dn}\ \forall d, n$. Then Bos chooses two of his many staffs, Alice and Bob, and lets each of them know a subset mentioned above, say, Alice knows S_A while Bob knows S_B. Suppose that Alice and Bob belong to two separate laboratories and one does not know what kind of job is assigned to the other one, so no one (except Bos, of course) knows the full detail of $|\psi\rangle$. Because only LOCC are allowed, Alice, Bob and Charlie must beforehand be 'connected' by some appropriate entangled channels. Here, different from [56], four high-dimensional

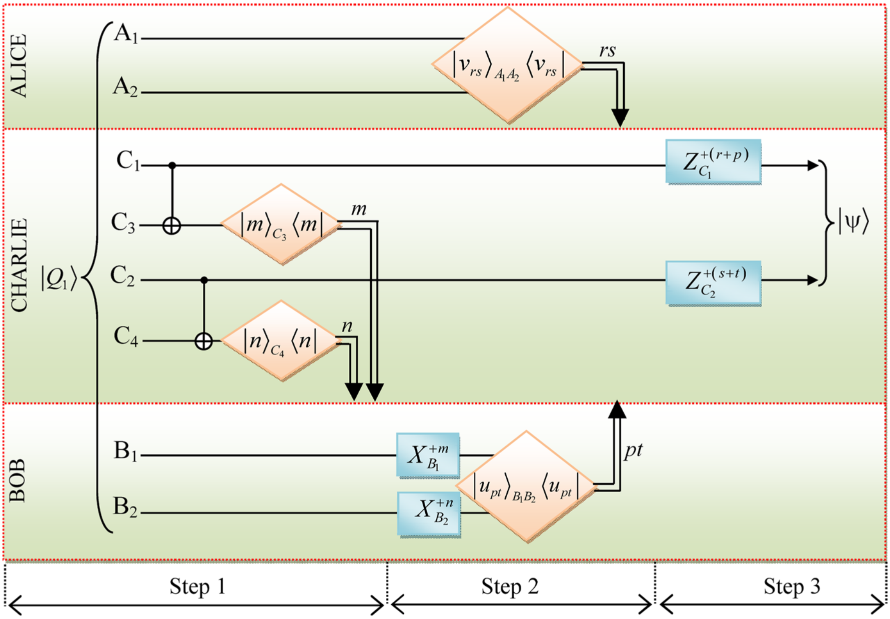

Fig. 1 Protocol with an active receiver. The state of the working quantum channel is given by Eq. (2), in which A_1, C_1, B_1 and C_3 are quDits, while A_2, C_2, B_2 and C_4 are quNits. Rectangles represent unitary transformations, diamonds measurements and double lines measurement outcomes

EPR pairs are served as the working quantum channels, two of which are quDit–quDit entanglements, while the other two are quNit–quNit ones. They can be written as

$$|Q_1\rangle_{A_1C_1A_2C_2B_1C_3B_2C_4} = |\mathrm{EPR}_D\rangle_{A_1C_1} \otimes |\mathrm{EPR}_N\rangle_{A_2C_2} \otimes |\mathrm{EPR}_D\rangle_{B_1C_3} \otimes |\mathrm{EPR}_N\rangle_{B_2C_4}, \tag{2}$$

where

$$|\mathrm{EPR}_M\rangle_{XY} = \frac{1}{\sqrt{M}} \sum_{k=0}^{M-1} |k, k\rangle_{XY} \tag{3}$$

is an M-dimensional EPR pair which is maximally entangled between quMits X and Y. In the state (2) Alice holds quDit A_1 and quNit A_2 , Bob holds quDit B_1 and quNit B_2, while quDits C_1, C_3 and quNits C_2, C_4 belong to Charlie. Of course during the process of sharing of the entangled channel an unauthorized party would attempt to eavesdrop or/and decoherence due to environment might occur, so the authorized parties should apply some effective means to detect the presence of eavesdropper or/and to distill the decohered channel. However, such delicate security checking procedures are beyond the scope of this work. Here, as in other quantum global protocols, we assume that we start to perform our task after we are securely provided with the desired quantum channel. With such an assumption our protocol is composed of three sequential steps, and the necessary actions in each step are illustrated in Fig. 1.

In the first step, Charlie first performs two high-dimensional controlled-NOT gates, $\text{CNOT}_{C_1C_3}$ on two quDits C_1, C_3 and $\text{CNOT}_{C_2C_4}$ on two quNits C_2, C_4: $\text{CNOT}_{C_1C_3}|i, j\rangle_{C_1C_3} = |i, i \oplus j \mod D\rangle_{C_1C_3}$ and $\text{CNOT}_{C_2C_4}|i, j\rangle_{C_2C_4} = |i, i \oplus j \mod N\rangle_{C_2C_4}$. Then he measures quDit C_3 in the basis $\{|m\rangle_{C_3}, m = 0, 1, \ldots, D-1\}$ and quNit C_4 in the basis $\{|n\rangle_{C_4}, n = 0, 1, \ldots, N-1\}$. If he finds $|m, n\rangle_{C_3C_4}$ (the classical outcome is mn, i.e., 1 dit and 1 nit), which happens with an equal probability of $1/(DN)$ for any possible outcome mn, state of the unmeasured quDits and quNits becomes

$$\begin{aligned}|Q_2\rangle_{A_1A_2B_1B_2C_1C_2} = \frac{1}{\sqrt{DN}} &\sum_{k_1=0}^{D-1}\sum_{k_2=0}^{N-1} |k_1, k_2\rangle_{A_1A_2} \\ &\otimes |k_1 \oplus m \mod D, k_2 \oplus n \mod N\rangle_{B_1B_2} \\ &\otimes |k_1, k_2\rangle_{C_1C_2}\,. \end{aligned} \tag{4}$$

The second step is turn to Alice and Bob. While Bob's action is adaptive in the sense that it is subjected to Charlie's measurement outcome in the first step, Alice's action is independent. More concretely, depending on the outcome mn mentioned above, Bob first applies $X_{B_1}^{\dagger m}$ and $X_{B_2}^{\dagger n}$ on B_1 and B_2, respectively, where

$$X_{B_1} = \sum_{l=0}^{D-1} |l \oplus 1 \mod D\rangle_{B_1} \langle l| \tag{5}$$

is the 'dit-flip' operator and

$$X_{B_2} = \sum_{l=0}^{N-1} |l \oplus 1 \mod N\rangle_{B_2} \langle l| \tag{6}$$

the 'nit-flip' one. Note that 'dit-flip' and 'nit-flip' are not an exact flip $|0\rangle \to |1\rangle$ as in the qubit case, but just implies 'formal flips' under the conventions $|l\rangle \to |l \oplus 1 \mod D\rangle$ and $|n\rangle \to |n \oplus 1 \mod N\rangle$, respectively. After the action of such $X_{B_1}^{\dagger m}$ and $X_{B_2}^{\dagger n}$, the state (4) is transformed to

$$|Q_3\rangle_{A_1A_2B_1B_2C_1C_2} = \frac{1}{\sqrt{DN}} \sum_{k_1=0}^{D-1}\sum_{k_2=0}^{N-1} |k_1, k_2\rangle_{A_1A_2} |k_1, k_2\rangle_{B_1B_2} |k_1, k_2\rangle_{C_1C_2}\,. \tag{7}$$

Then, Bob measures $|Q_3\rangle_{A_1A_2B_1B_2C_1C_2}$ in the basis $\{|u_{pt}\rangle_{B_1B_2}; p = 0, 1, \ldots, D-1; t = 0, 1, \ldots, N-1\}$,

$$\left|u_{pt}\right\rangle_{B_1B_2} = \frac{1}{\sqrt{DN}} \sum_{d'=0}^{D-1}\sum_{n'=0}^{N-1} \omega_D^{-pd'} \omega_N^{-tn'} e^{-ib_{d'n'}} \left|d', n'\right\rangle_{B_1B_2} \tag{8}$$

with $\omega_M = e^{2\pi i/M}$. There are in total DN equally possible outcomes, each is specified by pt corresponding to finding $|u_{pt}\rangle_{B_1B_2}$. Of importance is the fact that Bob and only Bob is able to do the above measurement since only he knows the data subset S_B mentioned at the beginning of this section and only he is in possession of the quDit B_1 and the quNit B_2.

As for Alice, she makes use of her knowledge of the data subset S_A to measure her quDit A_1 and quNit A_2 in the basis $\{|v_{rs}\rangle_{A_1A_2}\}$ defined as

$$|v_{rs}\rangle_{A_1A_2} = \frac{1}{\sqrt{DN}} \sum_{d''=0}^{D-1} \sum_{n''=0}^{N-1} \omega_D^{-rd''} \omega_N^{-sn''} e^{-ia_{d''n''}} \left|d'', n''\right\rangle_{A_1A_2} \tag{9}$$

with $r = 0, 1, \ldots, D-1$ and $s = 0, 1, \ldots, N-1$. Again, of the same degree of importance, no one else but Alice who is able to do the above measurement since only she knows the data subset S_A and only she is in possession of the quDit A_1 and the quNit A_2. A possible outcome is the finding of $|v_{rs}\rangle_{A_1A_2}$, which is denoted by rs and also happens with a probability of $1/(DN)$ for any rs. At the end of this step only quDit C_1 and quNit C_2 remain unmeasured whose state, however, due to the measurements by Charlie, Alice and Bob, is collapsed into

$$\left|\widetilde{\psi}\right\rangle_{C_1C_2} = \frac{1}{\sqrt{DN}} \sum_{d=0}^{D-1} \sum_{n=0}^{N-1} \omega_D^{(r+p)d} \omega_N^{(s+t)n} e^{ic_{dn}} |d, n\rangle_{C_1C_2}, \tag{10}$$

which entangles C_1 and C_2 though they were separable initially.

In the final step, Charlie collects all the outcomes of Alice's and Bob's measurements done in the second step, then applies the operators $Z_{C_1}^{\dagger(r+p)}$ on C_1 and $Z_{C_2}^{\dagger(s+t)}$ on C_2, where

$$Z_{C_1} = \sum_{l=0}^{D-1} \omega_D^l |l\rangle_{C_1} \langle l| \tag{11}$$

and

$$Z_{C_2} = \sum_{k=0}^{N-1} \omega_N^k |k\rangle_{C_2} \langle k| \tag{12}$$

are, respectively, D- and N-dimensional 'phase-flip' operators which are formal generalizations of the exact phase-flip $\{|k\rangle \to (-1)^k|k\rangle;\ k = 0, 1\}$ in the qubit case. Clearly,

$$Z_{C_2}^{\dagger(s+t)} Z_{C_1}^{\dagger(r+p)} \left|\widetilde{\psi}\right\rangle_{C_1C_2} = |\psi\rangle_{C_1C_2}, \tag{13}$$

implying that Charlie successfully and faithfully receives the desired hybrid state (1).

3 Protocol with a passive receiver

In the above-presented protocol, the receiver Charlie plays an active role. It is he who starts and completes the JRSP task. This is possible if Charlie is in the first step capable

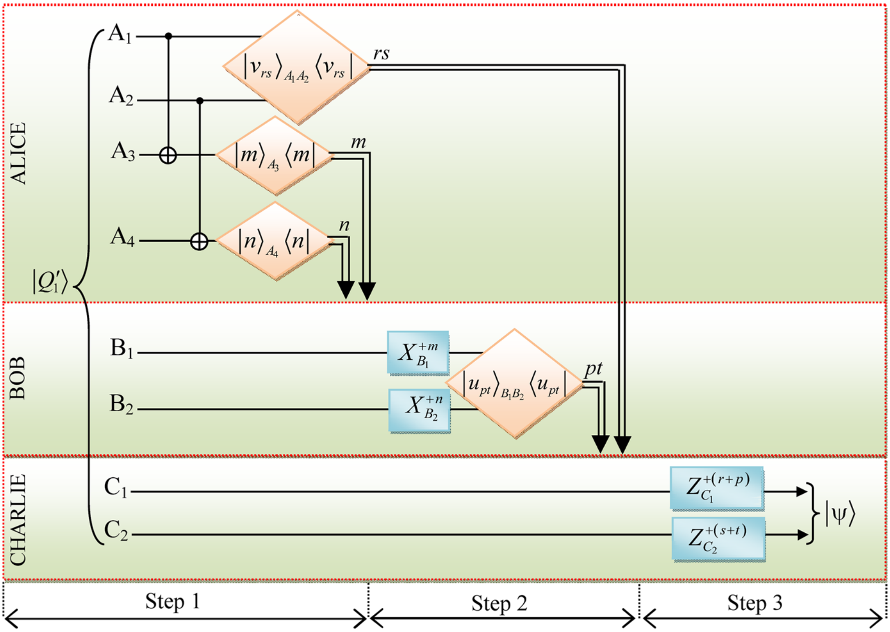

Fig. 2 Protocol with a passive receiver. The state of the working quantum channel is given by Eq. (14), in which A_1, C_1, A_3 and B_1 are quDits, while A_2, C_2, A_4 and B_2 are quNits. Rectangles represent unitary transformations, diamonds measurements and double lines measurement outcomes

of implementing the two quantum high-dimensional controlled-NOT gates which are commonly regarded as difficult technically. In realistic circumstances, Charlie may be not well-equipped, say, performing two-quMit gates is impossible in his laboratory. In that case, the protocol we shall present in this section will suit to overcome the mentioned technical difficulty.

Still four high-dimensional EPR pairs are used for the working quantum channels, but the entanglement distribution differs in this situation. Namely, instead of (2), now we need

$$|Q_1'\rangle_{A_1C_1A_2C_2A_3B_1A_4B_2} = |\text{EPR}_D\rangle_{A_1C_1} \otimes |\text{EPR}_N\rangle_{A_2C_2} \\ \otimes |\text{EPR}_D\rangle_{A_3B_1} \otimes |\text{EPR}_N\rangle_{A_4B_2}, \quad (14)$$

of which quDits A_1, A_3 and quNits A_2, A_4 are with Alice, quDit B_1 and quNit B_2 are with Bob, while Charlie holds only two (not four as in the protocol with an active receiver) particles, one quDit C_1 and one quNit C_2. An obvious difference is already seen when comparing the numbers of particles possessed by Alice, Bob and Charlie (in the protocol described in Sect. 2 those numbers are 2, 2 and 4, whereas in the protocol to be presented in this section they are 4, 2 and 2). Here, as in the previous section, we again assume that we are securely provided with the desired quantum channel $|Q_1'\rangle_{A_1C_1A_2C_2A_3B_1A_4B_2}$. Because the entanglement distribution is different, the way to execution also differs, though the number of steps remains three as sketched in Fig. 2.

Specifically, Alice (but not Charlie) initiates the first step by performing high-dimensional controlled-NOT gates $\text{CNOT}_{A_1A_3}$ and $\text{CNOT}_{A_2A_4}$ followed by two kinds of measurement. The first kind is to measure A_3 and A_4 like Charlie did for C_3 and C_4 in the protocol with an active receiver. That is, A_3 is measured in the basis $\{|m\rangle_{A_3}; m = 0, 1, \ldots, D-1\}$ and A_4 in the basis $\{|n\rangle_{A_4}; n = 0, 1, \ldots, N-1\}$, with the outcome mn if $|mn\rangle_{A_3A_4}$ is found. The second kind of measurement is exactly like what she did in the protocol with an active receiver. That is, A_1 and A_2 are collectively measured in the hybrid basis $\{|v_{rs}\rangle_{A_1A_2}\}$ defined in Eq. (9).

In the second step, Bob uses the outcome mn of Alice's first kind measurement and the data subset S_B to do the same things as in the protocol with an active receiver. That is, B_1 and B_2 are separately acted upon by $X_{B_1}^{\dagger m}$ and $X_{B_2}^{\dagger n}$, then together measured in the basis $\{|u_{pt}\rangle_{B_1B_2}\}$ defined in Eq. (8), with the outcome pt if $|pt\rangle_{B_1B_2}$ is found.

The final step is exactly the same as in the protocol with an active receiver. That is, Charlie, after hearing the outcomes rs and pt of Alice's and Bob's measurements, reconstructs the target hybrid state by applying the operator $Z_{C_1}^{\dagger(r+p)}$ on C_1 and $Z_{C_2}^{\dagger(s+t)}$ on C_2. Transparently, the total success probability is also one as in the protocol with an active receiver.

4 Discussion

As technological facilities vary from laboratory to laboratory, it is encouraging to devise for one and the same task several protocols each is suitable to an available technology level of a given laboratory. The task of concern here is JRSP of equatorial bipartite entangled states with hybrid dimensions. We assume that the preparers are well-equipped, but the receiver may or may not be so. Thus, two different protocols are proposed to fit the two 'opposite' situations. If the receiver is capable of performing quantum high-dimensional controlled-NOT gates (i.e., well-equipped), he is involved in both the first and the last step of a protocol, called protocol with an active receiver (as described in Sect. 2). On the contrary, if such controlled-NOT gates are beyond the reach of the receiver (i.e., not well-equipped), he can participate just with simple single-particle 'phase-flip' gates only in the last step of a protocol, called protocol with a passive receiver (as described in Sect. 3).

Both the protocols are deterministic thanks to a certain sequence of operations that the participants should comply with. The action of Bob on his B_1 and B_2 has to be executed in the second step, only after hearing the outcome mn of the measurement in the first step, because this is the key element to achieve unit success probability. However, there is a flexibility regarding the measurement of quDit A_1 and quNit A_2 in the basis $\{|v_{rs}\rangle_{A_1A_2}\}$. In Fig. 1 this measurement is shown to be carried out in the second step, while in Fig. 2 it is within the first step. Actually, in each of the two protocols, measuring A_1 and A_2 can be done either in the first or in the second step, but, compulsorily, this must be done before the third step.

In both the protocols not only the same amount of entanglement (i.e., four high-dimensional EPR pairs) is consumed, but also the same CCC (i.e., three dits plus three nits) is. The management of classical communication is the same as well. All the measurements outcomes should be publicly announced, but the data mn, the outcomes of measuring C_3, C_4 (or A_3, A_4) in the first (or second) protocol, should be announced right after finishing the measurement in the first step in order to proceed to the second step (in fact, it is this feed-forwarding measurement strategy that gives rise to unit total success probability), while announcement of the other data rs and pt, the outcomes of measuring A_1, A_2 and B_1, B_2, can be postponed until starting the last step.

One more subtle thing to be addressed is as follows. Up to now Charlie is a single party at a single location. Since $|\psi\rangle$ in Eq. (1) is the most general bipartite equatorial hybrid state, depending in the concrete values of the coefficients c_{dn} it may be an entangled state. In this case, it is surely better, for a later purpose, to prepare it among two parties, say, Charlie 1 and Charlie 2, who are at two remote locations. Interestingly, this is resolved simply in the second protocol (see Fig. 2) if C_1 is sent to Charlie 1 and C_2 to Charlie 2 during the entanglement distribution before execution of the JRSP. As for the first protocol (see Fig. 1) this can also be resolved by distributing C_1 and C_3 to Charlie 1, while C_2 and C_4 to Charlie 2, so that each Charlie is still able to perform a necessary high-dimensional controlled-NOT gate.

Last but not least, we would like to say some words about the feasibility of our protocols which mostly depends on technical availability in laboratories. The difficulty concerns the measurements in the bases (8) and (9) which are hybrid entangled bases of a quDit and a quNit. Here, we theoretically assume such measurements can be performed with unit probability. But in practice it is up to concrete measurement schemes and hardly to be of 100% success using current technology. As is well known, even two-qubit Bell state measurement cannot succeed with a probability larger than 50% by using passive linear optics and photodetectors. However, there have been considerable efforts to boost the Bell state measurement success probability resorting to different additional means (see, e.g., [59–61] and the references therein). These ideas or similar ones might be applied to the regime of measurements in high-dimensional hybrid entangled bases.

5 Conclusion

In summary, we have proposed two deterministic protocols for joint remote preparation of an equatorial quDit–quNit state living in two Hilbert spaces of different dimensions D and N via four high-dimensional EPR pairs combined with classical communication of three dits and three nits. One protocol works for a well-equipped active receiver, while the other one for a passive receiver who is not well-equipped. The determinacy (i.e., 100% success probability) is guaranteed by an adaptive measurement strategy (the action in the second step depends on the outcome of the measurement in the first step). By suitably modifying the distribution of the particles before performing the JRSP both the protocols can be used to establish hybrid entanglement between distant locations. The present protocols can be extended to those of joint remote preparation (with more than two preparers) and controlled joint remote preparation (with one or more than one controller) of the most general form of bipartite as well as multipartite entangled states with hybrid dimensions by using only high-dimensional EPR pairs. Effects of noises or/and non-maximal entangled quantum channels can also be straightforwardly accounted for.

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