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Functional integral method in quantum field theory of Dirac fermions in graphene

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Abstract

The purpose of this work is to elaborate the functional integral method in quantum field theory of Dirac fermions in the Dirac fermion gas of a graphene single layer at vanishing absolute temperature. The starting point to be assumed as the fundamental principle of the theory is the explicit expression of the action functional of this system. The efficient mathematical tool to be used in the study is the generating functional containing the Grassmann parameters anticommuting with the Dirac fermion field operators.

The analytical expression of the generating functional of free Dirac fermion system is exactly derived and efficiently used in the study of $2n$-point Green functions of free Dirac fermions. Then the celebrated Hubbard–Stratonovich transformation is applied to rewrite the functional integral of the interacting system of Dirac fermions in a new form expressing in terms of a scalar Hermitian quantum field describing the collective excitations in the interacting Dirac fermion gas and related to the graphene plasmons.

Keywords: functional integral, Dirac fermions, collective excitation, generating functional, Green functions
Classification numbers: 3.00, 5.15

1. Introduction

The discovery of graphene by Novoselov *et al* [1–4] has opened a new period in the development of condensed matter physics and materials science. Soon after this discovery a large number of basic and applied research works on graphene and graphene-based nanostructures has been performed [5, 6]. In the dynamical processes where the spin degree of freedom of electrons plays no role and therefore can be ignored, electrons can be considered as spinless fermions. In this case the quantum motion of charge carriers in single-layer graphene can be described by 2-component wave function satisfied Dirac equations in $(2 + 1)$-dimensional space-time and, therefore, they are called Dirac fermions [7].

The frequently applied method for the theoretical study of interaction processes between Dirac fermions as well as between Dirac fermions and the electromagnetic field is the perturbation theory with the use of Green functions. For example, explicit expressions of 2-point Green functions of free Dirac fermions can be used in the theoretical study of the generation of second order harmonics [8], third order harmonics [9] and high order harmonics [10] in graphene, the valley-dependent transport in graphene-based lateral quantum structures [11], the conductivity of gapped graphene [12], the photon-assited transport in bilayer graphene flakes [13], the scattering from spin-polarized charged impurities in graphene

[14], the effects of long range disorder and electronic interactions on the optical properties of graphene quantum dots [15], Landau level spectroscopy of electron–electron interactions in graphene [16] etc. The 2-point Green functions of Dirac fermions in graphene were studied in [17–19] by means of the conventional canonical quantization method of quantum field theory. However, the most universal and efficient method in quantum field theory is the functional integral method [20–22].

The purpose of this work is to present the basics of functional integral method in quantum field theory of Dirac fermion system in a graphene single layer. In the subsequent section 2 the notations and known formula for the physical quantities of the Dirac fermion system are introduced. In Particular, the explicit expression of the functional integral of the interacting system of Dirac fermions is presented. In section 3 the functional integral method is applied to the study of Green function of free Dirac fermion fields. We show that all they are expressed in terms of functional derivatives of the generating functional depending on Grassmann variables anticommuting with the Dirac fermion fields. From explicit expression of generating functional of free Dirac fermion field system it is straightforward to derive the formula of all $2n$-point Green functions and then to confirm the validity of the well-known Wick theorem in quantum theory of free fermion fields. The functional integral of the system of interacting Dirac fermion fields is studied in section 4. By using the celebrated Hubbard–Stratonovich transformation we demonstrate that the effective action functional of the interacting system of Dirac fermion field can be expressed in terms of the Green functions of free Dirac fermion fields and some quantum scalar field related with the plasmons in graphene single layer. Thus the mathematical tools for the study of plasmons in graphene is constructed. The conclusion and discussions are presented in section 5.

2. Notations and fundamental principles of the theory

Let us denote $\mathbf{x}$ the coordinate vector of a point in the plane of graphene and $x = \{\mathbf{x}, x_0\} = \{\mathbf{x}, t\}$ that of a point in the $(2+1)$-dimensional space-time. Quantum fields of Dirac fermions with momenta in the neighbours of two inequevalent Dirac points $\mathbf{K}$ and $\mathbf{K}'$ of the first Brillouin zone are described by two 2-component field operators $\psi^K(x)$ and $\psi^{K'}(x)$. A comprehensive review on dynamics of Dirac fermions in graphene was presented in [7]. In this work the authors showed that Hamiltonian of corresponding free Dirac fermions are

$$\begin{aligned} H^K &= v_F \boldsymbol{\tau}(-i\nabla), \\ H^{K'} &= v_F \boldsymbol{\tau}^*(-i\nabla). \end{aligned} \tag{1}$$

$\boldsymbol{\tau}$ being a 2D vector with components

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \tag{2}$$

The total action functional of the system of Dirac fermions in the presence of their Coulomb interaction has following expression

$$\begin{aligned} &I[\psi^K, \psi^{K'}, \overline{\psi}^K, \overline{\psi}^{K'}] \\ &= \int \mathrm{d}x\, \overline{\psi}^K(x) \left[\mathrm{i}\frac{\partial}{\partial x_0} - H^K \right] \psi^K(x) \\ &\quad + \int \mathrm{d}x\, \overline{\psi}^{K'}(x) \left[\mathrm{i}\frac{\partial}{\partial x_0} - H^{K'} \right] \psi^{K'}(x) \\ &\quad - \frac{1}{2} \int \mathrm{d}x \int \mathrm{d}y\, [\overline{\psi}^K(x) + \overline{\psi}^{K'}(x)] \quad [\psi^K(x) + \psi^{K'}(x)] \\ &\quad \times u(x-y)\, [\overline{\psi}^K(y) + \overline{\psi}^{K'}(y)]\, [\psi^K(y) + \psi^{K'}(y)], \end{aligned} \tag{3}$$

where

$$u(x-y) = \delta(x_0 - y_0) u(\mathbf{x} - \mathbf{y}),$$

$$u(\mathbf{x}-\mathbf{y}) = \frac{e^2}{|\mathbf{x} - \mathbf{y}|},$$

$$\int \mathrm{d}x = \int \mathrm{d}x_0 \int \mathrm{d}\mathbf{x} = \int \mathrm{d}x_0 \int \mathrm{d}x_1 \int \mathrm{d}x_2.$$

For simplifying formulae let us introduce 4-component spinor field

$$\psi = \begin{pmatrix} \psi^K \\ \psi^{K'} \end{pmatrix} \tag{4}$$

and consider 4×4 matrix

$$H = \begin{pmatrix} H^K & 0 \\ 0 & H^{K'} \end{pmatrix} \tag{5}$$

as the Hamiltonian of this 4-component spinor field. Then the total action functional of the interacting system of Dirac fermions becomes

$$\begin{aligned} I[\psi, \overline{\psi}] = &\int \mathrm{d}x\, \overline{\psi}(x) \left[\mathrm{i}\frac{\partial}{\partial x_0} - H \right] \psi(x) \\ &- \frac{1}{2} \int \mathrm{d}x \int \mathrm{d}y\, \overline{\psi}(x)\psi(x)u(x-y)\overline{\psi}(y)\psi(y). \end{aligned} \tag{6}$$

The key mathematical tool of the functional integral method in quantum field theory of interacting system of Dirac fermions is following functional integral [23]

$$\begin{aligned} Z^\psi &= \int [D\psi][D\overline{\psi}] \exp\{\mathrm{i}I[\psi; \overline{\psi}]\} \\ &= \int [D\psi][D\overline{\psi}] \exp\left\{ \mathrm{i} \int \mathrm{d}x\, \overline{\psi}(x) \left[\mathrm{i}\frac{\partial}{\partial x_0} - H \right] \psi(x) \right\} \\ &\quad \times \exp\left\{ -\frac{\mathrm{i}}{2} \int \mathrm{d}x \int \mathrm{d}y\, \overline{\psi}(x)\psi(x)u(x-y)\overline{\psi}(y)\psi(y) \right\}. \end{aligned} \tag{7}$$

3. Green functions of free Dirac fermions field

Consider now the case when the Coulomb interaction between Dirac fermions is neglected. In this case instead of Z^ψ we have

$$Z_0^\psi = \int [D\psi][D\overline{\psi}] \exp\left\{ \mathrm{i} \int \mathrm{d}x\, \overline{\psi}(x) \left[i\frac{\partial}{\partial x_0} - H \right] \psi(x) \right\}. \tag{8}$$

It is the functional of the system of free Dirac fermions. The statistical average, called also the expectation value, of the product of n pairs of components $\psi_{\alpha_i}(x_i)$ and $\overline{\psi}_{\beta_i}(y_i)$, $i = 1, 2, \ldots n$, of quantum fields of free Dirac fermions in the ground state $|0\rangle$ of the Dirac fermion gas at vanishing absolute temperature is determined by formula

$$\left\langle \psi_{\alpha_1}(x_1)\psi_{\alpha_2}(x_2)\ldots\psi_{\alpha_n}(x_n)\overline{\psi}_{\beta_n}(y_n)\ldots\overline{\psi}_{\beta_2}(y_2)\overline{\psi}_{\beta_1}(y_1)\right\rangle_0 = \frac{1}{Z_0^{\psi}}\int[D\psi][D\overline{\psi}]\exp\left\{\mathrm{i}\int \mathrm{d}x\ \overline{\psi}(x)\left[\mathrm{i}\frac{\partial}{\partial x_0} - H\right]\psi(x)\right\} \times \psi_{\alpha_1}(x_1)\psi_{\alpha_2}(x_2)\ldots\psi_{\alpha_n}(x_n)\overline{\psi}_{\beta_n}(y_n)\ldots\overline{\psi}_{\beta_2}(y_2)\overline{\psi}_{\beta_1}(y_1). \tag{9}$$

The $2n$-point green function of $2n$ components of free Dirac fermions are defined as follows:

$$G_{\alpha_1\alpha_2\ldots\alpha_n\beta_n\ldots\beta_2\beta_1}(x_1, x_2\ldots x_n; y_n, \ldots y_2, y_1) = \left\langle \psi_{\alpha_1}(x_1)\psi_{\alpha_2}(x_2)\ldots\psi_{\alpha_n}(x_n)\overline{\psi}_{\beta_n}(y_n)\ldots\overline{\psi}_{\beta_2}(y_2)\overline{\psi}_{\beta_1}(y_1)\right\rangle_0. \tag{10}$$

For establishing the functional integral method to the study of free Dirac fermion Green functional let us introduce 4-component Grassmann variables $\eta(x)$ and $\overline{\eta}(x)$ anticommuting with both free Dirac fermion fields $\psi(x)$ and $\overline{\psi}(x)$:

$$\{\eta_\alpha(x), \psi_\beta(y)\} = \{\eta_\alpha(x), \overline{\psi}_\beta(y)\} = \{\overline{\eta}_\alpha(x), \psi_\beta(y)\} = \{\overline{\eta}_\alpha(x), \overline{\psi}_\beta(y)\} = 0. \tag{11}$$

By definition they anticommute each with other:

$$\{\eta_\alpha(x), \eta_\beta(y)\} = \{\eta_\alpha(x), \overline{\eta}_\beta(y)\} = \{\overline{\eta}_\alpha(x), \eta_\beta(y)\} = \{\overline{\eta}_\alpha(x), \overline{\eta}_\beta(y)\} = 0. \tag{12}$$

The efficient mathematical tool for the study green functions of free Dirac fermion fields is the generating functional

$$Z_0^{\psi}[\eta,\overline{\eta}] = \int[D\psi][D\overline{\psi}] \times \exp\left\{\mathrm{i}\int \mathrm{d}x\ [\overline{\psi}(x)\eta(x) + \overline{\eta}(x)\psi(x)]\right\} \times \exp\left\{\mathrm{i}\int \mathrm{d}x\ \overline{\psi}(x)\left[\mathrm{i}\frac{\partial}{\partial x_0} - H\right]\psi(x)\right\}. \tag{13}$$

According to the definition (8) we have

$$Z_0^{\psi} = Z^{\psi}[0, 0]. \tag{14}$$

All $2n$-point Green functions free Dirac fermion fields (10) can be represented in terms of the functional derivatives of the functional (13) at $\eta = \overline{\eta} = 0$. For example, 2-point Green function

$$G_{\alpha\beta}(x; y) = \left\langle \psi_\alpha(x)\overline{\psi}_\beta(y)\right\rangle_0 \tag{15}$$

has following expression

$$G_{\alpha\beta}(x; y) = \frac{1}{Z_0^{\psi}}\frac{\delta^2 Z_0^{\psi}[\eta,\overline{\eta}]}{\delta\overline{\eta}_\alpha(x)\delta\eta_\beta(y)}\bigg|_{\eta=\overline{\eta}=0}. \tag{16}$$

Similarly, 4-point Green functions

$$G_{\alpha_1\alpha_2\beta_2\beta_1}(x_1, x_2; y_2, y_1) = \left\langle \psi_{\alpha_1}(x_1)\psi_{\alpha_2}(x_2)\overline{\psi}_{\beta_2}(y_2)\overline{\psi}_{\beta_1}(y_1)\right\rangle_0 \tag{17}$$

can be represented as follows

$$G_{\alpha_1\alpha_2\beta_2\beta_1}(x_1, x_2; y_2, y_1) = \frac{1}{Z_0^{\psi}}\frac{\delta^4 Z_0^{\psi}[\eta,\overline{\eta}]}{\delta\overline{\eta}_{\alpha_1}(x_1)\delta\overline{\eta}_{\alpha_2}(x_2)\delta\eta_{\beta_2}(y_2)\delta\eta_{\beta_1}(y_1)}\bigg|_{\eta=\overline{\eta}=0}. \tag{18}$$

It can be showed that in the general case of the $2n$-point Green function we have formula

$$G_{\alpha_1\alpha_2\ldots\alpha_n\beta_n\ldots\beta_2\beta_1}(x_1, x_2\ldots x_n; y_n, \ldots y_2, y_1) = \frac{1}{Z_0^{\psi}}\frac{\delta^{2n} Z_0^{\psi}[\eta,\overline{\eta}]}{\delta\overline{\eta}_{\alpha_1}(x_1)\delta\overline{\eta}_{\alpha_2}(x_2)\ldots\delta\overline{\eta}_{\alpha_n}(x_n)\delta\eta_{\beta_n}(y_n)\ldots\delta\eta_{\beta_2}(y_2)\delta\eta_{\beta_1}(y_1)}\bigg|_{\eta=\overline{\eta}=0}. \tag{19}$$

Now we establish the explicit formula of generating functional (13) in terms of the Grassmann parameters $\eta(x)$ and $\overline{\eta}(x)$. Denote $u^K_{\mathbf{k}\pm}(x)$ and $u^{K'}_{\mathbf{k}\pm}(x)$ the 2-component wave functions of free Dirac fermions with momentum $\mathbf{k}$ in the neighbours of Dirac points $\mathbf{K}$ and K', respectively, and energies $E_\pm(\mathbf{k}) = \pm v_F k$. They satisfy following 2D Dirac equations

$$v_F\boldsymbol{\tau}\left(-\mathrm{i}\frac{\partial}{\partial\mathbf{x}}\right)u^K_\pm(\mathbf{x}) = E_\pm(\mathbf{k})u^K_\pm(\mathbf{x}) \tag{20}$$

and

$$v_F\boldsymbol{\tau}^*\left(-\mathrm{i}\frac{\partial}{\partial\mathbf{x}}\right)u^{K'}_\pm(\mathbf{x}) = E_\pm(\mathbf{k})u^{K'}_\pm(\mathbf{x}). \tag{21}$$

Introduce 2×2 unit matrix τ_0 and 2×2 matrix functions $S^K(x, y) = S^K(\mathbf{x}, \mathbf{y}; x_0 - y_0)$ and $S^{K'}(x, y) = S^{K'}(\mathbf{x}, \mathbf{y}; x_0 - y_0)$ satisfying following inhomogeneous differential equations

$$\left[\mathrm{i}\frac{\partial}{\partial x_0} - v_F\boldsymbol{\tau}\left(-\mathrm{i}\frac{\partial}{\partial\mathbf{x}}\right)\right]S^K(\mathbf{x}, \mathbf{y}; x_0 - y_0) = \delta(x - y) = \delta(\mathbf{x}-\mathbf{y})\delta(x_0 - y_0)\tau_0 \tag{22}$$

and

$$\left[\mathrm{i}\frac{\partial}{\partial x_0} - v_F\boldsymbol{\tau}^*\left(-\mathrm{i}\frac{\partial}{\partial\mathbf{x}}\right)\right]S^{K'}(\mathbf{x}, \mathbf{y}; x_0 - y_0) = \delta(x - y) = \delta(\mathbf{x}-\mathbf{y})\delta(x_0 - y_0)\tau_0. \tag{23}$$

They are expressed in terms of the wave functions $u^K_{\mathbf{k}\sigma}(\mathbf{x})$ and $u^{K'}_{\mathbf{k}\sigma}(\mathbf{x})$, $\sigma = \pm$, as follows

$$S^{K,K'}(\mathbf{x},\mathbf{y}; x_0 - y_0) = \frac{1}{2\pi}\int \mathrm{d}\omega\,\mathrm{e}^{-\mathrm{i}\omega(x_0-y_0)}\frac{1}{(2\pi)^2}\int \mathrm{d}\mathbf{k}\sum_{\sigma=\pm}\frac{u^{K,K'}_{\mathbf{k}\sigma}(\mathbf{x})u^{K,K'}_{\mathbf{k}\sigma}(\mathbf{y})^+}{\omega - E_\sigma(\mathbf{k}) + \mathrm{i}0} + \frac{\mathrm{i}}{(2\pi)^2}\int \mathrm{d}\mathbf{k}\sum_{\sigma=\pm}C^{K,K'}_\sigma \mathrm{e}^{-E_\sigma(\mathbf{k})(x_0-y_0)}u^{K,K'}_{\mathbf{k}\sigma}(\mathbf{x})u^{K,K'}_{\mathbf{k}\sigma}(\mathbf{y})^+ \tag{24}$$

where $u^{K,K'}_{\mathbf{k},\sigma}(\mathbf{x})$ are the 2-component spinor wave functions of Dirac fermions in graphene [7], and the functions $C^{K,K'}_\sigma(\mathbf{k})$ are related to the characteristics of the free Dirac fermion gas. By means of the the same reasoning as those in [23] it can be shown that $C^{K,K'}_\sigma(\mathbf{k})$ are the occupation numbers $n^{K,K'}_\sigma(\mathbf{k})$ of the Dirac fermions at the quantum states with wave functions $u^{K,K'}_{\mathbf{k}\sigma}(\mathbf{x})$.

Consider the functional integral (8) of the free Dirac fermion gas. For subsequent calculations let us explicitly rewrite it as follows:

$$Z_0^{\psi} = Z_0^{\psi^K} Z_0^{\psi^{K'}}, \tag{25}$$

where

$$Z_0^{\psi^K} = \int [D\psi^K]\ [[D\overline{\psi}^K] \times \exp\left\{\mathrm{i}\int \mathrm{d}x \overline{\psi}^K(x)\left[\mathrm{i}\frac{\partial}{\partial x_0} - v_F\boldsymbol{\tau}\left(-\mathrm{i}\frac{\partial}{\partial \mathbf{x}}\right)\right]\psi^K(x)\right\}, \tag{26}$$

$$Z_0^{\psi^{K'}} = \int [D\psi^{K'}]\ [[D\overline{\psi}^{K'}] \times \exp\left\{\mathrm{i}\int \mathrm{d}x \overline{\psi}^{K'}(x)\left[\mathrm{i}\frac{\partial}{\partial x_0} - v_F\boldsymbol{\tau}^*\left(-\mathrm{i}\frac{\partial}{\partial \mathbf{x}}\right)\right]\psi^{K'}(x)\right\}. \tag{27}$$

Introducing Grassmann parameter $\eta^{K,K'}(x)$, $\overline{\eta}^{K,K'}(x)$ and performing following shift of functional integral variables $\psi^{K,K'}(x)$ and $\overline{\psi}^{K,K'}(x)$:

$$\begin{aligned}
\psi^{K,K'}(x) &\to \psi^{K,K'}(x) + \int dy\ S^{K,K'}(x,y)\eta^{K,K'}(y),\\
\overline{\psi}^{K,K'}(x) &\to \overline{\psi}^{K,K'}(x) + \int dy\ \overline{\eta}^{K,K'}(y)S^{K,K'}(y,x),
\end{aligned} \tag{28}$$

we obtain other formulae for $Z_0^{\psi^K}$ and $Z_0^{\psi^{K'}}$:

$$\begin{aligned}
Z_0^{\psi^K} = &\int [D\psi^K][D\overline{\psi}^K]\\
&\times \exp\left\{i\int \mathrm{d}x\ \overline{\psi}^K(x)\left[\mathrm{i}\frac{\partial}{\partial x_0} - v_F\boldsymbol{\tau}\left(-\mathrm{i}\frac{\partial}{\partial \mathbf{x}}\right)\right]\psi^K(x)\right\}\\
&\times \exp\left\{\mathrm{i}\int \mathrm{d}x\int \mathrm{d}y\ \overline{\psi}^K(x)\left[\mathrm{i}\frac{\partial}{\partial x_0} - v_F\boldsymbol{\tau}\left(-\mathrm{i}\frac{\partial}{\partial \mathbf{x}}\right)\right]S^K(x,y)\eta^K(y)\right\}\\
&\times \exp\left\{\mathrm{i}\int \mathrm{d}y\int \mathrm{d}x\ \overline{\eta}^K(y)S^K(y,x)\left[\mathrm{i}\frac{\partial}{\partial x_0} - v_F\boldsymbol{\tau}\left(-\mathrm{i}\frac{\partial}{\partial \mathbf{x}}\right)\right]\psi^K(x)\right\}\\
&\times \exp\left\{\mathrm{i}\int \mathrm{d}y\int \mathrm{d}x\int \mathrm{d}z\ \overline{\eta}^K(y)S^K(y,x)\left[\mathrm{i}\frac{\partial}{\partial x_0} - v_F\boldsymbol{\tau}\left(-\mathrm{i}\frac{\partial}{\partial \mathbf{x}}\right)\right]\right.\\
&\left.\times S^K(x,z)\eta^K(z)\right\},
\end{aligned} \tag{29}$$

$$\begin{aligned}
Z_0^{\psi^{K'}} = &\int [D\psi^{K'}][D\overline{\psi}^{K'}]\exp\left\{\mathrm{i}\int \mathrm{d}x\overline{\psi}^{K'}(x)\left[\mathrm{i}\frac{\partial}{\partial x_0} - v_F\boldsymbol{\tau}^*\left(-\mathrm{i}\frac{\partial}{\partial \mathbf{x}}\right)\right]\psi^{K'}(x)\right\}\\
&\times \exp\left\{\mathrm{i}\int \mathrm{d}x\int \mathrm{d}y\overline{\psi}^{K'}(x)\left[\mathrm{i}\frac{\partial}{\partial x_0} - v_F\boldsymbol{\tau}^*\left(-\mathrm{i}\frac{\partial}{\partial \mathbf{x}}\right)\right]S^{K'}(x,y)\eta^{K'}(y)\right\}\\
&\times \exp\left\{\mathrm{i}\int \mathrm{d}x\int \mathrm{d}y\overline{\eta}^{K'}(y)S^{K'}(y,x)\left[\mathrm{i}\frac{\partial}{\partial x_0} - v_F\boldsymbol{\tau}^*\left(-\mathrm{i}\frac{\partial}{\partial \mathbf{x}}\right)\right]\psi^{K'}(x)\right\}\\
&\times \exp\left\{\mathrm{i}\int \mathrm{d}x\int \mathrm{d}y\int \mathrm{d}z\overline{\eta}^{K'}(y)S^{K'}(y,x)\left[\mathrm{i}\frac{\partial}{\partial x_0} - v_F\boldsymbol{\tau}^*\left(-\mathrm{i}\frac{\partial}{\partial \mathbf{x}}\right)\right]\right.\\
&\left.\times S^{K'}(x,z)\eta^{K'}(z)\right\}.
\end{aligned} \tag{30}$$

Due to inhomogeneous differential equations (22) and (23) we have

$$\int \mathrm{d}x\int \mathrm{d}y\overline{\psi}^K(x)\left[\mathrm{i}\frac{\partial}{\partial x_0} - v_F\boldsymbol{\tau}\left(-\mathrm{i}\frac{\partial}{\partial \mathbf{x}}\right)\right]S^K(x,y)\eta^K(y) = \int \mathrm{d}x\overline{\psi}^K(x)\eta^K(x), \tag{31}$$

$$\int \mathrm{d}x\int \mathrm{d}y\int \mathrm{d}z\ \overline{\eta}^K(y)S^K(y,x)\left[\mathrm{i}\frac{\partial}{\partial x_0} - v_F\boldsymbol{\tau}\left(-\mathrm{i}\frac{\partial}{\partial \mathbf{x}}\right)\right]S^K(x,z)\eta^K(z) = \int \mathrm{d}x\int \mathrm{d}y\ \overline{\eta}^K(y)S^K(y,x)\eta^K(x) \tag{32}$$

and similarly

$$\int \mathrm{d}x\int \mathrm{d}y\ \overline{\psi}^{K'}(x)\left[\mathrm{i}\frac{\partial}{\partial x_0} - v_F\boldsymbol{\tau}^*\left(-\mathrm{i}\frac{\partial}{\partial \mathbf{x}}\right)\right]S^{K'}(x,y)\eta^{K'}(y) = \int \mathrm{d}x\ \overline{\psi}^{K'}(x)\ \eta^{K'}(x), \tag{33}$$

$$\int \mathrm{d}y\int \mathrm{d}x\int \mathrm{d}z\ \overline{\eta}^{K'}(y)S^{K'}(y,x)\left[\mathrm{i}\frac{\partial}{\partial x_0} - v_F\boldsymbol{\tau}^*\left(-\mathrm{i}\frac{\partial}{\partial \mathbf{x}}\right)\right]S^{K'}(x,z)\eta^{K'}(z) = \int \mathrm{d}y\int \mathrm{d}x\ \overline{\eta}^{K'}(y)S^{K'}(y,x)\eta^{K'}(x). \tag{34}$$

It remains to consider third exponential functi on in r.h.s. of formula (29) which contains the integral

$$\int \mathrm{d}xS^K(y,x)\left[\mathrm{i}\frac{\partial}{\partial x_0} - v_F\boldsymbol{\tau}\left(-\mathrm{i}\frac{\partial}{\partial \mathbf{x}}\right)\right]\psi^K(x) = \int \mathrm{d}x\left[-\mathrm{i}\frac{\partial S^K(y,x)}{\partial x_0} - \mathrm{i}v_F\frac{\partial S^K(y,x)}{\partial \mathbf{x}}\boldsymbol{\tau}\right]\psi^K(x) \tag{35}$$

and third exponential function in r.h.s. of formula (30) which contains the integral

$$\int \mathrm{d}xS^{K'}(y,x)\left[\mathrm{i}\frac{\partial}{\partial x_0} - v_F\boldsymbol{\tau}^*\left(-\mathrm{i}\frac{\partial}{\partial \mathbf{x}}\right)\right]\psi^{K'}(x) = \int \mathrm{d}x\left[-\mathrm{i}\frac{\partial S^{K'}(y,x)}{\partial x_0} - \mathrm{i}v_F\frac{\partial S^{K'}(y,x)}{\partial \mathbf{x}}\boldsymbol{\tau}^*\right]\psi^{K'}(x). \tag{36}$$

According to formula (24) for $S^K(y,x)$ we have

$$-\mathrm{i}\frac{\partial S^K(y,x)}{\partial x_0} = \mathrm{i}\frac{\partial S^K(y,x)}{\partial y_0} \tag{37}$$

and

$$\begin{aligned}
\mathrm{i}v_F\frac{\partial S^K(y,x)}{\partial \mathbf{x}}\boldsymbol{\tau} = &\frac{1}{2\pi}\int \mathrm{d}\omega \mathrm{e}^{-\mathrm{i}\omega(y_0-x_0)}\frac{1}{(2\pi)^2}\int \mathrm{d}\mathbf{k}\sum_{\sigma=\pm}\frac{1}{\omega - E_\sigma(\mathbf{k}) + i0}u^K_{k\sigma}(\mathbf{y})\\
&\times\left[v_F\boldsymbol{\tau}\left(-\mathrm{i}\frac{\partial}{\partial \mathbf{x}}\right)u^K_{\mathbf{k}\sigma}(\mathbf{x})\right]^+\\
&+ \frac{\mathrm{i}}{(2\pi)^2}\int \mathrm{d}\mathbf{k}\sum_{\sigma=\pm}C^K_\sigma(\mathbf{k})\mathrm{e}^{-\mathrm{i}E_\sigma(\mathbf{k})(y_0-x_0)}u^K_{\mathbf{k}\sigma}(\mathbf{y})\left[v_F\boldsymbol{\tau}\left(-\mathrm{i}\frac{\partial}{\partial \mathbf{x}}\right)u^K_{\mathbf{k}\sigma}(\mathbf{x})\right]^+.
\end{aligned} \tag{38}$$

Since

$$\begin{aligned}
v_F\boldsymbol{\tau}\left(-\mathrm{i}\frac{\partial}{\partial x}\right)u^K_{k\sigma}(\mathbf{x}) &= E_\sigma(\mathbf{k})u^K_{\mathbf{k}\sigma}(\mathbf{x}),\\
v_F\boldsymbol{\tau}\left(-\mathrm{i}\frac{\partial}{\partial \mathbf{y}}\right)u^K_{\mathbf{k}\sigma}(\mathbf{y}) &= E_\sigma(\mathbf{k})u^K_{\mathbf{k}\sigma}(\mathbf{y}),
\end{aligned} \tag{39}$$

we can rewrite relation (38) as follows:

$$\mathrm{i}v_F\frac{\partial S^K(y,x)}{\partial \mathbf{x}}\boldsymbol{\tau}=\frac{1}{2\pi}\int \mathrm{d}\omega \mathrm{e}^{-\mathrm{i}\omega(y_0-x_0)}\frac{1}{(2\pi)^2}\int \mathrm{d}\mathbf{k}$$
$$\times\sum_{\sigma=\pm}\frac{1}{\omega-E_\sigma(\mathbf{k})+\mathrm{i}0}v_F\boldsymbol{\tau}\left(-\mathrm{i}\frac{\partial}{\partial \mathbf{y}}\right)u^K_{\mathbf{k}\sigma}(\mathbf{y})u^K_{\mathbf{k}\sigma}(\mathbf{x})^+$$
$$+\frac{\mathrm{i}}{(2\pi)^2}\int \mathrm{d}\mathbf{k}\sum_{\sigma=\pm}C^K_\sigma(\mathbf{k})\mathrm{e}^{-\mathrm{i}E_\sigma(\mathbf{k})(y_0-x_0)}v_F\boldsymbol{\tau}\left(-\mathrm{i}\frac{\partial}{\partial \mathbf{y}}\right)u^K_{\mathbf{k}\sigma}(\mathbf{y})u^K_{\mathbf{k}\sigma}(\mathbf{x})^+. \tag{40}$$

This result means that

$$\mathrm{i}v_F\frac{\partial S^K(y,x)}{\partial \mathbf{x}}\boldsymbol{\tau}=v_F\boldsymbol{\tau}\left(-\mathrm{i}\frac{\partial}{\partial \mathbf{y}}\right)S^K(y,x). \tag{41}$$

Formula (35) becomes

$$\int \mathrm{d}x\, S^K(y,x)\left[\mathrm{i}\frac{\partial}{\partial x_0}-v_F\boldsymbol{\tau}\left(-\mathrm{i}\frac{\partial}{\partial \mathbf{x}}\right)\right]\psi^K(x)$$
$$=\int \mathrm{d}x\left[\mathrm{i}\frac{\partial}{\partial y_0}-v_F\boldsymbol{\tau}\left(-\mathrm{i}\frac{\partial}{\partial \mathbf{y}}\right)\right]S^K(y,x)\psi^K(x). \tag{42}$$

Similarly we have

$$\int \mathrm{d}x\, S^{K'}(y,x)\left[\mathrm{i}\frac{\partial}{\partial x_0}-v_F\boldsymbol{\tau}^*\left(-\mathrm{i}\frac{\partial}{\partial \mathbf{x}}\right)\right]\psi^{K'}(x)$$
$$=\int \mathrm{d}x\left[\mathrm{i}\frac{\partial}{\partial y_0}-v_F\boldsymbol{\tau}^*\left(-\mathrm{i}\frac{\partial}{\partial \mathbf{y}}\right)\right]S^{K'}(y,x)\psi^{K'}(x). \tag{43}$$

Using inhomogeneous differential equations (22) and (23), we obtain

$$\int \mathrm{d}y\int \mathrm{d}x\, \overline{\eta}^K(y)S^K(y,x)\left[\mathrm{i}\frac{\partial}{\partial x_0}-v_F\boldsymbol{\tau}\left(-\mathrm{i}\frac{\partial}{\partial x}\right)\right]\psi^K(x)$$
$$=\int \mathrm{d}x\, \overline{\eta}^K(x)\,\psi^K(x) \tag{44}$$

and

$$\int \mathrm{d}y\int \mathrm{d}x\, \overline{\eta}^{K'}(y)S^{K'}(y,x)\left[\mathrm{i}\frac{\partial}{\partial x_0}-v_F\boldsymbol{\tau}^*\left(-\mathrm{i}\frac{\partial}{\partial \mathbf{x}}\right)\right]\psi^{K'}(x)$$
$$=\int \mathrm{d}x\, \overline{\eta}^{K'}(x)\psi^{K'}(x). \tag{45}$$

Combining above presented results, we rewrite formulae (29) and (30) as follows:

$$Z_0^{\psi^K}=\int[D\psi^K][D\overline{\psi}^K]$$
$$\times\exp\left\{\mathrm{i}\int \mathrm{d}x\, \overline{\psi}^K(x)\left[\mathrm{i}\frac{\partial}{\partial x_0}-v_F\boldsymbol{\tau}\left(-\mathrm{i}\frac{\partial}{\partial x}\right)\right]\psi^K(x)\right\}$$
$$\times\exp\left\{\mathrm{i}\int \mathrm{d}x\, [\overline{\eta}^K(x)\psi^K(x)+\overline{\psi}^K(x)\eta^K(x)]\right\}$$
$$\times\exp\left\{\mathrm{i}\int \mathrm{d}x\int \mathrm{d}y\, \overline{\eta}^K(x)S^K(x,y)\eta^K(y)\right\}, \tag{46}$$

$$Z_0^{\psi^{K'}}=\int[D\psi^{K'}][D\overline{\psi}^{K'}]$$
$$\times\exp\left\{\mathrm{i}\int \mathrm{d}x\, \overline{\psi}^{K'}(x)\left[\mathrm{i}\frac{\partial}{\partial x_0}-v_F\boldsymbol{\tau}^*\left(-\mathrm{i}\frac{\partial}{\partial \mathbf{x}}\right)\right]\psi^{K'}(x)\right\}$$
$$\times\exp\left\{\mathrm{i}\int \mathrm{d}x\, [\overline{\eta}^{K'}(x)\psi^{K'}(x)+\overline{\psi}^{K'}(x)\eta^{K'}(x)]\right\}$$
$$\times\exp\left\{\mathrm{i}\int \mathrm{d}x\int \mathrm{d}y\, \overline{\eta}^{K'}(x)S^{K'}(x,y)\eta^{K'}(y)\right\}\cdot \tag{47}$$

Now consider generating functional (13). It can be represented as the product

$$Z_0^\psi[\eta,\overline{\eta}]=Z_0^{\psi^K}[\eta^K,\overline{\eta}^K]\,Z_0^{\psi^{K'}}[\eta^{K'},\overline{\eta}^{K'}], \tag{48}$$

where

$$Z_0^{\psi^K}[\eta^K,\overline{\eta}^K]=\int[D\psi^K][D\overline{\psi}^K]$$
$$\times\exp\left\{\mathrm{i}\int \mathrm{d}x\, \overline{\eta}^K(x)\psi^K(x)+\overline{\psi}^K(x)\eta^K(x)\right\}$$
$$\times\exp\left\{\mathrm{i}\int \mathrm{d}x\, \overline{\psi}^K(x)\left[\mathrm{i}\frac{\partial}{\partial x_0}-v_F\boldsymbol{\tau}\left(-\mathrm{i}\frac{\partial}{\partial \mathbf{x}}\right)\right]\psi^K(x)\right\}, \tag{49}$$

$$Z_0^{\psi^{K'}}[\eta^{K'},\overline{\eta}^{K'}]=\int[D\psi^{K'}][D\overline{\psi}^{K'}]$$
$$\times\exp\left\{\mathrm{i}\int \mathrm{d}x\, \overline{\eta}^{K'}(x)\psi^{K'}(x)+\overline{\psi}^{K'}(x)\eta^{K'}(x)\right\}$$
$$\times\exp\left\{\mathrm{i}\int \mathrm{d}x\, \overline{\psi}^{K'}(x)\left[\mathrm{i}\frac{\partial}{\partial x_0}-v_F\boldsymbol{\tau}^*\left(-\mathrm{i}\frac{\partial}{\partial \mathbf{x}}\right)\right]\psi^{K'}(x)\right\}. \tag{50}$$

From formulae (46) and (49), it follows that

$$Z_0^{\psi^K}[\eta^K,\overline{\eta}^K]=Z_0^{\psi^K}\exp\left\{-\mathrm{i}\int \mathrm{d}x\int \mathrm{d}y\, \overline{\eta}^K(x)S^K(x,y)\eta^K(y)\right\}. \tag{51}$$

Similarly, from formulae (47) and (50) we have

$$Z_0^{\psi^{K'}}[\eta^{K'},\overline{\eta}^{K'}]$$
$$=Z_0^{\psi^{K'}}\exp\left\{-\mathrm{i}\int \mathrm{d}x\int \mathrm{d}y\, \overline{\eta}^{K'}(x)S^{K'}(x,y)\eta^{K'}(y)\right\}\cdot \tag{52}$$

Using relation (48), (51) and (52), finally we obtain explicit formula of the generating functional (13)

$$Z_0^\psi[\eta,\overline{\eta}]=Z_0^\psi\exp\left\{-\mathrm{i}\int \mathrm{d}x\int \mathrm{d}y\, \overline{\eta}(x)S(x,y)\eta(y)\right\}. \tag{53}$$

In above presented reasonnings we have shown that 2-point Green function is expressed in terms of generating functional $Z_0^\psi[\eta,\overline{\eta}]$ through formula (16). Using formula (53) for $Z_0^\psi[\eta,\overline{\eta}]$, we obtain relation

$$G_{\alpha\beta}(x;y)=\mathrm{i}S_{\alpha\beta}(x,y). \tag{54}$$

Similarly, 4-point Green function is expressed in term of generating functional $Z_0^{\psi}[\eta, \overline{\eta}]$ through formula (18). Using expression (53) for $Z_0^{\psi}[\eta, \overline{\eta}]$ we obtain relation

$$G_{\alpha_1\alpha_2\beta_2\beta_1}(x_1x_2; y_2y_1) = G_{\alpha_1\beta_1}(x_1; y_1)G_{\alpha_2\beta_2}(x_2; y_2) - G_{\alpha_1\beta_2}(x_1; y_2)G_{\alpha_2\beta_1}(x_2; y_1) \tag{55}$$

which is the well-known Wick theorem for the 4-point Green function for the free Dirac fermion fields. It is straightforward to generalize above elaborated calculation method to verify the validity of the Wick theorem for Dirac fermion fields with any even positive integer n.

4. Functional integral of the interacting system of Dirac fermion fields

In this section we study the functional integral (7) of the system of Dirac fermion fields in the presence of the mutual Coulomb interaction between Dirac fermions. The last factor in expression (7) of functional integral Z^{ψ} of the interacting system of Dirac fermion fields contain a bilinear expression

$$\int \mathrm{d}x \int \mathrm{d}y \overline{\psi}(x)\psi(x)u(x-y)\overline{\psi}(y)\psi(y)$$

of the density $\overline{\psi}(x)\psi(x)$ of Dirac fermions. Let us linearize the factor

$$\exp\left\{-\frac{\mathrm{i}}{2}\int \mathrm{d}x \int \mathrm{d}y \overline{\psi}(x)\psi(x)u(x-y)\overline{\psi}(y)\psi(y)\right\}$$

in functional integral (7) with respect to the Dirac fermion density $\overline{\psi}(x)\psi(x)$. For this purpose we introduce a Hermitian scalar field $\varphi(x)$ and the functional integral

$$Z_0^{\varphi} = \int [D\varphi]\ \exp\left\{\frac{\mathrm{i}}{2}\int \mathrm{d}x \int \mathrm{d}y \varphi(x)u(x-y)\varphi(y)\right\}. \tag{56}$$

By shifting the functional integration variable

$$\varphi(x) \to \varphi(x) - \overline{\psi}(x)\psi(x), \tag{57}$$

we rewrite Z_0^{φ} in another form

$$\begin{aligned} Z_0^{\varphi} = \int [D\varphi]\ &\exp\left\{\frac{\mathrm{i}}{2}\int \mathrm{d}x \int \mathrm{d}y \varphi(x)u(x-y)\varphi(y)\right\} \\ &\times \exp\left\{-\mathrm{i}\int \mathrm{d}x \int \mathrm{d}y \varphi(x)u(x-y)\overline{\psi}(y)\psi(y)\right\} \\ &\times \exp\left\{\frac{\mathrm{i}}{2}\int \mathrm{d}x \int \mathrm{d}y \overline{\psi}(x)\psi(x)u(x-y)\overline{\psi}(y)\psi(y)\right\}. \end{aligned} \tag{58}$$

From this formula it follows the famous Hubbard–Stratonovich transformation

$$\begin{aligned} &\exp\left\{-\frac{\mathrm{i}}{2}\int \mathrm{d}x \int \mathrm{d}y \overline{\psi}(x)\psi(x)u(x-y)\overline{\psi}(y)\psi(y)\right\} \\ &\quad = \frac{1}{Z_0^{\varphi}}\int [D\varphi]\ \exp\left\{\frac{\mathrm{i}}{2}\int \mathrm{d}x \int \mathrm{d}y \varphi(x)u(x-y)\varphi(y)\right\} \\ &\quad\quad \times \exp\left\{-\mathrm{i}\int \mathrm{d}x \int \mathrm{d}y \varphi(x)u(x-y)\overline{\psi}(y)\psi(y)\right\}. \end{aligned} \tag{59}$$

Using this transformation we rewrite the functional integral Z^{ψ} of the interacting Dirac fermions in the form linearized with respect to the Dirac fermion density

$$\begin{aligned} Z^{\psi} = \frac{1}{Z_0^{\varphi}}&\int [D\varphi] \exp\left\{\frac{\mathrm{i}}{2}\int \mathrm{d}x \int \mathrm{d}y \varphi(x)u(x-y)\varphi(y)\right\} \\ &\times \int [D\psi]\ [D\overline{\psi}] \exp\left\{\mathrm{i}\int \mathrm{d}x \overline{\psi}(x)\left[\mathrm{i}\frac{\partial}{\partial x_0} - H\right]\psi(x)\right\} \\ &\times \exp\left\{-\mathrm{i}\int \mathrm{d}x \int \mathrm{d}y \overline{\psi}(x)\psi(x)u(x-y)\varphi(y)\right\}. \end{aligned} \tag{60}$$

Note that the expression

$$\exp\left\{-\mathrm{i}\int \mathrm{d}x \int \mathrm{d}y \overline{\psi}(x)\psi(x)u(x-y)\varphi(y)\right\}$$

is linear with respect to the Dirac fermion density $\overline{\psi}(x)\psi(x)$.

In terms of the statistical average (9) of the products of components of Dirac fermion quantum fields we have following new expression of the functional integral Z^{ψ} determined by formula (60)

$$\begin{aligned} Z^{\psi} = \frac{Z_0^{\psi}}{Z_0^{\varphi}}&\int [D\varphi]\ \exp\left\{\frac{\mathrm{i}}{2}\int \mathrm{d}x \int \mathrm{d}y \varphi(x)u(x-y)\varphi(y)\right\} \\ &\times \left\langle \exp\left\{-\mathrm{i}\int \mathrm{d}x \int \mathrm{d}x' \overline{\psi}(x)\psi(x)u(x-x')\varphi(x')\right\}\right\rangle. \end{aligned} \tag{61}$$

Expanding the exponential function

$$\exp\left\{-\mathrm{i}\int \mathrm{d}x \int \mathrm{d}x' \overline{\psi}(x)\psi(x)u(x-x')\varphi(x')\right\}$$

into functional power series of the function $\overline{\psi}(x)\psi(x)u(x-x')\varphi(x')$, we obtain

$$\begin{aligned} &\left\langle \exp\left\{-\mathrm{i}\int \mathrm{d}x \int \mathrm{d}x' \overline{\psi}(x)\psi(x)u(x-x')\varphi(x')\right\}\right\rangle \\ &\quad = 1 + \sum_{n=1}^{\infty} F^{(n)}[\varphi] \end{aligned} \tag{62}$$

where

$$F^{(n)}[\varphi] = \frac{1}{n!}\left\langle\left[-\mathrm{i}\int \mathrm{d}x \int \mathrm{d}x' \overline{\psi}(x)\psi(x)u(x-x')\varphi(x')\right]^n\right\rangle_0. \tag{63}$$

Explicit expressions of functional $F^{(n)}[\varphi]$ can be derived by means of the method presented in [23]. As the result we obtain following result

$$1 + \sum_{n=1}^{\infty} F^{(n)}[\varphi] = \exp\{\mathrm{i}I[\varphi]\ \} \tag{64}$$

with

$$I[\varphi] = \sum_{n=1}^{\infty} I^{(n)}[\varphi]\ , \tag{65}$$

$$I^{(1)}[\varphi] = -\int \mathrm{d}x \int \mathrm{d}x' \langle\overline{\psi}(x)\psi(x)\rangle_0 u(x-x')\varphi(x'), \tag{66}$$

$$\begin{aligned} I^{(2)}[\varphi] = \frac{\mathrm{i}}{2}\int \mathrm{d}x \int \mathrm{d}x' \int \mathrm{d}y \int \mathrm{d}y' S_{\alpha\beta}(x,y)S_{\beta\alpha}(y,x)u(x-x') \\ \times u(y-y')\varphi(x')\varphi(y'), \end{aligned} \tag{67}$$

$$I^{(3)}[\varphi] = \frac{\mathrm{i}}{3}\int \mathrm{d}x \int \mathrm{d}x' \int \mathrm{d}y \int \mathrm{d}y' \int \mathrm{d}z \int \mathrm{d}z' S_{\alpha\beta}(x, y) \times S_{\beta\gamma}(y, z) S_{\gamma\alpha}(z, x \times u(x - x') u(y - y') u(z - z') \varphi(x') \varphi(y') \varphi(z') \quad (68)$$

and so on, $S(x, y)$ being 4×4 matrices of the form

$$S(x, y) = \begin{pmatrix} S^K(x, y) & 0 \\ 0 & S^{K'}(x, y) \end{pmatrix}. \quad (69)$$

The Hermitian scalar field $\varphi(x)$ describes collective excitation in the interacting system of Dirac fermions, called also the Dirac fermion gas in graphene. This scalar field $\varphi(x)$ is related to the quantum fields of plasmons in graphene. The study of relationship between $\varphi(x)$ and quantum fields of plasmons in graphene is a very interesting scientific subject which would be done in the future.

5. Conclusion and discussions

In this work we have presented the basics of functional integral method for the study of interacting system of Dirac fermion gas in a graphene single layer. The fundamental principle of the theory is the assumption on the explicit expression of the action functional of the system. The efficient mathematical tool for the study is the generating functional containing Grassmann parameters anticommuting with Dirac fermion quantum fields. Explicit expression of the generating functional of free Dirac fermion fields was established and used for the study of $2n$-point Green functions of free Dirac fermions. The celebrated Hubbard–Stratonovich transformation was applied to express the functional integral of the interacting Dirac fermion system in terms of a Hermitian scalar field describing collective excitation in this system and related with graphene plasmons.

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