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Phenomenology of the $SU(3)_C \otimes SU(2)_L \otimes SU(3)_R \otimes U(1)_X$ gauge model

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We study the left-right asymmetric model based on the $SU(3)_C \otimes SU(2)_L \otimes SU(3)_R \otimes U(1)_X$ gauge group, which improves the theoretical and phenomenological aspects of the known left-right symmetric model. This new gauge symmetry yields that the fermion generation number is 3, and the tree-level flavor-changing neutral currents arise in both gauge and scalar sectors. Also, it can provide the observed neutrino masses, as well as dark matter, automatically. Further, we investigate the mass spectrum of the gauge and scalar fields. All the gauge interactions of the fermions and scalars are derived. We examine the tree-level contributions of the new neutral vector, Z'_R, and new neutral scalar, H_2, to flavor-violating neutral meson mixings, say $K-\bar{K}$, $B_d-\bar{B}_d$, and $B_s-\bar{B}_s$, which strongly constrain the new physics scale as well as the elements of the right-handed quark mixing matrices. The bounds for the new physics scale are in agreement with those coming from the ρ-parameter, as well as the mixing parameters between W, Z bosons and new gauge bosons.

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I. INTRODUCTION

In the standard model, the neutral currents of γ and Z conserve every flavor at the tree level, whereas the charged current of W changes quark flavors through the Cabibbo-Kobayashi-Maskawa (CKM) matrix (where lepton flavors are separately conserved). This directly leads to quark-flavor violating processes such as neutral meson mixings, $K-\bar{K}$, $D-\bar{D}$, $B_d-\bar{B}_d$, and $B_s-\bar{B}_s$, and rare meson decays, $B_s \to \mu^+\mu^-$, $B_s \to \varphi\mu^+\mu^-$, $B_d \to K(K^*)\mu^+\mu^-$, and others. All such standard model predictions have been experimentally tested so far, and that they are globally compatible with the existing data [1]. However, with the reduced experimental errors, as well as enhanced QCD and EW precision computations, a number of tensions have recently been found at $2-3\sigma$ levels corresponding to individual processes [2–6]. While some of them might be due to statistical fluctuations/errors, it does not exclude a possibility that they reveal some new physics. Further, the standard model cannot explain the small, nonzero neutrino masses and lepton-flavor mixings. It also fails to address dark matter that occupies roundly 25% of the mass-energy density of the Universe.

The minimal left-right symmetric model based on the $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ gauge group is one of the most attractive extensions of the standard model [7,8]. A motivation of the model is that the parity is exact but its asymmetry as seen in the weak interaction is due to the spontaneous breaking of $SU(2)_R$ at some large energy scale. It also plays an important role in developing the theories of neutrino masses, well-known as seesaw mechanisms, and that nonzero neutrino masses were suggested long before the experimental confirmations. Particularly, the phenomenological consequences of the new particles that contribute to the meson mixing systems, as well as rare meson decays, were studied in [9]. The contribution of the right current that addresses the V_{ub} problem was also discussed in [10]. Generally, the experimental bounds would require the left-right scale to be in the TeV region, and the explicit left-right asymmetries should be imposed in order to fit most, but not all of, the data, which may be well tested at the LHC run II. As the standard model, the minimal left-right symmetric model cannot solve the dark matter issue.

Furthermore, the minimal left-right symmetric model does not contain the necessary ingredients for solving the 750 GeV diphoton excess naturally [11]. Although the resonance was subsequently proved as statistical fluctuations

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[12], the guidance for going beyond this model to address new physics anomalies like that is still worth studying. In the literature, the proposals [13] that enlarged only the particle content are not considered here since they included the new fields by hand, and obviously it is not natural on both phenomenological and theoretical grounds. However, the proposals [14] that extended the gauge group can show alternative important results since they manifestly follow a gauge principle. Indeed, it was shown that the diphoton anomaly might be associated with fundamental left-right asymmetries, and thus the three theories that were proposed, corresponding to the gauge symmetries, $SU(3)_C \otimes SU(M)_L \otimes SU(N)_R \otimes U(1)_X$ $(3-M-N-1)$, for $(M,N) = (2,3), (3,2)$, and (3,3), respectively. Here, the left-right asymmetry is either explicitly recognized for $M \neq N$ or spontaneously produced after the gauge symmetry breaking for $M = N$. The diphoton excess was the new scalar fields, produced/decayed as mediated by the new fermions, which all transform as fundamental components, in the quotient space $[SU(M)_L \otimes SU(N)_R]/[SU(2)_L \otimes SU(2)_R]$, enlarged from those of the minimal left-right symmetric model.

However, the new physics scales were generally low, below a few TeVs, and the characteristic electric charge parameter was rarely big, in order to explain the large diphoton signal strength. Because the massive diphoton signals were absent, the new physics scales must be large, above those bounds, which are also needed to evade other constraints discussed hereafter (as also noted in [14]), and the electric charge parameter is not necessarily large beyond the usual electric charges. Simultaneously, as shown in this work, the fundamental left-right asymmetries as proposed provide automatically the tree-level flavor-changing neutral currents (FCNCs) through the gauge and Yukawa interactions, which may be the new source for addressing the B physics anomalies and others, which dominate over those loop-induced by the minimal left-right symmetric model. Additionally, the new gauge symmetries that reflect the left-right asymmetries can supply dark matter naturally by the means that dark matter candidates, along with their stability mechanism and relic density, automatically arise from the gauge principles.

In this work, we take the most simple theory among the three mentioned into account, which is given by the $SU(3)_C \otimes SU(2)_L \otimes SU(3)_R \otimes U(1)_X$ $(3-2-3-1)$ gauge symmetry. Note that the two others, namely, the $3-3-2-1$ and $3-3-3-1$ models, include an extension, $SU(2)_L \to SU(3)_L$, besides the corresponding enlargements of the weak hypercharge. Therefore, we see that the left-handed fermion content and symmetry are the same standard model and minimal left-right symmetric model, but the right sector is extended, explicitly violating a symmetry between the left and right, the so-called left-right asymmetry. This approach predicts the three fermion generations as observed as a result of $SU(3)_R$ anomaly cancellation and QCD asymptotic freedom. The new FCNCs come from two distinct sources: either loop induced by W_R and a charged Higgs boson or tree-level contributed by nonuniversal couplings of Z'_R and a neutral Higgs boson (H_2) with ordinary quarks. The former is similar to the minimal left-right symmetric model, which is negligible as suppressed by loop factors, whereas the latter may dominate and is in charge to interpret the mentioned flavor-physics anomalies. Additionally, the new gauge symmetry, $SU(3)_R \otimes U(1)_X$, may define a nontrivial W-parity as well as the W-odd matter content responsible for dark matter, which is quite similar to the $3-3-1-1$ model [15].

Let us stress that the interesting feature of the considered model is that the right-handed quarks of the first and second generations transform under the gauge symmetry differently from the third generation, which directly leads to the tree-level FCNCs caused by only the right-handed quarks when coupling to the H_2 scalar and Z'_R gauge boson. This property does not exist in the $3-3-1$ models or the other left-right theories. The former models, including the two remaining left-right asymmetric theories, have a similar property but caused by the left-handed quarks [14,16]. Unlike those theories, the model under consideration yields that the relevant observables depend only on the new energy scale and the right-handed quark mixing matrices, V_{uR} and V_{dR}, which are not constrained by the standard model (i.e., they act as arbitrary parameters). Further, this proposal implies that the neutrino masses are generated via seesaw mechanisms like the minimal left-right symmetric model. The electric charge operator is directly related to the baryon-minus-lepton charge ($B-L$), which is unlike the $3-3-1-1$ model [15]. The characteristic electric charge parameter of the model also defines the $B-L$ charge for the new particles, by which a class of wrong $B-L$ particles is naturally recognized, which transform nontrivially under a residual discrete gauge symmetry, called W-parity. The new gauge and Higgs bosons might also be the subjects for the dijet, Drell-Yan, and diboson searches by the LHC experiments. For all the purposes, we will identify the scalars and gauge bosons, as well as calculating all the necessary gauge interactions.

The rest of this paper is organized as follows: In Sec. II, we give a detailed review of the model with stress on dark matter and FCNCs. Sections III and IV study the mass spectra of the scalar and gauge boson fields, respectively. The gauge interactions of fermions and scalars are considered in Sec. V. Section VI is devoted to the FCNCs, which are directly mediated by the new neutral gauge and scalar fields. The mixing effects in the gauge and scalar sectors are also discussed therein. Finally, we summarize our results and conclude this work in Sec. VII.

II. THE MODEL

As mentioned, the gauge symmetry of the model is defined by $SU(3)_C \otimes SU(2)_L \otimes SU(3)_R \otimes U(1)_X$, where the first group factor is the ordinary QCD symmetry, while the last three are an extension of the electroweak symmetry, which contains that of the minimal left-right symmetric model as a subgroup. However, the considered model does not conserve a left-right symmetry, Z_2, that interchanges the left and right gauge groups as well as their corresponding field contents; i.e., the model presents an explicit left-right asymmetry.

The electric charge operator is embedded in the gauge symmetry as follows [14]:

$$Q = T_{3L} + T_{3R} + \beta T_{8R} + X, \tag{1}$$

where T_{aL} ($a = 1, 2, 3$), T_{iR} ($i = 1, 2, 3, \ldots, 8$), and X are the $SU(2)_L$, $SU(3)_R$, and $U(1)_X$ generators, respectively. β can be expressed via an electric charge parameter (q) as $\beta = -(2q+1)/\sqrt{3}$. The baryon-minus-lepton charge is embedded as $\frac{1}{2}(B-L) = \beta T_{8R} + X$. Hence, depending on the embedding parameter β (or q), this model may automatically provide dark matter candidates, which are stabilized by a W-parity,

$$P = (-1)^{3(B-L)+2s} = (-1)^{6(\beta T_{8R}+X)+2s}, \tag{2}$$

as residual gauge symmetry, similarly to the $3-3-1-1$ model [15] (see below).

The fermion content that is anomaly free is given by [14]

$$\psi_{aL} = \begin{pmatrix} \nu_{aL} \\ e_{aL} \end{pmatrix} \sim \left(1, 2, 1, -\frac{1}{2}\right),$$
$$\psi_{aR} = \begin{pmatrix} \nu_{aR} \\ e_{aR} \\ E^q_{aR} \end{pmatrix} \sim \left(1, 1, 3, \frac{q-1}{3}\right), \tag{3}$$

$$Q_{3L} = \begin{pmatrix} u_{3L} \\ d_{3L} \end{pmatrix} \sim \left(3, 2, 1, \frac{1}{6}\right),$$
$$Q_{3R} = \begin{pmatrix} u_{3R} \\ d_{3R} \\ J^{q+\frac{2}{3}}_{3R} \end{pmatrix} \sim \left(3, 1, 3, \frac{q+1}{3}\right), \tag{4}$$

$$Q_{\alpha L} = \begin{pmatrix} u_{\alpha L} \\ d_{\alpha L} \end{pmatrix} \sim \left(3, 2, 1, \frac{1}{6}\right),$$
$$Q_{\alpha R} = \begin{pmatrix} d_{\alpha R} \\ -u_{\alpha R} \\ J^{-q-\frac{1}{3}}_{\alpha R} \end{pmatrix} \sim \left(3, 1, 3^*, -\frac{q}{3}\right), \tag{5}$$

$$E^q_{aL} \sim (1, 1, 1, q),$$
$$J^{q+\frac{2}{3}}_{3L} \sim \left(3, 1, 1, q + \frac{2}{3}\right),$$
$$J^{-q-\frac{1}{3}}_{\alpha L} \sim \left(3, 1, 1, -q - \frac{1}{3}\right), \tag{6}$$

where $a = 1, 2, 3$ and $\alpha = 1, 2$ are generation indices. The numbers in the parentheses denote the quantum numbers based on the $3-2-3-1$ subgroups, respectively.

We see that the proposal of $SU(3)_R$ leads to not only the existence of the right-handed neutrinos which induce the neutrino masses via seesaw mechanisms, but also the new leptons E_a and exotic quarks J_a which might yield interesting phenomena. Indeed, note that E_a and J_a have a $B-L$ charge equal to two times their electric charges, i.e., $[B-L](E_a) = 2q$, $[B-L](J_3) = 2(q+2/3)$, and $[B-L](J_\alpha) = 2(-q-1/3)$. Therefore, the model recognizes a nontrivial W-parity for wrong $B-L$ particles that include E, J and others, called W-particles, responsible for dark matter if $q \neq (2m-1)/6 = \pm 1/6, \pm 1/2, \pm 5/6, \pm 7/6, \ldots$ for an m integer [15]. Here, W-particles have $P = P^+$ or P^-, with $P^\pm \equiv (-1)^{\pm(6q+1)} \neq 1$, while the remaining particles that include the standard model particles and some new ones have $P = 1$, called normal particles. Particularly, the model with ordinary charge $q = m/3 = 0, \pm 1/3, \pm 2/3, \pm 1, \ldots$ belongs to this class, which yields W-parity as R-parity and W-particles as R-odd particles (the simplest, but most realistic version is if $q = 0$).

Provided that the right-handed fermions are arranged in the fundamental representations of $SU(3)_R$, the $SU(3)_R$ anomaly cancellation demands that the number of triplets equal that of antitriplets. Thus, the generation number must be a multiple of 3. Since the extra quarks are included to complete the representations, the QCD asymptotic freedom requires the generation number to be less than or equal to 5. Hence, the generation number is 3, as expected. Furthermore, the right-handed quarks of the third generation transform differently from those of the first two generations. This leads to the tree-level quark FCNCs due to the interactions with the new gauge bosons of T_{8R} and X, as well as new neutral scalars (shown below). This demonstrates that the new dominant FCNCs all arise from such an explicit left-right symmetry violation.

To break the gauge symmetry and generate appropriate masses for the particles, the scalar multiplets are introduced as

$$S = \begin{pmatrix} S^0_{11} & S^+_{12} & S^{-q}_{13} \\ S^-_{21} & S^0_{22} & S^{-q-1}_{23} \end{pmatrix} \sim \left(1, 2, 3^*, -\frac{2q+1}{6}\right), \tag{7}$$

$$\phi = \begin{pmatrix} \phi^{-q}_1 \\ \phi^{-q-1}_2 \\ \phi^0_3 \end{pmatrix} \sim \left(1, 1, 3, -\frac{2q+1}{3}\right), \tag{8}$$

$$\Xi = \begin{pmatrix} \Xi^0_{11} & \frac{\Xi^-_{12}}{\sqrt{2}} & \frac{\Xi^q_{13}}{\sqrt{2}} \\ \frac{\Xi^-_{12}}{\sqrt{2}} & \Xi^{--}_{22} & \frac{\Xi^{q-1}_{23}}{\sqrt{2}} \\ \frac{\Xi^q_{13}}{\sqrt{2}} & \frac{\Xi^{q-1}_{23}}{\sqrt{2}} & \Xi^{2q}_{33} \end{pmatrix} \sim \left(1, 1, 6, \frac{2(q-1)}{3}\right), \tag{9}$$

which have the corresponding vacuum expectation values (VEVs),

$$\langle S\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} u & 0 & 0 \\ 0 & v & 0 \end{pmatrix},$$
$$\langle \phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 0 \\ w \end{pmatrix},$$
$$\langle \Xi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} \Lambda & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{10}$$

As mentioned in [14], if one introduces a scalar triplet Δ the neutrinos get masses through a combination of the type I and II seesaw mechanisms; otherwise, the only type I seesaw mechanism is presented. Since both cases can fit the data, we would not include Δ for simplicity. The W-fields include $\phi_{1,2}$, $S_{13,23}$, and $\Xi_{13,23}$. The other scalars are normal fields.

The gauge symmetry is broken via two steps,

$$\begin{gathered} SU(3)_C\otimes SU(2)_L\otimes SU(3)_R\otimes U(1)_X \\ \downarrow w,\Lambda \\ SU(3)_C\otimes SU(2)_L\otimes U(1)_Y\otimes P \\ \downarrow u,v \\ SU(3)_C\otimes U(1)_Q\otimes P. \end{gathered}$$

Here, the VEV of ϕ (w) provides the masses for new leptons and exotic quarks, while the VEV of Ξ (Λ) provides the Majorana masses for right-handed neutrinos. Both the VEVs w, Λ give the masses for new gauge bosons. The VEVs of S (u, v) generate the masses for ordinary charged leptons, quarks, and weak gauge bosons as well as Dirac masses for neutrinos. Subsequently, the small neutrino masses are induced via the seesaw mechanism as mentioned. Additionally, after the first step of symmetry breaking, the W-parity is defined along with the standard model symmetry due to the VEV Λ [15]. Note that w, u, v do not break $B-L$, whereas Λ breaks this charge, which defines the Majorana masses and W-parity. Thus, the observed neutrino masses and dark matter stability are strongly correlated, as originating from the $B-L$ gauge symmetry breaking. To be consistent with the standard model, we must impose $u, v \ll w, \Lambda$.

The total Lagrangian takes the form,

$$\mathcal{L} = \mathcal{L}_{\text{kinetic}} + \mathcal{L}_{\text{Yukawa}} - V_{\text{scalar}}, \tag{11}$$

where the first part includes kinetic terms and gauge interactions, which will be obtained later. The second and last parts correspond to the Yukawa Lagrangian and scalar potential, respectively, which are obtained by

$$\begin{aligned}\mathcal{L}_{\text{Yukawa}} = {}& h^l_{ab}\bar{\psi}_{aL}S\psi_{bR} + h^R_{ab}\bar{\psi}^c_{aR}\Xi^\dagger\psi_{bR} + h^q_{a3}\bar{Q}_{aL}SQ_{3R} \\ & + h^q_{\alpha\beta}\bar{\tilde{Q}}_{\alpha L}S^*Q_{\beta R} + h^E_{ab}\bar{E}_{aL}\phi^\dagger\psi_{bR} + h^J_{33}\bar{J}_{3L}\phi^\dagger Q_{3R} \\ & + h^J_{\alpha\beta}\bar{J}_{\alpha L}\phi^T Q_{\beta R} + \text{H.c.},\end{aligned} \tag{12}$$

$$\begin{aligned} V_{\text{scalar}} = {}& \mu_S^2\text{Tr}(S^\dagger S) + \lambda_{1S}[\text{Tr}(S^\dagger S)]^2 + \lambda_{2S}\text{Tr}(S^\dagger S S^\dagger S) \\ & + \mu_\Xi^2\text{Tr}(\Xi^\dagger\Xi) + \lambda_{1\Xi}[\text{Tr}(\Xi^\dagger\Xi)]^2 + \lambda_{2\Xi}\text{Tr}(\Xi^\dagger\Xi\Xi^\dagger\Xi) \\ & + \mu_\phi^2\phi^\dagger\phi + \lambda_\phi(\phi^\dagger\phi)^2 + \lambda_1(\phi^\dagger S^\dagger S\phi) \\ & + \lambda_2\text{Tr}(S^\dagger S\Xi\Xi^\dagger) + \lambda_3(\phi^\dagger\Xi\Xi^\dagger\phi) + \lambda_4(\phi^\dagger\phi)\text{Tr}(S^\dagger S) \\ & + \lambda_5(\phi^\dagger\phi)\text{Tr}(\Xi^\dagger\Xi) + \lambda_6\text{Tr}(\Xi^\dagger\Xi)\text{Tr}(S^\dagger S) \\ & + (fS\phi^* S + \text{H.c.}), \end{aligned} \tag{13}$$

where "Tr" is the trace operator. Here, note that $\tilde{Q}_L \equiv i\sigma_2 Q_L$ transforms as 2^* under $SU(2)_L$, i.e., $\tilde{Q}_L \to U_L^*\tilde{Q}_L$. Also, we have $S\to U_L S U_R^\dagger$, $\Xi \to U_R\Xi U_R^T$, and $Q_{\alpha R}\to U_R^* Q_{\alpha R}$, under $SU(2)_L\otimes SU(3)_R$. Observe that the third generation of quarks interacts with the scalars with the forms differently from those for the first two quark generations, which does not happen with leptons and is not analogous to the case of the minimal left-right symmetric model. As referred to in [14], the potential parameters have been redefined for easy reading, and the f, $\lambda_{1,2,3}$ couplings have been imposed for generalization, which were skipped in the previous study.

The gauge sector contains two W-fields as $X_R^{\pm q}$ and $Y_R^{\pm(q+1)}$, as coupled to $\frac{1}{\sqrt{2}}(T_{4R}\mp iT_{5R})$ and $\frac{1}{\sqrt{2}}(T_{6R}\mp iT_{7R})$, respectively. The other gauge bosons are normal fields. Summarizing all the W-fields, we see that the model provides dark matter candidates if $q = 0, \pm 1$ (note that the candidate must be electrically neutral). For the model with $q = 0$, the candidates are E^0 or X_R^0 or some combination of (ϕ_1^0, S_{13}^0, Ξ_{13}^0). For the model with $q = -1$, the candidates are Y_R^0 or some combination of (ϕ_2^0, S_{23}^0). For the model with $q = 1$, the candidates are only Ξ_{23}^0. The dark matter candidate must be the lightest W-particle, called the LWP, which is stabilized by W-parity. *Proof*: One has an interaction of r P^+-fields and s P^--fields, where r and s are integers. Since P is conserved, it follows $(P^+)^r(P^-)^s = 1$, which is valid only if $r = s$. In other words, P^+ and P^- always appear in pairs in interactions, which is analogous to superparticles in supersymmetry.

A detailed study of the three mentioned versions for dark matter phenomenologies is out of the scope of this work, which should be published elsewhere [17]. In the following, we identify physical particles, calculate all interactions, and present selected phenomena for the general model.

III. SCALAR SECTOR

Let us expand the neutral scalar fields ($S_{11}^0, S_{22}^0, \phi_3^0, \Xi_{11}^0$) around the mentioned VEVs as

$$S = \begin{pmatrix} \frac{u+S_1+iA_1}{\sqrt{2}} & S_{12}^+ & S_{13}^{-q} \\ S_{21}^- & \frac{v+S_2+iA_2}{\sqrt{2}} & S_{23}^{-q-1} \end{pmatrix}, \quad (14)$$

$$\phi = \begin{pmatrix} \phi_1^{-q} \\ \phi_2^{-q-1} \\ \frac{w+S_3+iA_3}{\sqrt{2}} \end{pmatrix},$$

$$\Xi = \begin{pmatrix} \frac{\Lambda+S_4+iA_4}{\sqrt{2}} & \frac{\Xi_{12}^-}{\sqrt{2}} & \frac{\Xi_{13}^q}{\sqrt{2}} \\ \frac{\Xi_{12}^-}{\sqrt{2}} & \Xi_{22}^{--} & \frac{\Xi_{23}^{q-1}}{\sqrt{2}} \\ \frac{\Xi_{13}^q}{\sqrt{2}} & \frac{\Xi_{23}^{q-1}}{\sqrt{2}} & \Xi_{33}^{2q} \end{pmatrix}. \quad (15)$$

To find the potential minimization and scalar mass spectrum, we correspondingly expand the original potential terms up to the second-order terms of the component fields given above and then sum all the resulting terms that have the same order in fields. Therefore, the scalar potential is divided into $V(S,\phi,\Xi) = V_{\text{min}} + V_{\text{linear}} + V_{\text{mass}} + V_{\text{interaction}}$, where all the interactions are grouped to $V_{\text{interaction}}$, which we need not determine. V_{min} is the minimum of the potential, which is independent of the fields and only contributes to the vacuum energy. V_{linear} contains all the terms that depend linearly on the fields, and the gauge invariance requires $V_{\text{linear}} = 0$, which leads to the minimization conditions as follows:

$$\mu_S^2 u + (\lambda_{1S} + \lambda_{2S})u^3 - \sqrt{2} f v w + \frac{1}{2} u[2\lambda_{1S} v^2 + \lambda_4 w^2 + (\lambda_2 + \lambda_6)\Lambda^2] = 0, \quad (16)$$

$$\mu_S^2 v + (\lambda_{1S} + \lambda_{2S})v^3 - \sqrt{2} f u w + \frac{1}{2} v(2\lambda_{1S} u^2 + \lambda_4 w^2 + \lambda_6 \Lambda^2) = 0, \quad (17)$$

$$\mu_\phi^2 w + \lambda_\phi w^3 - \sqrt{2} f u v + \frac{1}{2} w[\lambda_4(u^2 + v^2) + \lambda_5 \Lambda^2] = 0, \quad (18)$$

$$\mu_\Xi^2 + (\lambda_{1\Xi} + \lambda_{2\Xi})\Lambda^2 + \frac{1}{2}[(\lambda_2 + \lambda_6)u^2 + \lambda_{\Xi S} v^2 + \lambda_5 w^2] = 0. \quad (19)$$

V_{mass} consists of the terms that depend quadratically on the fields, given in the form, $V_{\text{mass}} = V_{\text{mass}}^S + V_{\text{mass}}^A + V_{\text{mass}}^{\text{charged}}$, where the first two terms describe the CP-even and CP-odd scalar fields, respectively, while the last one contains the charged scalar fields.

After we substitute the minimization conditions into the scalar potential, V_{mass}^S is given by

$$V_{\text{mass}}^S = \frac{1}{2}\begin{pmatrix} S_1 & S_2 & S_3 & S_4 \end{pmatrix} M_S^2 \begin{pmatrix} S_1 & S_2 & S_3 & S_4 \end{pmatrix}^T, \quad (20)$$

where M_S^2 is

$$\begin{pmatrix} 2(\lambda_{1S} + \lambda_{2S})u^2 + \frac{\sqrt{2} f v w}{u} & 2\lambda_{1S} u v - \sqrt{2} f w & -\sqrt{2} f v + \lambda_4 u w & (\lambda_2 + \lambda_6) u \Lambda \\ 2\lambda_{1S} u v - \sqrt{2} f w & 2(\lambda_{1S} + \lambda_{2S}) v^2 - \lambda_{2S} u^2 + \frac{\lambda_2 u^2 \Lambda^2}{2(v^2 - u^2)} & -\sqrt{2} f u + \lambda_4 v w & \lambda_6 v \Lambda \\ -\sqrt{2} f v + \lambda_4 u w & -\sqrt{2} f u + \lambda_4 v w & \frac{\sqrt{2} f u v}{w} + 2\lambda_\phi w^2 & \lambda_5 w \Lambda \\ (\lambda_2 + \lambda_6) u \Lambda & \lambda_6 v \Lambda & \lambda_5 w \Lambda & 2(\lambda_{1\Xi} + \lambda_{2\Xi})\Lambda^2 \end{pmatrix}.$$

Note that f is a mass parameter satisfying

$$f = -\frac{\lambda_{2S} u v}{\sqrt{2} w} - \frac{\lambda_2 u v \Lambda^2}{2\sqrt{2}(u^2 - v^2) w}, \quad (21)$$

which is derived from the minimization conditions (16) and (17). Because of $u, v \ll w, \Lambda$, the parameter f is large in the w, Λ scale. At the leading order, $u, v \ll w, \Lambda, f$, the above mass matrix implies a massless scalar field, $H_1 = \frac{u S_1 + v S_2}{\sqrt{u^2 + v^2}}$, and three heavy scalar fields with masses given by

$$H_2 = \frac{-vS_1 + uS_2}{\sqrt{u^2+v^2}}, \qquad m^2_{H_2} = \frac{\lambda_2(u^2+v^2)\Lambda^2}{2(v^2-u^2)},$$
$$H_3 = c_\varphi S_3 - s_\varphi S_4, \qquad m^2_{H_3} = \lambda_\phi w^2 + (\lambda_{1\Xi}+\lambda_{2\Xi})\Lambda^2 - \sqrt{[(\lambda_{1\Xi}+\lambda_{2\Xi})\Lambda^2 - \lambda_\phi w^2]^2 + \lambda_5^2 w^2\Lambda^2},$$
$$H_4 = s_\varphi S_3 + c_\varphi S_4, \qquad m^2_{H_4} = \lambda_\phi w^2 + (\lambda_{1\Xi}+\lambda_{2\Xi})\Lambda^2 + \sqrt{[(\lambda_{1\Xi}+\lambda_{2\Xi})\Lambda^2 - \lambda_\phi w^2]^2 + \lambda_5^2 w^2\Lambda^2},$$

where we have denoted $c_\varphi = \cos\varphi$, $s_\varphi = \sin\varphi$, and so forth, with

$$t_{2\varphi} = \frac{\lambda_5 w\Lambda}{(\lambda_{1\Xi}+\lambda_{2\Xi})\Lambda^2 - \lambda_\phi w^2}. \tag{22}$$

At the next-to-leading order, the Higgs masses, $m^2_{H_i}$ ($i = 1, 2, 3, 4$), are contributed by the u^2, v^2 terms. Particularly, the H_1 mass is approximated as

$$m^2_{H_1} = 2(\lambda_{1S}+\lambda_{2S})u^2 - \lambda_{2S}v^2. \tag{23}$$

It is easily realized that the light state, H_1, is identical to the standard model Higgs boson, whereas the heavy states, $H_{2,3,4}$, are new particles with the masses as given in the w, Λ scales.

The mass terms of the pseudoscalars, A_1, A_2, A_3, A_4, are given by

$$V^A_{\mathrm{mass}} = \frac{1}{2}\begin{pmatrix} A_1 & A_2 & A_3 & A_4 \end{pmatrix} M^2_A \begin{pmatrix} A_1 & A_2 & A_3 & A_4 \end{pmatrix}^T, \tag{24}$$

where

$$M^2_A = \begin{pmatrix} \frac{v^2(-2\lambda_{2S}u^2+2\lambda_{2S}v^2-\lambda_2\Lambda^2)}{2(u^2-v^2)} & -\frac{uv[2\lambda_{2S}(u^2-v^2)+\lambda_2\Lambda^2]}{2(u^2-v^2)} & \frac{uv^2[2\lambda_{2S}(u^2-v^2)+\lambda_2\Lambda^2]}{2(u^2-v^2)w} & 0 \\ -\frac{uv[2\lambda_{2S}(u^2-v^2)+\lambda_2\Lambda^2]}{2(u^2-v^2)} & -\frac{u^2[2\lambda_{2S}(u^2-v^2)+\lambda_2\Lambda^2]}{2(u^2-v^2)} & \frac{u^2v[2\lambda_{2S}(u^2-v^2)+\lambda_2\Lambda^2]}{2(u^2-v^2)w} & 0 \\ \frac{uv^2[2\lambda_{2S}(u^2-v^2)+\lambda_2\Lambda^2]}{2(u^2-v^2)w} & \frac{u^2v[2\lambda_{2S}(u^2-v^2)+\lambda_2\Lambda^2]}{2(u^2-v^2)w} & -\frac{u^2v^2[2\lambda_{2S}(u^2-v^2)+\lambda_2\Lambda^2]}{2(u^2-v^2)w^2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{25}$$

The above mass matrix provides only a combination of the pseudoscalars as a physical pseudoscalar, called $\mathcal{A}$, with mass, $m^2_{\mathcal{A}}$, obtained by

$$\mathcal{A} = \frac{vwA_1 + uwA_2 - uvA_3}{\sqrt{(u^2+v^2)w^2 + u^2v^2}}, \qquad m^2_{\mathcal{A}} = -\frac{[v^2w^2 + u^2(v^2+w^2)][2\lambda_{2S}(u^2-v^2)+\lambda_2\Lambda^2]}{2(u^2-v^2)w^2}, \tag{26}$$

which is in w, Λ scales. The remainders are three massless pseudoscalars,

$$G_Z = \frac{-uA_1 + vA_2}{\sqrt{u^2+v^2}}, \qquad G_{\mathcal{Z}_1} = A_4, \qquad G_{\mathcal{Z}'_1} = \frac{uv^2A_1 + u^2vA_2 + w(u^2+v^2)A_3}{\sqrt{(u^2+v^2)(u^2v^2+w^2u^2+w^2v^2)}}, \tag{27}$$

which are the Goldstone bosons of the neutral gauge bosons, $Z, \mathcal{Z}_1$, and $\mathcal{Z}'_1$, respectively.

For the charged scalar sector, $\Xi^{\pm\pm}_{22}$, $\Xi^{\pm(q-1)}_{23}$, and $\Xi^{\pm 2q}_{33}$ do not mix and are physical fields by themselves with masses

$$m^2_{\Xi^{\pm\pm}_{22}} = \frac{\lambda_2(v^2-u^2) - 2\lambda_{2\Xi}\Lambda^2}{2}, \tag{28}$$

$$m^2_{\Xi^{\pm(q-1)}_{23}} = \frac{\lambda_2(v^2-2u^2) + \lambda_3 w^2 - 4\lambda_{2\Xi}\Lambda^2}{4}, \tag{29}$$

$$m^2_{\Xi^{\pm 2q}_{33}} = \frac{\lambda_3 w^2 - \lambda_2 u^2 - 2\lambda_{2\Xi}\Lambda^2}{2}, \tag{30}$$

which are all in w, Λ scales. The remaining charged scalars mix in terms of

$$V_{\text{mass}}^{\text{charged}} \supset \begin{pmatrix} S_{12}^{+} & S_{21}^{+} & \Xi_{12}^{+} \end{pmatrix} M_{C_1}^2 \begin{pmatrix} S_{12}^{-} \\ S_{21}^{-} \\ \Xi_{12}^{-} \end{pmatrix} + \begin{pmatrix} S_{13}^{q} & \phi_1^{q} & \Xi_{13}^{q} \end{pmatrix} M_{C_q}^2 \begin{pmatrix} S_{13}^{-q} \\ \phi_1^{-q} \\ \Xi_{13}^{-q} \end{pmatrix} + \begin{pmatrix} S_{23}^{(q+1)} & \phi_2^{(q+1)} \end{pmatrix} M_{C_{(q+1)}}^2 \begin{pmatrix} S_{23}^{-(q+1)} \\ \phi_2^{-(q+1)} \end{pmatrix}, \tag{31}$$

where $M_{C_1}^2$, $M_{C_q}^2$, and $M_{C_{(q+1)}}^2$ are mass-squared matrices for the singly, q, and $(q+1)$ charged scalars, respectively (as shown below).

First, we derive

$$M_{C_1}^2 = \begin{pmatrix} \frac{\lambda_2 u^2\Lambda^2}{2(v^2-u^2)} & \frac{\lambda_2 uv\Lambda^2}{2(v^2-u^2)} & \frac{\lambda_2 u\Lambda}{2\sqrt{2}} \\ \frac{\lambda_2 uv\Lambda^2}{2(v^2-u^2)} & \frac{\lambda_2 v^2\Lambda^2}{2(v^2-u^2)} & \frac{\lambda_2 u\Lambda}{2\sqrt{2}} \\ \frac{\lambda_2 u\Lambda}{2\sqrt{2}} & \frac{\lambda_2 u\Lambda}{2\sqrt{2}} & \frac{\lambda_2(v^2-u^2)}{4} \end{pmatrix}. \tag{32}$$

This leads to a physical, singly-charged field, with mass in the w, Λ scales,

$$H_5^{\pm} = \frac{\sqrt{2}u\Lambda S_{12}^{\pm} + \sqrt{2}v\Lambda S_{21}^{\pm} + (v^2-u^2)\Xi_{12}^{\pm}}{\sqrt{2(u^2+v^2)\Lambda^2 + (v^2-u^2)^2}}, \qquad m_{H_5^{\pm}}^2 = \frac{\lambda_2}{4}\left[v^2 - u^2 + \frac{2(u^2+v^2)\Lambda^2}{v^2-u^2}\right]. \tag{33}$$

Two remaining states are massless, combined of

$$G_{W_1}^{\pm} = \frac{-vS_{12}^{\pm} + uS_{21}^{\pm}}{\sqrt{u^2+v^2}}, \qquad G_{W_2}^{\pm} = \frac{u(u^2-v^2)S_{12}^{\pm} + v(u^2-v^2)S_{21}^{\pm} + \sqrt{2}(u^2+v^2)\Lambda\Xi_{12}^{\pm}}{\sqrt{(u^2-v^2)^2(u^2+v^2) + 2(u^2+v^2)^2\Lambda^2}}, \tag{34}$$

which are the Goldstone bosons of $W_1^{\pm}, W_2^{\pm}$ gauge bosons, respectively.

The mass matrix for q charged scalars is

$$M_{C_q}^2 = \begin{pmatrix} \frac{\sqrt{2}fvw}{u} + \frac{1}{2}(\lambda_1 w^2 - \lambda_2\Lambda^2) & \sqrt{2}fv + \frac{\lambda_1 uw}{2} & \frac{\lambda_2 u\Lambda}{2\sqrt{2}} \\ \sqrt{2}fv + \frac{\lambda_1 uw}{2} & \frac{1}{2}\left(\lambda_1 u^2 + \lambda_3\Lambda^2 + \frac{2\sqrt{2}fuv}{w}\right) & \frac{\lambda_3 w\Lambda}{2\sqrt{2}} \\ \frac{\lambda_2 u\Lambda}{2\sqrt{2}} & \frac{\lambda_3 w\Lambda}{2\sqrt{2}} & \frac{1}{4}(\lambda_3 w^2 - \lambda_2 u^2) \end{pmatrix}. \tag{35}$$

We obtain a massless state, $G_X^{\pm q} = \frac{uS_{13}^{\pm q} - w\phi_1^{\pm q} + \sqrt{2}\Lambda\Xi_{13}^{\pm q}}{\sqrt{u^2+w^2+2\Lambda^2}}$, as the Goldstone boson of the $X^{\pm q}$ gauge boson. To find the remaining states, we define

$$H_6'^{\pm q} = \frac{wS_{13}^{\pm q} + u\phi_1^{\pm q}}{\sqrt{u^2+w^2}}, \qquad H_7'^{\pm q} = \frac{-\sqrt{2}u\Lambda S_{13}^{\pm q} + \sqrt{2}w\Lambda\phi_1^{\pm q} + (u^2+w^2)\Xi_{13}^{\pm q}}{\sqrt{(u^2+w^2)(u^2+w^2+2\Lambda^2)}}, \tag{36}$$

which are orthogonal to $G_X^{\pm q}$. The corresponding physical fields as the combinations of $H_6'^{\pm q}$, $H_7'^{\pm q}$ with masses are given by

$$H_6^{\pm q} = c_{\varphi_q} H_6'^{\pm q} - s_{\varphi_q} H_7'^{\pm q}, \quad m_{H_6^{\pm q}}^2 \simeq \frac{\lambda_1(u^2-v^2)w^2 - \lambda_2 u^2\Lambda^2}{2(u^2-v^2)}, \quad H_7^{\pm q} = s_{\varphi_q} H_6'^{\pm q} + c_{\varphi_q} H_7'^{\pm q}, \quad m_{H_7^{\pm q}}^2 \simeq \frac{\lambda_3(w^2+2\Lambda^2)}{4}, \tag{37}$$

where the $H_6'^{\pm q} - H_7'^{\pm q}$ mixing angle, called φ_q, is defined by

$$t_{2\varphi_q} \simeq \frac{2(\lambda_2+\lambda_3)u\Lambda\sqrt{2(\Lambda^2+w^2)}}{-2\lambda_1 w^3+\lambda_3 w(w^2+2\Lambda^2)+\frac{2\lambda_2 w u^2\Lambda^2}{u^2-v^2}}, \quad (38)$$

which is small due to $u,\, v \ll w, \Lambda$, implying that such states slightly mix.

Lastly, there remain two $(q+1)$ charged scalars. One of them is massless to be identified as the Goldstone boson of the $Y^{\pm(q+1)}$ gauge boson,

$$G_Y^{\pm(q+1)} = \frac{-vS_{23}^{\pm(q+1)}+w\phi_2^{\pm(q+1)}}{\sqrt{v^2+w^2}}. \quad (39)$$

The field that is orthogonal to it is heavy with mass in the w, Λ scales, given by

$$H_8^{\pm(q+1)} = \frac{wS_{23}^{\pm(q+1)}+v\phi_2^{\pm(q+1)}}{\sqrt{v^2+w^2}},$$

$$m^2_{H_8^{\pm(q+1)}} = -\frac{(v^2+w^2)[(u^2-v^2)(2\lambda_{2S}u^2-\lambda_1 w^2)+\lambda_2 u^2\Lambda^2]}{2(u^2-v^2)w^2}. \quad (40)$$

In summary, the model contains 12 massive Higgs fields, $H^0_{1,2,3,4}$, $\mathcal{A}^0$, $H_5^{\pm}$, $H_{6,7}^{\pm q}$, $H_8^{\pm(q+1)}$, $\Xi_{22}^{\pm\pm}$, $\Xi_{23}^{\pm(q-1)}$, and $\Xi_{33}^{\pm 2q}$, in which H_1 is the standard-model-like Higgs boson with mass in the weak scale, while the others are new, heavy Higgs bosons with masses in w, Λ scales. Also, there are 11 massless Goldstone bosons, which are correspondingly eaten by the 11 massive gauge bosons (where the conjugated fields are also counted). At the leading order, the physical scalar states are related to those in the gauge basis as

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} \simeq \begin{pmatrix} c_{\alpha_1} & s_{\alpha_1} \\ -s_{\alpha_1} & c_{\alpha_1} \end{pmatrix}\begin{pmatrix} S_1 \\ S_2 \end{pmatrix},$$

$$\begin{pmatrix} H_3 \\ H_4 \end{pmatrix} \simeq \begin{pmatrix} c_{\varphi} & -s_{\varphi} \\ s_{\varphi} & c_{\varphi} \end{pmatrix}\begin{pmatrix} S_3 \\ S_4 \end{pmatrix},$$

$$\begin{pmatrix} \mathcal{A} \\ G_Z \\ G_{\mathcal{Z}_1} \\ G_{\mathcal{Z}'_1} \end{pmatrix} \simeq \begin{pmatrix} s_{\alpha_1} & c_{\alpha_1} & -\frac{u}{w}s_{\alpha_1} & 0 \\ -c_{\alpha_1} & s_{\alpha_1} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{v}{2w}s_{2\alpha_1} & \frac{u}{2w}s_{2\alpha_1} & 1 & 0 \end{pmatrix}\begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{pmatrix},$$

$$\begin{pmatrix} H_5^{\pm} \\ G_{W_1}^{\pm} \\ G_{W_2}^{\pm} \end{pmatrix} \simeq \begin{pmatrix} c_{\alpha_1} & s_{\alpha_1} & \frac{v^2-u^2}{\sqrt{2}\sqrt{u^2+v^2}\Lambda} \\ -s_{\alpha_1} & c_{\alpha_1} & 0 \\ \frac{u}{\sqrt{2}\Lambda}c_{2\alpha_1} & \frac{v}{\sqrt{2}\Lambda}c_{2\alpha_1} & 1 \end{pmatrix}\begin{pmatrix} S_{12}^{\pm} \\ S_{21}^{\pm} \\ \Xi_{12}^{\pm} \end{pmatrix},$$

$$\begin{pmatrix} G_X^{\pm q} \\ H_6^{\pm q} \\ H_7^{\pm q} \end{pmatrix} \simeq \begin{pmatrix} \frac{u}{w}s_{\alpha_2} & -s_{\alpha_2} & c_{\alpha_2} \\ c_{\varphi_q} & \frac{u}{w}c_{\varphi_q}-c_{\alpha_2}s_{\varphi_q} & -s_{\alpha_2}s_{\varphi_q} \\ s_{\varphi_q}-\frac{u}{w}c_{\alpha_2}c_{\varphi_q} & c_{\alpha_2}c_{\varphi_q} & s_{\alpha_2}c_{\varphi_q} \end{pmatrix}\begin{pmatrix} S_{13}^{\pm q} \\ \phi_1^{\pm q} \\ \Xi_{13}^{\pm q} \end{pmatrix},$$

$$\begin{pmatrix} G_Y^{\pm(q+1)} \\ H_8^{\pm(q+1)} \end{pmatrix} \simeq \begin{pmatrix} -\frac{v}{w} & 1 \\ 1 & \frac{v}{w} \end{pmatrix}\begin{pmatrix} S_{23}^{\pm(q+1)} \\ \phi_2^{\pm(q+1)} \end{pmatrix}, \quad (41)$$

where the $\alpha_{1,2}$ angles have been introduced, defined by $t_{\alpha_1} = v/u$ and $t_{\alpha_2} = w/\sqrt{2}\Lambda$, respectively.

IV. GAUGE SECTOR

Let us investigate the mass spectrum of the gauge bosons in the considering model. When the scalars develop the VEVs, the gauge bosons get masses as derived from

$$\mathcal{L}_s = \mathrm{Tr}[(D_\mu S)^\dagger(D^\mu S)+(D_\mu\Xi)^\dagger(D^\mu\Xi)] + (D_\mu\phi)^\dagger(D^\mu\phi), \quad (42)$$

where the covariant derivatives are defined by

$$D_\mu S = \partial_\mu S + ig_L\frac{\sigma_a}{2}A_{aL\mu}S - ig_R S\frac{\lambda_i}{2}A_{iR\mu} + ig_X X_S B_\mu S, \quad (43)$$

$$D_\mu\Xi = \partial_\mu\Xi + ig_R\frac{\lambda_i}{2}A_{iR\mu}\Xi + ig_R\Xi\frac{\lambda_i^*}{2}A_{iR\mu} + ig_X X_\Xi B_\mu\Xi, \tag{44}$$

$$D_\mu\phi = \partial_\mu\phi + ig_R\frac{\lambda_i}{2}A_{iR\mu}\phi + ig_X X_\phi B_\mu\phi. \tag{45}$$

Here, σ_a are the Pauli matrices, and λ_i are the Gell-Mann matrices. g_L, g_R, and g_X are the gauge coupling constants of $SU(2)_L$, $SU(3)_R$, and $U(1)_X$, respectively. $X_{S,\Xi,\phi}$ stand for the $U(1)_X$ charges of the corresponding scalar multiplets.

Substituting the VEVs for S, Ξ, ϕ, we obtain the mass Lagrangian,

$$\begin{aligned}\mathcal{L}_{\text{mass}}^{\text{gauge}} &= \frac{g_L^2}{8}\left\{\left[A_{3L}^\mu - t_R A_{3R}^\mu - \frac{t_R}{\sqrt{3}}A_{8R}^\mu - \frac{(1+2q)t_X}{3}B^\mu\right]^2 + 2[W_L^{\mu+}W_{L\mu}^- + t_R^2(W_R^{\mu+}W_{R\mu}^-\right.\\
&\quad \left.+X_{R\mu}^q X_R^{-q\mu})]\right\}u^2 + \frac{g_L^2}{8}\left\{\left[A_{3L}^\mu - t_R A_{3R}^\mu + \frac{t_R}{\sqrt{3}}A_{8R}^\mu + \frac{(1+2q)t_X}{3}B^\mu\right]^2 + 2[W_L^{\mu+}\right.\\
&\quad \left.\times W_{L\mu}^- + t_R^2(W_R^{\mu+}W_{R\mu}^- + Y_{R\mu}^{q+1}Y_R^{-(q+1)\mu})]\right\}v^2 - \frac{g_L^2}{2}(W_{L\mu}^- W_R^{\mu+} + W_{L\mu}^+ W_R^{\mu-})t_R uv\\
&\quad + \frac{g_L^2}{2}\left\{\left[t_R A_{3R}^\mu + \frac{t_R}{\sqrt{3}}A_{8R}^\mu + \frac{2t_X}{3}(q-1)B^\mu\right]^2 + t_R^2(W_{R\mu}^+ W_R^{\mu-} + X_{R\mu}^q X_R^{-q\mu})\right\}\Lambda^2\\
&\quad + \frac{g_L^2}{18}\left\{[\sqrt{3}t_R A_{8R}^\mu + t_X(1+2q)B^\mu]^2 + \frac{9}{2}t_R^2(X_{R\mu}^q X_R^{-q\mu} + Y_{R\mu}^{q+1}Y_R^{-(q+1)\mu})\right\}w^2,\\
&= \frac{g_L^2 t_R^2}{4}(v^2+w^2)Y_{R\mu}^{-(q+1)}Y_R^{(q+1)\mu} + \frac{g_L^2 t_R^2}{4}(u^2+w^2+2\Lambda^2)X_{R\mu}^{-q}X_R^{q\mu} + (W_L^{\mu+}W_R^{\mu+})\\
&\quad \times M_W^2(W_{L\mu}^- W_{R\mu}^-)^T + \frac{1}{2}(A_{3L}^\mu A_{3R}^\mu A_{8R}^\mu B^\mu)M_0^2(A_{3L\mu}A_{3R\mu}A_{8R\mu}B_\mu)^T,\end{aligned} \tag{46}$$

where we have denoted $t_X = \frac{g_X}{g_L}$, $t_R = \frac{g_R}{g_L}$, and the non-Hermitian gauge bosons as

$$W_{L\mu}^\pm = \frac{1}{\sqrt{2}}(A_{1L\mu} \mp iA_{2L\mu}),$$
$$W_{R\mu}^\pm = \frac{1}{\sqrt{2}}(A_{1R\mu} \mp iA_{2R\mu}), \tag{47}$$

$$Y_{R\mu}^{\pm(q+1)} = \frac{1}{\sqrt{2}}(A_{6R\mu} \pm iA_{7R\mu}),$$
$$X_{R\mu}^{\pm q} = \frac{1}{\sqrt{2}}(A_{4R\mu} \pm iA_{5R\mu}). \tag{48}$$

The mass Lagrangian in (46) has be rewritten in terms of the matrix forms, where M_W and M_0 define the mass matrices of the left-right W and neutral gauge bosons, respectively.

We see that the gauge bosons, $X_{R\mu}^{\pm q}$ and $Y_{R\mu}^{\pm(q+1)}$, by themselves are physical with masses,

$$m_{X_R}^2 = \frac{g_R^2}{4}(u^2+w^2+2\Lambda^2),$$
$$m_{Y_R}^2 = \frac{g_R^2}{4}(v^2+w^2). \tag{49}$$

The left-right W bosons mix via a mass matrix as given by

$$M_W^2 = \frac{g_L^2}{4}\begin{pmatrix} u^2+v^2 & -2t_R uv \\ -2t_R uv & t_R^2(u^2+v^2+2\Lambda^2)\end{pmatrix}. \tag{50}$$

Diagonalizing this matrix, we obtain two physical states,

$$W_{1\mu}^\pm = c_\xi W_{L\mu}^\pm - s_\xi W_{R\mu}^\pm,$$
$$W_{2\mu}^\pm = s_\xi W_{L\mu}^\pm + c_\xi W_{R\mu}^\pm, \tag{51}$$

where the $W_L - W_R$ mixing angle (ξ) is obtained by $t_{2\xi} = \tan 2\xi = \frac{-4t_R uv}{2t_R^2\Lambda^2 + (t_R^2-1)(u^2+v^2)}$. The corresponding masses are

$$m_{W_1}^2 \simeq \frac{g_L^2}{4}\left[u^2+v^2 - \frac{4t_R^2u^2v^2}{2t_R^2\Lambda^2+(t_R^2-1)(u^2+v^2)}\right],$$
$$m_{W_2}^2 \simeq \frac{g_R^2}{4}\left[u^2+v^2+2\Lambda^2 + \frac{4u^2v^2}{2t_R^2\Lambda^2+(t_R^2-1)(u^2+v^2)}\right]. \tag{52}$$

Because of the condition, $u, v \ll w, \Lambda$, the W_1 boson has a small mass in the weak scales (u, v) which is identical to the standard model W boson, whereas the W_2 boson is a new, heavy charged gauge boson with a mass proportional to the Λ scale. The mixing between these two fields is small since $\xi \to 0$ due to the above condition.

The diagonalization of the neutral gauge boson sector is more complicated, because all four gauge fields generally mix. Indeed, the mass matrix is given by

$$M_0^2 = \frac{g_L^2}{4}\begin{pmatrix} u^2+v^2 & -t_R(u^2+v^2) & -\frac{t_R}{\sqrt{3}}(u^2-v^2) & \frac{\beta t_X}{\sqrt{3}}(u^2-v^2) \\ -t_R(u^2+v^2) & t_R^2(u^2+v^2+4\Lambda^2) & \frac{t_R^2}{\sqrt{3}}(u^2-v^2+4\Lambda^2) & m_{42}^2 \\ -\frac{t_R}{\sqrt{3}}(u^2-v^2) & \frac{t_R^2}{\sqrt{3}}(u^2-v^2+4\Lambda^2) & \frac{t_R^2}{3}[u^2+v^2+4(w^2+\Lambda^2)] & m_{43}^2 \\ \frac{\beta t_X}{\sqrt{3}}(u^2-v^2) & m_{42}^2 & m_{43}^2 & m_{44}^2 \end{pmatrix}, \tag{53}$$

where

$$m_{42}^2 = -\frac{t_R t_X}{3}[\sqrt{3}\beta(u^2-v^2+4\Lambda^2)+12\Lambda^2],$$
$$m_{43}^2 = \frac{-t_R t_X}{3}[\beta(u^2+v^2+4w^2+4\Lambda^2)+4\sqrt{3}\Lambda^2],$$
$$m_{44}^2 = \frac{t_X^2}{3}[(u^2+v^2+4w^2)\beta^2+4(\sqrt{3}+\beta)^2\Lambda^2].$$

First of all, from the mass matrix, we can always obtain a zero eigenvalue (i.e., photon mass) with the corresponding eigenstate (i.e., photon field) as

$$A_\mu = \frac{t_R t_X}{\sqrt{t_R^2+t_X^2(1+\beta^2+t_R^2)}}\left(A_{3L\mu}+\frac{1}{t_R}A_{3R\mu}+\frac{\beta}{t_R}A_{8R\mu}+\frac{1}{t_X}B_\mu\right), \tag{54}$$

which is independent of the VEVs as a consequence of the electric charge conservation [18]. Next, we can determine electromagnetic interactions following the standard procedure in [18], and thus the Weinberg's angle (θ_W) is identified as

$$s_W = \frac{t_R t_X}{\sqrt{t_R^2+t_X^2(1+\beta^2+t_R^2)}}. \tag{55}$$

Note that $s_W = \sin\theta_W$, $c_W = \cos\theta_W$, and so forth. With this at hand, the photon field is rewritten in terms of

$$A_\mu = s_W A_{3L\mu} + c_W\left(\frac{t_W}{t_R}A_{3R\mu}+\beta\frac{t_W}{t_R}A_{8R\mu}+\frac{t_W}{t_X}B_\mu\right), \tag{56}$$

where the parentheses present the field as coupled to the weak hypercharge $Y = T_{3R}+\beta T_{8R}+X$.

The standard model Z boson is orthogonal to the photon field as usual,

$$Z_\mu = c_W A_{3L\mu} - s_W\left(\frac{t_W}{t_R}A_{3R\mu}+\beta\frac{t_W}{t_R}A_{8R\mu}+\frac{t_W}{t_X}B_\mu\right). \tag{57}$$

The model under consideration contains two new neutral gauge bosons, called Z_R and Z'_R, which are given orthogonally to the hypercharge field in the parentheses (i.e., orthogonal to both the photon and Z fields). Thus, they are obtained by

$$Z'_{R\mu} = \frac{1}{\sqrt{t_R^2+\beta^2 t_X^2}}(t_R A_{8R\mu} - \beta t_X B_\mu), \tag{58}$$

$$Z_{R\mu} = \frac{1}{\sqrt{(t_R^2+\beta^2t_X^2)[t_R^2+(1+\beta^2)t_X^2]}}[-(t_R^2+\beta^2t_X^2)A_{3R\mu}+\beta t_X^2 A_{8R\mu}+t_X t_R B_\mu], \tag{59}$$

where $t_X = \frac{s_W t_R}{\sqrt{t_R^2-(1+\beta^2+t_R^2)s_W^2}}$, and these new states must be heavy.

Next, let us change to the new basis consisting of A_μ, Z_μ, $Z'_{R\mu}$, and $Z_{R\mu}$ by the transformation, $(A_{3L\mu}A_{3R\mu}A_{8R\mu}B_\mu)^T = U(A_\mu Z_\mu Z'_{R\mu} Z_{R\mu})^T$, where

$$U = \begin{pmatrix} s_W & c_W & 0 & 0 \\ \frac{s_W}{t_R} & -\frac{s_W t_W}{t_R} & 0 & -\frac{t_R^2+\beta^2 t_X^2}{\sqrt{(t_R^2+\beta^2 t_X^2)[t_R^2+(1+\beta^2)t_X^2]}} \\ \frac{\beta s_W}{t_R} & -\frac{\beta s_W t_W}{t_R} & \frac{t_R}{\sqrt{t_R^2+\beta^2 t_X^2}} & \frac{\beta t_X^2}{\sqrt{(t_R^2+\beta^2 t_X^2)[t_R^2+(1+\beta^2)t_X^2]}} \\ \frac{s_W}{t_X} & -\frac{s_W t_W}{t_X} & -\frac{\beta t_X}{\sqrt{t_R^2+\beta^2 t_X^2}} & \frac{t_X t_R}{\sqrt{(t_R^2+\beta^2 t_X^2)[t_R^2+(1+\beta^2)t_X^2]}} \end{pmatrix}. \tag{60}$$

Correspondingly, the mass matrix M_0^2 is changed to

$$M_0'^2 = U^T M_0^2 U = \begin{pmatrix} 0 & 0 \\ 0 & M'^2 \end{pmatrix}. \tag{61}$$

We see that the photon field, A_μ, is decoupled as a physical massless field, while the other states $(Z_\mu, Z'_{R\mu}, Z_{R\mu})$ mix by themselves via a 3×3 mass matrix found to be

$$M'^2 = \frac{g_L^2}{4}\begin{pmatrix} \frac{u^2+v^2}{c_W^2} & -\frac{(u^2-v^2)\kappa c_W}{\sqrt{3}[t_R^2+t_X^2(1+\beta^2)]} & \frac{t_R(u^2+v^2)\kappa c_W}{[t_R^2+t_X^2(1+\beta^2)]^{3/2}} \\ -\frac{(u^2-v^2)\kappa c_W}{\sqrt{3}[t_R^2+t_X^2(1+\beta^2)]} & \frac{(t_R^2+\beta^2 t_X^2)^2(u^2+v^2+4w^2)+4\kappa'\Lambda^2}{3(t_R^2+\beta^2 t_X^2)} & \frac{\kappa''(v^2-u^2)}{\sqrt{3}} - \frac{4t_R^2\sqrt{\kappa'}\Lambda^2}{\sqrt{3}\kappa''} \\ \frac{t_R(u^2+v^2)\kappa c_W}{[t_R^2+t_X^2(1+\beta^2)]^{3/2}} & \frac{\kappa''(v^2-u^2)}{\sqrt{3}} - \frac{4t_R^2\sqrt{\kappa'}\Lambda^2}{\sqrt{3}\kappa''} & t_R^2\left(\frac{u^2+v^2}{\kappa'''} + 4\kappa'''\Lambda^2\right) \end{pmatrix},$$

where we have conveniently denoted

$$\kappa = [t_R^2(1+t_X^2) + (1+\beta^2)t_X^2]\sqrt{t_R^2+\beta^2 t_X^2}, \qquad \kappa' = [t_R^2 + (\sqrt{3}+\beta)\beta t_X^2]^2,$$
$$\kappa'' = t_R(t_R^2+\beta^2 t_X^2)/\sqrt{t_R^2+(1+\beta^2)t_X^2}, \qquad \kappa''' = [t_R^2+(1+\beta^2)t_X^2]/(t_R^2+\beta^2 t_X^2). \tag{62}$$

Because of the condition, $u, v \ll w, \Lambda$, the first row and first column of M'^2 consist of the elements that are much smaller than those of the remaining entries. Hence, we can diagonalize M'^2 by using the familiar seesaw formula. We introduce a basis $(\mathcal{Z}_\mu, \mathcal{Z}'_{R\mu}, \mathcal{Z}_{R\mu})$ in such a way as to separate the light $\mathcal{Z}_\mu$ boson from the two heavy $\mathcal{Z}'_{R\mu}, \mathcal{Z}_{R\mu}$ bosons. This basis is related to the previous basis $(Z_\mu, Z'_{R\mu}, Z_{R\mu})$ by a unitary transformation as $(Z_\mu Z'_{R\mu} Z_{R\mu})^T = \mathcal{U}(\mathcal{Z}_\mu \mathcal{Z}'_{R\mu}\mathcal{Z}_{R\mu})^T$. Correspondingly, the mass matrix, M'^2, is changed to

$$\mathcal{M}'^2 = \mathcal{U}^T M'^2 \mathcal{U} = \begin{pmatrix} m_{\mathcal{Z}}^2 & 0 \\ 0 & \mathcal{M}_{2\times 2}^2 \end{pmatrix}. \tag{63}$$

Using the seesaw approximation, we obtain

$$\mathcal{U} \simeq \begin{pmatrix} 1 & \epsilon_1 & \epsilon_2 \\ -\epsilon_1 & 1 & 0 \\ -\epsilon_2 & 0 & 1 \end{pmatrix},$$
$$m_{\mathcal{Z}}^2 \simeq \frac{g_L^2}{4}\left\{\frac{u^2+v^2}{c_W^2} + \frac{\epsilon_1(u^2-v^2)\kappa c_W}{\sqrt{3}[t_R^2+t_X^2(1+\beta^2)]} - \frac{\epsilon_2 t_R^2(u^2+v^2)\kappa c_W}{[t_R^2+t_X^2(1+\beta^2)]^{\frac{3}{2}}}\right\},$$
$$\mathcal{M}_{2\times 2}^2 \simeq \frac{g_L^2}{4}\begin{pmatrix} \frac{(t_R^2+\beta^2 t_X^2)^2(u^2+v^2+4w^2)+4\kappa'\Lambda^2}{3(t_R^2+\beta^2 t_X^2)} & \frac{\kappa''(v^2-u^2)}{\sqrt{3}} - \frac{4t_R^2\sqrt{\kappa'}\Lambda^2}{\sqrt{3}\kappa''} \\ \frac{\kappa''(v^2-u^2)}{\sqrt{3}} - \frac{4t_R^2\sqrt{\kappa'}\Lambda^2}{\sqrt{3}\kappa''} & t_R^2\left(\frac{u^2+v^2}{\kappa'''} + 4\kappa'''\Lambda^2\right) \end{pmatrix},$$

where $\epsilon_{1,2}$ are defined as

$$\epsilon_1 = \frac{\sqrt{3}\kappa c_W}{4\kappa'''}\left\{\frac{-(u^2+v^2)}{[t_R^2+\beta t_X^2(\sqrt{3}+\beta)][t_R^2+(1+\beta^2)t_X^2]\Lambda^2} - \frac{(u^2-v^2)}{(t_R^2+\beta^2 t_X^2)^2 w^2 + [t_R^2+(\sqrt{3}+\beta)\beta t_X^2]^2\Lambda^2}\right\}, \quad (64)$$

$$\epsilon_2 = \frac{\kappa c_W}{4\kappa''}\left\{\frac{u^2-v^2}{t_R[t_R^2+\beta t_X^2(\sqrt{3}+\beta)][t_R^2+t_X^2(1+\beta^2)]\Lambda^2} + \frac{u^2+v^2}{t_R[t_R^2+(1+\beta^2)t_X^2]^2\Lambda^2}\right\}. \quad (65)$$

Note that $\mathcal{M}^2_{2\times 2}$ describes two heavy states, $\mathcal{Z}_R \simeq Z_R$ and $\mathcal{Z}'_R \simeq Z'_R$, as given at the leading order. The mixing between Z and these heavy states is very suppressed, $\epsilon_1, \epsilon_2 \ll 1$, due to $u, v \ll w, \Lambda$. The $\mathcal{Z}_\mu$ boson is identical to the standard model Z boson with mass, $m^2_{\mathcal{Z}} \simeq \frac{g_L^2}{4c_W^2}(u^2+v^2)$.

Finally, the states $\mathcal{Z}'_R$ and $\mathcal{Z}_R$ still mix. Diagonalizing their mass matrix, we obtain the corresponding physical states

$$\mathcal{Z}_1 = c_\epsilon \mathcal{Z}'_R - s_\epsilon \mathcal{Z}_R, \qquad \mathcal{Z}'_1 = s_\epsilon \mathcal{Z}'_R + c_\epsilon \mathcal{Z}_R, \quad (66)$$

with masses,

$$m^2_{\mathcal{Z}_1} \simeq \frac{g_L^2}{6}\{t_R^2(w^2+4\Lambda^2) + t_X^2[\beta^2 w^2 + (\sqrt{3}+\beta)^2\Lambda^2] - \sqrt{[t_R^2(w^2+4\Lambda^2)+t_X^2(\beta^2 w^2+(\sqrt{3}+\beta)^2\Lambda^2)]^2 - 12t_R^2[t_R^2+(1+\beta^2)t_R^2]w^2\Lambda^2}\}, \quad (67)$$

$$m^2_{\mathcal{Z}'_1} \simeq \frac{g_L^2}{6}\{t_R^2(w^2+4\Lambda^2) + t_X^2[\beta^2 w^2 + (\sqrt{3}+\beta)^2\Lambda^2] + \sqrt{[t_R^2(w^2+4\Lambda^2)+t_X^2(\beta^2 w^2+(\sqrt{3}+\beta)^2\Lambda^2)]^2 - 12t_R^2[t_R^2+(1+\beta^2)t_R^2]w^2\Lambda^2}\}, \quad (68)$$

which are all in w, Λ scales. Above, the $\mathcal{Z}'_R - \mathcal{Z}_R$ mixing angle, ϵ, is obtained by

$$t_{2\epsilon} \simeq \frac{2\sqrt{3}t_R^2(t_R^2+\beta^2 t_X^2)[t_R^2+\beta(\beta+\sqrt{3})t_X^2]\Lambda^2}{\kappa''\{(t_R^2+\beta^2 t_X^2)^2 w^2 - [2t_R^4 + (\sqrt{3}-\beta)^2 t_R^2 t_X^2 - (\sqrt{3}+\beta)^2\beta^2 t_X^4]\Lambda^2\}}, \quad (69)$$

which is generally finite due to $w \sim \Lambda$.

To summarize, the physical neutral gauge bosons are related to the gauge states as $(A_{3L}A_{3R}A_{8R}B)^T = V(A\mathcal{Z}\mathcal{Z}_1\mathcal{Z}'_1)^T$, with

$$V = U\mathcal{U}U_\epsilon \simeq UU_\epsilon = \begin{pmatrix} s_W & c_W & 0 & 0 \\ \frac{s_W}{t_R} & -\frac{s_W^2}{t_R c_W} & \frac{\sqrt{t_R^2+t_X^2\beta^2}s_\epsilon s_W}{t_R t_X c_W} & -\frac{\sqrt{t_R^2+t_X^2\beta^2}c_\epsilon s_W}{t_R t_X c_W} \\ \frac{\beta s_W}{t_R} & -\frac{\beta s_W^2}{t_R c_W} & \frac{t_R^2 c_\epsilon c_W - \beta t_X s_\epsilon s_W}{t_R c_W\sqrt{t_R^2+t_X^2\beta^2}} & \frac{t_R^2 s_\epsilon c_W + \beta t_X c_\epsilon s_W}{t_R c_W\sqrt{t_R^2+t_X^2\beta^2}} \\ \frac{s_W}{t_X} & -\frac{s_W^2}{t_X c_W} & \frac{-\beta t_X c_\epsilon c_W - s_\epsilon s_W}{c_W\sqrt{t_R^2+t_X^2\beta^2}} & \frac{-\beta t_X s_\epsilon c_W + c_\epsilon s_W}{c_W\sqrt{t_R^2+t_X^2\beta^2}} \end{pmatrix}, \quad (70)$$

where $\mathcal{U} \simeq 1$ due to $\epsilon_{1,2} \ll 1$, and

$$U_\epsilon = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_\epsilon & s_\epsilon \\ 0 & 0 & -s_\epsilon & c_\epsilon \end{pmatrix}. \quad (71)$$

For the following calculations, we will use V as approximated; thus, $\mathcal{Z} = Z$, and $\mathcal{Z}_1$, $\mathcal{Z}'_1$ are directly related to Z'_R, Z_R by an expression like (66) since $\mathcal{Z}_R = Z_R$ and $\mathcal{Z}'_R = Z'_R$.

V. INTERACTIONS

A. Fermion–gauge boson interactions

The gauge interactions of fermions arise from the Lagrangian,

$$\mathcal{L}_f = \bar{\Psi} i\gamma^\mu D_\mu \Psi = \bar{\Psi} i\gamma^\mu \partial_\mu \Psi - g_L \bar{\Psi}_L \gamma^\mu (P^{CC}_{L\mu} + P^{NC}_{L\mu})\Psi_L - g_R \bar{\Psi}_R \gamma^\mu (P^{CC}_{R\mu} + P^{NC}_{R\mu})\Psi_R, \quad (72)$$

where the covariant derivative is $D_\mu = \partial_\mu + ig_L T_{aL} A_{aL\mu} + ig_R T_{iR} A_{iR\mu} + ig_X X B_\mu$ and the gauge vectors relevant to the charged and neutral currents are obtained as

$$P^{CC}_L = T_{1L}A_{1L} + T_{2L}A_{2L}, \qquad P^{NC}_L = T_{3L}A_{3L} + t_X X_{\Psi_L} B,$$
$$P^{CC}_R = \sum_{i=1,2,4,5,6,7} T_{iR}A_{iR}, \qquad P^{NC}_R = T_{3R}A_{3R} + T_{8R}A_{8R} + \frac{t_X}{t_R} X_{\Psi_R} B.$$

Above, Ψ_L and Ψ_R run on all the left-handed and right-handed fermion multiplets of the model, respectively. Note also that the interactions of fermions with gluons have the common form, which are easily determined and thus have been omitted.

Using (47) and (48), as well as (51) for (72), we derive the interactions of the physical charged gauge bosons with fermions as

$$\begin{aligned}\mathcal{L}_{CC} &= -g_L \bar{\Psi}_L \gamma^\mu P^{CC}_{L\mu}\Psi_L - g_R \bar{\Psi}_R \gamma^\mu P^{CC}_{R\mu}\Psi_R \\ &= J^{-\mu}_{1W} W^+_{1\mu} + J^{-\mu}_{2W} W^+_{2\mu} + J^{-q\mu}_X X^q_{R\mu} + J^{-(q+1)\mu}_Y Y^{q+1}_{R\mu} + \text{H.c.},\end{aligned} \quad (73)$$

where the charged currents $J^{-\mu}_{1W}$, $J^{-\mu}_{2W}$, $J^{-q\mu}_X$, and $J^{-(q+1)\mu}_Y$ are, respectively, defined as

$$\begin{aligned}J^{-\mu}_{1W} &= -\frac{g_L c_\xi}{\sqrt{2}}(\bar{\nu}_{aL}\gamma^\mu e_{aL} + \bar{u}_{aL}\gamma^\mu d_{aL}) + \frac{g_R s_\xi}{\sqrt{2}}(\bar{\nu}_{aR}\gamma^\mu e_{aR} + \bar{u}_{aR}\gamma^\mu d_{aR}),\\ J^{-\mu}_{2W} &= -\frac{g_L s_\xi}{\sqrt{2}}(\bar{\nu}_{aL}\gamma^\mu e_{aL} + \bar{u}_{aL}\gamma^\mu d_{aL}) - \frac{g_R c_\xi}{\sqrt{2}}(\bar{\nu}_{aR}\gamma^\mu e_{aR} + \bar{u}_{aR}\gamma^\mu d_{aR}),\\ J^{-q\mu}_X &= -\frac{g_R}{\sqrt{2}}(\bar{E}_{aR}\gamma^\mu \nu_{aR} - \bar{d}_{\alpha R}\gamma^\mu J_{\alpha R} + \bar{J}_{3R}\gamma^\mu u_{3R}),\\ J^{-(q+1)\mu}_Y &= -\frac{g_R}{\sqrt{2}}(\bar{E}_{aR}\gamma^\mu e_{aR} + \bar{u}_{\alpha R}\gamma^\mu J_{\alpha R} + \bar{J}_{3R}\gamma^\mu d_{3R}).\end{aligned} \quad (74)$$

Using the physical neutral gauge bosons defined by (70), $P^{NC}_{L\mu}$ and $P^{NC}_{R\mu}$ become

$$P^{NC}_{L\mu} = s_W Q_{\Psi_L} A_\mu + \frac{1}{c_W}(T_{3L} - s^2_W Q_{\Psi_L})Z_\mu + \frac{t_X(T_{3L} - Q_{\Psi_L})}{c_W\sqrt{t^2_R + t^2_X\beta^2}}[(\beta t_X c_\epsilon c_W + s_\epsilon s_W)\mathcal{Z}_{1\mu} + (\beta t_X s_\epsilon c_W - c_\epsilon s_W)\mathcal{Z}'_{1\mu}],$$

$$\begin{aligned}t_R P^{NC}_{R\mu} &= s_W Q_{\Psi_R} A_\mu - \frac{1}{c_W} s^2_W Q_{\Psi_R} Z_\mu + \left\{\frac{c_W(t_X\beta c_\epsilon t_W + t^2_R s_\epsilon)T_{3R} - t_W(\beta t_X c_\epsilon c_W + s_\epsilon s_W)Q_{\Psi_R}}{t^{-1}_X s_W\sqrt{t^2_R + t^2_X\beta^2}} + c_\epsilon\sqrt{t^2_R + t^2_X\beta^2}T_{8R}\right\}\mathcal{Z}_{1\mu}\\ &\quad + \left\{\frac{c_W(t_X\beta s_\epsilon t_W - t^2_R c_\epsilon)T_{3R} - t_W(\beta t_X s_\epsilon c_W - c_\epsilon s_W)Q_{\Psi_R}}{t^{-1}_X s_W\sqrt{t^2_R + t^2_X\beta^2}} + s_\epsilon\sqrt{t^2_R + t^2_X\beta^2}T_{8R}\right\}\mathcal{Z}'_{1\mu},\end{aligned}$$

where $Q_{\Psi_L} = T_{3L} + X_{\Psi_L}$ and $Q_{\Psi_R} = T_{3R} + \beta T_{8R} + X_{\Psi_R}$.

Hence, we have the neutral current interactions from (72) such that

$$\begin{aligned}\mathcal{L}_{NC} &= -g_L \bar{\Psi}_L \gamma^\mu P^{NC}_{L\mu}\Psi_L - g_R \bar{\Psi}_R \gamma^\mu P^{NC}_{R\mu}\Psi_R\\ &= -eQ(f)\bar{f}\gamma^\mu f A_\mu - \frac{g_L}{2c_W}\bar{f}\gamma^\mu[g^Z_V(f) - g^Z_A(f)\gamma_5]f Z_\mu\\ &\quad - \frac{g_L}{2c_W}\bar{f}\gamma^\mu[g^{\mathcal{Z}_1}_V(f) - g^{\mathcal{Z}_1}_A(f)\gamma_5]f\mathcal{Z}_{1\mu} - \frac{g_L}{2c_W}\bar{f}\gamma^\mu[g^{\mathcal{Z}'_1}_V(f) - g^{\mathcal{Z}'_1}_A(f)\gamma_5]f\mathcal{Z}'_{1\mu},\end{aligned} \quad (75)$$

where f indicates every fermion of the model, $Q(f_L) = Q(f_R) = Q(f)$, and $e = g_L s_W$. The vector and axial-vector couplings $g_{V,A}^{Z,\mathcal{Z}_1,\mathcal{Z}'_1}(f)$ can be directly obtained from the corresponding chiral couplings in the expressions of $P^{NC}_{L,R\mu}$ above to yield

$$\begin{aligned}
&g_V^Z(f) = T_{3L}(f_L) - 2s_W^2 Q(f), \qquad g_A^Z(f) = T_{3L}(f_L),\\
&g_V^{\mathcal{Z}_1}(f) = \frac{t_W^2(\beta t_X c_\epsilon c_W + s_\epsilon s_W)[T_{3L}(f_L) - 2Q(f)] + s_W(t_R^2 s_\epsilon + t_X t_W \beta c_\epsilon)T_{3R}(f_R)}{t_X^{-1} t_W^2 \sqrt{t_R^2 + t_X^2\beta^2}} + c_\epsilon c_W \sqrt{t_R^2 + t_X^2\beta^2}\, T_{8R}(f_R),\\
&g_A^{\mathcal{Z}_1}(f) = \frac{t_W^2(\beta t_X c_\epsilon c_W + s_\epsilon s_W)T_{3L}(f_L) - s_W(t_R^2 s_\epsilon + t_W t_X \beta c_\epsilon)T_{3R}(f_R)}{t_X^{-1} t_W^2 \sqrt{t_R^2 + t_X^2\beta^2}} - c_\epsilon c_W \sqrt{t_R^2 + t_X^2\beta^2}\, T_{8R}(f_R),\\
&g_{V,A}^{\mathcal{Z}'_1} = g_{V,A}^{\mathcal{Z}_1}(c_\epsilon \to s_\epsilon, s_\epsilon \to -c_\epsilon).
\end{aligned} \tag{76}$$

The first term in (75) yields electromagnetic interactions, as usual. The second term in (75) determines the neutral current coupled to the Z boson, which is consistent with the standard model. Note that the couplings of $\mathcal{Z}'_1$ can be obtained from those of $\mathcal{Z}_1$ by replacing $c_\epsilon \to s_\epsilon, s_\epsilon \to -c_\epsilon$, and vice versa. All the vector and axial-vector couplings of $Z, \mathcal{Z}_1, \mathcal{Z}'_1$ with fermions are explicitly calculated as collected in the Appendix.

B. Scalar–gauge boson interactions

The interactions of gauge bosons with scalars arise from (42). First note that there is no strong interaction for the scalars since they are colorless. Next, expand the scalar fields around their VEVs as in (14) and (15). Substituting the physical scalar states from (41) and the physical gauge states from (48), (51), and (70) into the mentioned Lagrangian, we get desirable interactions according to the vertex types between a gauge boson and two scalars, a scalar and two gauge bosons, and two scalars and two gauge bosons in the model.[1] Consequently, all the standard model interactions between the Higgs boson and the gauge fields are consistently recovered at the leading order.

VI. NEW PHYSICS EFFECTS AND CONSTRAINTS

In Ref. [14], the 750 GeV diphoton excess reported by the ATLAS and CMS experiments was studied, and the ATLAS diboson anomalies were briefly discussed too. Since these signals disappeared in the early search results of the LHC run II, the new physics scales should be high enough and their masses should be correspondingly large, above several TeVs, to escape the detections. Of course, the electric charge parameter q could be kept compatible to the usual ones. Retaining these conditions, in this work we will pay attention to alternative, interesting new-physics features that include the mixing effects in gauge and scalar sectors, as well as the tree-level FCNCs.

A. ρ and mixing parameters

The new-physics contribution to the ρ-parameter starts from the tree level due to both mixings of the standard model Z and W bosons with new gauge bosons. It is evaluated as

$$\begin{aligned}
\Delta\rho \equiv \rho - 1 &= \frac{m_{W_1}^2}{c_W^2 m_{\mathcal{Z}}^2} - 1\\
&\simeq \epsilon_2 \frac{t_R^2 c_W^3 \kappa}{[t_R^2 + t_X^2(1+\beta^2)]^{3/2}}\\
&\quad + \epsilon_1 \frac{(v^2 - u^2)c_W^3\kappa}{\sqrt{3}(u^2+v^2)[t_R^2 + t_X^2(1+\beta^2)]}\\
&\quad - \frac{2u^2v^2}{(u^2+v^2)\Lambda^2},
\end{aligned} \tag{77}$$

which is suppressed due to $u, v \ll w, \Lambda$.

The W mass implies $u^2 + v^2 = (246\ \text{GeV})^2$. Further, we take $t_R = g_R/g_L = 1$ and thus $t_X = s_W/\sqrt{1-(2+\beta^2)s_W^2}$. Note also that $|\beta| < \sqrt{1/s_W^2 - 2} \simeq 1.5261$, provided $s_W^2 \simeq 0.231$. From the global fit, the ρ-parameter is constrained by $\rho = 1.0004 \pm 0.00024$, which is 1.7σ deviating from the standard model prediction, $\rho = 1$ [1]. If the data imply a potential new physics, it sets the corresponding new-physics scale via $0.00016 < \Delta\rho < 0.00064$ at 95% C.L. Otherwise, when the measured central value is due to statistic errors, it induces a lower bound on the new physics scale of the interested model via $\Delta\rho < 0.00064$ at 95% C.L. For this case, note that an upper bound on the new-physics scale is not presented. In the following, we take the first interpretation into account. We make a contour (long and short dashed line) for $\Delta\rho$ as a function of $\Lambda = w = 1$–20 TeV and $u = 0$–246 GeV for three cases $\beta = -1/\sqrt{3}$, 0, and $1/\sqrt{3}$ as in Figs. 1, 2, and 3,

[1]See Appendix B in the first version of the arXiv posting of this article, https://arxiv.org/abs/1609.03444v1, for the detailed derivations of the Feynman rules corresponding to the various vertices between the scalar and gauge fields and the associated couplings, which were appropriately listed from Table IV to Table XXIII.

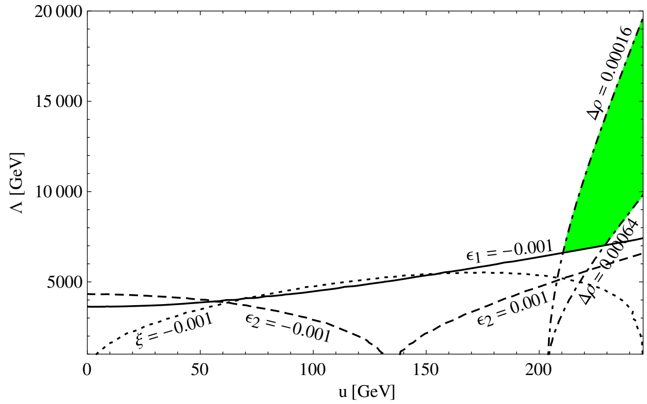

FIG. 1. The viable new physics regime (green) as constrained by $0.00016 < \Delta\rho < 0.00064$, $\xi = \epsilon_1 = \epsilon_2 = \pm 10^{-3}$ for the case $\beta = -1/\sqrt{3}$.

respectively. The available parameter space is bounded by both lines of the respective $\Delta\rho$ values.

The mixing of W, Z bosons with the new gauge bosons also modifies the well-measured couplings of W, Z with fermions. This new-physics effect is safe if one imposes the mixing parameters ξ, $\epsilon_{1,2}$ in 10^{-3} [1]. (Recall that these parameters are very suppressed due to the condition, $u, v \ll w, \Lambda$, too.) In Figs. 1, 2, and 3, we make contours (solid line for ϵ_1, dashed line for ϵ_2, and short dashed line for ξ) for $|\xi| = |\epsilon_{1,2}| = 10^{-3}$ in terms of ($\Lambda = w$, u), using the above inputs. The available parameter space lies above these three lines.

Combining all the constraints, we show the new-physics regime in green in Figs. 1, 2, and 3 for the three cases of β aforementioned. Consequently, Λ (thus $w = \Lambda$) is bounded by $4.6\ \text{TeV} < \Lambda < 13.7\ \text{TeV}, 5.5\ \text{TeV} < \Lambda < 16.3\ \text{TeV}$, and $6.6\ \text{TeV} < \Lambda < 19.4\ \text{TeV}$ for $\beta = 1/\sqrt{3}, 0$, and $-1/\sqrt{3}$, respectively. The weak scale regime for u [thus $v = \sqrt{(246\ \text{GeV})^2 - u^2}$ is followed] is narrow, as limited

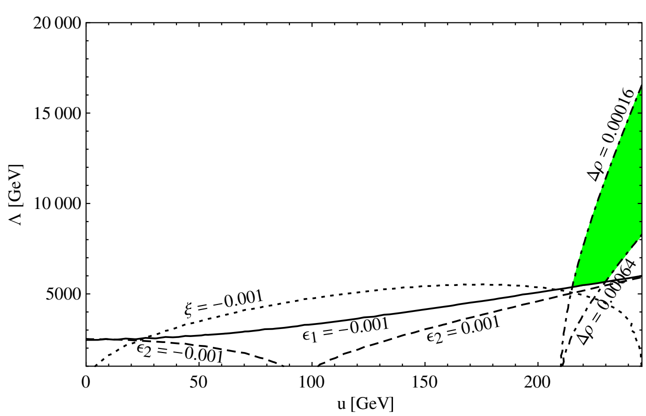

FIG. 2. The viable new physics regime (green) as constrained by $0.00016 < \Delta\rho < 0.00064$, $\xi = \epsilon_1 = \epsilon_2 = \pm 10^{-3}$ for the case $\beta = 0$.

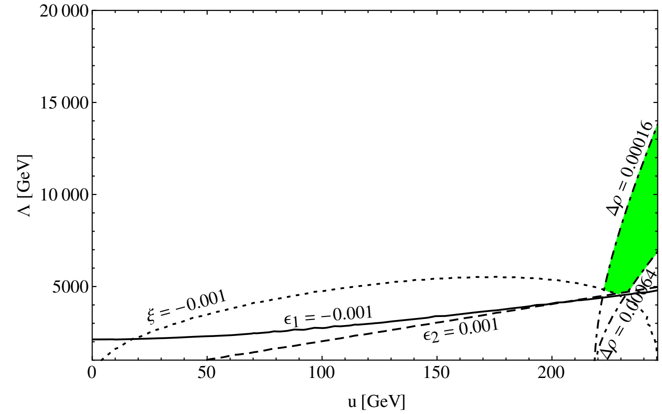

FIG. 3. The viable new-physics regime (green) as constrained by $0.00016 < \Delta\rho < 0.00064$, $\xi = \epsilon_1 = \epsilon_2 = \pm 10^{-3}$ for the case $\beta = 1/\sqrt{3}$.

by $u < 246$ GeV, and $u > 222.3$, 215, and 210.4 GeV, corresponding to the β values as mentioned.

Particularly, the bounds for the $3-2-3-1$ scales (w, Λ) that come from the ρ-parameter depend significantly on the weak scale, u, and they may even relax to zero for certain values of u with regard to the corresponding β values. But, it is not the complete story that we hope to close the $3-2-3-1$ symmetry at the weak scale, like the $3-3-1$ models investigated in [19]. As a matter of fact, although $\Delta\rho$ is proportional to $\epsilon_{1,2}$ and $\xi \sim uv/\Lambda^2$ (with the finite coefficients) according to the mixing of Z with (Z_R, Z'_R) and the mixing of W_L with W_R, the new physics is not decoupled from the standard model when w, Λ tend to the weak scale or even to zero. Indeed, the mixing effects and thus the W, Z-coupling corrections diverge when $(w, \Lambda) \to 0$ along the $\Delta\rho$ bounds, as possibly seen from the corresponding figures for $\epsilon_{1,2}$ and ξ (even though these mixing effects cancel out in the $\Delta\rho$ expression). Apparently, this property also emerges at loop levels because the good custodial symmetry $SU(2)_{L+R}$, if imposed, only protects ρ from the large contributions due to the effective cancellations, but it does not preserve any individual mixing effect from the divergences. The above judgment is also valid for arbitrary β and the $w - \Lambda$ relation. Therefore, closing new gauge symmetries at the weak scale as observed in the $3-3-1$ models would be lost due to the contribution of several new gauge bosons (not one) to the ρ-parameter.

It is easily checked that when (w, Λ) go to infinity, we have $\rho \to 1$ since $\epsilon_{1,2}$ and ξ, as well as the loop effects of the new gauge boson and scalar doublets, are suppressed by $(u^2, v^2)/(w^2, \Lambda^2)$, as the mass-squared splittings of the doublet components are. The standard-model-like fields and masses, as well as couplings, are restored to the standard model. Therefore, the $3-2-3-1$ model has a decoupling limit at a high scale for (w, Λ), not at the small scale as analyzed above.

The running of the gauge couplings and scalar self-couplings along with the energy scale may potentially present a upper bound on the $3-2-3-1$ breaking scales, such as the Landau pole at which some gauge coupling becomes infinity or the metastable scale at which some scalar self-coupling becomes negative. This model predicts the Weinberg angle as

$$s_W^2 = t_R^2 t_X^2/[t_R^2 + t_X^2(1+\beta^2+t_R^2)] < t_R^2/(1+\beta^2+t_R^2), \quad (78)$$

where g_L and g_R lightly change as $t_R = g_R/g_L$ does, while g_X and thus $t_X = g_X/g_L$ significantly increase, with the increasing of the energy scale. Therefore, the model encounters a Landau pole (M) at which $s_W^2(M) = t_R^2/(1+\beta^2+t_R^2) < 1$ or $g_X(M) = \infty$. Of course, the consistent condition of the theory is $w, \Lambda < M$. For simplicity, we assume $t_R = 1$, thus $s_W^2(M) = 1/(2+\beta^2)$, without much loss of generality ($t_R = 1$ is possibly protected by a minimal left-right symmetry, and this value is being also used around the text). The condition (78), $s_W^2(M) > s_W^2 \simeq 0.231$ at the weak scale, implies $|\beta| < 1.5261$, which translates to $-1.821 < q < 0.821$, which is very constrained, but spans every elementary charge as observed. Although the q charge is arbitrary in its range, the model predicts only integer charges for $q = 0, -1$. When q coincides with its bounds $q = -1.821$ or 0.821, the Landau pole lies at the weak scale $M \sim v_{\mathrm{weak}}$. Since the new physics is not decoupled as shown, the model in this case is inconsistent. For the half integer charges $q = 0.5, -1.5$ which are near the corresponding bounds, the Landau pole is greatly increased, $M \sim 10$ TeV, by which the new physics can be explored with great interest at the current colliders. For $q = 0, -1/2, -1$, which are being taken throughout the text, the Landau pole may be higher than the Planck scale. The above conclusions are analogous to the case of the $3-3-1$ models as studied in [20].

B. FCNC

As described above, after the spontaneous symmetry breaking, the Yukawa Lagrangian yields the masses for the fermions. Therefore, we will extract the quark mass terms from (12). The exotic quarks get large masses at the w scale,

$$\mathcal{L}^J_{\mathrm{mass}} = \bar{J}_{3L}\frac{h^J_{33}w}{\sqrt{2}}J_{3R} + \bar{J}_{\alpha L}\frac{h^J_{\alpha\beta}w}{\sqrt{2}}J_{\beta R} + \mathrm{H.c.}, \quad (79)$$

which are physical and decoupled (i.e., do not mix with the ordinary quarks and can be integrated out). However, the ordinary quarks mix by themselves with a mass Lagrangian given by

$$\mathcal{L}^{u,d}_{\mathrm{mass}} = -\sum_{a,b}\bar{u}_{aL}\mathcal{M}^U_{ab}u_{bR} - \sum_{a,b}\bar{d}_{aL}\mathcal{M}^D_{ab}d_{bR} + \mathrm{H.c.}, \quad (80)$$

where

$$\mathcal{M}^U = \{\mathcal{M}^U_{ab}\} = -\frac{1}{\sqrt{2}}\begin{pmatrix} h^q_{11}v & h^q_{12}v & h^q_{13}u \\ h^q_{21}v & h^q_{22}v & h^q_{23}u \\ h^q_{31}v & h^q_{32}v & h^q_{33}u \end{pmatrix}, \quad (81)$$

$$\mathcal{M}^D = \{\mathcal{M}^D_{ab}\} = -\frac{1}{\sqrt{2}}\begin{pmatrix} h^q_{11}u & h^q_{12}u & h^q_{13}v \\ h^q_{21}u & h^q_{22}u & h^q_{23}v \\ h^q_{31}u & h^q_{32}u & h^q_{33}v \end{pmatrix}, \quad (82)$$

which are generally complex-valued matrices and correlated due to $u \neq v$.

By applying biunitary transformations, we can diagonalize the mass matrices, $\mathcal{M}^U$ and $\mathcal{M}^D$, separately, such that

$$V^\dagger_{dL}\mathcal{M}^D V_{dR} = M^D, \qquad V^\dagger_{uL}\mathcal{M}^U V_{uR} = M^U, \quad (83)$$

where M^U, M^D are diagonal matrices and $V_{uL,R}, V_{dL,R}$ are unitary matrices. The mass eigenstates and gauge states are related by

$$d_{L,R} = V_{dL,R}d'_{L,R}, \qquad u_{L,R} = V_{uL,R}u'_{L,R}, \quad (84)$$

where we use the notations, the gauge states for up-quarks $u = (u_1, u_2, u_3)^T$, for down-quarks $d = (d_1, d_2, d_3)^T$, and the mass eigenstates $u' = (u, c, t)^T$, $d' = (d, s, b)^T$. The CKM matrix is defined as $V_{\mathrm{CKM}} = V^\dagger_{uL}V_{dL}$. Note also that although the up and down quark mass matrices differ by only a relation, $u \neq v$, the realistic masses for the quarks can be achieved by choosing appropriate parameters. Even if h^q_{ab} is flavor diagonal, we need only $u \gg v$ and $h^q_{33} \gg h^q_{11,22}$. In this case, there are only two unsuitably small masses corresponding to u, c as well as the small quark mixing angles, which can be radiatively induced.

We would like to emphasize that two of the three right-handed quark multiplets transform differently from the remainder under $SU(3)_R$. This causes the FCNCs at the tree level for the ordinary quarks due to the following:

(i) *Nonuniversal gauge (Z'_R) couplings*: The flavors of ordinary quarks such as $\{u_a\}$ and $\{d_a\}$ differ in T_{8R} as well as X charges (note that all the lepton flavors $\{\nu_a\}$, $\{e_a\}$, $\{E_a\}$ and exotic quark flavors $\{J_\alpha\}$ do not have this property since the corresponding left or right flavors in each group are identical under every neutral gauge charge; also, there is no flavor changing associated with Q, $T_{3L,R}$ since each of them couples universally to every left or right flavor group, as mentioned previously). Since X is related to T_{8R}, the FCNCs are mediated only by the extra neutral gauge boson, Z'_R, which couples to T_{8R}.

(ii) *Nonuniversal Higgs (H_2) couplings*: Although the Higgs doublets are unified in S, the FCNCs associated with the ordinary quarks arise due to the nonuniversal arrangement of quark generations under the gauge symmetry. This can be seen from the Yukawa interactions for S and quarks. Similarly to the previous case, there is no flavor changing associated with the other fermions, as well as other neutral scalars. A combination of S_{11} and S_{22} is just the standard model Higgs boson, H_1, which conserves flavors since its Yuakwa couplings are proportional to the corresponding quark mass matrices. However, the new Higgs state, H_2, which is directly orthogonal to H_1, changes flavors.

First, let us consider the FCNCs induced from quark and scalar interactions. The same Yukawa terms in (12) that yield the quark masses also bring FCNCs into the up and down quark sectors,

$$\begin{aligned}\mathcal{L}_{\text{int}}^{u,d} &= h^q_{a3}\bar{d}_{aL}S^0_{22}d_{3R} + h^q_{a\beta}\bar{d}_{aL}S^0_{11}d_{\beta R} + h^q_{a3}\bar{u}_{aL}S^0_{11}u_{3R} + h^q_{a\beta}\bar{u}_{aL}S^0_{22}u_{\beta R} + \text{H.c.}\\ &= h^q_{a3}\bar{d}_{aL}\frac{uH_2+vH_1}{\sqrt{2(u^2+v^2)}}d_{3R} + h^q_{a\beta}\bar{d}_{aL}\frac{uH_1-vH_2}{\sqrt{2(u^2+v^2)}}d_{\beta R} + h^q_{a3}\bar{u}_{aL}\frac{uH_1-vH_2}{\sqrt{2(u^2+v^2)}}u_{3R} + h^q_{a\beta}\bar{u}_{aL}\frac{uH_2+vH_1}{\sqrt{2(u^2+v^2)}}u_{\beta R} + \text{H.c.}\\ &= -\bar{d}'_L\frac{M^D}{\sqrt{(u^2+v^2)}}d'_R H_1 + \frac{v}{u}\bar{d}'_L\frac{M^D}{\sqrt{(u^2+v^2)}}d'_R H_2 - \bar{u}'_L\frac{M^U}{\sqrt{(u^2+v^2)}}u'_R H_1 - \frac{u}{v}\bar{u}'_L\frac{M^U}{\sqrt{(u^2+v^2)}}u'_R H_2\\ &\quad -\frac{\sqrt{u^2+v^2}}{u^2}\bar{d}'_{iL}(V^\dagger_{dL}V_{uL})_{ik}(M^U)_{km}(V^*_{uR})_{3m}(V_{dR})_{3j}d'_{jR}H_2 + \frac{\sqrt{u^2+v^2}}{v^2}\bar{u}'_{iL}(V^\dagger_{uL}V_{dL})_{ik}(M^D)_{km}(V^*_{dR})_{3m}(V_{uR})_{3j}u'_{jR}H_2\\ &\quad + \text{H.c.}\end{aligned} \tag{85}$$

We see that the Higgs boson, H_1, couples to quarks, even charged leptons, similarly to the Higgs couplings in the standard model, which is a feature validating this model [21]. H_2 is a new heavy Higgs boson, which changes quark flavors, as desirable, presented by the nonzero off-diagonal elements ($i \neq j$) in the last two terms of (85). Therefore, the tree-level FCNC processes might appear due to the contribution of H_2 as mediators. Conventionally, we rewrite the relevant couplings as follows:

$$\mathcal{L}^{H_2}_{\text{FCNC}} = \bar{d}'_{iL}\Gamma^d_{ij}d'_{jR}H_2 + \bar{u}'_{iL}\Gamma^u_{ij}u'_{jR}H_2 + \text{H.c.}, \tag{86}$$

where

$$\begin{aligned}\Gamma^d_{ij} &= -\frac{\sqrt{u^2+v^2}}{u^2}(V^\dagger_{dL}V_{uL})_{ik}(M^U)_{km}(V^*_{uR})_{3m}(V_{dR})_{3j},\\ \Gamma^u_{ij} &= \frac{\sqrt{u^2+v^2}}{v^2}(V^\dagger_{uL}V_{dL})_{ik}(M^D)_{km}(V^*_{dR})_{3m}(V_{uR})_{3j}.\end{aligned} \tag{87}$$

Second, we consider the FCNCs due to the fermion and gauge boson interactions. As mentioned, the FCNCs associated with Z'_R are due to the third generation of quarks that transforms differently from the first two under the gauge symmetry. Here, the FCNCs occur in the right-handed quark sector and with the gauge bosons, A_{8R} and B, which couple to T_{8R} and X, respectively. Since $X = Q - T_{3L} - T_{3R} - \beta T_{8R}$, the source for the FCNCs is only T_{8R}. Indeed, considering the interacting Lagrangian of neutral gauge bosons with fermions and using the expression for X, we come to the relevant interaction,

$$\begin{aligned}\mathcal{L}_8 &= -\sum_{a=1}^{3}\bar{Q}_{aR}\gamma^\mu T_{8R}Q_{aR}(g_R A_{8R\mu} - \beta g_X B_\mu)\\ &= -g_L\sqrt{t_R^2+\beta^2 t_X^2}\sum_{a=1}^{3}\bar{Q}_{aR}\gamma^\mu T_{8R}Q_{aR}Z'_{R\mu}\\ &\supset -g_L\sqrt{t_R^2+\beta^2 t_X^2}(\bar{u}_R\gamma^\mu T_u u_R + \bar{d}_R\gamma^\mu T_d d_R)Z'_{R\mu}\\ &= -g_L\sqrt{t_R^2+\beta^2 t_X^2}(\bar{u}'_R\gamma^\mu(V^\dagger_{uR}T_u V_{uR})u'_R\\ &\quad + \bar{d}'_R\gamma^\mu(V^\dagger_{dR}T_d V_{dR})d'_R)Z'_{R\mu},\end{aligned} \tag{88}$$

where $T_u = T_d = \frac{1}{2\sqrt{3}}\text{diag}(-1,-1,1)$ includes T_{8R} values of up or down quark flavors. The tree-level FCNC associated with the field Z'_R is obtained by

$$\mathcal{L}^{Z'_R}_{\text{FCNC}} = -\Theta^{Z'_R}_{ij}\bar{q}'_{iR}\gamma^\mu q'_{jR}Z'_{R\mu} \tag{89}$$

with $i \neq j$, where q' is denoted as either u' or d', and $\Theta^{Z'_R}_{ij}$ is defined as

$$\Theta^{Z'_R}_{ij} = \frac{g_L}{\sqrt{3}}\sqrt{t_R^2+\beta^2 t_X^2}(V^*_{qR})_{3i}(V_{qR})_{3j}. \tag{90}$$

In the following, we will calculate the contribution of the new physics to the meson mixing systems as mediated by the neutral scalar H_2 and neutral gauge boson Z'_R. For the case of the $K^0 - \bar{K}^0$ mixing, the relevant effective Lagrangian is given after integrating out H_2 and Z'_R,

$$\mathcal{L}^{\Delta S=2}_{\text{effective}} = -\frac{(\Theta^{Z'_R}_{12})^2}{m^2_{Z'_R}}(\bar{d}_R\gamma^\mu s_R)^2 + \frac{(\Gamma^d_{12})^2}{m^2_{H_2}}(\bar{d}_L s_R)^2 + \frac{(\Gamma^{d*}_{21})^2}{m^2_{H_2}}(\bar{d}_R s_L)^2 + \frac{\Gamma^{d*}_{21}\Gamma^d_{12}}{m^2_{H_2}}(\bar{d}_L s_R)(\bar{d}_R s_L) + \frac{\Gamma^{d*}_{21}\Gamma^d_{12}}{m^2_{H_2}}(\bar{d}_R s_L)(\bar{d}_L s_R). \tag{91}$$

This yields the contribution to the $K^0 - \bar{K}^0$ mixing parameter or mass difference Δm_K as

$$\Delta m_K = 2\text{Re}\langle \bar{K}^0| - \mathcal{L}^{\Delta S=2}_{\text{eff}}|K^0\rangle. \tag{92}$$

Using the matrix elements [22]

$$\langle \bar{K}^0|(\bar{d}_R\gamma^\mu s_R)^2|K^0\rangle = \langle \bar{K}^0|(\bar{d}_L\gamma^\mu s_L)^2|K^0\rangle = \frac{1}{3}m_K f_K^2,$$
$$\langle \bar{K}^0|(\bar{d}_L s_R)^2|K^0\rangle = \langle \bar{K}^0|(\bar{d}_R s_L)^2|K^0\rangle = -\frac{5}{24}\left(\frac{m_K}{m_s+m_d}\right)^2 m_K f_K^2,$$
$$\langle \bar{K}^0|(\bar{d}_L s_R)(\bar{d}_R s_L)|K^0\rangle = \langle \bar{K}^0|(\bar{d}_R s_L)(\bar{d}_L s_R)|K^0\rangle = \left[\frac{1}{24}+\frac{1}{4}\left(\frac{m_K}{m_s+m_d}\right)^2\right] m_K f_K^2,$$

the $K^0 - \bar{K}^0$ mixing parameter Δm_K is obtained by

$$\Delta m_K = \text{Re}\left\{\frac{2}{3}\frac{(\Theta^{Z'_R}_{12})^2}{m^2_{Z'_R}} + \frac{5}{12}\left(\frac{(\Gamma^{d*}_{21})^2}{m^2_{H_2}} + \frac{(\Gamma^d_{12})^2}{m^2_{H_2}}\right)\left(\frac{m_K}{m_s+m_d}\right)^2 - \frac{\Gamma^{d*}_{21}\Gamma^d_{12}}{m^2_{H_2}}\left[\frac{1}{6}+\left(\frac{m_K}{m_s+m_d}\right)^2\right]\right\} m_K f_K^2. \tag{93}$$

Similarly, we obtain $B^0_{d,s} - \bar{B}^0_{d,s}$ mixing parameters, Δm_{B_d} and Δm_{B_s}, as

$$\Delta m_{B_d} = \text{Re}\left\{\frac{2}{3}\frac{(\Theta^{Z'_R}_{13})^2}{m^2_{Z'_R}} + \frac{5}{12}\left(\frac{(\Gamma^{d*}_{31})^2}{m^2_{H_2}} + \frac{(\Gamma^d_{13})^2}{m^2_{H_2}}\right)\left(\frac{m_{B_d}}{m_b+m_d}\right)^2 - \frac{\Gamma^{d*}_{31}\Gamma^d_{13}}{m^2_{H_2}}\left[\frac{1}{6}+\left(\frac{m_{B_d}}{m_b+m_d}\right)^2\right]\right\} m_{B_d} f_{B_d}^2, \tag{94}$$

$$\Delta m_{B_s} = \text{Re}\left\{\frac{2}{3}\frac{(\Theta^{Z'_R}_{23})^2}{m^2_{Z'_R}} + \frac{5}{12}\left(\frac{(\Gamma^{d*}_{32})^2}{m^2_{H_2}} + \frac{(\Gamma^d_{23})^2}{m^2_{H_2}}\right)\left(\frac{m_{B_s}}{m_b+m_s}\right)^2 - \frac{\Gamma^{d*}_{32}\Gamma^d_{23}}{m^2_{H_2}}\left[\frac{1}{6}+\left(\frac{m_{B_s}}{m_b+m_s}\right)^2\right]\right\} m_{B_s} f_{B_s}^2. \tag{95}$$

Let us numerically study the mixing parameters, Δm_K and $\Delta m_{B_{d,s}}$, by using the following input parameters (mass parameters are measured in MeV) [1,23,24]:

$$m_d = 4.73,\quad m_s = 93.4,\quad m_b = 4190,\quad m_t = 173\times 10^3,\quad f_K = 156.1,$$
$$m_K = 497.614,\quad f_{B_d} = 188,\quad m_{B_d} = 5279.5,\quad f_{B_s} = 225,\quad m_{B_s} = 5366.3,$$
$$(V_{\text{CKM}})_{31} = 0.00886,\quad (V_{\text{CKM}})_{32} = 0.0405,\quad (V_{\text{CKM}})_{33} = 0.99914. \tag{96}$$

Referring to the above results for the weak scales, we take $u = 230$ GeV, and thus v is followed from $u^2 + v^2 = (246\ \text{GeV})^2$. Also, $t_R = 1$, $t_X = s_W/\sqrt{1-(2+\beta^2)s_W^2}$, and $s_W^2 = 0.231$ as given before are used. For the above β values (i.e., $\beta = 0, \pm 1/\sqrt{3}$), t_X and $\Theta^{Z'_R}_{ij}$ slightly change. So, we can take $|\beta| = 1/\sqrt{3}$ for further calculations. We have $g_L = \sqrt{4\pi\alpha/s_W^2}$, with $\alpha = 1/128$. For the right-handed quark mixing matrices, $V_{qR}(q = u, d)$, the elements that enter the meson mass differences, $\Delta m_{K,B_d,B_s}$, are $(V_{uR})_{33}$, $(V_{dR})_{31}$, $(V_{dR})_{32}$, and $(V_{dR})_{33}$. Since $\Delta m_{K,B_d,B_s}$ depend symmetrically on $(V_{dR})_{31}$ and $(V_{dR})_{32}$, one can assume $(V_{dR})_{31} = (V_{dR})_{32} \equiv V_{dR}$ without loss of generality. Thus, $(V_{dR})^2_{33} = 1-2V^2_{dR}$ due to the unitarity. We also label $(V_{uR})_{33} \equiv V_{uR}$ for simplicity. As seen, the contributions of H_2 and Z'_R are compatible. So, let us simply take $m_{H_2} = m_{Z'_R} \equiv M$, which are commonly called the new-physics scale, as entering the flavor-changing processes.

The standard model contributions to the meson mass differences are given by [25]

$$(\Delta m_K)_{\rm SM} = 0.467\times 10^{-2}/ps,\quad (\Delta m_{B_d})_{\rm SM} = 0.528/ps,\quad (\Delta m_{B_s})_{\rm SM} = 18.3/ps, \tag{97}$$

whereas the experimental values are [25]

$$(\Delta m_K)_{\rm Exp} = 0.5292\times 10^{-2}/ps,\quad (\Delta m_{B_d})_{\rm Exp} = 0.5055/ps,\quad (\Delta m_{B_s})_{\rm Exp} = 17.757/ps. \tag{98}$$

Note that the meson mass differences of the considered model are given by

$$(\Delta m_{K,B_d,B_s})_{\rm tot} = (\Delta m_{K,B_d,B_s})_{\rm SM} + \Delta m_{K,B_d,B_s}, \tag{99}$$

where the last terms are due to the new-physics contributions, which have been obtained above. These total contributions will be compared with the experimental values. We require the theory to produce the data for the kaon mass difference within 30% due to the potential long-range uncertainties, whereas it is within 5% for the B-meson mass differences, namely,

$$0.37044\times 10^{-2}/ps < (\Delta m_K)_{\rm tot} < 0.68796\times 10^{-2}/ps, \tag{100}$$

$$0.480225/ps < (\Delta m_{B_d})_{\rm tot} < 0.530775/ps, \tag{101}$$

$$16.8692/ps < (\Delta m_{B_s})_{\rm tot} < 18.6449/ps. \tag{102}$$

In Fig. 4, we make contours for the mass differences, Δm_K, Δm_{B_d}, and Δm_{B_s}, as functions of the right-handed quark mixing matrix elements (V_{uR}, V_{dR}) for the new-physics scale $M = 5$ TeV and $M = 10$ TeV, respectively. The M values have been chosen consistently with the bounds previously given. The available region for Δm_K is the whole frame. The two separated regions are for Δm_{B_d}. A lower half region is for Δm_{B_s}. Hence, the available parameter space for $\Delta m_{K,B_d,B_s}$ is only the (darkest) region in the lower left corner of each panel. From the allowed regimes, we obtain constraints for the right-handed quark mixing matrix elements as $|V_{uR}| < 0.08$ and $|V_{dR}| < 0.0015$ for $M = 5$ TeV, while $|V_{uR}| < 0.2$ and $|V_{dR}| < 0.003$ are for $M = 10$ TeV.

Considering $V_{uR} = 0.05$, 0.1, and 0.15, we make contours for Δm_K, Δm_{B_d}, and Δm_{B_s} as functions of (M, V_{dR}) in Fig. 5, respectively. The viable parameter space is the (darkest) region bounded in the upper left corner of each panel. We obtain $M > 2.8$ TeV for $V_{uR} = 0.05$ (left panel), $M > 5.7$ TeV for $V_{uR} = 0.1$ (middle panel), and $M > 8.2$ TeV for $V_{uR} = 0.15$ (right panel). Thus the new-physics scale M is low when V_{uR} is low, and vice versa.

We see that the bounds for the H_2 and Z'_R masses are consistent with the new-physics scale given in the previous subsection.

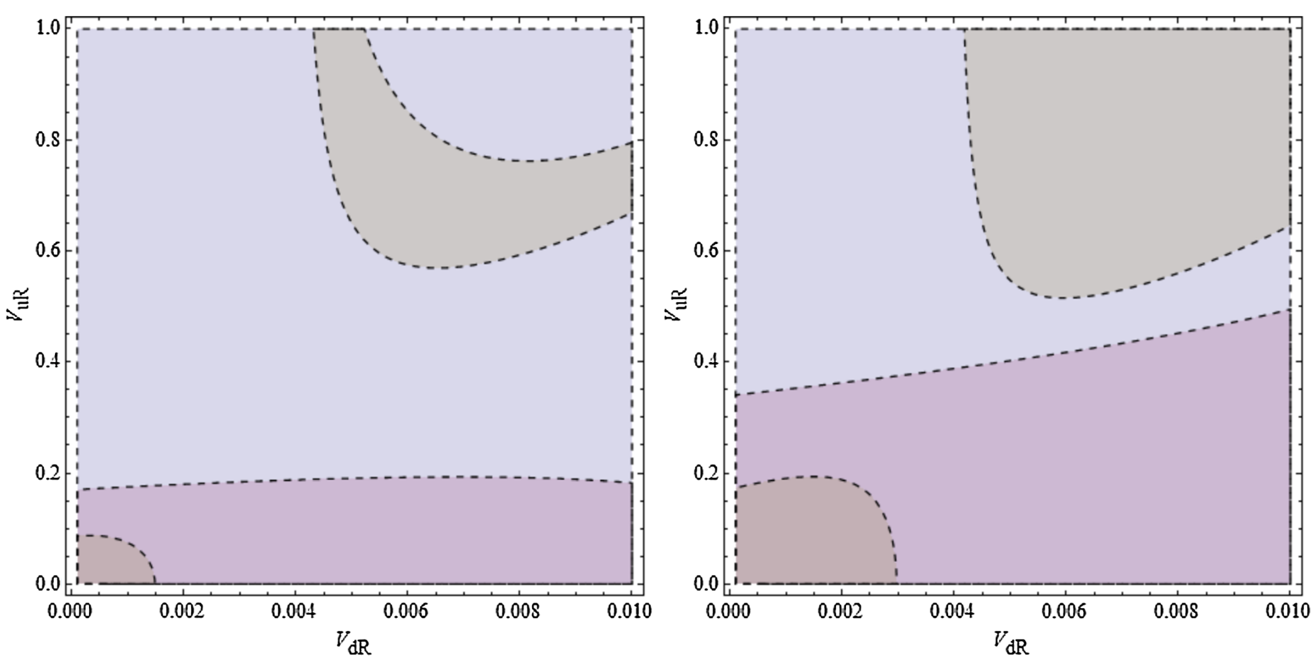

FIG. 4. The left panel presents constraints for (V_{uR}, V_{dR}) coming from the meson mass differences, $\Delta m_{K,B_d,B_s}$, with respect to the new-physics scale, $M = 5$ TeV, while the right panel is those for the new-physics scale, $M = 10$ TeV.

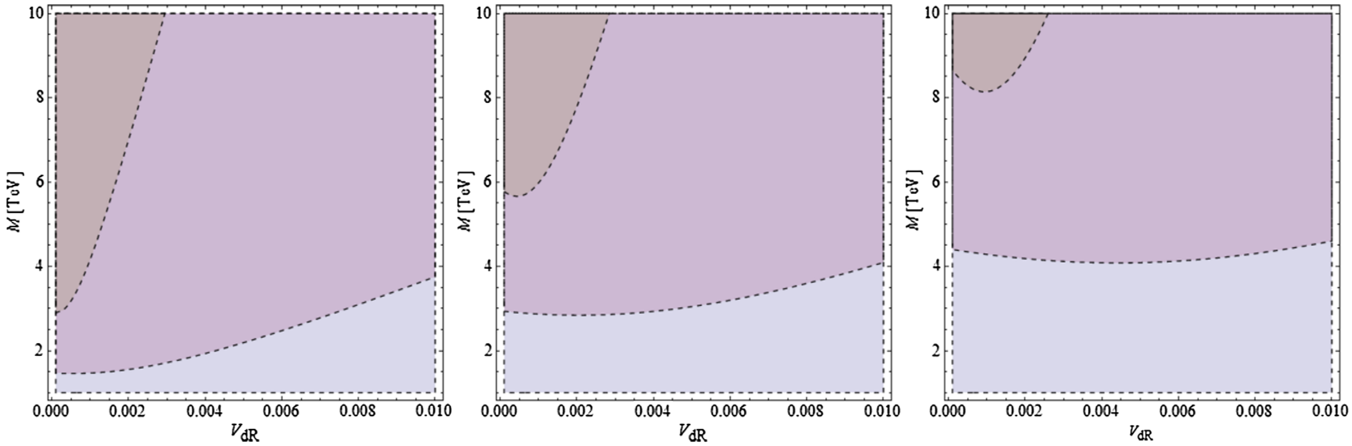

FIG. 5. The left, middle, right panels present bounds for (M, V_{dR}) coming from the meson mass differences, $\Delta m_{K,B_d,B_s}$, corresponding to $V_{uR} = 0.05$, 0.1, and 0.15, respectively.

VII. CONCLUSION

We have shown that the left-right asymmetric model with the $SU(3)_C \otimes SU(2)_L \otimes SU(3)_R \otimes U(1)_X$ gauge group naturally provides the new, tree-level FCNCs through both gauge and Yukawa interactions as a result of the nonuniversal fermion generations, which is different from the minimal left-right symmetric model. The new gauge symmetry contains automatically not only the right-handed neutrinos but also the wrong $B-L$ particles which induce the observed neutrino masses and dark matter candidates as a result of the gauge symmetry breaking. Particularly, the W-parity is naturally realized as a residual gauge symmetry, which is actually larger than Z_2 and stabilizing the dark matter. In other words, they all arise from the gauge principles.

The scalar sector has been explicitly diagonalized. The number of Goldstone bosons matches the number of the massive gauge bosons. There are 12 physical scalar fields, one of which is the standard model Higgs boson and the others are new and heavy. Because of the condition, $u, v \ll w, \Lambda$, the standard model Higgs boson gains a mass at the leading order in the electroweak scale, and it slightly mixes with the new neutral Higgs bosons. The gauge sector has been explicitly diagonalized too. The model contains five new heavy gauge bosons, $\mathcal{Z}_1, \mathcal{Z}'_1, W_2^\pm, X_R^{\pm q}, Y_R^{\pm(q+1)}$, besides the standard-model-like gauge bosons, $A, \mathcal{Z}, W_1^\pm$. The charged gauge bosons, $W_{L,R}^\pm$, mix via a small angle, $t_\xi \propto \frac{uv}{\Lambda^2}$. Also, the neutral gauge boson Z_L slightly mixes with the new neutral gauge bosons, Z_R and Z'_R, which are suppressed by $u, v \ll w, \Lambda$ too. In the $B_{d,s}$ and K mass differences, the model can have box diagrams due to the contributions of $W_2^\pm$, $H_5^\pm$, and even other new particles, in addition to the standard model boxes, but such contributions are more suppressed by $u, v \ll w, \Lambda, f$. Furthermore, the new FCNCs that come from the tree-level interactions with Z'_R and H_2 are larger than the mentioned ones by loop factors.

All the interactions of the gauge bosons with fermions and scalars have been derived. The standard model interactions are successfully recovered. The new interactions play important roles: they change quark flavors and set dark matter observables, besides others. We have concentrated on the first kind of interaction, as induced by Z'_R and H_2, and obtained their contributions to the neutral meson mass differences, $\Delta m_{K,B_d,B_s}$, which depend on the new particle masses and the elements of the right-handed quark mixing matrices. The mixing effects also modify the ρ-parameter, as well as the well-measured couplings of W, Z bosons, which are determined by the mixing parameters $\xi, \epsilon_1, \epsilon_2$. In agreement with electroweak precision measurements, the parameters ρ, ξ, and $\epsilon_{1,2}$ set the new-physics scale (assuming $w = \Lambda$) as $4.6\ \text{TeV} < \Lambda < 13.7\ \text{TeV}, 5.5\ \text{TeV} < \Lambda < 16.3\ \text{TeV}$, and $6.6\ \text{TeV} < \Lambda < 19.4\ \text{TeV}$, for $\beta = 1/\sqrt{3}, 0$, and $-1/\sqrt{3}$, respectively. They also set the narrow regimes for the weak scales such as $u > 222.3$, 215, and 210.4 GeV for those respective β values [the v scale is thus followed from $u^2 + v^2 = (246\ \text{GeV})^2$, and noting that $u, v < 246$ GeV]. The mass differences yield that when the new-physics masses are fixed, the right-handed quark mixing elements are constrained, such that $|V_{uR}| < 0.08$ and $|V_{dR}| < 0.0015$ for $M = 5$ TeV, while $|V_{uR}| < 0.2$ and $|V_{dR}| < 0.003$ for $M = 10$ TeV, assuming that $m_{Z'_R} = m_{H_2} \equiv M$, $(V_{dR})_{31} = (V_{dR})_{32} \equiv V_{dR}$, and $(V_{uR})_{33} \equiv V_{uR}$ for short. In the other case, fixing $V_{uR} = 0.05$, 0.1, and 0.15, we obtain $M > 2.8$ TeV, $M > 5.7$ TeV, and $M > 8.2$ TeV, respectively, where V_{dR} is free to float. The results yield that the new-physics scale is more sensitive to V_{uR}. The conclusion is that the two kinds of bounds are compatible, and the new physics scale should be in 5–10 TeV order.

This model may predict the quantization of charges such as the electric charge and $B-L$. It belongs to a class of the model that provides dark matter naturally without supersymmetry. All these are worth exploring and to be published elsewhere [17].

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APPENDIX: VECTOR AND AXIAL-VECTOR COUPLINGS

This section obtains all the couplings of fermions with the neutral gauge bosons Z, $\mathcal{Z}_1$, and $\mathcal{Z}'_1$ as displayed in Tables I, II, and III, respectively.

TABLE I. The couplings of Z with fermions.

f	$g_V^Z(f)$	$g_A^Z(f)$	f	$g_V^Z(f)$	$g_A^Z(f)$
ν_a	$\frac{1}{2}$	$\frac{1}{2}$	e_a	$-\frac{1}{2}+2s_W^2$	$-\frac{1}{2}$
E_a	$-2s_W^2 q$	0	u_a	$\frac{1}{2}-\frac{4}{3}s_W^2$	$\frac{1}{2}$
d_a	$-\frac{1}{2}+\frac{2}{3}s_W^2$	$-\frac{1}{2}$	J_α	$2s_W^2(q+\frac{1}{3})$	0
J_3	$-2s_W^2(q+\frac{2}{3})$	0	No data	No data	No data

TABLE II. The couplings of $\mathcal{Z}_1$ with fermions.

f	$g_V^{\mathcal{Z}_1}(f)$	$g_A^{\mathcal{Z}_1}(f)$
ν_a	$\frac{t_X[t_R^2+\beta t_X^2(2\sqrt{3}+\beta)]c_\epsilon c_W+\sqrt{3}[t_R^2+t_X^2(2+\beta^2)]s_\epsilon s_W}{2\sqrt{3}t_X\sqrt{t_R^2+t_X^2\beta^2}}$	$-\frac{\sqrt{t_R^2+t_X^2\beta^2}(t_Xc_\epsilon c_W+\sqrt{3}s_\epsilon s_W)}{2\sqrt{3}t_X}$
e_a	$\frac{t_X[t_R^2+\beta t_X^2(2\sqrt{3}+\beta)]c_\epsilon c_W-\sqrt{3}[t_R^2+t_X^2(\beta^2-2)]s_\epsilon s_W}{2\sqrt{3}t_X\sqrt{t_R^2+t_X^2\beta^2}}$	$-\frac{\sqrt{t_R^2+t_X^2\beta^2}(t_Xc_\epsilon c_W-\sqrt{3}s_\epsilon s_W)}{2\sqrt{3}t_X}$
E_a	$\frac{-(t_R^2+\beta^2t_X^2)c_\epsilon c_W-2\sqrt{3}qt_X(t_X\beta c_\epsilon c_W+s_\epsilon s_W)}{\sqrt{3}\sqrt{t_R^2+t_X^2\beta^2}}$	$\frac{\sqrt{t_R^2+t_X^2\beta^2}c_\epsilon c_W}{\sqrt{3}}$
u_α	$\frac{-t_X[\sqrt{3}t_R^2+\beta t_X^2(2+\sqrt{3}\beta)]c_\epsilon c_W+[3t_R^2+t_X^2(3\beta^2-2)]s_\epsilon s_W}{6t_X\sqrt{t_R^2+t_X^2\beta^2}}$	$\frac{\sqrt{t_R^2+t_X^2\beta^2}(t_Xc_\epsilon c_W-\sqrt{3}s_\epsilon s_W)}{2\sqrt{3}t_X}$
u_3	$\frac{t_X[\sqrt{3}t_R^2+\beta t_X^2(\sqrt{3}\beta-2)]c_\epsilon c_W+[3t_R^2+t_X^2(3\beta^2-2)]s_\epsilon s_W}{6t_X\sqrt{t_R^2+t_X^2\beta^2}}$	$-\frac{\sqrt{t_R^2+t_X^2\beta^2}(t_Xc_\epsilon c_W+\sqrt{3}s_\epsilon s_W)}{2\sqrt{3}t_X}$
d_α	$\frac{-t_X[\sqrt{3}t_R^2+\beta t_X^2(2+\sqrt{3}\beta)]c_\epsilon c_W-[3t_R^2+t_X^2(3\beta^2+2)]s_\epsilon s_W}{6t_X\sqrt{t_R^2+t_X^2\beta^2}}$	$\frac{\sqrt{t_R^2+t_X^2\beta^2}(t_Xc_\epsilon c_W+\sqrt{3}s_\epsilon s_W)}{2\sqrt{3}t_X}$
d_3	$\frac{t_X[\sqrt{3}t_R^2+\beta t_X^2(\sqrt{3}\beta-2)]c_\epsilon c_W-[3t_R^2+t_X^2(3\beta^2+2)]s_\epsilon s_W}{6t_X\sqrt{t_R^2+t_X^2\beta^2}}$	$-\frac{\sqrt{t_R^2+t_X^2\beta^2}(t_Xc_\epsilon c_W-\sqrt{3}s_\epsilon s_W)}{2\sqrt{3}t_X}$
J_α	$\frac{[\sqrt{3}t_R^2+\beta t_X^2(2+6q+\sqrt{3}\beta)]c_\epsilon c_W+2(1+3q)t_Xs_\epsilon s_W}{3\sqrt{t_R^2+t_X^2\beta^2}}$	$-\frac{\sqrt{t_R^2+t_X^2\beta^2}c_\epsilon c_W}{\sqrt{3}}$
J_3	$\frac{-\sqrt{3}(t_R^2+\beta^2t_X^2)c_\epsilon c_W-2(2+3q)t_X(t_X\beta c_\epsilon c_W+s_\epsilon s_W)}{3\sqrt{t_R^2+t_X^2\beta^2}}$	$\frac{\sqrt{t_R^2+t_X^2\beta^2}c_\epsilon c_W}{\sqrt{3}}$

TABLE III. The couplings of $\mathcal{Z}'_1$ with fermions.

f	$g_V^{\mathcal{Z}'_1}(f)$	$g_A^{\mathcal{Z}'_1}(f)$
ν_a	$\frac{t_X[t_R^2+\beta t_X^2(2\sqrt{3}+\beta)]s_\epsilon c_W-\sqrt{3}[t_R^2+t_X^2(2+\beta^2)]c_\epsilon s_W}{2\sqrt{3}t_X\sqrt{t_R^2+t_X^2\beta^2}}$	$-\frac{\sqrt{t_R^2+t_X^2\beta^2}(t_Xs_\epsilon c_W-\sqrt{3}c_\epsilon s_W)}{2\sqrt{3}t_X}$
e_a	$\frac{t_X[t_R^2+\beta t_X^2(2\sqrt{3}+\beta)]s_\epsilon c_W+\sqrt{3}[t_R^2+t_X^2(\beta^2-2)]c_\epsilon s_W}{2\sqrt{3}t_X\sqrt{t_R^2+t_X^2\beta^2}}$	$-\frac{\sqrt{t_R^2+t_X^2\beta^2}(t_Xs_\epsilon c_W+\sqrt{3}c_\epsilon s_W)}{2\sqrt{3}t_X}$
E_a	$\frac{-(t_R^2+\beta^2t_X^2)s_\epsilon c_W-2\sqrt{3}qt_X(t_X\beta s_\epsilon c_W-c_\epsilon s_W)}{\sqrt{3}\sqrt{t_R^2+t_X^2\beta^2}}$	$\frac{\sqrt{t_R^2+t_X^2\beta^2}s_\epsilon c_W}{\sqrt{3}}$
u_α	$\frac{-t_X[\sqrt{3}t_R^2+\beta t_X^2(2+\sqrt{3}\beta)]s_\epsilon c_W-[3t_R^2+t_X^2(3\beta^2-2)]c_\epsilon s_W}{6t_X\sqrt{t_R^2+t_X^2\beta^2}}$	$\frac{\sqrt{t_R^2+t_X^2\beta^2}(t_Xs_\epsilon c_W+\sqrt{3}c_\epsilon s_W)}{2\sqrt{3}t_X}$
u_3	$\frac{t_X[\sqrt{3}t_R^2+\beta t_X^2(\sqrt{3}\beta-2)]s_\epsilon c_W-[3t_R^2+t_X^2(3\beta^2-2)]c_\epsilon s_W}{6t_X\sqrt{t_R^2+t_X^2\beta^2}}$	$-\frac{\sqrt{t_R^2+t_X^2\beta^2}(t_Xs_\epsilon c_W-\sqrt{3}c_\epsilon s_W)}{2\sqrt{3}t_X}$
d_α	$\frac{-t_X[\sqrt{3}t_R^2+\beta t_X^2(2+\sqrt{3}\beta)]s_\epsilon c_W+[3t_R^2+t_X^2(3\beta^2+2)]c_\epsilon s_W}{6t_X\sqrt{t_R^2+t_X^2\beta^2}}$	$\frac{\sqrt{t_R^2+t_X^2\beta^2}(t_Xs_\epsilon c_W-\sqrt{3}c_\epsilon s_W)}{2\sqrt{3}t_X}$
d_3	$\frac{t_X[\sqrt{3}t_R^2+\beta t_X^2(\sqrt{3}\beta-2)]s_\epsilon c_W+[3t_R^2+t_X^2(3\beta^2+2)]c_\epsilon s_W}{6t_X\sqrt{t_R^2+t_X^2\beta^2}}$	$-\frac{\sqrt{t_R^2+t_X^2\beta^2}(t_Xs_\epsilon c_W+\sqrt{3}c_\epsilon s_W)}{2\sqrt{3}t_X}$
J_α	$\frac{[\sqrt{3}t_R^2+\beta t_X^2(2+6q+\sqrt{3}\beta)]s_\epsilon c_W-2(1+3q)t_Xc_\epsilon s_W}{3\sqrt{t_R^2+t_X^2\beta^2}}$	$-\frac{\sqrt{t_R^2+t_X^2\beta^2}s_\epsilon c_W}{\sqrt{3}}$
J_3	$\frac{-\sqrt{3}(t_R^2+\beta^2t_X^2)s_\epsilon c_W-2(2+3q)t_X(t_X\beta s_\epsilon c_W-c_\epsilon s_W)}{3\sqrt{t_R^2+t_X^2\beta^2}}$	$\frac{\sqrt{t_R^2+t_X^2\beta^2}s_\epsilon c_W}{\sqrt{3}}$

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