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GAUGE BOSONS IN THE 3-3-1 MODEL WITH THREE NEUTRINO SINGLETS

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Abstract. *We show that the mass matrix of electrically neutral gauge bosons in the recent proposed model based on* $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X$ *group with three neutrino singlets [9] has two exact eigenvalues and corresponding eigenvectors. Hence the neutral non-Hermitian gauge boson* X^0_μ *is properly determined. With extra vacuum expectation values of the Higgs fields, there are mixings among charged gauge bosons* $W^\pm$ *and* $Y^\pm$ *as well as among neutral gauge bosons* Z, Z' *and* X^0.

Keywords: neutrino mass, beyond the standard model.

I. INTRODUCTION

At present, it is well known that neutrinos are massive that contradicts the Standard Model (SM). The experimental data [1] show that masses of neutrinos are tiny small and neutrinos mix with special pattern in approximately tribimaximal form [2]. The neutrino masses, dark matter and the baryon asymmetry of Universe (BAU) are the facts requiring extension of the SM.

Among the extensions beyond the SM, the models based on $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X$ gauge group (called 3-3-1 for short) [3, 4] have some interesting features including the ability to explain the generation problem [3, 4] and the electric charge quantization [5]. Concerning the content in lepton triplet, the exist two main versions of 3-3-1 models: the minimal version [3] without extra lepton and the model with right-handed neutrinos [4] without exotic charged particles. Due to the fact that particles with different lepton numbers lie in the same triplet, the lepton number is violated and it is better to deal with a new conserved charge $\mathscr{L}$ commuting with the gauge symmetry [6]

$$L = \frac{4}{\sqrt{3}} T_8 + \mathscr{L}\,. \tag{1}$$

In the framework of the 3-3-1 models, almost issues concerning neutrino physics are solvable. In the framework of the minimal 3-3-1 model where perturbative regime is trustable until 4-5 TeV, to realize idea of seesaw, the effective dimension-5 operator is used [7]. In regard to the 3-3-1 model with right-handed neutrinos, effective-5 operators are sufficient to generate light neutrino masses. The effective dimension-5 operator may be realized through a kind of type-II seesaw mechanism implemented by a sextet of scalars belonging to the GUT scale [8]. There are two ways to explain smallness of neutrino masses: the radiative mechanism and the seesaw one. The seesaw mechanism is the most easy and elegant way of generating small neutrino masses by using the Majorana neutrinos with mass belonging to GUT scale. With such high scale, the Majorana neutrinos are unavailable for laboratory searches. There are attempts to improve the situation.

In the recently proposed model [9], the authors have introduced three neutrino singlets and used radiative mechanism to get model, where the seesaw mechanism is realized at quite low scale of few TeV scale. We remind that in the 3-3-1 model with right-handed neutrinos, there are two scalar triplets η,χ containing two neutral components lying at top and bottom of triplets: η_1^0,χ_1^0 and η_3^0,χ_3^0. In the previous version [4], only η_1^0 and χ_3^0 have VEVs, however, in new version, the η_3^0 carrying lepton number 2 has larger VEV of new physics scale. This leads to the mixing in both charged and neutral gauge bosons sectors. In the neutral gauge boson sector, the mass mixing matrix is 4×4. In general, the diagonalizing process for 4×4 matrix is approximate only. However, in this paper, we show that two exact eigenvalues and eigenstates, consequently the diagonalization is exact!

In this paper we study in details the above mentioned model. In Sect. II, we briefly give particle content of the model.

II. THE MODEL

As usual [4], the left-handed leptons are assigned to the triplet representation of $SU(3)_L$

$$f_L^\ell = \left(\nu_\ell\,,\ell^-\,,N_\ell^c\right)_L^T \sim \left(1,3,-\frac{1}{3},\frac{1}{3}\right),\ell_R \sim (1,1,-1,1) \tag{2}$$

where $\ell = 1,2,3 \equiv e,\,\mu,\,\tau$. The numbers in bracket are assignment in $SU(3)_C, SU(3)_L, U(1)_X$ and $\mathscr{L}$.

The third quark generation is in triplet

$$\begin{aligned} Q_L^3 &= (t\,,b\,,T)_L^T \sim \left(3,3,\frac{1}{3},-\frac{2}{3}\right),\, T_R \sim \left(3,1,\frac{2}{3},-2\right), \\ t_R &\sim \left(3,1,\frac{2}{3},0\right),\, b_R \sim \left(3,1,-\frac{1}{3},0\right) \end{aligned} \tag{3}$$

Two first quark generations are in anti triplet

$$\begin{aligned} Q_L^i &= (d_i\,,-u_i\,,D_i)_L^T \sim \left(3,3^*,0,-\frac{2}{3}\right),i=1,2, \\ D_{iR} &\sim \left(3,1,-\frac{1}{3},2\right),\, u_{iR} \sim \left(3,1,\frac{2}{3},0\right),\, d_{iR} \sim \left(3,1,-\frac{1}{3},0\right) \end{aligned} \tag{4}$$

In addition to the new two-component neutral fermions present in the lepton triplet $N_L^c \equiv (N^c)_L \equiv (\nu_R)^c$ where $\psi^c = -C\overline{\psi}^T$ and C is the charge conjugation matrix, ones introduce new sequential lepton-number-carrying gauge singlets $S = \{S_1, S_2, S_3\}$ with the following number [9]

$$S_i \sim (1, 1, 0, -1).$$

With the above $\mathscr{L}$ assignment the electric charge operator is given in terms of the $U(1)_X$ generator X and the diagonal generators of the $SU(3)_L$ as

$$Q = T_3 - \frac{1}{\sqrt{3}} T_8 + X\,. \tag{5}$$

In order to spontaneously break the weak gauge symmetry, we introduce three scalar triplets

$$\chi = \left(\chi'^0, \chi^-, \chi^0\right)^T \sim \left(1, 3, -\frac{1}{3}, \frac{4}{3}\right); \langle\chi\rangle = (0\,, 0\,, n_1)^T \tag{6}$$

$$\eta = \left(\eta'^0, \eta^-, \eta^0\right)^T \sim \left(1, 3, -\frac{1}{3}, -\frac{2}{3}\right); \langle\eta\rangle = (k_2\,, 0\,, n_2)^T \tag{7}$$

$$\rho = \left(\rho^+, \rho^0, \rho'^+\right)^T \sim \left(1, 3, \frac{2}{3}, -\frac{2}{3}\right); \langle\rho\rangle = (0\,, k_1\,, 0)^T\,. \tag{8}$$

Note that the third component of η carries two units of lepton number, and the k_1 and k_2 VEVs are at the electroweak scale and correspond to the VEV of the $SU(2)_L \subset SU(3)_L$ doublets. The VEVs n_1 and n_2 are isosinglet VEVs that characterize the $SU(3)_L$ breaking scale. Note that while η takes VEV in both electrically neutral directions, the second VEV of χ is neglected, so that lepton number is broken only by $SU(2)_L$ singlets. This pattern gives the simplest consistent neutrino mass spectrum, avoiding the linear seesaw contribution. As we discuss below, this structure of VEVs leads to mixing not only among charged gauge bosons, but also among electrically neutral gauge bosons.

The spontaneous symmetry breaking follows the pattern

$$SU(3)_L \otimes U(1)_X \xrightarrow{n_{1,2}} SU(2)_L \otimes U(1)_Y \xrightarrow{k_{1,2}} U(1)_Q.$$

III. GAUGE BOSON SECTOR

The kinetic term for the scalar fields is

$$\mathscr{L}_{\text{Kin}} = \sum_{H=\chi,\eta,\rho} (D^\mu H)^\dagger (D_\mu H)\,, \tag{9}$$

where covariant derivative is defined as

$$D_\mu = \partial_\mu - igA_{\mu a}T_a - ig'XB_\mu T_9, \tag{10}$$

here X is the $U(1)_X$ charge of the field, $A_{\mu a}$ and B_μ are the gauge bosons of $SU(3)_L$ and $U(1)_X$, respectively. The above equation applies for triplet: $T_a \to \lambda_a/2\,, T_9 \to \lambda_9/2$ where λ_i are the Gell-Mann matrices, and $\lambda_9 = \sqrt{\frac{2}{3}}\,\text{diag}\,(1,1,1)$. The matrix $\mathbf{A}_\mu \equiv \sum_a A^a_\mu \lambda_a$ is

$$\mathbf{A}^\mu = \begin{pmatrix} A^\mu_3 + \frac{1}{\sqrt{3}}A^\mu_8 & \sqrt{2}W^{\mu+}_{12} & A^\mu_4 - iA^\mu_5 \\ \sqrt{2}W^{\mu-}_{12} & -A^\mu_3 + \frac{1}{\sqrt{3}}A^\mu_8 & \sqrt{2}W^{\mu-}_{67} \\ A^\mu_4 + iA^\mu_5 & \sqrt{2}W^{\mu+}_{67} & -\frac{2}{\sqrt{3}}A^\mu_8 \end{pmatrix}.$$

The charged states are defined as

$$W_{12}^{\mu\pm} = \frac{1}{\sqrt{2}}(A_1^\mu \mp iA_2^\mu), \quad W_{67}^{\mu\pm} = \frac{1}{\sqrt{2}}(A_6^\mu \pm iA_7^\mu). \tag{11}$$

The mass Lagrangian for gauge fields is given by

$$\mathscr{L}_{\text{mass}} = \sum_{H=\chi,\eta,\rho} (D^\mu \langle H \rangle)^\dagger (D_\mu \langle H \rangle). \tag{12}$$

In charged gauge boson sector, the mass Lagrangian in (12) gives one decoupled A_5^μ with mass

$$m_{A_5}^2 = \frac{g^2}{4}(n_1^2 + n_2^2 + k_2^2), \tag{13}$$

and two others with the mass matrix given in the basis of (W_{12}^μ, W_{67}^μ) as

$$M_{charged} = \frac{g^2}{2} \begin{pmatrix} k_1^2 + k_2^2 & n_2 k_2 \\ n_2 k_2 & n_1^2 + n_2^2 + k_1^2 \end{pmatrix}. \tag{14}$$

The matrix in Eq. (14) has two eigenvalues

$$\lambda_{1,2} - k_1^2 = \frac{1}{2}\left(n_1^2 + n_2^2 + k_2^2 \pm \sqrt{\Delta}\right), \tag{15}$$

where

$$\Delta = (n_1^2 + n_2^2 - k_2^2)^2 + 4n_2^2 k_2^2. \tag{16}$$

In the limit $n_1 \sim n_2 \gg k_1 \sim k_2$, one has

$$\begin{aligned} \sqrt{\Delta} &\simeq (n_1^2 + n_2^2)\left[1 - 2\frac{k_2^2}{(n_1^2 + n_2^2)} + \frac{4n_2^2 k_2^2}{(n_1^2 + n_2^2)^2}\right]^{\frac{1}{2}} \\ &= n_1^2 + n_2^2 + k_2^2 - \frac{2n_1^2 k_2^2}{(n_1^2 + n_2^2)}. \end{aligned} \tag{17}$$

We will identify the light eigenvalue with square mass of the SM W boson, while the heavy one with that of the new charged gauge boson Y:

$$\begin{aligned} m_W^2 &= \frac{g^2}{2}\lambda_1 \simeq \frac{g^2}{2}\left[k_1^2 + \frac{n_1^2 k_2^2}{(n_1^1 + n_2^2)}\right] \\ &\simeq \frac{g^2}{2}\left(k_1^2 + \frac{k_2^2}{2}\right), \end{aligned} \tag{18}$$

$$\begin{aligned} m_Y^2 &= \frac{g^2}{2}\lambda_2 \simeq \frac{g^2}{2}\left[n_1^2 + n_2^2 + k_1^2 + k_2^2 - \frac{n_1^2 k_2^2}{(n_1^1 + n_2^2)}\right] \\ &\simeq \frac{g^2}{2}(n_1^2 + n_2^2). \end{aligned} \tag{19}$$

Note that in the limit $n_1 \sim n_2 \gg k_1 \sim k_2$, our result is consistent with that in [9].

Two physical bosons are determined as [10]

$$\begin{aligned} W_\mu^- &= \cos\theta\, W_{\mu 12}^- - \sin\theta\, W_{\mu 67}^-, \\ Y_\mu^- &= \sin\theta\, W_{\mu 12}^- + \cos\theta\, W_{\mu 67}^-, \end{aligned} \tag{20}$$

where the $W-Y$ mixing angle θ charaterizing lepton number violation is given by

$$\tan 2\theta \equiv \varepsilon \sim \frac{2n_2k_2}{n_1^2+n_2^2-k_2^2}. \tag{21}$$

Now we turn to the electrically neutral gauge boson sector. Five neutral fields, namely $A_3^\mu, A_8^\mu, B^\mu, A_4^\mu$ mix

$$M^2 = \frac{g^2}{4}\begin{pmatrix} k_1^2+k_2^2 & \frac{1}{\sqrt{3}}(k_2^2-k_1^2) & -t\sqrt{\frac{2}{27}}(k_2^2+2k_1^2) & n_2k_2 \\ & \frac{1}{3}[4(n_1^2+n_2^2)+(k_1^2+k_2^2)] & M_{23} & -\frac{1}{\sqrt{3}}n_2k_2 \\ & & M_{33} & -2t\sqrt{\frac{2}{27}}n_2k_2 \\ & & & n_1^2+n_2^2+k_2^2 \end{pmatrix}, \tag{22}$$

where $M_{23} \equiv \frac{\sqrt{2}}{9}t[2(n_1^2+n_2^2)+2k_1^2-k_2^2]$, $M_{33} \equiv \frac{2t^2}{27}(n_1^2+n_2^2+4k_1^2+k_2^2)$ and t is given (see the last paper in Ref. [4])

$$t = \frac{g'}{g} = \frac{3\sqrt{2}\sin\theta_W(m'_Z)}{\sqrt{3-4\sin^2\theta_W(m'_Z)}}.$$

The matrix (22) has one massless state

$$A_\mu = \frac{1}{\sqrt{18+4t^2}}\left(\sqrt{3}tA_{3\mu} - tA_{8\mu} + 3\sqrt{2}B_\mu\right), \tag{23}$$

which is identified to the photon. The second eigenvalue of (22) is defined

$$m^2_{A'_4} = \frac{g^2}{2}(n_1^2+n_2^2+k_2^2), \tag{24}$$

with eigenstate

$$\begin{aligned} A'_{4\mu} &= \frac{n_2k_2}{n_1^2+n_2^2-k_2^2}A_{3\mu} + \frac{\sqrt{3}n_2k_2}{n_1^2+n_2^2-k_2^2}A_{8\mu} + A_{4\mu} \\ &= \frac{t_{2\theta}}{2\sqrt{1+4t_{2\theta}^2}}A_{3\mu} + \frac{\sqrt{3}t_{2\theta}}{2\sqrt{1+4t_{2\theta}^2}}A_{8\mu} + \frac{1}{\sqrt{1+4t_{2\theta}^2}}A_{4\mu}, \end{aligned} \tag{25}$$

where $t_{2\theta} \equiv \tan 2\theta$ is determined *the same* as in (21).

Comparing (13) with (24) we see that two components of W_{45} have, as expected, the same mass. Hence we can identify

$$X_\mu^0 = \frac{1}{\sqrt{2}}(A'_{4\mu} - iA_{5\mu}) \tag{26}$$

as physical electrically neutral non-Hermitian gauge boson. It is easy to see that this gauge boson X_μ^0 carries lepton number two, hence it is called bilepton gauge boson.

With this exact eigenvalues, we return to the diagonalization of the matrix in (22). It is difficult to find two states which are orthogonal simultaneously to two states A_μ and $A_{4\mu}$. Thus we follow the method in [10]. The diagonalization separates into three steps. Firstly, let us take

$$\begin{aligned}
A_\mu &= s_W W_{3\mu} + c_W\left(-\frac{t_W}{\sqrt{3}}W_{8\mu} + \sqrt{1-\frac{t_W^2}{3}}B_\mu\right),\\
Z_\mu &= c_W W_{3\mu} - s_W\left(-\frac{t_W}{\sqrt{3}}W_{8\mu} + \sqrt{1-\frac{t_W^2}{3}}B_\mu\right),\\
Z'_\mu &= \sqrt{1-\frac{t_W^2}{3}}W_{8\mu} + \frac{t_W}{\sqrt{3}}B_\mu\\
A_{4\mu} &= A_{4\mu}.
\end{aligned} \tag{27}$$

or in matrix form

$$\begin{pmatrix} A_\mu \\ Z_\mu \\ Z'_\mu \\ A_{4\mu} \end{pmatrix} = \begin{pmatrix} s_w & -\frac{s_w}{\sqrt{3}} & \sqrt{\frac{3-4s_w^2}{3}} & 0 \\ c_w & \frac{s_w^2}{c_w\sqrt{3}} & -t_w\sqrt{\frac{3-4s_w^2}{3}} & 0 \\ 0 & \frac{1}{c_w}\sqrt{\frac{3-4s_w^2}{3}} & \frac{t_w}{\sqrt{3}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} A_{3\mu} \\ A_{8\mu} \\ B_\mu \\ A_{4\mu} \end{pmatrix}. \tag{28}$$

Then

$$\begin{pmatrix} A_{3\mu} \\ A_{8\mu} \\ B_\mu \\ A_{4\mu} \end{pmatrix} = S^T \begin{pmatrix} A_\mu \\ Z_\mu \\ Z'_\mu \\ A_{4\mu} \end{pmatrix}. \tag{29}$$

Let us denote

$$U_1 = S^T = \begin{pmatrix} s_w & c_w & 0 & 0 \\ -\frac{s_w}{\sqrt{3}} & \frac{s_w^2}{c_w\sqrt{3}} & \frac{1}{c_w}\sqrt{\frac{3-4s_w^2}{3}} & 0 \\ \sqrt{\frac{3-4s_w^2}{3}} & -t_w\sqrt{\frac{3-4s_w^2}{3}} & \frac{t_w}{\sqrt{3}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

then

$$\begin{aligned}
M'^2 = U_1^T M U_1 &= \frac{g^2}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{k_1^2+k_2^2}{c_w^2} & \frac{k_2^2(1-2s_w^2)-k_1^2}{c_w^2\sqrt{3-4s_w^2}} & \frac{n_2k_2}{c_w} \\ 0 & \frac{k_2^2(1-2s_w^2)-k_1^2}{c_w^2\sqrt{3-4s_w^2}} & -\frac{k_1^2+c_{2w}^2k_2^2+4c_W^4(n_1^2+n_2^2)}{c_w^2(3-4s_w^2)} & \frac{n_2k_2}{c_w\sqrt{3-4s_w^2}} \\ 0 & \frac{n_2k_2}{c_w} & \frac{n_2k_2}{c_w\sqrt{3-4s_w^2}} & n_1^2+n_2^2+k_2^2 \end{pmatrix}\\
&= \begin{pmatrix} 0 & 0 \\ 0 & \mathscr{M}33 \end{pmatrix},
\end{aligned} \tag{30}$$

where we have denoted

$$\mathscr{M}33 \equiv \begin{pmatrix} m^2_Z & m^2_{ZZ'} & m^2_{ZA_4} \\ m^2_{ZZ'} & m^2_{Z'} & m^2_{Z'A_4} \\ m^2_{ZA_4} & m^2_{Z'A'_4} & m^2_{A_4} \end{pmatrix}. \tag{31}$$

We turn to the second step. To see explicitly that the following basis is orthogonal and normalized, comparing with (25), let us put

$$s_{\theta'} \equiv \frac{t_{2\theta}}{c_w\sqrt{1+4t^2_{2\theta}}}, \tag{32}$$

which leads to

$$A'_{4\mu} = s_{\theta'} Z_\mu + c_{\theta'}\left[t_{\theta'}\sqrt{3-4s^2_w} Z'_\mu + \sqrt{1-t^2_{\theta'}(3-4s^2_w)} A_{4\mu}\right]. \tag{33}$$

Note that the mixing angle in this step θ' is the same order as the mixing angle in the charged gauge boson sector. Taking into account [1] $s^2_W \simeq 0.231$, from (32) we get $s_{\theta'} \simeq 2.28 s_\theta$. It is now easy to choose two remaining gauge vectors

$$\begin{aligned} \mathscr{Z}_\mu &= c_{\theta'} Z_\mu - s_{\theta'}\left[t_{\theta'}\sqrt{3-4s^2_w} Z'_\mu + \sqrt{1-t^2_{\theta'}(3-4s^2_w)} A_{4\mu}\right], \\ \mathscr{Z}'_\mu &= \sqrt{1-t^2_{\theta'}(3-4s^2_w)} Z'_\mu - t_{\theta'}\sqrt{3-4s^2_w} A_{4\mu}. \end{aligned} \tag{34}$$

Therefore, in the base of $(A_\mu, \mathscr{Z}_\mu, Z'_\mu, A'_{4\mu})$ the matrix is quasi-diagonal by matrix

$$\begin{pmatrix} A_\mu \\ \mathscr{Z}_\mu \\ \mathscr{Z}'_\mu \\ A'_{4\mu} \end{pmatrix} = S_2 \begin{pmatrix} A_\mu \\ Z_\mu \\ Z'_\mu \\ A'_{4\mu} \end{pmatrix}, \tag{35}$$

where

$$S_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\theta'} & -s_{\theta'} t_{\theta'}\sqrt{3-4s^2_w} & -s_{\theta'}\sqrt{1-t^2_{\theta'}(3-4s^2_w)} \\ 0 & 0 & \sqrt{1-t^2_{\theta'}(3-4s^2_w)} & -t_{\theta'}\sqrt{3-4s^2_w} \\ 0 & s_{\theta'} & c_{\theta'} t_{\theta'}\sqrt{3-4s^2_w} & c_{\theta'}\sqrt{1-t^2_{\theta'}(3-4s^2_w)} \end{pmatrix} \tag{36}$$

Let us denote

$$U_2 = S_2^T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\theta'} & 0 & s_{\theta'} \\ 0 & -s_{\theta'} t_{\theta'}\sqrt{3-4s^2_w} & \sqrt{1-t^2_{\theta'}(3-4s^2_w)} & c_{\theta'} t_{\theta'}\sqrt{3-4s^2_w} \\ 0 & -s_{\theta'}\sqrt{1-t^2_{\theta'}(3-4s^2_w)} & -t_{\theta'}\sqrt{3-4s^2_w} & c_{\theta'}\sqrt{1-t^2_{\theta'}(3-4s^2_w)} \end{pmatrix},$$

then the mass matrix M'^2 has a quasi-diagonal form

$$M''^2 = U_2^T M'^2 U_2 = \frac{g^2}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & m^2_{\mathscr{Z}} & m^2_{\mathscr{Z}\mathscr{Z}'} & 0 \\ 0 & m^2_{\mathscr{Z}\mathscr{Z}'} & m^2_{\mathscr{Z}'} & 0 \\ 0 & 0 & 0 & (n_1^2+n_2^2+k_2^2) \end{pmatrix}, \tag{37}$$

where

$$\begin{aligned} m^2_{\mathscr{Z}} &= \frac{1}{4(4c_W^2-s_{2\theta}^2)}[(4-s_{2\theta}^2)k_2^2+4k_1^2-s_{2\theta}^2(n_1^2+n_2^2)] \ , \\ m^2_{\mathscr{Z}\mathscr{Z}'} &= \frac{1}{4(4c_W^2-s_{2\theta}^2)\sqrt{4c_W^2-1}}\{[4c_{2\theta}c_{2W}+s_{2\theta}t_{2\theta}(2c_{2W}+1)]k_2^2-4c_{2\theta}k_1^2 \\ &\quad -s_{2\theta}t_{2\theta}(n_1^2+n_2^2)\} \ , \\ m^2_{\mathscr{Z}'} &= \frac{1}{4(4c_W^2-s_{2\theta}^2)(4c_W^2-1)}[(4c_W^2-s_{2\theta}^2)k_2^2+4c_{2\theta}^2k_1^2+(16c_W^4-s_{2\theta}^2)(n_1^2+n_2^2)] \ . \end{aligned} \tag{38}$$

As expected, we get mass of $A_{4\mu}$ as given in (24). The final step is determined as

$$\begin{pmatrix} A_\mu \\ Z_{1\mu} \\ \mathscr{Z}'_\mu \\ A'_{4\mu} \end{pmatrix} = U_3 \begin{pmatrix} A_\mu \\ Z_{1\mu} \\ Z_{2\mu} \\ A_{4\mu} \end{pmatrix}, \tag{39}$$

where

$$U_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_\zeta & s_\zeta & 0 \\ 0 & -s_\zeta & c_\zeta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{40}$$

and

$$M'''^2 = U_3^T M\text{''}^2 U_3 = diag(0, m^2_{Z_1}, m^2_{Z_2}, m^2_{A'_4}) \tag{41}$$

The mixing angle is defined as

$$\tan 2\zeta = \frac{2m^2_{ZZ'}}{m^2_{Z'}-m^2_Z} \ . \tag{42}$$

Note that in the limit $n_1 \sim n_2 \gg k_1 \sim k_2$, two angles are in the same order, i.e., $\theta \propto \zeta$. Masses of heavy physical bosons are given by

$$m^2_{Z_1} = \frac{g^2}{2}\frac{1}{27}\left[(n_1^2+n_2^2)(18+t^2)+k_2^2(18+4t^2)+k_1^2(18+t^2)-\sqrt{\Delta'}\right] \tag{43}$$

$$m^2_{Z_2} = \frac{g^2}{2}\frac{1}{27}\left[(n_1^2+n_2^2)(18+t^2)+k_2^2(18+4t^2)+k_1^2(18+t^2)+\sqrt{\Delta'}\right], \tag{44}$$

where

$$\begin{aligned}
\Delta' &= [(n_1^2+n_2^2+k_2^2)(18+t^2)+2k_1^2(9+2t^2)]^2-108(9+2t^2)\left[n_1^2k_2^2+(n_1^2+n_2^2+k_2^2)k_1^2\right] \\
&\simeq (n_1^2+n_2^2)^2(18+t^2)^2\left\{1+2\frac{k_2^2}{(n_1^2+n_2^2)}+k_1^2\frac{4(9+2t^2)}{(n_1^2+n_2^2)(18+t^2)}\right. \\
&- \left. 108\frac{(9+2t^2)}{(n_1^2+n_2^2)(18+t^2)^2}\left[k_1^2+\frac{n_1^2k_2^2}{(n_1^2+n_2^2)}\right]\right\}. \qquad (45)
\end{aligned}$$

Then

$$\begin{aligned}
\sqrt{\Delta'} &\simeq (n_1^2+n_2^2)(18+t^2)+k_2^2(18+t^2)+2(9+2t^2)k_1^2 \\
&- 54\frac{(9+2t^2)}{(18+t^2)}\left[k_1^2+\frac{n_1^2k_2^2}{(n_1^2+n_2^2)}\right]\Bigg\} \\
&= \frac{54}{(3-4s_w^2)}\left\{(n_1^2+n_2^2+k_2^2)c_w^2+k_1^2-\frac{(3-4s_w^2)}{2c_w^2}\left[k_1^2+\frac{n_1^2k_2^2}{(n_1^2+n_2^2)}\right]\right\}. \qquad (46)
\end{aligned}$$

Substituting (46) into (43) yields

$$\begin{aligned}
m_{Z_1}^2 &\simeq \frac{g^2}{54}\left\{3t^2(k_2^2-k_1^2)+\frac{54(9+2t^2)}{18+t^2}\left[k_1^2+\frac{n_1^2k_2^2}{(n_1^2+n_2^2)}\right]\right\} \\
&= g^2\left\{\frac{s_w^2}{(3-4s_w^2)}(k_2^2-k_1^2)+\frac{1}{2c_w^2}\left[k_1^2+\frac{n_1^2k_2^2}{(n_1^2+n_2^2)}\right]\right\}. \qquad (47)
\end{aligned}$$

Similarly, for the heavy extra neutral gauge boson Z_2, one obtains

$$\begin{aligned}
m_{Z_2}^2 &\simeq \frac{g^2}{54}\left\{2(n_1^2+n_2^2+k_1^2+k_2^2)(18+t^2)+3t^2(k_2^2+k_1^2)\right. \\
&- \left.\frac{54(9+2t^2)}{(18+t^2)}\left[k_1^2+\frac{n_1^2k_2^2}{(n_1^2+n_2^2)}\right]\right\} \\
&= \frac{g^2}{(3-4s_w^2)}\left\{2(n_1^2+n_2^2+k_1^2+k_2^2)c_w^2+(k_1^2+k_2^2)s_w^2\right. \\
&- \left.\frac{(3-4s_w^2)}{2c_w^2}\left[k_1^2+\frac{n_1^2k_2^2}{(n_1^2+n_2^2)}\right]\right\} \qquad (48)
\end{aligned}$$

In summary, the physical vector fields relate to the gauge states as

$$\begin{pmatrix} A_{3\mu} \\ A_{8\mu} \\ B_\mu \\ A_{4\mu} \end{pmatrix} = U \begin{pmatrix} A_\mu \\ Z_{1\mu} \\ Z_{2\mu} \\ A'_{4\mu} \end{pmatrix}, \qquad (49)$$

where

$$
\begin{aligned}
U &= U_1 U_2 U_3 \\
&= \begin{pmatrix}
s_w & c_w c_{\theta'} c_\zeta & c_w c_{\theta'} s_\zeta & c_w s_{\theta'} \\
\frac{s_w}{3} & M_{22} & M_{23} & \frac{3 c_w s_{\theta'}}{\sqrt{3}} \\
\frac{\sqrt{3-4s_w^2}}{\sqrt{3}} & M_{32} & M_{33} & 0 \\
0 & M_{42} & M_{43} & c_{\theta'}\sqrt{1-t_{\theta'}^2(3-4s_w^2)}
\end{pmatrix}
\end{aligned}
\tag{50}
$$

with

$$
\begin{aligned}
M_{22} &= \frac{1}{\sqrt{3}c_w}\left[\frac{c_\zeta(s_w^2-3c_w^2 s_{\theta'}^2)}{c_{\theta'}} - s_\zeta\sqrt{3-4s_w^2}\sqrt{1-t_{\theta'}^2(3-4s_w^2)}\right], \\
M_{23} &= \frac{1}{\sqrt{3}c_w}\left[\frac{s_\zeta(s_w^2-3c_w^2 s_{\theta'}^2)}{c_{\theta'}} + c_\zeta\sqrt{3-4s_w^2}\sqrt{1-t_{\theta'}^2(3-4s_w^2)}\right], \\
M_{32} &= -\frac{t_w}{\sqrt{3}}\left(\frac{c_\zeta}{c_{\theta'}}\sqrt{3-4s_w^2} - s_\zeta\sqrt{1-t_{\theta'}^2(3-4s_w^2)}\right), \\
M_{33} &= -\frac{t_w}{\sqrt{3}}\left(\frac{s_\zeta}{c_{\theta'}}\sqrt{3-4s_w^2} - c_\zeta\sqrt{1-t_{\theta'}^2(3-4s_w^2)}\right), \\
M_{42} &= -c_\zeta s_{\theta'}\sqrt{1-t_{\theta'}^2(3-4s_w^2)} + s_\zeta t_{\theta'}\sqrt{3-4s_w^2}, \\
M_{43} &= -s_\zeta s_{\theta'}\sqrt{1-t_{\theta'}^2(3-4s_w^2)} - c_\zeta t_{\theta'}\sqrt{3-4s_w^2}.
\end{aligned}
$$

Note that $s_{\theta'} \sim s_\zeta \ll 1$, then the matrix in (50) becomes

$$
\begin{aligned}
U_p &\simeq U(s_{\theta'} \sim s_\zeta \ll 1) \\
&= \begin{pmatrix}
s_w & c_w c_{\theta'} & c_w s_\zeta & c_w s_{\theta'} \\
\frac{s_w}{3} & \frac{1}{\sqrt{3}c_w}\left(\frac{c_\zeta s_w^2}{c_{\theta'}} - s_\zeta\sqrt{3-4s_w^2}\right) & \frac{1}{\sqrt{3}c_w}\left(\frac{s_\zeta s_w^2}{c_{\theta'}} + c_\zeta\sqrt{3-4s_w^2}\right) & \frac{3c_w s_{\theta'}}{\sqrt{3}} \\
\frac{\sqrt{3-4s_w^2}}{\sqrt{3}} & -\frac{t_w}{\sqrt{3}}\left(\frac{c_\zeta}{c_{\theta'}}\sqrt{3-4s_w^2} - s_\zeta\right) & -\frac{t_w}{\sqrt{3}}\left(\frac{s_\zeta}{c_{\theta'}}\sqrt{3-4s_w^2} - c_\zeta\right) & 0 \\
0 & -c_\zeta s_{\theta'} & -c_\zeta t_{\theta'}\sqrt{3-4s_w^2} & c_{\theta'}
\end{pmatrix}
\end{aligned}
\tag{51}
$$

The matrix U_p in (51) is very useful for practical calculation in consideration of phenomenology.

IV. CONCLUSION

In this paper, we have showed that the mass matrix of electrically neutral gauge bosons in the recent proposed 3-3-1 model with three neutrino singlets [9] has two exact eigenvalues and corresponding eigenvectors. Hence the 4×4 mass matrix is diagonalized exactly. Two components of neutral bilepton boson X^0_μ have the same mass, hence the neutral non-Hermitian gauge boson X^0_μ is properly determined. This contradicts to previous analysis in Ref. [9]. With extra vacuum expectation values of the Higgs fields, there are mixings among charged gauge bosons $W^\pm$ and $Y^\pm$ as well as among neutral gauge bosons Z, Z' and X^0. Due to these mixings, the lepton number violating interactions exist in leptonic currents not only in bileptons Y and X^0 but also in both SM W and Z bosons. The scale of new physics was estimated to be in range of few TeVs. With this limit, masses of the exotic quarks are also not high, in the range of few TeVs.

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