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2016 J. Phys.: Conf. Ser. 726 012023

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On the fluctuation induced mass enhancement

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Abstract. The effective mass induced by the background fluctuation on particles is considered. The analytical results show that the effective mass depends only on the properties of fluctuation, and takes non-zero value when and only when fluctuation mean value is non-zero. The possible applications of the obtained results to complex systems such as biology and ecology where environmental factors lead to the changes of the information exchange ranges from long to short one are discussed, i.e. the possibility of using physical modeling techniques to investigate macroscopic behaviors of some complex systems under consideration.

1. Introduction

In conventional understanding of modern physics, motion of a particle is characterized by its dispersion law $\mathcal{E}(p)$ where the dependence of $\mathcal{E}(p)$ on p reflects whether particle is massive or massless.

In one hand, the massless particle has dispersion law in form

$$\mathcal{E}(p) = cp, \quad (1)$$

and in other hand, the massive particle m_0 has the dispersion law written in the relativistic form[1, 2] as

$$\mathcal{E}(p) = \sqrt{(mc^2)^2 + (cp)^2}, \quad (2)$$

which tends to additive law of energy

$$\mathcal{E}(p) = mc^2 + \frac{1}{2m}p^2 + \underbrace{V(p)}_{o(p^4)}, \quad (3)$$



when momentum p approaches to zero, where c is some maximal velocity of the massless motion in given media. It is important to think of a possibility that the coordinate representation of function $V(p)$

$$V(r) = \frac{1}{\sqrt{V_\Omega}} \int_{\Omega} dp e^{ipr} V(p), \quad (4)$$

would contain some physical insights of potential affecting on the motion of a massive particle in the medium, or in other words, interaction between massive particle and surrounding environment.

Following the standard mathematical routine found in textbooks, the mass of a moving particle with given dispersion law $\mathcal{E}(p)$ is determined by second order derivative of $\mathcal{E}(p)$ with respect to p [3, 4] as

$$\frac{1}{m} = \left. \frac{d^2}{dp^2} \mathcal{E}(p) \right|_{p=0}. \quad (5)$$

When a particle moves under influence of some fluctuations which changes the dispersion law of the particle, energy of massive particle reads[5, 6, 7]

$$\mathcal{E}(p, \epsilon) = \sqrt{(mc^2)^2 + c^2p^2 + \epsilon^2}, \quad (6)$$

then the energy of moving particle is changed, and obtained by including all contributions of fluctuations via integrating out with fluctuation distribution function $\mathcal{D}(\epsilon)$

$$\mathcal{E}[cp; \mathcal{D}] = \int_{\Omega} d\epsilon \mathcal{D}(\epsilon) \sqrt{(mc^2)^2 + c^2p^2 + \epsilon^2}. \quad (7)$$

The effective mass of moving particle will be determined by the definition (5).

2. The Effective Mass

To determine the fluctuation induced mass [3, 4] of initial particle, the second order derivative of dispersion law $\mathcal{E}(cp; \mathcal{D})$ with respect to momentum should be calculated, and the result reads

$$\begin{aligned} \frac{1}{m^*} &= \left. \frac{d^2}{dp^2} \mathcal{E}[cp; \mathcal{D}] \right|_{p=0} \\ &= c^2 \int_{\Omega} d\epsilon \mathcal{D}(\epsilon) \frac{1}{\sqrt{(mc^2)^2 + \epsilon^2}}. \end{aligned} \quad (8)$$

The impression enlightened by second derivative of the integral is that fluctuations of the medium would cause a change of mass of particle moving in it. However, to see more insights, some simple type of fluctuation will be investigated.

2.1. Simplest single mode fluctuation – Dirac distribution

In single mode fluctuation, the distribution function $\mathcal{D}(\epsilon)$ is nothing but the Dirac δ -function[10]

$$\mathcal{D}(\epsilon) = \delta(\epsilon - \epsilon_0). \quad (9)$$

Inserting fluctuation distribution function into the integral and performing integrating over random variable ϵ , the result reads

$$m^* = \left(\frac{d^2}{dp^2} \mathcal{E} [cp; \mathcal{D}] \Big|_{p=0} \right)^{-1} = \frac{1}{c^2} \sqrt{(mc^2)^2 + \epsilon_0^2}. \tag{10}$$

By introducing new mass denoted by $m_\epsilon = \frac{\epsilon_0}{c^2}$ characterizing the fluctuations, the effective mass m^* of a moving particle will be rewritten as follows

$$m^* = \sqrt{m^2 + m_\epsilon^2}. \tag{11}$$

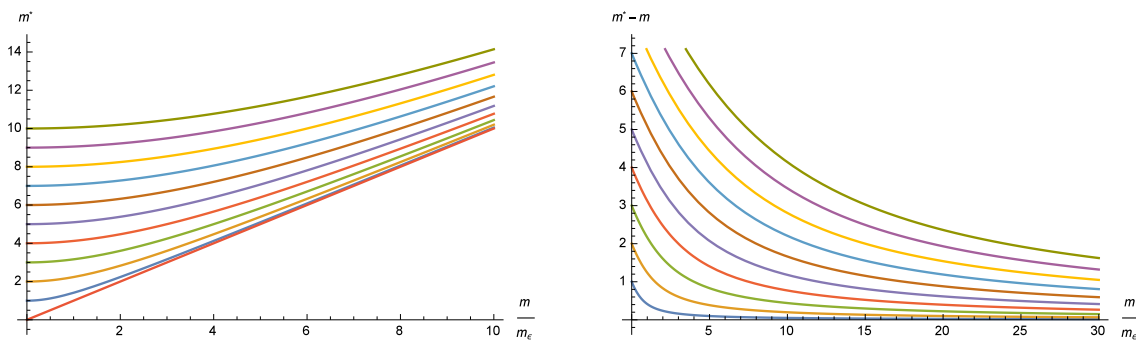


Figure 1. The graph of the effective mass under the influence of fluctuations in single mode fluctuation.

2.2. Boltzmann fluctuation

In the Boltzmann mode of fluctuation, the Boltzmann distribution function[9]

$$\mathcal{D}(\epsilon) = \frac{1}{2\lambda} \exp \left\{ -\frac{|\epsilon - \epsilon_0|}{\lambda} \right\}, \tag{12}$$

will be used as distribution function of media fluctuations. The corresponding effective mass takes the form

$$m^* = \left(\frac{d^2}{dp^2} \mathcal{E} [cp; \mathcal{D}] \Big|_{p=0} \right)^{-1} = \frac{1}{c^2} \left(\frac{\pi \left(\mathbf{H}_0 \left(\frac{mc^2}{\lambda} \right) - Y_0 \left(\frac{mc^2}{\lambda} \right) \right)}{2\lambda} \right)^{-1}, \tag{13}$$

where $\mathbf{H}_n(z)$ is the Struve function, and $Y_n(z)$ is the Bessel function of the second kind.

2.3. Gaussian fluctuation

The media with white noise provides the Gaussian fluctuation which is expressed by normal distribution function [8, 9, 11]

$$\mathcal{D}(\epsilon) = \frac{1}{\sqrt{2\pi}\sigma} \exp \left\{ -\frac{(\epsilon - \epsilon_0)^2}{2\sigma^2} \right\}. \tag{14}$$

Moving in this kind of media, the effective mass of a particle is

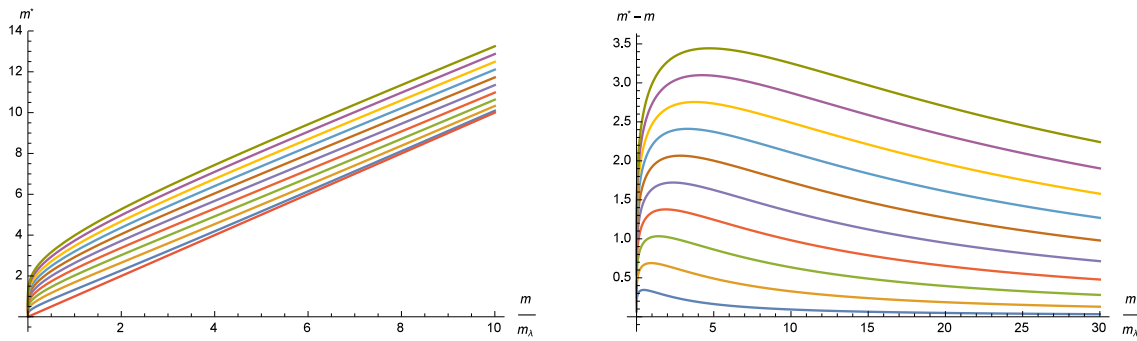


Figure 2. The graph of the effective mass under the influence of fluctuations in Boltzmann fluctuation, where $m_\lambda = \frac{\lambda}{c^2}$ is characteristic mass of Boltzmann fluctuation.

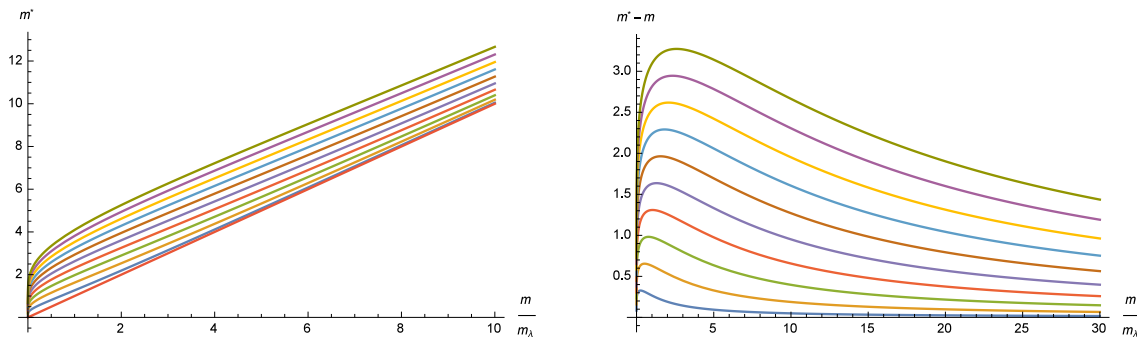


Figure 3. The graphs of the effective mass under the influence of fluctuations in Gaussian fluctuation, where $m_\sigma = \frac{\sigma}{c^2}$ is characteristic mass of Gaussian fluctuation.

$$\begin{aligned}
 m^* &= \left(\frac{d^2}{dp^2} \mathcal{E} [cp; \mathcal{D}] \Big|_{p=0} \right)^{-1} \\
 &= \frac{1}{c^2} \exp \left\{ -\frac{c^4 m^2}{4\sigma^2} \right\} \left(\frac{K_0 \left(\frac{c^4 m^2}{4\sigma^2} \right)}{\sqrt{2\pi}\sigma} \right)^{-1},
 \end{aligned} \tag{15}$$

where $K_n(z)$ is the modified Bessel function of the second kind.

2.4. Discussion

When a particle moves in the fluctuations of an environment, the environmental impact will produce the change of particle mass which is not linear. Furthermore, a light particle, $\frac{m}{m_\epsilon} \ll 1$, becomes much heavier within fluctuation of the media, but less influence of fluctuation is observed for heavy particles, $\frac{m}{m_\epsilon} \gg 1$.

For all kinds of fluctuations under consideration the effective mass is heavier than the mass of particle, so that the effect would be called the mass enhancement under influence of media fluctuations. As a result of the effect, an observer will see slower movement of initial particle, or in other words, the kinetic energy of the particle decreases when it gets moving in any environmental fluctuations.

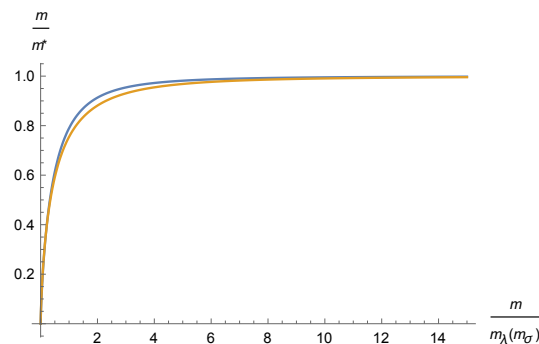


Figure 4. The graph of the ratio of effective mass and mass under the influence of fluctuations in Gaussian and Boltzmann fluctuations.

One of the most important issues appeared in the investigation is the mass of massless particle moving in noisy background. This problem can be formulated in following theorem

Theorem The fluctuation induced mass on a massless particle

$$m^* = \left(c^2 \int_{\Omega} d\epsilon \frac{\mathcal{D}(\epsilon)}{|\epsilon|} \right)^{-1}, \quad (16)$$

depends only on the nature of the fluctuations, and is non-zero when and only when fluctuation mean value gets non-zero value.

However, the proof of the theorem will be left to future research.

3. Conclusion

In conclusion, by considering different probability density functions of background fluctuation, the change of effective mass of a moving massive and massless particles has been studied. In the case of massless particle, the effective mass gets non-zero value when and only when the expectation value of background fluctuation gets non-zero. In the case of massive particle, the dependence of effective mass on the ratio of mass and characteristic mass of background fluctuations, m_{σ} and m_{λ} , respectively has been analytically found.

The obtained results are possibly applicable to study many collective behaviors of complex systems, where environmental factors would change the information exchange ranges from long to short one.

Acknowledgments

This research is funded by Vietnam National Foundation for Science and Technology Development (NAFOSTED) under grant number 103.01-2015.42.

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