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2016 J. Phys.: Conf. Ser. 726 012017

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Cooper pair of superconductivity in the coordinate representation and q -deformed harmonic oscillator

Man Van Ngu¹, Ngo Gia Vinh², Nguyen Tri Lan³, Luu Thi Kim Thanh⁴ and Nguyen Ai Viet³

¹ Hung Yen Industrial College, Hung Yen Province, Vietnam

² Bac Ninh Department of Education and Training, Bac Ninh Province, Vietnam

³ Institute of Physics, Vietnam Academy of Science and Technology, 10 Dao Tan street, Ba Dinh, Hanoi, Vietnam

⁴ Hanoi Pedagogical University No.2, Vinh Phuc Province, Vietnam

E-mail: nguqlkt@yahoo.com

Abstract. In this work we study the similarity between the wave functions of q -deformed harmonic oscillator and wave functions of Cooper pair. The wave functions of Cooper pairs in coordinate-space have an “onion-like” layered structure with exponent decay (Boltzmann) envelope modulation. The ground state wave function of q -deformed harmonic oscillator has the form of oscillate functions with Gaussian decay envelope modulation. The corresponding between Boltzmann and Gaussian forms of envelope functions and their quantum similarity are discussed.

1. Introduction

In the last decades, the mathematical methods based on quantum group theory and its q -deformed algebras have found their applications in many fields of interest from physics to chemistry, and even to other sciences of complexity [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. The main idea of the q -deformed algebras applicability is of its possibility to transform the perturbative methods approach to the non-perturbative ones, which would simplify mathematical representations and open new physical insights of the systems under consideration. In general panorama of quantum group theory and its q -deformed algebras, it would remind us the pioneering research of W. Heisenberg who has provided the famous commutation relations in quantum mechanics and developed q -deformed formalism. In q -deformed algebras, the key parameter is q that can take any value in the range from 0 to 1 in which the formalism falls back to non-deformed when parameter q approaches to one.

In our previous work [13] we have investigated the relations between first, second quantization representations of quantum mechanics and q -deformed algebra. In the case of harmonic oscillation, the axiom of first quantization (the commutation relation between coordinate and momentum operators) and the axiom of second quantization (the commutation relation between creation and annihilation operators) are equivalent, so that harmonic oscillator would be considered as a good object for investigating the link of two these representations in quantum mechanics. We have also shown that in the case of q -deformed harmonic oscillation, the breakdown of the axiom of second quantization leads to the breakdown of the axiom of first



quantization, and visa versa. Using the coordinate representation we study fine structures of the vacuum state wavefunction depending on the deformation parameter q . A comparison with fine structures of Cooper pair in the coordinate representation [11] is also performed. We have shown that the real part $\text{Re}\Psi_0$ of ground state wavefunction Ψ_0 of q -deformed harmonic oscillator is very similar the wave function Ψ_C of Cooper pair in superconductivity.

In present work, we continue to develop the main idea of investigating the relation between wavefunctions of q -deformed harmonic oscillator and of Cooper pairs in space representations, and to answer the question which deformation parameter q will be equivalent to Cooper pairs? We have explored the change of the envelope functions and studied the physical meaning of that given transition.

To simplify the mathematical expression, we will use the system of natural units with $\hbar = c = 1$.

2. q -deformed harmonic oscillator

In this section, we recall some basic of q -deformed harmonic oscillator [13] that would provide the consistency of ongoing representation. We start with the q -deformed harmonic oscillator which is represented via Hamiltonian as

$$H = \frac{\omega}{2} (aa^\dagger + a^\dagger a), \quad (1)$$

where the creation and annihilation operators in the Hamiltonian (1) satisfy the commutation relations

$$\begin{aligned} [a, a^\dagger]_q &= aa^\dagger - qa^\dagger a \\ &= 1, \end{aligned} \quad (2)$$

in which q - deformation parameter takes values in the range $[0, 1]$.

It is easy to obtain the energy spectrum of q -deformed harmonic oscillator taking the form

$$E_n = \frac{\omega}{2} ([n]_q + [n+1]_q),$$

where

$$[n]_q = \frac{1 - q^n}{1 - q},$$

are the q -integers that differ from natural numbers. The properties of q -numbers are out of the scope of this work.

The values of ratio $\frac{[n]_q}{n}$ depend on deformation parameter q are presented in the figure 1.

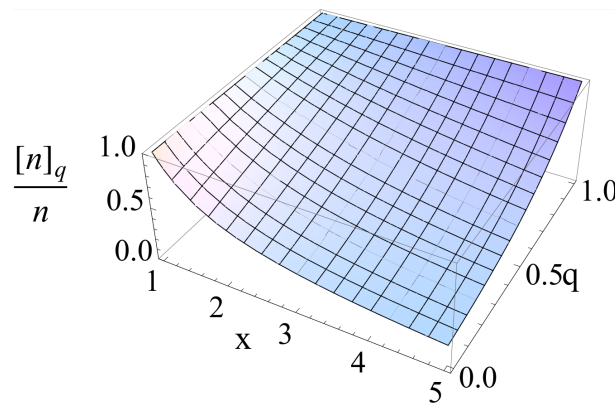


Figure 1. The values of ratio $\frac{[n]_q}{n}$ depend on deformation parameter q .

This ratio tends to zero as

$$\lim_{q \rightarrow 0} \frac{[n]_q}{n} = 0. \tag{3}$$

The maximum value of $[n]_{qmax} \rightarrow \infty$ when $q \rightarrow 1$ and we return to case of standard harmonic oscillator. By introducing new deformation parameter α

$$\alpha = \sqrt{-\frac{1}{2} \ln q}, \tag{4}$$

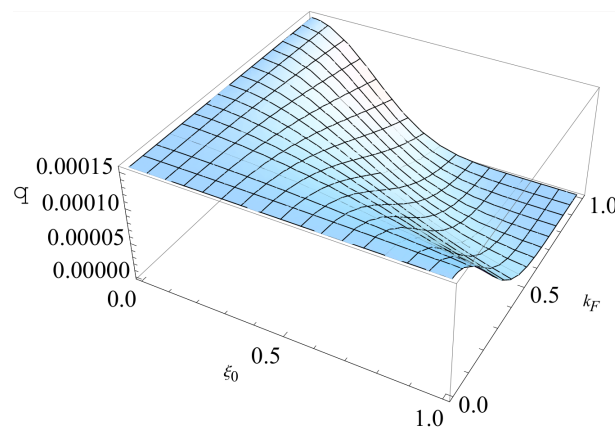


Figure 2. Drawing of deformed parameter q as the function of new one $\alpha = \frac{3}{2}a$, where a is defined in the second line of equation (7), $q = \exp(-2\alpha^2)$.

taking values in the interval $[0, \infty]$, the ground state wave function of q -deformed harmonic oscillator is found and given in the form

$$\Psi_D(x) = C_D \exp\left(-\frac{x^2}{2} + i\frac{3}{2}\alpha x\right), \tag{5}$$

where C_D is the normalized constant. It is obvious that in the case of $q \rightarrow 1$, i.e. $\alpha \rightarrow 0$ standard harmonic oscillator is recovered.

The density operator of q -deformed harmonic oscillator is defined by

$$\rho(x) = \Psi_0^+(x) \Psi_0(x),$$

so that the expectation value of ground state is

$$\langle 0 | \rho | 0 \rangle = \langle 0 | \rho_0 | 0 \rangle = \frac{1}{2}.$$

Values of density vacuum fluctuations for both harmonic oscillator and q -deformed harmonic oscillator are the same. Real and imaginary parts of ground state wave function are presented in figure 3.

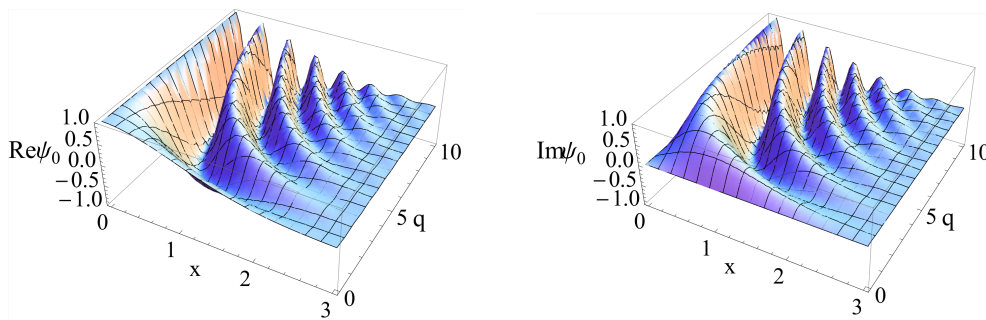


Figure 3. Real part Re and imaginary part Im of ground state wave function.

As expected, when $q = 1$, the wave function falls back to harmonic oscillator case with real part of Gaussian form, and imaginary part of zero.

3. Comparison with Cooper pairs in superconductivity

From condensed matter physics we have learn that the real part of wave function plays important role. In this section we try to explore the physical meaning of real part of ground state wave function of q -deformed harmonic oscillator. For comparison we take the well known case of Cooper pairs in superconductivity. In a long history of the BCS theory, the Cooper pair usually analyzed in the momentum space. The first investigation of Cooper pair in coordinate space was done in the work [11], where it was shown that this leads to a spherically symmetrical quasi-atomic wave function with an identical “onion-like” layered structure for each of the electrons constituting the Cooper pair.

The internal structure of Cooper pair wave function (also called the singlet pairing function or the Gorkov’s wave function) is given by

$$\Psi_C(r) = C_C \cos(k_F r) K(r/\pi\xi_0), \tag{6}$$

where C_C is the normalized constant, k_F is the Fermi wave vector at the surface of the Fermi sea and K_0 is the zero-order modified Bessel function with an asymptotic form that is similar to an exponential for large x .

Denote dimensionless variable x and parameter a

$$\begin{aligned} x &= \frac{r}{\pi\xi_0}, \\ a &= \pi k_F \xi_0, \end{aligned} \tag{7}$$

the Cooper pair wave function can be rewritten as

$$\Psi_C(r) = C_C \cos(ax) K_0(x). \tag{8}$$

The Cooper wave function in coordinate representation corresponds to the standing waves with a spatial modulation α . Typically values of these parameters are $1/k_F \sim 0.1nm$ and $\xi_0 \sim 100nm$. These waves rapidly oscillate around k_F , modulate slowly varying envelope with a characteristic scale off ξ_0 . Taking the value $k_F\xi_0 = 20/\pi$, or $a = \pi k_F\xi_0 = 20$, and corresponding $C_C \simeq$, the wave function of Cooper pair is plotted in the figure 4.

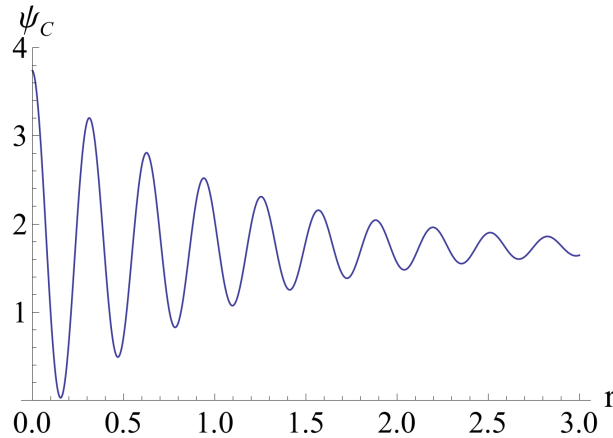


Figure 4. The wave function of Cooper pair in superconductivity.

Graphically, we could realize that the real part of ground state wave function of q -deformed harmonic oscillator (see figure 3) is very similar with the wave function of Cooper pair.

The real part Re of ground state wave function of q -deformed harmonic oscillator with $\alpha = 3a/2 = 30$ and exponential envelope function $\lambda = 10/3$ is presented in figure 5.

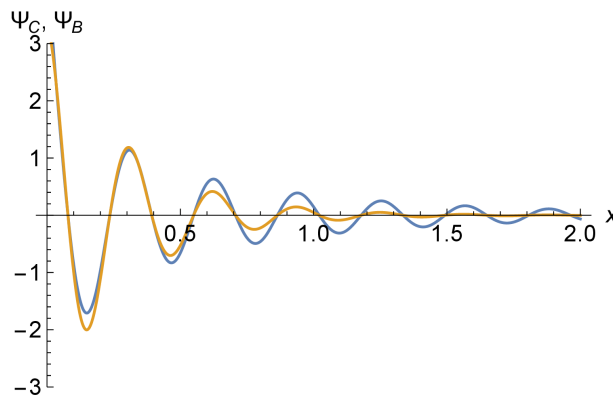


Figure 5. The real part Re of ground state wave function of q -deformed harmonic oscillator with $\alpha = 3a/2 = 30$ and exponential envelope function $\lambda = 10/3$.

The similarity between real parts of wave functions of q -deformed harmonic oscillator and of Cooper pair seems to be not simply accidental, that it supports a serious question of why the forms of the wave functions are so similar? We are going to find the reasons of this graphical similarity in below.

4. Transition between Gaussian and Boltzmann forms of envelope functions

As an attempt to extract physical meanings of q -deformation algebra, we, in this section, study the similarity of envelop functions in different cases by mean of graphical comparison. First we realize that the zero-order modified Bessel function K_0 is very look like the exponential function with $\lambda \simeq 10/3$ [13]. In following figures 6, 7 and 8, the Cooper pair's $F_C(x)$, Boltzmann-like $F_B(x)$ and Gaussian-like $F_G(x)$ envelope functions are pairwise presented.

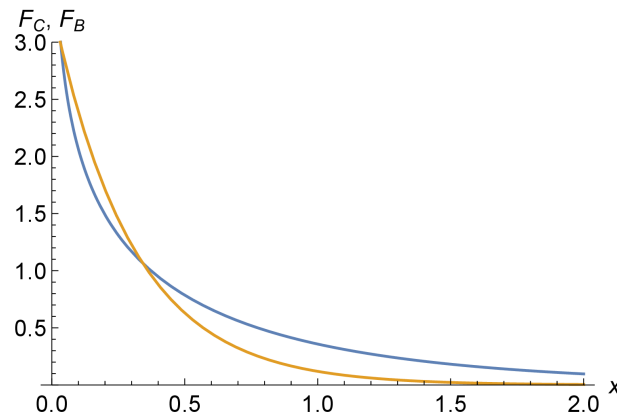


Figure 6. The Cooper pair's $F_C(x)$ and Boltzmann-like $F_B(x)$ envelope functions.

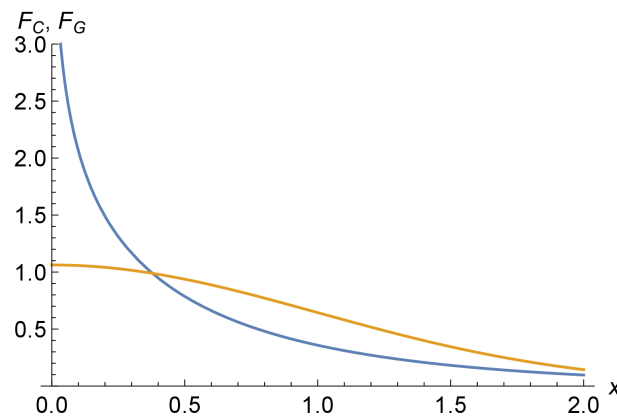


Figure 7. The Cooper pair's $F_C(x)$ and Gaussian-like $F_G(x)$ envelope functions.

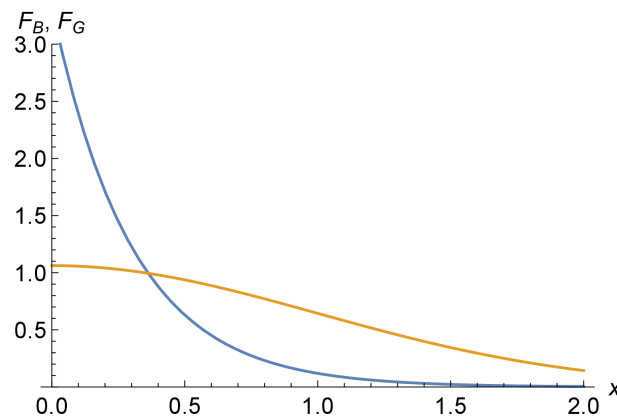


Figure 8. The Gaussian-like $F_G(x)$ and Boltzmann-like $F_B(x)$ envelope functions.

The phenomena of Boltzmann–Gaussian transition of the envelope functions are quite common in study of complexity (for example in economy see [14, 15]). That might be the links to the symmetry of the system where the Boltzmann form of the envelope function is corresponding to the translation symmetry of the system and Gaussian one corresponds to other symmetry. It is well-known in the superconducting state, the Cooper pair is formed and can move in system as free quasi-particle without resistance, i.e. some translation symmetry for Cooper pair is occurred. This interesting topic would be the subject of our next research.

5. Discussion

In current work, we have presented the relation between wave functions of q -deformed harmonic oscillator and Cooper pairs in space representations and the interchangeability of their set-parameters, $\{q, \alpha\} \longleftrightarrow \{k_F, \xi_0\}$. We have also found that a strong deformation where $q \simeq 0$ is corresponding to Copper pairs in superconductivity.

Different types of the envelope functions such as Cooper-, Boltzmann-, Gaussian-, Mean Gaussian-likes have been investigated and compared. We have explored the transition from Boltzmann form to Gaussian form of the envelope functions of normal and superconducting states, and pointed out some possible physical meanings and underlying physics of that transition.

We consider necessity to rewrite the Hamiltonian of deformed harmonic oscillator to describe the phenomenon of phase transitions. The connections between the q -deformed harmonic oscillators, coherent states, and the physical meanings of the envelope functions will be the topic of our future consideration.

Acknowledgments

This research is funded by Vietnam National Foundation for Science and Technology Development (NAFOSTED) under grant number 103.01-2015.42.

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