Generalized Bogoliubov Polariton Model: An Application to Stock Exchange Market

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Abstract. A generalized Bogoliubov method for investigation non-simple and complex systems was developed. We take two branch polariton Hamiltonian model in second quantization representation and replace the energies of quasi-particles by two distribution functions of research objects. Application to stock exchange market was taken as an example, where the changing the form of return distribution functions from Boltzmann-like to Gaussian-like was studied.

1. Introduction

Bogoliubov method[1] is frequently used in superconductivity[2, 3] systems and condensed matter physics. It is a well-known method, used as a filter for reducing special hamiltonians of interacting particles to almost noninteracting quasi-particles[4, 5, 6, 7]. In this investigation, Bogoliubov method is used to describe the transition of two branches polariton Hamiltonian model in second quantization. This method will be developed in the case of boson gases and plasmon-polariton. Then it will be applied in stock exchange market, as an example of complex system.

Quasi-particles are replaced by complex systems, described by statistical functions. It will appear then two branches as in boson or plasmon-polariton case. But in stock exchange market, we know only one system corresponding to one branch, the other branch is unknown. The transition of distribution function in stock exchange market corresponds to one branch. The unknown duality system remains as a challenge for our next investigation.

2. Bogoliubov transformation

Consider a system containing two quasi-particles a and b, their interaction is described by second quantized Hamiltonian

$$H = \sum_k \left\{ \varepsilon_a\left(k\right) a_k^+ a_k + \varepsilon_b\left(k\right) b_k^+ b_k + g_k \left[a_k^+ b_k + b_k^+ a_k \right] \right\}, \tag{1}$$

in which $\varepsilon_a\left(k\right)$, $\varepsilon_b\left(k\right)$ are quasi-particle energy, represented by creation operators a_k^+, b_k^+ and annihilation operators a_k^-, b_k^-, interaction constant g_k and pulse k. Interacting particle system could be transformed to non-interacting particle system via Bogoliubov transformation by annihilation operators A, B, with corresponding energy E_A and E_B. For simplicity, we use only real parameters. After diagonalized, Hamiltonian of system becomes:

$$H = \sum_k \left\{ E_A(k) A_k^+ A_k + E_B(k) B_k^+ B_k \right\}. \tag{2}$$

Bogoliubov direct transformation

$$A_k = u_k a_k + v_k b_k, \ B_k = -v_k a_k + u_k b_k. \tag{3}$$

Bogoliubov inverse transformation

$$a_k = u_k A_k - v_k B_k, \ b_k = v_k A_k + u_k B_k. \tag{4}$$

Normalization condition

$$u_k^2 + v_k^2 = 1, \tag{5}$$

With given Hamiltonian with three parameters $\{\varepsilon_a, \varepsilon_b, g\}$, we need to transform $\{u, v\}$ in order to get Hamiltonian of non-interacting quasi-particle system

$$\{\varepsilon_a, \varepsilon_b, g\} \Longleftrightarrow \{E_A, E_B, u\}, \tag{6}$$

with $\{E_A, E_B\}$ parameters of non-interacting quasi-particle system.

2.1. Boson case
In case that a and b are bosons, commutation relation for bosonic creation and annihilation operators

$$\left[a_k, a_k^+\right] = \left[b_k, b_k^+\right] = 1, \tag{7}$$

other commutations of a and b are 0. It is easy to see that A and B are also bosons because

$$\left[A_k, A_k^+\right] = \left[B_k, B_k^+\right] = 1, \tag{8}$$

other commutations of A and B are 0.

Dispersion law of new quasi-particles is

$$E_{A,B}(k) = \frac{1}{2} \left\{ [\varepsilon_a(k) + \varepsilon_b(k)] \pm \sqrt{[\varepsilon_a(k) + \varepsilon_b(k)]^2 + 4g_k^2} \right\}. \tag{9}$$

Transformation parameters are

$$\begin{cases} u_k^2 &= \frac{1}{2} \left\{ 1 + \frac{|\varepsilon_a(k) - \varepsilon_b(k)|}{\sqrt{[\varepsilon_a(k) + \varepsilon_b(k)]^2 + 4g_k^2}} \right\}, \\ v_k^2 &= \frac{1}{2} \left\{ 1 - \frac{|\varepsilon_a(k) - \varepsilon_b(k)|}{\sqrt{[\varepsilon_a(k) + \varepsilon_b(k)]^2 + 4g_k^2}} \right\}. \end{cases} \tag{10}$$

2.2. Plasmon polariton
In case of plasmon-polariton, quasi-particle a is photon γ, quasi-particle b is plasmon, dispersion law

$$\varepsilon_a = \varepsilon_\gamma(k) = \frac{ck}{\sqrt{\epsilon_0}}, \ \varepsilon_b = \varepsilon_P(k) = \hbar\omega_P = \frac{ck_P}{\sqrt{\epsilon_0}}, \tag{11}$$

in which c is speed of light, ϵ_0 is dielectric constant , ω_P is plasmon frequency. Consider $\hbar = c = 1$ and $\epsilon_0 = 1$.

Dispersion law of plasmon-polariton up and down are described in Fig.1.

Transformation parameters u^2, v^2 are described in Fig.2.

When k increases, in the upper line plasmon becomes photon, while photon become plasmon in the lower line.

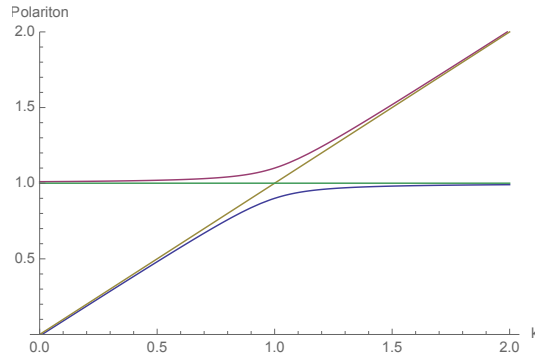This property could be applied to complex systems.
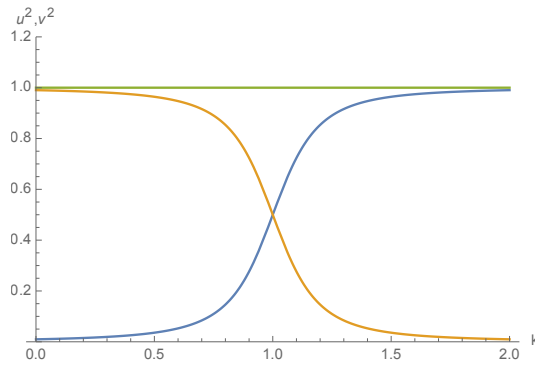
Figure 1. Plasmon polariton dispersion law.

Figure 2. Transformation parameters.

3. Application for financial market

Consider two complex systems α and β , their quasi-interaction is described by second quantized quasi-Hamiltonian

$$\hat{H} = \sum_{\kappa} \left\{ [\varepsilon_\alpha(\kappa) + \varepsilon_{\alpha 0}] \, \alpha_\kappa^+ \alpha_\kappa + [\varepsilon_\beta(\kappa) + \varepsilon_{\beta 0}] \, \beta_\kappa^+ \beta_\kappa + G_\kappa \left[\alpha_\kappa^+ \beta_\kappa + \beta_\kappa \alpha_\kappa \right] \right\}, \tag{12}$$

in which $\varepsilon_\alpha(\kappa)$, $\varepsilon_\beta(\kappa)$ are quasi-energy of complex systems, calculated from quasi-vacuum $\varepsilon_{\alpha 0}, \varepsilon_{\beta 0}$; quasi-annihilation operator $\alpha_\kappa, \beta_\kappa$ quasi-creation operator $\alpha_\kappa^+, \beta_\kappa^+$, action coordinates quasi-pulse κ ; and $G(\kappa)$ quasi-interacting constant.

Statistical parameter x is expended another space dimension. $P_\alpha(x), P_\beta(x)$ are x distribution functions corresponding to complex systems, satisfy normalized condition on X

$$\begin{aligned} \int_X P_\alpha(x)\,dx &= 1, \\ \int_X P_\beta(x)\,dx &= 1, \end{aligned} \tag{13}$$

quasi- Hamiltonian is rewritten

$$\hat{H} = \sum_{\kappa} \int_X dx \left\{ [\varepsilon_\alpha(\kappa) + \varepsilon_{\alpha 0} P_\alpha(x)] \, \alpha_\kappa^+ \alpha_\kappa + [\varepsilon_\beta(\kappa) + \varepsilon_{\beta 0} P_\beta(x)] \, \beta_\kappa^+ \beta_\kappa + G_\kappa(x) \left[\alpha_\kappa^+ \beta_\kappa + \beta_\kappa \alpha_\kappa \right] \right\}. \tag{14}$$

Bogoliubov method could be applied to diagonalize quasi-Hamiltonian of interacting complex systems to non-interacting complex systems, described by quasi-annihilation operators $\hat{A}$ and

$\hat{B}$ ($\hat{B}^+$, $\hat{B}^+$ are corresponding quasi-creation operators) at quasi-energy Ω_A and Ω_B. In this investigation, all parameters are real. After diagonalization, Hamiltonian becomes

$$\hat{H} = \sum_{\kappa} \int_X dx \left\{ \Omega_A(\kappa,x)\,\hat{A}_\kappa^+\hat{A}_\kappa + \Omega_B(\kappa,x)\,\hat{B}_\kappa^+\hat{B}_\kappa \right\} = \sum_{\kappa} \int_X dx\,\hat{H}_\kappa(x) \tag{15}$$

Bogoliubov direct transformation

$$\hat{A}_\kappa = U_\kappa \alpha_\kappa + V_\kappa \beta_\kappa,\ \hat{B}_\kappa = -U_\kappa \alpha_\kappa + V_\kappa \beta_\kappa. \tag{16}$$

Bogoliubov inverse transformation

$$\alpha_\kappa = U_\kappa \hat{A}_\kappa - V_\kappa \hat{B}_\kappa,\ \beta_\kappa = V_\kappa \hat{A}_\kappa + U_\kappa \hat{B}_\kappa. \tag{17}$$

Normalization condition

$$U_\kappa^2 + V_\kappa^2 = 1, \tag{18}$$

For a given quasi Hamiltonian of interacting complex systems $\{\varepsilon_\alpha,\ \varepsilon_\beta,\ G\}$, $\{U,\ V\}$ need to be changed with normalization condition $U^2 + V^2 = 1$ to get the new quasi Hamiltonian of non-interacting complex system $\{\Omega_A,\ \Omega_B\}$

$$\begin{aligned}\hat{H}_\kappa(x) &= [\varepsilon_\alpha(\kappa) + \varepsilon_{\alpha 0}P_\alpha(x)]\alpha_\kappa^+\alpha_\kappa + [\varepsilon_\beta(\kappa) + \varepsilon_{\beta 0}P_\beta(x)]\beta_\kappa^+\beta_\kappa + G_\kappa(x)[\alpha_\kappa^+\beta_\kappa + \beta_\kappa\alpha_\kappa] \\ &= \Omega_A(\kappa,x)\,\hat{A}_\kappa^+\hat{A}_\kappa + \Omega_B(\kappa,x)\,\hat{B}_\kappa^+\hat{B}_\kappa.\end{aligned} \tag{19}$$

In the mathematical point of view, it is a transformation

$$\{\varepsilon_\alpha,\ \varepsilon_\beta,\ G\} \Longleftrightarrow \{\Omega_A,\ \Omega_B\,U\}. \tag{20}$$

Dispersion law of the new system

$$\begin{aligned}\Omega_{\hat{A},\hat{B}}(\kappa,x) &= \tfrac{1}{2}[\varepsilon_\alpha(k) + \varepsilon_{\alpha 0}P_\alpha(x) + \varepsilon_\beta(k) + \varepsilon_{\alpha 0}P_\alpha(x)] \\ &\quad \pm \tfrac{1}{2}\sqrt{[\varepsilon_\alpha(k) + \varepsilon_{\alpha 0}P_\alpha(x) - \varepsilon_\beta(k) - \varepsilon_{\alpha 0}P_\alpha(x)]^2 + 4G_\kappa^2(x)},\end{aligned} \tag{21}$$

with transformation factor

$$\begin{aligned}U_\kappa^2(x) &= \tfrac{1}{2}\left\{ 1 + \frac{|\varepsilon_\alpha(k) + \varepsilon_{\alpha 0}P_\alpha(x) - \varepsilon_\beta(k) - \varepsilon_{\alpha 0}P_\alpha(x)|}{\sqrt{[\varepsilon_\alpha(k) + \varepsilon_{\alpha 0}P_\alpha(x) + \varepsilon_\beta(k) + \varepsilon_{\alpha 0}P_\alpha(x)]^2 + 4G_k^2(x)}} \right\} \\ &= 1 - V_\kappa^2(x).\end{aligned} \tag{22}$$

Increasing κ in the lower line, α becomes β, while in the upper line, β becomes α.

In financial market, returns distribution function x transform from Boltzman-like $P_B(x) = (1/2\pi\lambda)\exp(-x/\lambda)$ to Gaussian-like $P_G(x) = (1/\sqrt{2\pi}\sigma)\exp(-x^2/4\sigma^2)$ with corresponding statistical factors λ, σ, in this case, action coordinates κ is time t. Apply Bogoliubov method, Hamiltonian can be written as

$$\begin{aligned}\hat{H}_t(x) &= [\varepsilon_\alpha(t) + \varepsilon_{\alpha 0}P_\alpha(t)]\alpha_t^+\alpha_t + [\varepsilon_\beta(t) + \varepsilon_{\beta 0}P_\beta(t)]\beta_t^+\beta_t + G_t(x)[\alpha_t^+\beta_t + \beta_t\alpha_t] \\ &= \Omega_U(t,x)\,\hat{A}_\kappa^+\hat{A}_\kappa + \Omega_D(t,x)\,\hat{B}_\kappa^+\hat{B}_\kappa,\end{aligned} \tag{23}$$

in which α_t, β_t are quasi-annihilation operators, which describes impaction with quasi-energy $\varepsilon_\alpha(t) = bt$ to financial market at quasi-static energy $\varepsilon_\beta(t) = \varepsilon_M$. The evolution of returns distribution from Boltzmann like to Gaussian like is described in Fig. 3 and Fig. 4

In econophysics systems, quasi-energy of complex systems is identified by market returns. As observed in our previous works[8, 9], there exists the fluctuation of market returns distribution. Most of the published works investigated only on Gaussian-Boltzmann distribution transformation but not the reverse process. This works showed that there might exist Boltzmann-Gaussian distribution transformation also.
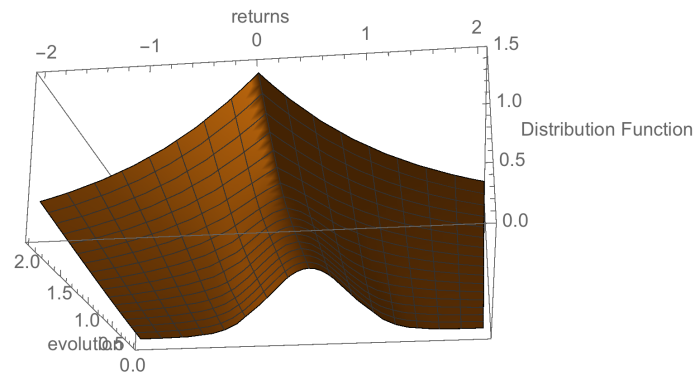
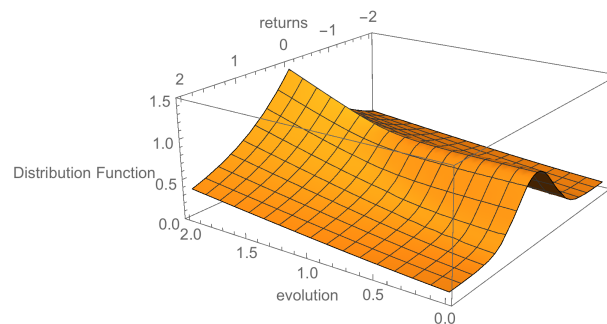**Figure 3.** Returns distribution evolution.

Figure 4. Returns distribution evolution.

4. Conclusion

After a brief presentation of general Bogoliubov method, investigations in Bosonic system and plasmon-polariton system are made. A transition between two lines of dispersion is observed.

In previous publications, a transition of market returns distribution function is realized in some investigations. Some explanations are made, with the hope to understand the dynamic of this transition. In this investigation, Bogoliubov method is used to describe the transition, as another view of this phenomenon. Beyond that, the result of this work opens a possibility to understand that there exist the vice-versa transformation, which is never ever been mentioned before.

References

[1] Bogolubov N 1947 *J. Phys. USSR* **11** 23–32
[2] Bogoliubov N 1958 *Sov. Phys. - JETP* **7** 41–46
[3] Bogoliubov N 1958 *Nuovo Cim.* **7** 794–805
[4] Dodonov V, Dremin I, Man'ko O, Man'ko V and Polynkin P 1998 *J. Russ. Laser Res.* **19** 427–463
[5] Honegger R and Rieckers A 1996 *J. Math. Phys.* **37** 4292–4309
[6] Nieto M and Truax D 1997 *Fortsch.Phys.* **45** 145–156
[7] Shumovskii A 1991 *Theor. Math. Phys.* **89** 1323–1329
[8] Chu T A, Do H L and Nguyen A V 2013 *Communications in Physics* **23** 185–191
[9] Anh C T, Lan N T and Viet N A 2014 *Journal of Physics: Conference Series* **537** 012005