Controlling heat flows among three reservoirs asymmetrically coupled to two two-level systems

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We study heat flows among three thermal reservoirs via two two-level systems (TLSs). Two reservoirs are coupled to one TLS and the third reservoir to the second TLS. The two TLSs are also coupled to each other, thus bridging the third reservoir with the two other reservoirs. We show that the magnitudes and directions of the reservoirs' heat currents can be controlled by varying the various damping rates of the two TLSs due to coupling with the corresponding reservoirs. First, it is shown that by changing the damping rate due to one reservoir, magnitudes of heat currents of the other two reservoirs can behave in completely different manners, namely, although one may be enhanced, the other may instead be suppressed, and vice versa. Second, the sign of the heat current of one reservoir may change (i.e., crossover from heat absorption to heat release, or vice versa) if a damping rate or the coupling strength between the two TLSs is swept through a critical value, which depends on the temperature settings for the three reservoirs. Due to the asymmetric couplings of the two TLSs to the three reservoirs, the thermal rectification occurs without introducing any additional asymmetry to the systems.

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I. INTRODUCTION

Understanding classical thermodynamics at the quantum level has given birth to the subject of quantum thermodynamics [1], which provides a new approach to explore the microscopic world as well as a microscopic alternative to test the fundamental laws of thermodynamics. Quantum thermodynamics has attracted more and more interest from different perspectives and has achieved considerable progress. By means of quantum resources, various quantum analogues [2–11] of classical heat engines have been constructed, such as Carnot engines [8] and Otto cycles [9–11]. Based only on the temperature differences of the heat baths, without using external sources of work and prescribed unitary transformations, a scheme of a self-contained quantum refrigerator has been proposed [12] and extensively studied [13–17].

In addition to the quantum heat engine and refrigerator, the quantum devices that are based on the controlling of thermal transport, such as the heat rectifiers [18–23], heat transistors [24,25], heat logical gates [26], thermal memory [27], and thermal ratchet [28], have also become one of the goals of recent research in quantum thermodynamics. The thermal rectifiers are components which exhibit an asymmetric flux when the temperatures at their ends are inverted. In Ref. [18], Zhang *et al.* studied thermal rectification in quantum spin-chain systems by means of quantum master equations and found that the sign of rectification can be changed when one changes the magnetic field, temperature, anisotropy, and system size. In Ref. [19], Roy proposed an optical diode that works at low intensity of light in the fully quantum regime. In Ref. [20], Fratini *et al.* studied the one-dimensional Fabry-Perot interferometer built with a pair of two-level systems (TLSs), which can operate as a microscopic integrated optical rectifier. In Ref. [21], an optimal rectification in the ultrastrong-coupling regime of two TLSs was proposed by Werlang *et al.*, which under a certain condition allows heat flow in one direction but completely forbids heat flow in the opposite direction. The authors of Ref. [21] stressed the necessity of the strong-coupling formalism over the phenomenological approach for predicting optimal rectification. In Ref. [22], Zhang *et al.* studied ballistic thermal transport in three-terminal atomic nanojunctions. In a recent work [24], Joulain *et al.* designed a quantum thermal transistor made up of three interacting TLSs, each of which is in turn coupled to a thermal reservoir. It was shown that a small variation of the heat current in one TLS can considerably amplify the variations of the other two in a wide range of energy parameters and temperatures [24]. Those quantum devices closely rely on the ability to manage the heat flow through a set of coupled quantum systems. Moreover, the control of heat flow in interacting quantum systems is also a fundamental question since open quantum systems are always exposed to their environments, in particular to the thermal baths [29–33]. Therefore, it is meaningful from both aspects of theory and application to investigate the controlling of the magnitudes as well as directions of heat flows among thermal reservoirs out of equilibrium.

In most studies (e.g., [29–33]) of heat transport, only two thermal baths are involved, each of which is coupled to a terminal site of the coupled systems so that the absorbed heat of the cold bath is always equal to the released heat of the hot bath and their variations are synchronous. If one more thermal bath is added in the end of the coupled systems, namely, three heat baths are asymmetrically coupled to the systems, then the behaviors of heat currents regarding the three baths, in particular their magnitudes' variations and direction switch, become rich and remain unclear. For this purpose, in this work, we consider the system comprised of two interacting TLSs A and B. The TLS A is coupled at the same time

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with two thermal reservoirs $R_A^{(1)}$ and $R_A^{(2)}$, while the TLS B is coupled only to a single reservoir R_B. In such an asymmetric coupling regime, we are interested in the control of both the magnitudes and directions of the heat currents of each of the three reservoirs by adjusting the strengths of the different existent couplings. It is found that (i) increasing the magnitude of one heat current does not always promote the other two: one of them may instead be suppressed, e.g., in the situation in which two of three reservoirs have an equal temperature, and (ii) when the three reservoirs possess different temperatures, the one with intermediate temperature may experience a crossover from heat release to heat absorption, or vice versa (i.e., an inversion of heat current direction occurs). Moreover, due to the inherent asymmetry of the model, the thermal rectification occurs without introducing additional asymmetry into the system. As we shall show, the heat transfer is more favorable from the TLS B to A, while restrained in the opposite direction.

The paper is organized as follows. In Sec. II, we present the model and construct a quantum master equation to describe the dynamics of the two TLSs in their eigenstate representation. By virtue of the steady-state solution of the master equation, we discuss the control of the magnitudes and directions of the three reservoirs' heat currents in Secs. III A and III B, respectively, and demonstrate the thermal rectification effect in Sec. III C. Finally, we conclude in Sec. IV.

II. THE MODEL

Consider two interacting TLSs A and B governed by the Hamiltonian (with $\hbar = 1$)

$$\hat{H}_S = \frac{\omega_A}{2}\hat{\sigma}_z^A + \frac{\omega_B}{2}\hat{\sigma}_z^B + g(\hat{\sigma}_+^A\hat{\sigma}_-^B + \hat{\sigma}_-^A\hat{\sigma}_+^B), \tag{1}$$

where $\hat{\sigma}_z^\mu = |1\rangle_\mu\langle 1| - |0\rangle_\mu\langle 0|$ is a Pauli operator, $\hat{\sigma}_+^\mu = |1\rangle_\mu\langle 0|$ ($\hat{\sigma}_-^\mu = |0\rangle_\mu\langle 1|$) is the raising (lowering) operator for a TLS $\mu = A,B$ with transition frequency ω_μ, and g denotes the interaction strength between the two TLSs.

In the usual consideration, each TLS is assumed to be coupled to a single thermal reservoir and heat currents may arise between the reservoirs in the stationary state scenario. Here, we study the problem of how the heat will flow when one of the two TLSs is instead coupled to multiple independent reservoirs. To be definite, let us simultaneously couple the TLS A to two boson heat reservoirs $R_A^{(1)}$ and $R_A^{(2)}$ with temperatures $T_A^{(1)}$ and $T_A^{(2)}$, respectively, while allowing the TLS B to only interact with a single boson heat reservoir R_B with temperature T_B (see Fig. 1). The reservoirs can be described by the Hamiltonian

$$\hat{H}_R = \sum_l \omega_{al}\hat{a}_l^\dagger\hat{a}_l + \sum_m \omega_{bm}\hat{b}_m^\dagger\hat{b}_m + \sum_n \omega_{cn}\hat{c}_n^\dagger\hat{c}_n, \tag{2}$$

with the creation and annihilation operators $\hat{a}_l^\dagger$ ($\hat{b}_m^\dagger$, $\hat{c}_n^\dagger$) and $\hat{a}_l$ ($\hat{b}_m$, $\hat{c}_n$) describing the lth (mth, nth) boson mode with frequency ω_{al} (ω_{bm}, ω_{cn}) of the reservoir $R_A^{(1)}$ ($R_A^{(2)}$, R_B). The interaction Hamiltonian of the two TLSs with their respective

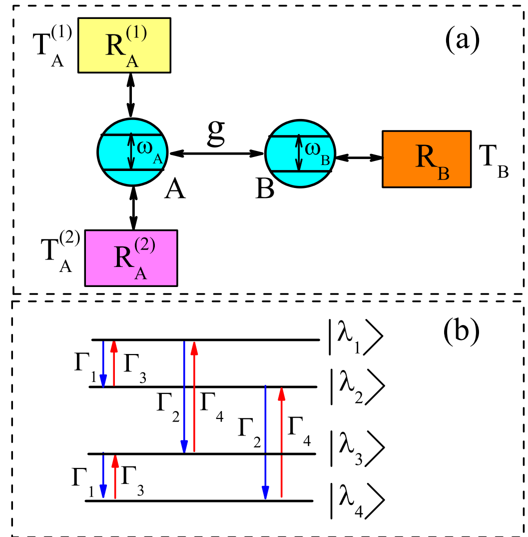

FIG. 1. (a) Schematic diagram of the physical model under consideration. Two TLSs A and B are coupled to each other with a strength g. The TLS A is coupled simultaneously to two heat reservoirs $R_A^{(1)}$ and $R_A^{(2)}$ with temperatures $T_A^{(1)}$ and $T_A^{(2)}$, respectively, while B is coupled to a single heat reservoir R_B with temperature T_B. (b) The levels of the four eigenstates $|\lambda_i\rangle$ ($i = 1,2,3,4$) of Hamiltonian $\hat{H}_S$, given by Eq. (1), for the two coupled TLSs. In the presence of the baths, there exist bath-induced exciting and damping processes among the four eigenstates. The effective transition rates are given by Γ_1, Γ_2, Γ_3, and Γ_4 in Eq. (12).

reservoirs reads

$$\hat{H}_I = \sum_l \kappa_{A,l}^{(1)}(\hat{\sigma}_+^A\hat{a}_l + \hat{\sigma}_-^A\hat{a}_l^\dagger) + \sum_m \kappa_{A,m}^{(2)}(\hat{\sigma}_+^A\hat{b}_m + \hat{\sigma}_-^A\hat{b}_m^\dagger) + \sum_n \kappa_{B,n}(\hat{\sigma}_+^B\hat{c}_n + \hat{\sigma}_-^B\hat{c}_n^\dagger), \tag{3}$$

where $\kappa_{A,l}^{(1)}$ ($\kappa_{A,m}^{(2)}$) represent the coupling strengths of A and the lth (mth) mode of the reservoir $R_A^{(1)}$ ($R_A^{(2)}$), while $\kappa_{B,n}$ denote that of B and the nth mode of R_B.

In the present model, the heat transports include the direct one between $R_A^{(1)}$ and $R_A^{(2)}$ through the common TLS A and the indirect ones between $R_A^{(1)}$, $R_A^{(2)}$, and R_B through both the TLSs A and B. The interaction strength g between A and B thus becomes an important parameter that affects the heat currents. In the strong A-B coupling regime, the dissipation of each TLS depends not only on its coupling with the corresponding reservoir(s), which is assumed to be weak, but also on the coupling between themselves. Therefore, we should construct the master equation to describe the system evolution in the representation of the eigenstates of the full system Hamiltonian $\hat{H}_S$. The eigenstates (eigenenergies) of $\hat{H}_S$, given by Eq. (1), are $|\lambda_1\rangle = |11\rangle$ [$E_1 = \frac{1}{2}(\omega_A + \omega_B)$], $|\lambda_2\rangle = \sin(\theta/2)|01\rangle + \cos(\theta/2)|10\rangle$ ($E_2 = \alpha$), $|\lambda_3\rangle = \cos(\theta/2)|01\rangle - \sin(\theta/2)|10\rangle$ ($E_3 = -\alpha$), and $|\lambda_4\rangle = |00\rangle$ [$E_4 = -\frac{1}{2}(\omega_A + \omega_B)$], with $\alpha = \sqrt{g^2 + \frac{1}{4}(\omega_A - \omega_B)^2}$ and $\tan\theta = 2g/(\omega_A - \omega_B)$. In terms of the eigenstates of $\hat{H}_S$, the total Hamiltonian $\hat{H} = \hat{H}_S + \hat{H}_R + \hat{H}_I$

can be rewritten as

$$\hat{H} = \sum_{i=1}^{4} E_i|\lambda_i\rangle\langle\lambda_i| + \hat{H}_R + \sum_j \hat{H}'_{I,j}, \tag{4}$$

where

$$\begin{aligned}\hat{H}'_{I,j} = &\sum_l \kappa^{(1)}_{A,l}[V^\dagger_{A,j}(\omega_j)\hat{a}_l + V_{A,j}(\omega_j)\hat{a}^\dagger_l] \\ &+\sum_m \kappa^{(2)}_{A,m}[V^\dagger_{A,j}(\omega_j)\hat{b}_m + V_{A,j}(\omega_j)\hat{b}^\dagger_m] \\ &+\sum_n \kappa_{B,n}[V^\dagger_{B,j}(\omega_j)\hat{c}_n + V_{B,j}(\omega_j)\hat{c}^\dagger_n].\end{aligned} \tag{5}$$

In Eq. (5), $V_{\mu,j}(\omega_j)$ denotes the eigenoperators of the Hamiltonian H_S, such that $[H_S,V_{\mu,j}(\omega_j)] = -\omega_j V_{\mu,j}(\omega_j)$, and ω_j stands for the eigenfrequency $\omega_1 = \frac{1}{2}(\omega_A+\omega_B)-\alpha$ corresponding to transitions $|\lambda_1\rangle \leftrightarrow |\lambda_2\rangle$ and $|\lambda_3\rangle \leftrightarrow |\lambda_4\rangle$, while $\omega_2 = \frac{1}{2}(\omega_A+\omega_B)+\alpha$ corresponds to transitions $|\lambda_1\rangle \leftrightarrow |\lambda_3\rangle$ and $|\lambda_2\rangle \leftrightarrow |\lambda_4\rangle$. Explicitly, the forms of $V_{\mu,j}(\omega_j)$ are obtained as follows:

$$\begin{aligned}V_{A,1}(\omega_1) &= \sin\frac{\theta}{2}(|\lambda_2\rangle\langle\lambda_1| - |\lambda_4\rangle\langle\lambda_3|), \\ V_{A,2}(\omega_2) &= \cos\frac{\theta}{2}(|\lambda_3\rangle\langle\lambda_1| + |\lambda_4\rangle\langle\lambda_2|), \\ V_{B,1}(\omega_1) &= \cos\frac{\theta}{2}(|\lambda_2\rangle\langle\lambda_1| + |\lambda_4\rangle\langle\lambda_3|), \\ V_{B,2}(\omega_2) &= \sin\frac{\theta}{2}(-|\lambda_3\rangle\langle\lambda_1| + |\lambda_4\rangle\langle\lambda_2|).\end{aligned} \tag{6}$$

In the above, we have presented the forms of $V_{\mu,j}(\omega_{\mu,j})$ when $\omega_j > 0$, otherwise $V_{\mu,j}(-\omega_j) = V^\dagger_{\mu,j}(\omega_j)$.

Because the coupling between a TLS and its reservoir(s) is weak, the equation of motion for the two coupled TLSs can be derived within the framework of the Born-Markov approximation as

$$\dot{\rho} = -i[H_S,\rho] + \mathcal{L}^{(1)}_A[\rho] + \mathcal{L}^{(2)}_A[\rho] + \mathcal{L}_B[\rho], \tag{7}$$

where the Lindblad operators $\mathcal{L}^{(k)}_A[\rho]$ $(k = 1,2)$ and $\mathcal{L}_B[\rho]$ are given by

$$\begin{aligned}\mathcal{L}^{(k)}_A[\rho] = &\sum_j \Gamma^{(k)}_A(\omega_j)\big\{\big[\bar{n}^{(k)}_A(\omega_j)+1\big][2V_{A,j}(\omega_j)\rho V^\dagger_{A,j}(\omega_j) \\ &- V^\dagger_{A,j}(\omega_j)V_{A,j}(\omega_j)\rho - \rho V^\dagger_{A,j}(\omega_j)V_{A,j}(\omega_j)] \\ &+\bar{n}^{(k)}_A(\omega_j)[2V^\dagger_{A,j}(\omega_j)\rho V_{A,j}(\omega_j) \\ &- V_{A,j}(\omega_j)V^\dagger_{A,j}(\omega_j)\rho - \rho V_{A,j}(\omega_j)V^\dagger_{A,j}(\omega_j)]\big\},\end{aligned} \tag{8}$$

and

$$\begin{aligned}\mathcal{L}_B[\rho] = &\sum_j \Gamma_B(\omega_j)\{[\bar{n}_B(\omega_j)+1][2V_{B,j}(\omega_j)\rho V^\dagger_{B,j}(\omega_j) \\ &- V^\dagger_{B,j}(\omega_j)V_{B,j}(\omega_j)\rho - \rho V^\dagger_{B,j}(\omega_j)V_{B,j}(\omega_j)] \\ &+\bar{n}_B(\omega_j)[2V^\dagger_{B,j}(\omega_j)\rho V_{B,j}(\omega_j) - V_{B,j}(\omega_j) \\ &\times V^\dagger_{B,j}(\omega_j)\rho - \rho V_{B,j}(\omega_j)V^\dagger_{B,j}(\omega_j)]\},\end{aligned} \tag{9}$$

where $\Gamma^{(k)}_A(\omega_j)$ and $\Gamma_B(\omega_j)$ are spectral functions which depend on the coupling strengths $\kappa^{(k)}_{A,l}$ and $\kappa_{B,n}$ and characterize damping rates of the system of two TLSs due to interaction with the reservoirs $R^{(k)}_A$ and R_B, respectively. For simplicity, we suppose that $\Gamma^{(k)}_A(\omega_j) = \Gamma^{(k)}_A$ and $\Gamma_B(\omega_j) = \Gamma_B$ are frequency independent throughout the paper. The average photon numbers $\bar{n}^{(k)}_A(\omega_j)$ and $\bar{n}_B(\omega_j)$ of the reservoir $R^{(k)}_A$ and R_B depend on the temperature of the reservoirs and take the forms

$$\bar{n}^{(k)}_A(\omega_j) = \frac{1}{\exp\big[\frac{\omega_j}{T^{(k)}_A}\big]-1}, \quad \bar{n}_B(\omega_j) = \frac{1}{\exp\big[\frac{\omega_j}{T_B}\big]-1}. \tag{10}$$

III. THE CONTROL OF HEAT CURRENTS

In order to reveal the heat flow when the system reaches the stationary state, we should derive the steady-state solution ρ^S of the master equation (7). By letting $\dot{\rho}^S = 0$ in Eq. (7), we obtain that all off-diagonal elements $\rho^S_{ii'} = \langle\lambda_i|\rho^S|\lambda_{i'}\rangle$ with $i \neq i'$ are zero, while the diagonal ones are as follows:

$$\begin{aligned}\rho^S_{11} &= \frac{\Gamma_1\Gamma_2}{(\Gamma_1+\Gamma_3)(\Gamma_2+\Gamma_4)}, \\ \rho^S_{22} &= \frac{\Gamma_2\Gamma_3}{(\Gamma_1+\Gamma_3)(\Gamma_2+\Gamma_4)}, \\ \rho^S_{33} &= \frac{\Gamma_1\Gamma_4}{(\Gamma_1+\Gamma_3)(\Gamma_2+\Gamma_4)}, \\ \rho^S_{44} &= \frac{\Gamma_3\Gamma_4}{(\Gamma_1+\Gamma_3)(\Gamma_2+\Gamma_4)},\end{aligned} \tag{11}$$

where Γ_1, Γ_2, Γ_3, and Γ_4 denote the net transition rates among the eigenstates $\{|\lambda_i\rangle\}$ shown in Fig. 1(b), which are given by

$$\begin{aligned}\Gamma_1 &= \sin^2(\theta/2)\big[J^{(1)}_A(\omega_1) + J^{(2)}_A(\omega_1)\big] + \cos^2(\theta/2)J_B(\omega_1), \\ \Gamma_2 &= \cos^2(\theta/2)\big[J^{(1)}_A(\omega_2) + J^{(2)}_A(\omega_2)\big] + \sin^2(\theta/2)J_B(\omega_2), \\ \Gamma_3 &= \sin^2(\theta/2)\big[J^{(1)}_A(-\omega_1) + J^{(2)}_A(-\omega_1)\big] \\ &\quad + \cos^2(\theta/2)J_B(-\omega_1), \\ \Gamma_4 &= \cos^2(\theta/2)\big[J^{(1)}_A(-\omega_2) + J^{(2)}_A(-\omega_2)\big] \\ &\quad + \sin^2(\theta/2)J_B(-\omega_2),\end{aligned} \tag{12}$$

with

$$\begin{aligned}J^{(k)}_A(-\omega_j) &= \Gamma^{(k)}_A(\omega_j)\big[\bar{n}^{(k)}_A(\omega_j)+1\big], \\ J^{(k)}_A(\omega_j) &= \Gamma^{(k)}_A(\omega_j)\bar{n}^{(k)}_A(\omega_j), \\ J_B(-\omega_j) &= \Gamma_B(\omega_j)[\bar{n}_B(\omega_j)+1], \\ J_B(\omega_j) &= \Gamma_B(\omega_j)\bar{n}_B(\omega_j).\end{aligned} \tag{13}$$

In this work, we are interested in the heat flow among the three reservoirs after the total system has reached the stationary state. The heat currents associated with the reservoirs $R^{(k)}_A$ and R_B can be defined, respectively, as [34,35]

$$Q^{(k)}_A = Tr\big\{\mathcal{L}^{(k)}_A[\rho^S]H_S\big\}, \quad Q_B = Tr\{\mathcal{L}_B[\rho^S]H_S\}. \tag{14}$$

From the perspective of reservoir, a positive heat current means heat release from the reservoir, while a negative value implies heat absorption by the reservoir. Therefore, a change of sign of the heat current can be used as a witness for a crossover between heat absorption and heat release, or vice versa.

A. Magnitude modulation of the heat current

When each of two interacting TLSs is coupled only to a single thermal reservoir, the released heat (if any) of one reservoir should be equal to the absorbed heat of another one in the steady-state regime due to the energy conservation. Therefore, if the output (input) heat is increased for one reservoir, e.g., by increasing the coupling strength of the reservoir to the system, then the input (output) heat from another reservoir is also increased, and vice versa. By contrast, here the two TLSs are asymmetrically coupled to three thermal reservoirs, namely, the TLS A is coupled at the same time to two reservoirs $R_A^{(1)}$ and $R_A^{(2)}$, while the TLS B is coupled just to the single one R_B. So there exist three heat currents $Q_A^{(1)}$, $Q_A^{(2)}$, and Q_B associated with these three reservoirs. In this connection, a question arises: how does the variation of the magnitude (i.e., the absolute value) of one heat current (e.g., $|Q_A^{(1)}|$) affect that of the other two (e.g., $|Q_A^{(2)}|$ and $|Q_B|$)? In particular, do they vary in the same manner, namely, all of them increase or decrease simultaneously? Here, we shall show that an increase (decrease) of one heat current, e.g., via changing the corresponding damping rate, does not always lead to the simultaneous increases (decreases) of the other two, namely, one of the latter can instead be reduced (amplified).

To be explicit and without loss of generality, let us take the case of $T_A^{(1)} = T_A^{(2)} \neq T_B$ as an example to illustrate the unsynchronized variations of the heat currents. If we change the damping rate Γ_B, we get the same variation trends with respect to the magnitudes of the heat currents $|Q_A^{(1)}|$, $|Q_A^{(2)}|$, and $|Q_B|$, namely, $|Q_A^{(1)} = |Q_A^{(2)}| = |Q_B| = 0$ when $\Gamma_B = 0$, but they all increase with increasing Γ_B, similar to the usual situation involving only two reservoirs as stated above. Nevertheless, if we alter $\Gamma_A^{(1)}$ or $\Gamma_A^{(2)}$, the variations of the three heat currents no longer synchronize.

The dependence of heat currents and their magnitudes (i.e., $|Q_A^{(1)}|$, $|Q_A^{(2)}|$, and $|Q_B|$) on the values of $\Gamma_A^{(2)}$ is shown in Fig. 2(a) and its inset, where we set $T_A^{(1)} = T_A^{(2)} < T_B$. We observe that $|Q_A^{(1)}| = Q_B > 0$ and $|Q_A^{(2)}| = 0$ at $\Gamma_A^{(2)} = 0$, but an increase in $\Gamma_A^{(2)}$ causes increases in both $|Q_A^{(2)}|$ and $|Q_B|$, which are accompanied by a decrease in $|Q_A^{(1)}|$. This reveals the fact that the heat amount absorbed by $R_A^{(1)}$ in the presence of $R_A^{(2)}$ is smaller than that without $R_A^{(2)}$. In other words, adding an additional cold source $R_A^{(2)}$ and increasing its coupling strength to the TLS A favor the heat release of the hot source R_B and, at the same time, suppresses the heat absorption of the other cold source $R_A^{(1)}$.

We now reset the parameters to $T_A^{(1)} = T_A^{(2)} > T_B$, i.e., the reservoirs $R_A^{(1)}$ and $R_A^{(2)}$ are hot sources serving equally as a heat supplier and R_B is the cold source playing the role of a heat absorber. Although the variations of each of $Q_A^{(1)}$, $Q_A^{(2)}$, and Q_B displayed in Fig. 2(b) are different from those displayed in Fig. 2(a), we still observe similar behaviors of the magnitudes of the heat currents [see the inset of Fig. 2(b)] as in the previous case with $T_A^{(1)} = T_A^{(2)} < T_B$, which should,

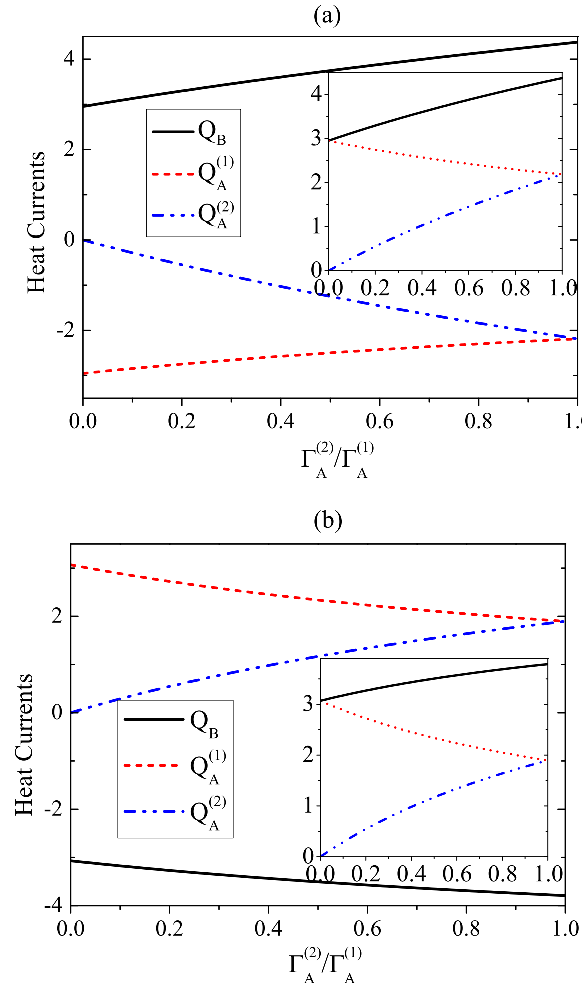

FIG. 2. Heat currents $Q_A^{(1)}$ (dashed line), $Q_A^{(2)}$ (dash-dotted line), and Q_B (solid line) as a function of the scaled damping rates $\Gamma_A^{(2)}/\Gamma_A^{(1)}$. The insets show the variations of $|Q_A^{(1)}|$, $|Q_A^{(2)}|$, and $|Q_B|$. The temperatures are set as (a) $T_A^{(1)} = T_A^{(2)} = \Gamma_A^{(1)} < T_B = 10\Gamma_A^{(1)}$ and (b) $T_A^{(1)} = T_A^{(2)} = 10\Gamma_A^{(1)} > T_B = \Gamma_A^{(1)}$. The other parameters are set as $\omega_A = 10\Gamma_A^{(1)}$, $\omega_B = 9\Gamma_A^{(1)}$, $g = \Gamma_A^{(1)}$, and $\Gamma_B = \Gamma_A^{(1)}$.

however, be interpreted as follows: adding an additional hot source $R_A^{(2)}$ and increasing its coupling strength to the TLS A favor the heat absorption by the cold source R_B and, at the same time, reduces the heat release from the other hot source $R_A^{(1)}$.

In order to understand physical reasons for the different behaviors of the heat current magnitudes depicted above, we derive from Eq. (14) the explicit expressions of the heat currents of the reservoirs $R_A^{(k)}$ and R_B in terms of steady-state populations as

$$Q_A^{(k)} = x_{11}^{(k)}\rho_{11}^S + x_{22}^{(k)}\rho_{22}^S + x_{33}^{(k)}\rho_{33}^S + x_{44}^{(k)}\rho_{44}^S, \tag{15}$$

with

$$x_{11}^{(k)} = 2\left[\sin^2\left(\frac{\theta}{2}\right)J_A^{(k)}(-\omega_1)E_{21} + \cos^2\left(\frac{\theta}{2}\right)J_A^{(k)}(-\omega_2)E_{31}\right],$$

$$x_{22}^{(k)} = 2\left[\sin^2\left(\frac{\theta}{2}\right)J_A^{(k)}(\omega_1)E_{12} + \cos^2\left(\frac{\theta}{2}\right)J_A^{(k)}(-\omega_2)E_{42}\right],$$

$$x_{33}^{(k)} = 2\left[\cos^2\left(\frac{\theta}{2}\right)J_A^{(k)}(\omega_2)E_{13} + \sin^2\left(\frac{\theta}{2}\right)J_A^{(k)}(-\omega_1)E_{43}\right],$$
$$x_{44}^{(k)} = 2\left[\cos^2\left(\frac{\theta}{2}\right)J_A^{(k)}(\omega_2)E_{24} + \sin^2\left(\frac{\theta}{2}\right)J_A^{(k)}(\omega_1)E_{34}\right]; \tag{16}$$

and

$$Q_B = y_{11}\rho_{11}^S + y_{22}\rho_{22}^S + y_{33}\rho_{33}^S + y_{44}\rho_{44}^S, \tag{17}$$

with

$$y_{11} = 2\left[\cos^2\left(\frac{\theta}{2}\right)J_B(-\omega_1)E_{21} + \sin^2\left(\frac{\theta}{2}\right)J_B(-\omega_2)E_{31}\right],$$
$$y_{22} = 2\left[\cos^2\left(\frac{\theta}{2}\right)J_B(\omega_1)E_{12} + \sin^2\left(\frac{\theta}{2}\right)J_B(-\omega_2)E_{42}\right],$$
$$y_{33} = 2\left[\sin^2\left(\frac{\theta}{2}\right)J_B(\omega_2)E_{13} + \cos^2\left(\frac{\theta}{2}\right)J_B(-\omega_1)E_{43}\right],$$
$$y_{44} = 2\left[\sin^2\left(\frac{\theta}{2}\right)J_B(\omega_2)E_{24} + \cos^2\left(\frac{\theta}{2}\right)J_B(\omega_1)E_{34}\right], \tag{18}$$

in which $E_{ii'} = E_i - E_{i'}$ $(i,i' = 1,2,3,4)$ denotes the energy differences between the two eigenstates $|\lambda_i\rangle$ and $|\lambda_{i'}\rangle$. The above Eqs. (15)–(18) demonstrate all the contributions of transitions between energy levels of the TLSs to the heat currents $Q_A^{(k)}$ and Q_B, respectively. Since E_{21} and E_{31} (E_{24} and E_{34}) are always negative (positive), the first (last) terms of the right-hand side of (15) and (17) contribute to heat absorption (release) of the corresponding reservoirs. As for the middle two terms, their signs are determined by the concrete parameters. It is worth pointing out that the magnitudes of heat currents are solely determined by the populations ρ_{11}^S, ρ_{22}^S, ρ_{33}^S, and ρ_{44}^S once the coefficients $x_{ii}^{(k)}$ and y_{ii} are given.

In Fig. 2(a), we have considered the dependences of magnitudes of the three heat currents on the damping rate $\Gamma_A^{(2)}$ of reservoir $R_A^{(2)}$ for $T_A^{(1)} = T_A^{(2)} < T_B$, and shown that although $|Q_A^{(2)}|$ grows with $\Gamma_A^{(2)}$, $|Q_A^{(1)}|$ is instead reduced. Since increasing $\Gamma_A^{(2)}$ leads to the stronger coupling between the reservoir $R_A^{(2)}$ and the TLSs, it is easy to understand that in this case, $R_A^{(2)}$ absorbs more heat from Q_B with an increase of $|Q_A^{(2)}|$. However, it remains unclear why $|Q_A^{(1)}|$ is decreased, namely, its heat absorption is suppressed. For the parameters used in Fig. 2(a), the first three terms in (15) are all negative with respect to $Q_A^{(1)}$, which implies that ρ_{11}^S, ρ_{22}^S, ρ_{33}^S contribute to the heat absorption and ρ_{44}^S contributes to the heat release of the reservoir $R_A^{(1)}$, therefore $|Q_A^{(1)}| = |x_{11}^{(k)}|\rho_{11}^S + |x_{22}^{(k)}|\rho_{22}^S + |x_{33}^{(k)}|\rho_{33}^S - x_{44}^{(k)}\rho_{44}^S$. From the variations of the populations with $\Gamma_A^{(2)}$, shown in Fig. 3(a), we observe that ρ_{44}^S dominates among the populations and increases with $\Gamma_A^{(2)}$, while the values of ρ_{11}^S, ρ_{22}^S, and ρ_{33}^S retain small values and slightly decrease. As a result, $|Q_A^{(1)}|$ is reduced with an increase of $\Gamma_A^{(2)}$.

When the temperatures are reset to $T_A^{(1)} = T_A^{(2)} > T_B$, as considered in Fig. 2(b), we still observe the different behaviors of $|Q_A^{(2)}|$ and $|Q_A^{(1)}|$ with $\Gamma_A^{(2)}$, namely, the former is increased while the latter is decreased. Here, for the used parameters, the first three terms in (15) are all negative and only the last term is positive, and, at the same time, for this temperature setting, $Q_A^{(1)} > 0$ in the whole range of $\Gamma_A^{(2)}$; therefore, we have $|Q_A^{(1)}| = -|x_{11}^{(k)}|\rho_{11}^S - |x_{22}^{(k)}|\rho_{22}^S - |x_{33}^{(k)}|\rho_{33}^S + x_{44}^{(k)}\rho_{44}^S$. The dependences of the populations on $\Gamma_A^{(2)}$ in Fig. 3(b) show that ρ_{44}^S decreases with $\Gamma_A^{(2)}$, while ρ_{11}^S, ρ_{22}^S, and ρ_{33}^S retain small values and slightly increase. Consequently, $|Q_A^{(1)}|$ is reduced with an increase of $\Gamma_A^{(2)}$.

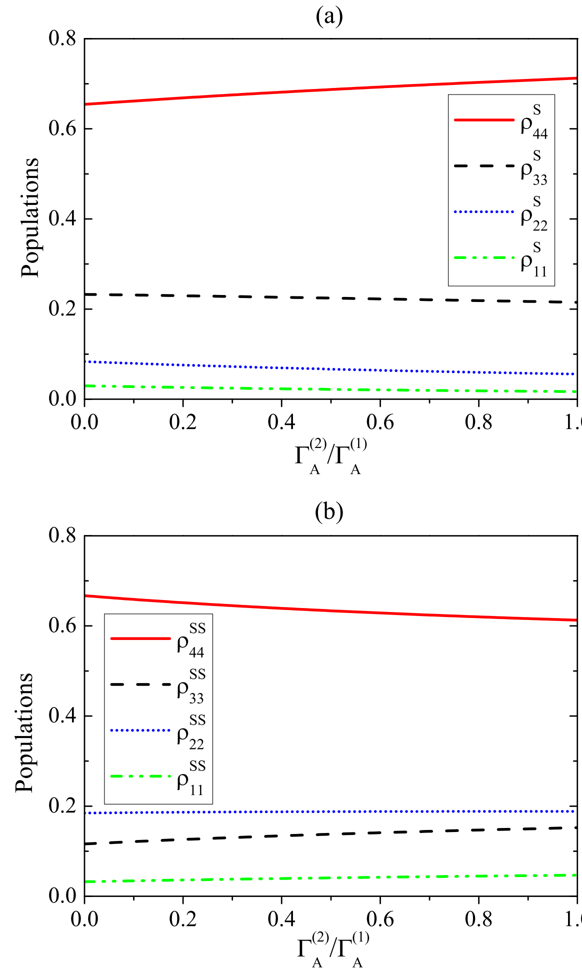

FIG. 3. Population variations with the scaled damping rate $\Gamma_A^{(2)}/\Gamma_A^{(1)}$ for the same parameters used in Figs. 2(a) and 2(b).

B. Direction switch of the heat current

So far, we have discussed the situation when the temperatures of two among the three reservoirs are equal and have demonstrated behaviors of the magnitudes of the heat currents by changing the damping rate of the joint two TLSs A and B due to an individual reservoir. In this section, we deal with the situation when the temperature of each of the three reservoirs is distinct. We are interested in controlling the heat transfer among the reservoirs, in particular the inversion of heat current direction by changing one damping rate at a time or the coupling strength between the two TLSs. We show that due to the competition between the heat current

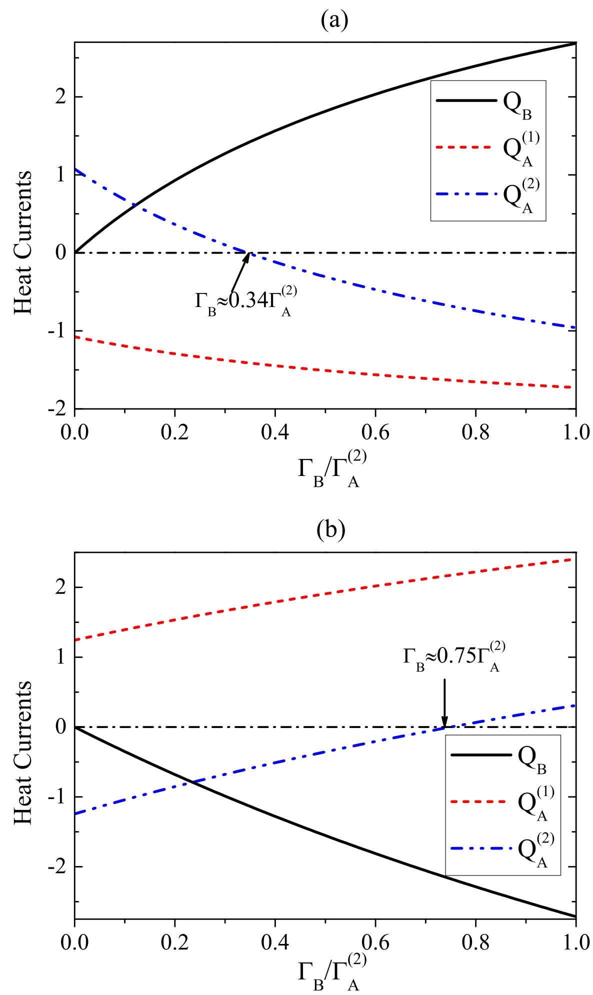

FIG. 4. Heat currents $Q_A^{(1)}$ (dashed line), $Q_A^{(2)}$ (dash-dotted line), and Q_B (solid line) as a function of the scaled damping rates $\Gamma_B/\Gamma_A^{(2)}$. The temperatures are chosen as (a) $T_A^{(1)}=\Gamma_A^{(2)}, T_A^{(2)}=6\Gamma_A^{(2)}$, $T_B=10\Gamma_A^{(2)}$ and (b) $T_A^{(1)}=10\Gamma_A^{(2)}, T_A^{(2)}=6\Gamma_A^{(2)}, T_B=\Gamma_A^{(2)}$. The other parameters are chosen as $\Gamma_A^{(1)}=0.5\Gamma_A^{(2)}, g=4\Gamma_A^{(2)}, \omega_A=10\Gamma_A^{(2)}$, and $\omega_B=9\Gamma_A^{(2)}$. The horizontal line at zero is added to guide the eye for the heat crossover.

to the low-temperature reservoir and the heat current from the high-temperature reservoir, the reservoir with intermediate temperature can experience an inversion of heat current direction.

We consider the variations of heat currents $Q_A^{(1)}$, $Q_A^{(2)}$, and Q_B when Γ_B is changed, in Fig. 4(a), for $T_B > T_A^{(2)} > T_A^{(1)}$ and, in Fig. 4(b), for $T_B < T_A^{(2)} < T_A^{(1)}$, namely, the reservoir R_A^2 is set possessing intermediate temperature. As is seen from Fig. 4(a), when Γ_B is increased from zero, R_B is connected to the others and releases heat (i.e., $Q_B > 0$), but the released heat by R_B is still small so the absorbed heat by $R_A^{(2)}$ from R_B is still less than that which it releases to $R_A^{(1)}$, resulting in a positive $Q_A^{(2)}$. In the course of increasing Γ_B, there appears a critical value at which the heat amount transferred from R_B to $R_A^{(2)}$ is in balance with the heat amount transferred from $R_A^{(2)}$ to $R_A^{(1)}$, leading to $Q_A^{(2)}=0$ [for the parameters used in Fig. 4(a), the critical value is $\Gamma_B\approx 0.34\Gamma_A^{(2)}$]. For any further increase in Γ_B, the heat amount coming into $R_A^{(2)}$ is larger than that going out, so $Q_A^{(2)}$ becomes negative. This picture indicates a crossover from heat release to heat absorption experienced by $R_A^{(2)}$, the reservoir with intermediate temperature. Interestingly, if the temperature order is reversed, the reservoir with intermediate temperature may also exhibit a crossover in the "reversed direction," i.e., from heat absorption to heat release, as shown in Fig. 4(b), in which the critical value for crossover is $\Gamma_B\approx 0.75\Gamma_A^{(2)}$. It can be verified that a crossover of heat current always occurs for the reservoir with intermediate temperature when the coupling between the two TLSs and a reservoir varies, no matter whether it is $R_A^{(1)}$, $R_A^{(2)}$, or R_B.

From the above discussion, we know that the reservoir with intermediate temperature would undergo a heat crossover at some critical values of the damping rates as they are increasing from zero. What do such critical values depend on? We shall now show that they depend on the setting of temperatures of the reservoirs. To do that, let us introduce the following notations: $\Delta_M=T_>-T_=$ and $\Delta_m=T_=-T_<$, with $T_>$, $T_=$, and $T_<$ being the highest, intermediate, and lowest temperature possessed by the reservoirs. First, consider the situation when R_B is the hot, $R_A^{(2)}$ the cool, and $R_A^{(1)}$ the cold reservoir (i.e., $T_B > T_A^{(2)} > T_A^{(1)}$). Clearly, from Fig. 5(a), at $\Gamma_B=0$, the reservoir $R_A^{(2)}$ only releases its heat to $R_A^{(1)}$ so its heat current $Q_A^{(2)}>0$. For $\Gamma_B>0$, this reservoir also absorbs heat from R_B so, as a result, it will experience a heat crossover from $Q_A^{(2)}>0$ to $Q_A^{(2)}<0$, if the heat amount $Q_{B\to A_2}$ transferred from R_B to $R_A^{(2)}$ becomes equal to $Q_{A_2\to A_1}$, the amount of heat delivered to $R_A^{(1)}$ from $R_A^{(2)}$. From the fact that $Q_{B\to A_2}\propto\Gamma_B\Delta_M$ and $Q_{A_2\to A_1}\propto\Delta_m$, it follows that the critical value $\Gamma_{B,cr}\propto\Delta_m/\Delta_M$. This means that the larger the value of Δ_m (i.e., the temperature difference between the cool and cold reservoirs) and/or the smaller the value of Δ_M (the temperature difference between the hot and cool reservoirs), the higher the value of $\Gamma_{B,cr}$ which is needed to trigger such a crossover. This is perfectly reflected in Fig. 5(a) where we can see that a higher $T_A^{(2)}$, which corresponds to a bigger ratio Δ_m/Δ_M, leads to a larger critical value $\Gamma_{B,cr}$. If the temperatures of R_B and $R_A^{(1)}$ are reversed (that is, $T_A^{(1)} > T_A^{(2)} > T_B$), we then have $Q_{A_2\to B}\propto\Gamma_B\Delta_m$ and $Q_{A_1\to A_2}\propto\Delta_M$ so that $\Gamma_{B,cr}\propto\Delta_M/\Delta_m$. This is also nicely reflected in Fig. 5(b) where, contrary to what happens in Fig. 5(a), we see that a higher $T_A^{(2)}$, which corresponds to a smaller ratio Δ_M/Δ_m, leads to a smaller critical value $\Gamma_{B,cr}$ at which the sign of $Q_A^{(2)}$ changes from negative to positive.

Now we turn to the coupling between the two TLSs A and B, which plays an essential role in the present model because without it the reservoirs $R_A^{(1)}$, $R_A^{(2)}$ and the reservoir R_B are not connected so the heat currents obey the trivial equation $Q_B=Q_A^{(1)}+Q_A^{(2)}=0$. In the remaining part of this work, we shall clarify the question: how can we control the heat currents of different reservoirs, in particular the inversion of heat current direction, by engineering g? As is well known, a reservoir with the highest (lowest) temperature always releases (absorbs) heat; hence, its heat current remains positive (negative) for the whole range of variation of g. As for

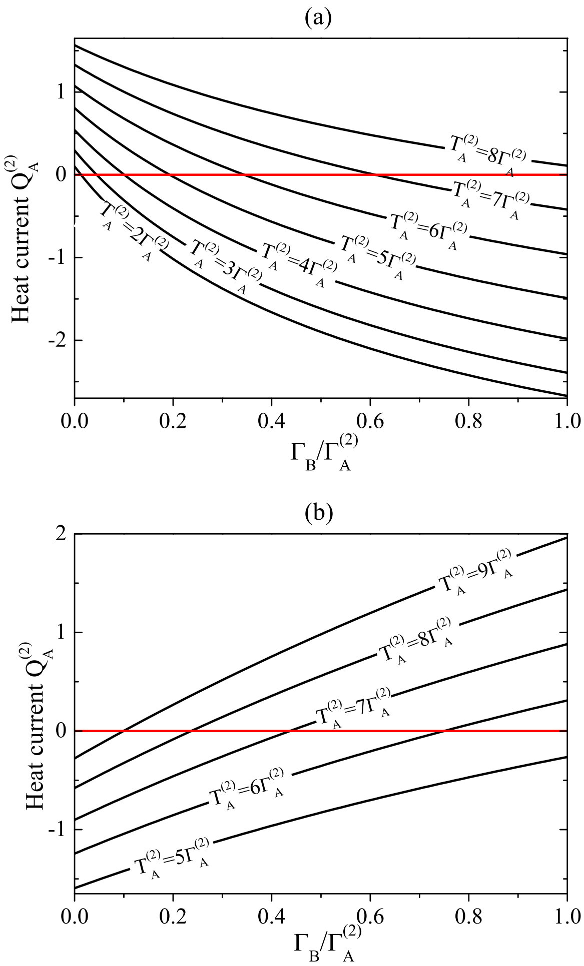

FIG. 5. (a) $Q_A^{(2)}$ vs $\Gamma_B/\Gamma_A^{(2)}$ with various temperatures $T_A^{(2)}$ between $T_A^{(1)}$ and T_B for $T_A^{(1)} = \Gamma_A^{(2)}$ and $T_B = 10\Gamma_A^2$. (b) Same as in (a), with various temperatures $T_A^{(2)}$ between T_B and $T_A^{(1)}$ for $T_B = \Gamma_A^{(2)}$ and $T_A^{(1)} = 10\Gamma_A^{(2)}$. The other parameters are chosen as in Fig. 4. The horizontal line at zero is added to guide the eye for the heat crossover.

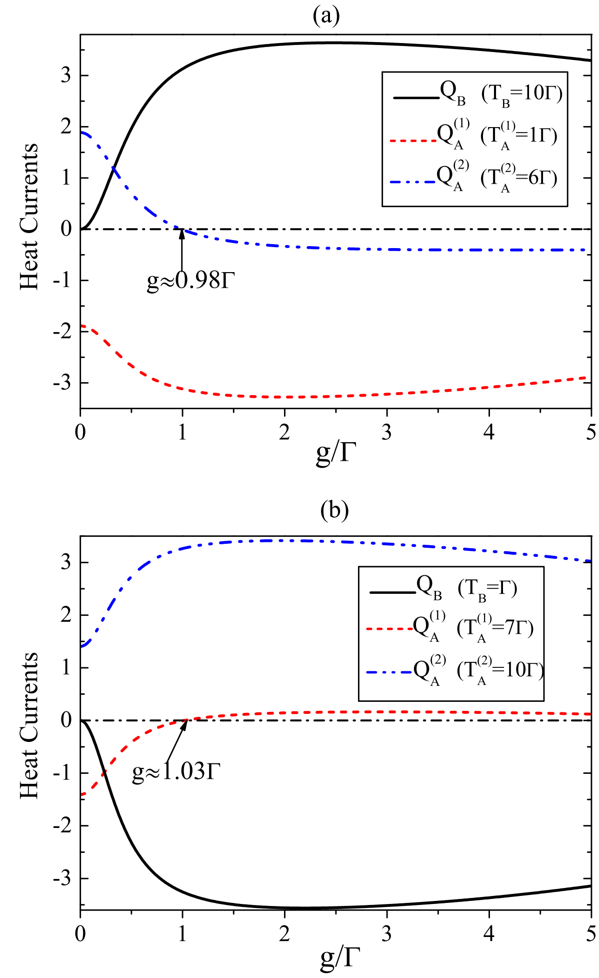

FIG. 6. Heat currents $Q_A^{(1)}$ (dashed line), $Q_A^{(2)}$ (dash-dotted line), and Q_B (solid line) as a function of the scaled coupling strength g/Γ of TLSs A and B for different temperature settings. The other parameters are set as $\omega_A = 10\Gamma$, $\omega_B = 9\Gamma$, and $\Gamma_A^{(1)} = \Gamma_A^{(2)} = \Gamma_B = \Gamma$. The horizontal line at zero is added to guide the eye for the heat crossover.

a reservoir with intermediate temperature, it may at the same time absorb heat from a hotter reservoir and release heat to a colder one. The net value of its heat current depends on whether heat absorption or heat release prevails and, as a consequence, the heat current may change its sign (i.e., experience a heat crossover) when g is varied. For example, a crossover of the sign of heat current of the reservoir $R_A^{(2)}$ ($R_A^{(1)}$) from "positive" to "negative" ("negative" to "positive") appears at $g = g_{cr} = 0.98\Gamma$ (1.03Γ) for the setting $T_B > T_A^{(2)} > T_A^{(1)}$ ($T_B < T_A^{(1)} < T_A^{(2)}$), as illustrated in Fig. 6(a) [Fig. 6(b)] with detailed data given in the figure caption.

The critical value $g = g_{cr}$ at which a crossover occurs also depends on the temperatures of the different reservoirs. Figure 7(a) plots the heat current $Q_A^{(2)}$ versus g/Γ for several values of $T_A^{(2)}$ satisfying the inequality $T_B > T_A^{(2)} > T_A^{(1)}$. This figure clearly indicates the increasing of g_{cr} with $T_A^{(2)}$, in accordance with a theoretical estimate $g_{cr} \propto \Delta_m/\Delta_M \propto T_A^{(2)}$. For another temperature setting, say, $T_A^{(2)} > T_A^{(1)} > T_B$, the critical value g_{cr} decreases when the temperature of the reservoir $R_A^{(1)}$ (i.e., the one with intermediate temperature) increases, as seen from Fig. 7(b), which is also in good agreement with a theoretical estimate for this situation, $g_{cr} \propto \Delta_M/\Delta_m \propto 1/T_A^{(1)}$.

C. Thermal rectification

In the following, we would like to discuss the thermal rectification of our model. The rectification efficiency R can be quantified as $R = (Q_{BA} - Q_{AB})/\max\{Q_{BA}, Q_{AB}\}$, where Q_{BA} (Q_{AB}) is the heat current from B to A (from A to B) when the reservoir R_B is set as the higher temperature $T_>$ (the lower temperature $T_<$). For convenience, we assume the reservoirs $R_A^{(1)}$ and $R_A^{(2)}$ that are in contact with the TLS A have the same temperature so that the thermal rectification can be demonstrated via an exchange of the temperatures of the reservoir R_B and that of R_A^1 and R_A^2. Generally, if one introduces asymmetry to the nonlinear system, then it may show the rectification effect [18]. In Fig. 8(a), the situation of $\omega_A = 10\Gamma \neq \omega_B = 4\Gamma$ (with $\Gamma_A^{(1)} = \Gamma_A^{(2)} = \Gamma_B = \Gamma$) is considered, where we can see that the rectification R changes

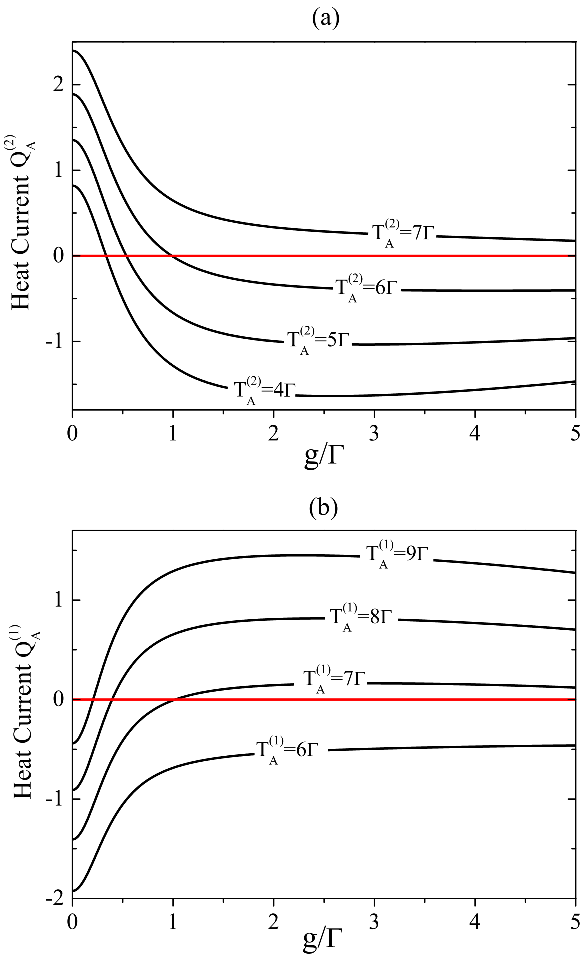

FIG. 7. (a) $Q_A^{(2)}$ vs g/Γ with different values of temperatures $T_A^{(2)}$ for $T_A^{(1)} = \Gamma < T_A^{(2)} < T_B = 10\Gamma$; (b) $Q_A^{(1)}$ vs g/Γ with different values of temperatures $T_A^{(1)}$ for $T_B = \Gamma < T_A^{(1)} < T_A^{(2)} = 10\Gamma$. The other parameters used here are the same as in Fig. 6. The horizontal line at zero is added to guide the eye for the heat crossover.

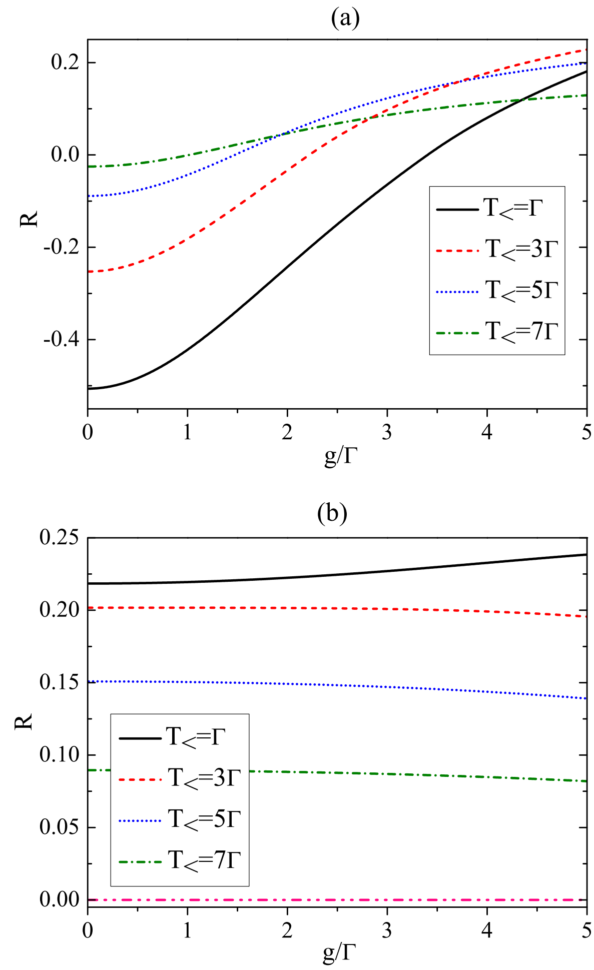

FIG. 8. Rectification as a function of scaled coupling strength g/Γ for different lower temperature $T_<$ with a given higher temperature $T_> = 10\Gamma$. Two nonresonant TLSs are considered in (a) with $\omega_A = 10\Gamma$ and $\omega_B = 4\Gamma$, while two identical TLSs are considered in (b) with $\omega_A = \omega_B = 10\Gamma$. The horizontal line at zero in (b) denotes the rectification in the two-reservoir case, namely, each of two identical TLSs being coupled to a single reservoir.

sign when the coupling strength g between the TLSs A and B increases. R can be positive, zero, or negative, depending on the values of g, and retains similar behaviors for different lower temperatures $T_<$ with the given higher temperature $T_>$. However, this rectification induced by the detuning of the two TLSs is not the unique character of our model, which happens also for the usual two-reservoir scenario where each TLS is equally coupled to a single reservoir. Our model with three reservoirs actually possesses inherent asymmetry for the heat conduction due to the asymmetric couplings of the TLSs A and B to the thermal reservoirs. In Fig. 8(b), we show the rectification as a function of g/Γ for two identical TLSs, namely, $\omega_A = \omega_B$. We observe that R remain positive and slightly vary with g/Γ for all the considered temperature differences, which implies that the heat transfer is favorable from B to A, while restrained in the opposite direction. Here, the rectification with identical TLSs is an important character of our model since for two identical TLSs being equally coupled to two reservoirs, the rectification R will remain zero, as shown by the horizontal line at zero in Fig. 8(b).

IV. CONCLUSION

In conclusion, we have studied the possibility of controlling both the magnitudes and directions of heat currents of three thermal reservoirs that are asymmetrically coupled to two interacting TLSs. More precisely, two of the three reservoirs are simultaneously coupled to one TLS, but the remaining reservoir is coupled to another TLS. In terms of the eigenstates of the two interacting TLSs, we have constructed a master equation to describe the dynamics of the two TLSs and derived the solution in the steady-state regime. First, we have considered the situation when two of the three reservoirs possess an equal temperature which is different from that of the third one, and have analyzed the control of the magnitudes of the reservoirs' heat currents under this situation. We have shown that increasing (decreasing) the damping rate regarding one reservoir can increase (decrease) the magnitude of the heat current associated with this reservoir, but leads to completely opposite behaviors of those of the other two reservoirs, namely, the magnitude of heat current of one reservoir may be increased (decreased) as well, while that of the other one may instead be decreased (increased). In other words, adding an additional thermal reservoir and increasing its coupling strength to one TLS can favor heat transfer of one of the two other reservoirs, while suppress that of the remaining one. Then, the situation when each of the three reservoirs has a different temperature has also been considered. In this situation, we have demonstrated the inversion of the direction of the heat current (i.e., the crossover between heat absorption and release) of the reservoir with intermediate temperature by varying the damping rate regarding one of the other two reservoirs (i.e., the ones with the highest and lowest temperatures) or by varying the coupling strength of the two TLSs. The critical values of the damping rate and the coupling strength that trigger the crossover are found related to the temperature settings of the three reservoirs. We have also discussed the thermal rectification of our model due to the inherent asymmetric couplings of the two TLSs to the three reservoirs.

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