Flexible controlled joint remote preparation of an arbitrary two-qubit state via non-maximally

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Abstract

Transferring a quantum state from one location to another without physically sending the state itself through open space is a special global task that can only be carried out thanks to the laws of nature, namely the principles of quantum mechanics. In this work, we devise protocols for two senders to jointly prepare the most general two-qubit state for a receiver under the supervision of a controller by using three different types of quantum channels, all of which are non-maximally entangled. First, we propose the schemes to produce the quantum channels concerned, and then we present the concrete steps required to execute the protocols, highlighting the issue of why shared non-maximal entanglement is intentionally used instead of maximal entanglement.

Keywords: controlled joint remote state preparation, arbitrary two-qubit state, non-maximally entangled quantum channel, averaged fidelity
Classification numbers: 3.01

1. Introduction

One of the most specific and also most important concepts in quantum information sciences is quantum entanglement [1] which Schrödinger singled out many decades ago as 'the characteristic trait of quantum mechanics'. It lies at the heart of quantum mechanics and is now viewed as a useful resource for quantum information processing and quantum computing. With its help, a number of classically impossible tasks can be done quantumly (i.e. in a way based on the postulates of quantum mechanics), such as unconditionally secure cryptography [2], superdense coding [3], quantum secret sharing [4], quantum dialogue [5] and so on. Most notably, in 1993 Bennett *et al* proposed for the first time a highly intriguing

quantum protocol called teleportation [6] which allows one to securely and faithfully transfer an unknown single-qubit state from a sender to a distant receiver, without direct physical transportation of the state itself across space, by employing an Einstein–Podolsky–Rosen (EPR) pair [7] as the shared quantum channel. Soon after this, related works appeared widely and achieved great advancements in both theoretical [8] and experimental [9] aspects. As extensions of the teleportation scheme, two recognized versions are the remote state preparation (RSP) scheme [10], in which the sender knows the full identity of the to-be-prepared state, and the joint remote state preparation (JRSP) scheme [11], in which there is more than one sender and each sender is allowed to know only partial information of the state to be prepared, thus ensuring the secrecy of the target state since neither sender is able to learn the state identity. In addition, a set of controllable schemes such as controlled teleportation [12], controlled RSP [13] and controlled JRSP [14] have also been introduced to enhance the security. The essence of a

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controlled protocol is that a given task cannot be perfectly completed without permission of the controller, although they are not required to know the details of the state of concern. All of them can be performed with unit success probability and unit fidelity when maximal entanglement is used as the shared quantum channel along with the so-called feed-forward measurement strategy. If the quantum channel is non-maximally entangled, the fidelity may remain at one, but the success probability is often less than one [15]. Nevertheless, in certain circumstances non-maximal entanglement turns out to be necessary and, surprisingly, sometimes less entanglement proves to be more useful (see [16] and the references therein). As demonstrated in reference [17–19], using particular non-maximally entangled states can still provide both unit success probability and unit fidelity for controlled teleportation and controlled RSP. Such perfect features have also been achieved recently for controlled JRSP of an arbitrary single-qubit state [20]. Developing this idea, in this work we use three different types of non-maximally entangled states as the shared quantum channels for controlled JRSP of an arbitrary two-qubit state. The first, most trivial, quantum channel is two four-qubit non-maximally entangled states as considered in reference [20]. The second type of non-local resource comprises one Greenberger–Horne–Zeilinger (GHZ) state [21] and one four-qubit non-maximally entangled state. The last quantum channel type is a single seven-qubit non-maximally entangled state. We shall describe in detail how to produce the above-mentioned quantum channels and how to employ them to perform our task. Furthermore, we shall also consider a case when the controller is disabled by some objective reasons and analyze the dependence of the averaged fidelity on the parameters of the quantum channels concerned, indicating flexibility of the protocols.

This paper is structured as follows. After the introduction, section 2 considers three different non-maximally entangled states as the working quantum channels shared among the authorized participants. The main content is presented in section 3 where perfect protocols for controlled JRSP of the most general two-qubit state are devised. Section 4 provides discussion on a possible unwanted situation that prevents the protocols from being perfect, but retains *a priori* intended demand. Finally, we summarize our work in section 5.

2. The working quantum channels

The task we are concerned with involves four parties: two senders (Alice and Bob), one receiver (Charlie) and one controller (David). All four authorized participants are arbitrarily far apart from each other. The two-qubit state which needs be prepared at the receiver's location has the most general form

$$|\psi\rangle = x|00\rangle + ye^{i\varphi_1}|01\rangle + ze^{i\varphi_2}|10\rangle + te^{i\varphi_3}|11\rangle, \quad (1)$$

with real parameters x, y, z, t, φ_1, φ_2 and φ_3 satisfying the normalization condition $x^2 + y^2 + z^2 + t^2 = 1$ and $|pq\rangle \equiv |p\rangle \otimes |q\rangle$, $p, q \in \{0,1\}$, denotes a product state of

two qubits, one of which is in the state $|p\rangle$ and the other in $|q\rangle$. To keep the state $|\psi\rangle$ secret, neither sender is permitted to know the full set of x, y, z, t, φ_1, φ_2 and φ_3. Instead, Alice knows only x, y, z and t, while Bob only φ_1, φ_2 and φ_3. Moreover, the four parties are allowed to carry out only local actions (unitary operations and/or measurements) combined with reliable classical communication. They, therefore, should safely share some entangled quantum channel beforehand. The quantum channels we have chosen to fulfill our task in this work are three new non-maximally entangled states, labeled $|Q_1\rangle$, $|Q_2\rangle$ and $|Q_3\rangle$, which have the following forms:

$$|Q_1\rangle_{A_1B_1C_1D_1A_2B_2C_2D_2} = |q(\theta_1)\rangle_{A_1B_1C_1D_1} \otimes |q(\theta_2)\rangle_{A_2B_2C_2D_2}, \quad (2)$$

$$|Q_2\rangle_{A_1B_1C_1A_2B_2C_2D} = |\text{GHZ}\rangle_{A_1B_1C_1} \otimes |q(\theta)\rangle_{A_2B_2C_2D}, \quad (3)$$

with

$$|q(\varphi)\rangle_{1234} = \frac{1}{\sqrt{2}}$$
$$\times (|0000\rangle + \cos\varphi |1110\rangle - \sin\varphi |1111\rangle)_{1234} \quad (4)$$

and

$$|\text{GHZ}\rangle_{123} = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{123}, \quad (5)$$

while

$$|Q_3\rangle_{A_1A_2B_1B_2C_1C_2D} = \frac{1}{2}(|0000000\rangle + \cos\theta |0101010\rangle$$
$$- \sin\theta |0101011\rangle + |1010100\rangle$$
$$- \cos\theta |1111110\rangle$$
$$+ \sin\theta |1111111\rangle)_{A_1A_2B_1B_2C_1C_2D}. \quad (6)$$

In the above quantum channels Alice holds two qubits A_1, A_2, Bob two qubits B_1, B_2, and Charlie two qubits C_1, C_2, while David holds two qubits D_1, D_2 in the case of $|Q_1\rangle_{A_1B_1C_1D_1A_2B_2C_2D_2}$, but just one qubit D in the cases of $|Q_2\rangle_{A_1A_2B_1B_2C_1C_2D}$ or $|Q_3\rangle_{A_1A_2B_1B_2C_1C_2D}$. As is clear from equations (2), (3) and (6), the quantum channel states are characterized by the angles θ_1, θ_2 or θ. The entanglement between the senders and the receiver is always present independent of θ_1, θ_2, θ. However, the entanglement between the controller and the others is sensitive to θ_1, θ_2, θ. For the values of those angles for which the *sine* is equal to 0 the qubit(s) of David is (are) totally disentangled from the other ones, thus annulling his role as a 'quantum' controller. On the other hand, for other values of the angles, the states (2), (3) and (6) possess some degree of entanglement between David and the remaining group of participants, justifying him as a 'quantum' controller. In particular, for the values of the angles for which the *sine* is equal to ± 1, the degree of that entanglement is maximal. Here, the relevant values of $\theta_{1,2}$ and θ are such that the *sine* is different from 0 and ± 1. Of importance is the fact that the values of $\theta_{1,2}$ or θ are consciously assigned only to the controller David. This obviously enhances the security level because even if the qubits of David and Charlie are unexpectedly captured by Eve, a malicious enemy, the identity of
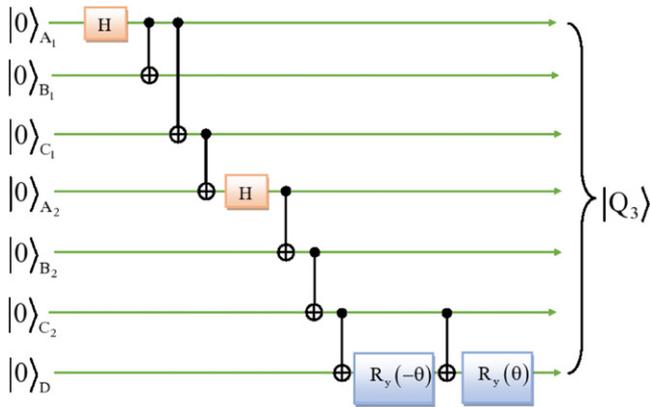
Figure 1. Quantum circuit for producing the seven-qubit non-maximally entangled state in equation (6). The circuit is read from left to right and each single line denotes a qubit.

the target state (1) remains undiscovered by Eve as she has no idea about the angles and hence is unable to decide the correct recovery actions (see later). Production of the state $|GHZ\rangle_{123}$, equation (5), had long been well-known [21] and that of the state $|q(\theta)\rangle_{1234}$, equation (4), was devised in reference [20]. So the states $|Q_1\rangle_{A_1B_1C_1D_1A_2B_2C_2D_2}$ and $|Q_2\rangle_{A_1A_2B_1B_2C_1C_2D}$ in equations (2) and (3) are readily producible. As for the state $|Q_3\rangle_{A_1A_2B_1B_2C_1C_2D}$, equation (6), we propose to produce it using a quantum circuit shown in figure 1 by means of single-qubit Hadamard gates:

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \qquad (7)$$

single-qubit rotations:

$$R_y(\varphi) = \begin{pmatrix} \cos(\varphi/2) & -\sin(\varphi/2) \\ \sin(\varphi/2) & \cos(\varphi/2) \end{pmatrix}, \qquad (8)$$

both of which are written in the qubit's computational basis $\{|0\rangle, |1\rangle\}$, and two-qubit controlled-NOT gates:

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \qquad (9)$$

which is written in the qubits' computational basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$. We are now in the position to employ the above-listed quantum channels for our purpose.

3. The controlled JRSP protocols

In order to obtain unit success probability, our schemes, which we shall describe in detail in this section using each of the three above-mentioned quantum channels, are performed by four sequential steps as follows.

In the first step, Alice applies to her two qubits A_1 and A_2 a unitary operator U, which for the quantum channel states (2)

and (3) is defined by

$$U(x, y, z, t) = \begin{pmatrix} x & y & z & t \\ y & -x & -t & z \\ z & t & -x & -y \\ t & -z & y & -x \end{pmatrix}, \qquad (10)$$

while for the quantum channel state (6) it reads

$$U(x, y, z, t) = \begin{pmatrix} x & y & z & t \\ y & -x & t & -z \\ z & -t & -x & y \\ -t & -z & y & x \end{pmatrix}, \qquad (11)$$

both are written in the computational basis of two qubits A_1 and A_2: $\{|00\rangle_{A_1A_2}, |01\rangle_{A_1A_2}, |10\rangle_{A_1A_2}, |11\rangle_{A_1A_2}\}$. Then, she measures them in that basis to find a state $|kl\rangle_{A_1A_2}$ with $k, l \in \{0, 1\}$. After measuring, she publicly announces the obtained outcomes kl.

The second step is tricky in the sense that the feed-forward measurement strategy will be exploited. Concretely, in this step Bob's correct action depends essentially on Alice's measurement outcomes kl. Namely, if $kl = 00$, he needs to apply to his qubits B_1, B_2 the following unitary operator:

$$V = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ e^{i\varphi_1} & -e^{i\varphi_1} & -e^{i\varphi_1} & e^{i\varphi_1} \\ e^{i\varphi_2} & -e^{i\varphi_2} & e^{i\varphi_2} & -e^{i\varphi_2} \\ e^{i\varphi_3} & e^{i\varphi_3} & -e^{i\varphi_3} & -e^{i\varphi_3} \end{pmatrix}, \qquad (12)$$

which is written in the computational basis $\{|00\rangle_{B_1B_2}, |01\rangle_{B_1B_2}, |10\rangle_{B_1B_2}, |11\rangle_{B_1B_2}\}$. However, the correct unitary operator should be

$$V = \frac{1}{2}\begin{pmatrix} e^{i\varphi_1} & e^{i\varphi_1} & e^{i\varphi_1} & e^{i\varphi_1} \\ 1 & -1 & -1 & 1 \\ e^{i\varphi_3} & -e^{i\varphi_3} & e^{i\varphi_3} & -e^{i\varphi_3} \\ e^{i\varphi_2} & e^{i\varphi_2} & -e^{i\varphi_2} & -e^{i\varphi_2} \end{pmatrix}, \qquad (13)$$

if $kl = 01$,

$$V = \frac{1}{2}\begin{pmatrix} e^{i\varphi_2} & e^{i\varphi_2} & e^{i\varphi_2} & e^{i\varphi_2} \\ e^{i\varphi_3} & -e^{i\varphi_3} & -e^{i\varphi_3} & e^{i\varphi_3} \\ 1 & -1 & 1 & -1 \\ e^{i\varphi_1} & e^{i\varphi_1} & -e^{i\varphi_1} & -e^{i\varphi_1} \end{pmatrix}, \qquad (14)$$

if $kl = 01$ and">

$$V = \frac{1}{2}\begin{pmatrix} e^{i\varphi_3} & e^{i\varphi_3} & e^{i\varphi_3} & e^{i\varphi_3} \\ e^{i\varphi_2} & -e^{i\varphi_2} & -e^{i\varphi_2} & e^{i\varphi_2} \\ e^{i\varphi_1} & -e^{i\varphi_1} & e^{i\varphi_1} & -e^{i\varphi_1} \\ 1 & 1 & -1 & -1 \end{pmatrix}, \qquad (15)$$

if $kl = 11$. Afterwards, Bob measures his qubits in their computational basis to find a state $|mn\rangle_{B_1B_2}$ with $m, n \in \{0, 1\}$. The obtained outcomes mn will also be publicly announced after the measurement.

The third step will be done by David which is subject to the quantum channel used. For the quantum channel (2) David first applies $R_y(-\theta_1)$ to qubit D_1 and $R_y(-\theta_2)$ to qubit D_2 (that is, he rotates qubit D_1 by an angle $-\theta_1$ and qubit D_2 by an angle $-\theta_2$ around the y-axis), then measures those qubits in their computational basis $\{|rs\rangle_{D_1D_2}; r, s \in \{0,1\}\}$. Yet, in the case of using the states (3) or (6) as quantum

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channels, David just rotates qubit D by an angle $-\theta$ around the y-axis followed by measurement of it in the computational basis $\{|r\rangle_D ; r \in \{0,1\} \}$. It is worth noting that only David can do the aforementioned rotation(s) because, as devised, nobody else except him knows the exact values of the rotation angle(s). This is the figure of merit to be highlighted regarding the use of non-maximally entangled states (2), (3) or (6), which yield a security level higher than that of using maximally entangled quantum channels. Of course, as a rule, after finishing the measurement David is required to inform Charlie of his outcomes rs or r (or simply make them public through a classical communication channel), depending on which quantum channel was actually in use.

Finally, in the fourth step, after collecting the outcomes of all the necessary measurements from Alice, Bob and David, Charlie is able to transform the state of her qubits to the target one by applying the recovery operators R_1 and R_2 to the qubits C_1 and C_2, respectively. The correct recovery operators differ depending on the quantum channels used. For our protocols we have been able to work out explicit and compact formulae for the recovery operations. Precisely, if the quantum channel is via the state (2), then

$$R_1 = X^k Z^{r\oplus n \oplus k}, \ R_2 = X^l Z^{s\oplus n \oplus m \oplus l \oplus k}. \tag{16}$$

However, if it is the state (3), then

$$R_1 = X^k Z^{n\oplus k}, \ R_2 = X^l Z^{r\oplus n \oplus m \oplus l \oplus k}, \tag{17}$$

and otherwise (i.e. the state (6) serves as the quantum channel)

$$R_1 = X^k Z^{m\oplus l \oplus k}, \ R_2 = X^l Z^{r\oplus n \oplus m \oplus l}. \tag{18}$$

In the above recovery operators $\oplus$ stands for addition mod 2, while

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{19}$$

and

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{20}$$

are the well-known Pauli bit-flip and phase-flip operators. The fidelity is clearly one. The total success probability is also one because Charlie is able to obtain the desired state for any possible set of the measurement outcomes via whatever quantum channel. Our protocols are therefore perfect despite using non-maximally entangled states as the working quantum channel. The steps for implementing our controlled JRSP protocols are schematically sketched in figure 2 when the state (2) is used and in figure 3 when the state (3) or the state (6) serves as the shared quantum channel.

Although anyone can obtain the broadcasted measurement outcomes, not everyone can benefit from the classical information, thus guaranteeing the security of our protocols.
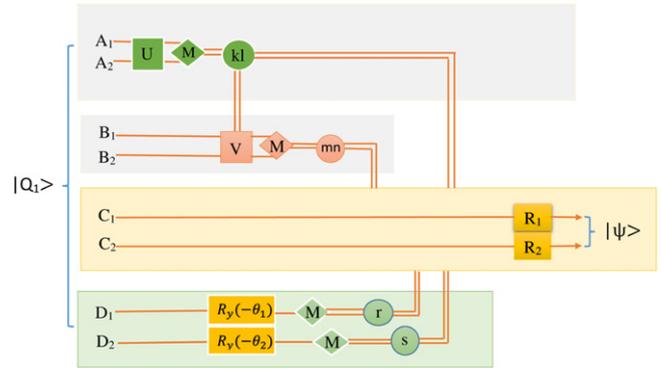

Figure 2. Scheme for the controlled joint remote preparation of a general two-qubit state by using the quantum channel in equation (2). Each single line denotes a qubit, while a double line denotes the publicly broadcasted classical communication which costs two bits.
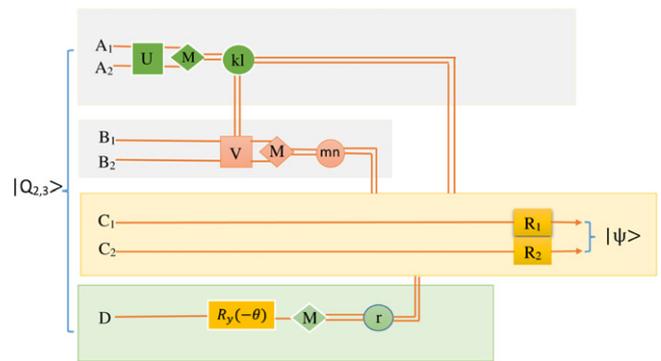

Figure 3. Scheme for the controlled joint remote preparation of a general two-qubit state by using the quantum channel in equations (3) or (6). Each single line denotes a qubit, while a double line denotes the publicly broadcasted classical communication which costs two bits.

4. Discussion

As presented above, our controlled JRSP protocols are perfect (with unit fidelity, unit success probability and absolute security). Nevertheless, there are many situations that prevent the protocols from obtaining perfection. In this section we shall address a situation that might occur in practice. Suppose that David's qubits D_1, D_2 (or D) are, for some reason, lost or captured by an unauthorized party during the entanglement distribution process and the authorized parties are aware of this accident. In this case one may ask: 'how close to the target state could the state at Charlie's location be?' Let us analyze this. The above-mentioned accident can be treated as the removal of the controller's participation. Mathematically, this is tantamount to tracing out over the controller's qubits. As a consequence, what will be at hand to Charlie is a mixed state specified by a reduced density matrix $\rho_{C_1 C_2} \neq |\psi\rangle\langle\psi|$ which Charlie would obtain via the steps described in the previous section, but without David's participation (i.e. the third step is skipped). The fidelity f of the state at hand to Charlie's as compared to the target state $|\psi\rangle$ in equation (1) is

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given by

$$f = \langle \psi \, | \rho_{C_1 C_2} | \, \psi \rangle. \tag{21}$$

Our calculations yield

$$
\begin{aligned}
f = x^4 + y^4 + z^4 + t^4 &+ 2(y^2 t^2 + x^2 z^2)\cos\theta_1 \\
&+ 2(z^2 t^2 + x^2 y^2)\cos\theta_2 \\
&+ 2(x^2 t^2 + y^2 z^2)\cos\theta_1\cos\theta_2
\end{aligned} \tag{22}
$$

when using the quantum channel (2) and

$$
\begin{aligned}
f = x^4 + y^4 + z^4 + t^4 &\\
+ 2(z^2 t^2 + x^2 y^2 + x^2 t^2 + y^2 z^2)&\cos\theta \\
+ 2(x^2 z^2 + y^2 t^2)&
\end{aligned} \tag{23}
$$

when the quantum channel (3) or (6) is used. From equations (22) and (23) we see that the fidelities depend not only on the quantum channel, but also on the state to be prepared. Of interest then is the averaged fidelity f_{av} (i.e. the fidelity which is averaged over all the possible input states). To obtain an explicit analytical expression, we set $x = \cos\eta_3$, $y = \sin\eta_3\cos\eta_2$, $z = \sin\eta_3\sin\eta_2\cos\eta_1$ and $t = \sin\eta_3\sin\eta_2\sin\eta_1$, with new angles η_1, η_2 and η_3. The averaged fidelity can then be calculated using the following formula [22]:

$$f_{av} = \frac{1}{\mu}\prod_{k=1}^{3}\int_0^{\pi/2}\cos\eta_k(\sin\eta_k)^{2k-1}d\eta_k\prod_{k=1}^{3}\int_0^{2\pi}d\varphi_k f, \tag{24}$$

where $\eta_k \in [0, \pi/2]$, $\varphi_k \in [0, 2\pi)$, $k = \{1, 2, 3\}$ and $\mu = \frac{\pi^3}{3!}$ is the total volume of the manifold of pure states [22]. Substituting f from equations (22) or (23) into equation (24) and carrying out the integrations we obtain

$$f_{av} = \frac{1}{5}(2 + \cos\theta_1 + \cos\theta_2 + \cos\theta_1\cos\theta_2) \tag{25}$$

for the quantum channel (2) and

$$f_{av} = \frac{1}{5}(3 + 2\cos\theta) \tag{26}$$

for the quantum channel (3) or (6). The expressions (25) and (26) imply that the closeness between $\rho_{C_1 C_2}$ and $|\psi\rangle\langle\psi|$ can be tailored by the angles θ or $\theta_{1,2}$. The dependence of f_{av} on $\theta_{1,2}$ and θ are displayed in figures 4 and 5, respectively. At that point we notice that the best value of the fidelity that can be obtained only by classical means is equal to $f_{av}^{cl} = 2/5$ [23]. Now, as in the visual from figures 4 and 5, there are domains of $\theta_{1,2}$ or θ in which $f_{av} \geqslant f_{av}^{cl}$ or $f_{av} < f_{av}^{cl}$. So, the values of $\theta_{1,2}$, θ could be on-purpose chosen properly in order that, even if the qubits $D_{1,2}$ or D are accidentally lost or captured by an unauthorized party, the fidelity f_{av} is greater than, equal to or smaller than f_{av}^{cl}, depending on a pre-established demand.

Returning to our protocols we see that the classical communication cost needed to broadcast the measurement outcomes is 6 bits when using the quantum channel (2) and 5 bits when using the channel (3) or the channel (6). As for the quantum resource, the channel (2) requires 8 qubits, 2
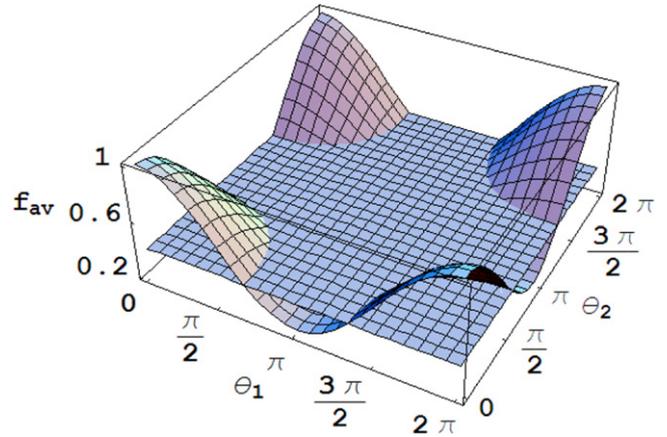

Figure 4. The dependence of the averaged fidelity f_{av}, equation (25), on the angles θ_1 and θ_2. The plane at $2/5$ represents the classical limit of the averaged fidelity.
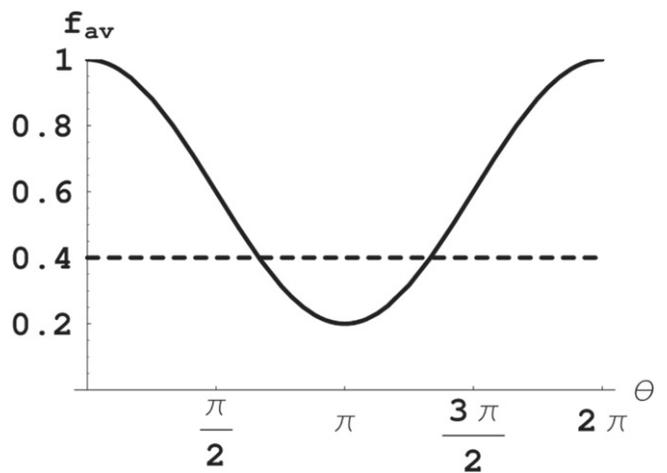

Figure 5. The dependence of the averaged fidelity f_{av}, equation (26), on the angles θ. The horizontal line at $2/5$ represents the classical limit of the averaged fidelity.

Hadamard gates, 8 CNOTs and 4 rotations, the channel (3) requires 7 qubits, 2 Hadamard gates, 5 CNOTs and 2 rotations, while the channel (6) requires 7 qubits, 2 Hadamard gates, 7 CNOTs and 2 rotations. An overall assessment, especially taking into account the fact that the CNOT gate is much more expensive than the Hadamard gate and rotation, indicates that use of the quantum channel (3) proves to be cheaper than that of the other two.

5. Conclusion

To summarize, we have considered three distinct non-maximally entangled quantum channels, equations (2), (3) or (6), proposed their production schemes and devised protocols for two parties to jointly prepare the most general two-qubit state for a third party under 'quantum' control of a fourth party. This is a truly global quantum task because all the participants may be space-like. We have shown that these protocols in ideal conditions are perfect in the sense that both the fidelity

and success probability are equal to one. The advantages of using non-maximally entangled quantum channels are an enhancement of security level and provide the flexibility of choosing the parameters of the quantum channels so that a pre-demanded averaged fidelity can be achieved even in case when the controller's qubits are lost en route. In this sense our protocols are flexible. Furthermore, we have also assessed the overall cost of each of the protocols and found that the quantum channel (3) is the most advantageous.

Acknowledgments

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