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A prediction of δ_{CP} for a normal neutrino mass hierarchy in an extended standard model with an A4 flavour symmetry

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Abstract. A method of diagonalization of a general neutrino mass matrix in an extended standard model with an A4 flavour symmetry is used. The method allows us to determine a relation between the mixing angles $(\theta_{12}, \theta_{23}, \theta_{13})$ and the CP-violation Dirac phase (δ_{CP}). The current prediction of δ_{CP} is quite good, near the 1σ region of the best fit of the experimental data at a normal neutrino mass hierarchy.

1. Introduction

The neutrino masses and mixings are among the phenomena calling for extending the standard model (SM) [1, 2]. One of the extensions of the SM is to add a flavour symmetry. A popular flavor symmetry intensively investigated in literature is A4 (see, for instance, [3, 4]) which allows to obtain a tribi-maximal neutrino mixing (TBM). The recent experimental data [5] showing, however, a non-zero mixing angle θ_{13} and a possible CP-violation Dirac phase δ_{CP}, rejects the TBM. There have been many attempts to explain these new phenomena. Here, using a perturbation method we can diagonalize a general neutrino mass matrix and derive a relation between δ_{CP} and the mixing angles θ_{ij}. The latter allows us to calculate δ_{CP} numerically using experimental data on the mixing angles.

We will make in the next section a quick introduction to the representations of A4 and their application to a simple model. In Sect. 3 we will deal with the diagonalization of the neutrino mass matrix obtained for this model from which we can get a mass spectrum and a relation between δ_{CP} and θ_{ij}.

2. Extended standard model with A4 flavour symmetry

2.1. Representations of A4 in brief

Here we give a brief information on representations of A4 [6, 7]. A4 group has 12 elements and it can be generated by two basic permutations S and T

$$S^2 = T^3 = (ST)^3 = 1. \tag{1}$$

A4 has three one-dimensional unitary representations 1, $1^{'}$ and $1^{''}$ generated by

$$\begin{aligned} 1: \ & S = 1 \quad T = 1, \\ 1^{'}: \ & S = 1 \quad T = e^{i2\pi/3} \equiv \omega, \\ 1^{''}: \ & S = 1 \quad T = e^{i4\pi/3} \equiv \omega^2. \end{aligned} \tag{2}$$

and a three-dimensional unitary representation with the generators

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}. \tag{3}$$

In applications of a group we often need to know the multiplication rule of its (irreducible) representations which in the case of A4 are

$$\begin{aligned} & 1 \times 1 = 1, \\ & 1^{'} \times 1^{''} = 1, \\ & 3 \times 3 = 1 + 1^{'} + 1^{''} + 3_S + 3_{AS}. \end{aligned} \tag{4}$$

If we have two triplets $3_a \sim (a_1, a_2, a_3)$ and $3_b \sim (b_1, b_2, b_3)$, their direct product can be decomposed into irreducible representations as follows

$$\begin{aligned} & 1 = a_1b_1 + a_2b_2 + a_3b_3, \\ & 1^{'} = a_1b_1 + \omega^2 a_2b_2 + \omega a_3b_3, \\ & 1^{''} = a_1b_1 + \omega a_2b_2 + \omega^2 a_3b_3, \\ & 3_1 \sim (a_2b_3, a_3b_1, a_1b_2), \\ & 3_2 \sim (a_3b_2, a_1b_3, a_2b_1). \end{aligned} \tag{5}$$

The above given information is used in constructing the Lagrangian of a considered model with an A4 symmetry, for example, the Lagrangian (6).

2.2. A simple model

In any model with three active Majorana neutrinos, the most general neutrino mass matrix is symmetric and parametrized by 6 independent parameters. As an example, we consider here a model studied in [8, 9, 10]. The lepton sector (the quark sector is not considered here) of this model includes an A4 triplet N which is an $SU(2)_L$ singlet (called also right-handed neutrino), and the SM leptons among which the left-handed leptons l_L, $l = e, \mu, \tau$, transform as A4 triplets, while the right-handed ones e_R, μ_R and τ_R transform as A4 singlets 1, $1^{'}$ and $1^{''}$, resp. Besides the SM Higgs ϕ_h which is a singlet under A4, the scalar sector of the model has five additional

$SU(2)_L$-singlet scalars: two A4-triplets ϕ_E, ϕ_N and three A4-singlets ξ, $\xi^{'}$, $\xi^{''}$. The transformation rules under A4 and $SU(2)_L$ of the leptons and the scalars in this model are summarized in the table below.

	l_L	e_R	μ_R	τ_R	N	ϕ_E	ϕ_N	ξ	$\xi^{'}$	$\xi^{''}$	ϕ_h
A4	3	1	$1^{'}$	$1^{''}$	3	3	3	1	$1^{'}$	$1^{''}$	1
$SU(2)_L$	2	1	1	1	1	1	1	1	1	1	2

The new Yukawa term of the Lagrangian of this model is

$$-\mathcal{L}_Y^{new} = \left[\lambda_e \bar{l}_L \phi_h e_R \frac{\phi_E}{\Lambda} + \lambda_\mu \left(\bar{l}_L \phi_h\right)^{''} \mu_R \frac{\phi_E}{\Lambda} + \lambda_\tau \left(\bar{l}_L \phi_h\right)^{'} \tau_R \frac{\phi_E}{\Lambda} + \lambda_N \bar{l}_L \tilde{\phi}_h N + H.c.\right]$$
$$+ c_N N^T N \phi_N + c_\xi N^T N \xi + c_{\xi^{'}} \left(N^T N\right)^{''} \xi^{'} + c_{\xi^{''}} \left(N^T N\right)^{'} \xi^{''} + H.c. \quad (6)$$

Denoting the VEVs of the scalars as

$$\langle \xi \rangle = u_a,\ \langle \xi^{'} \rangle = u_b,\ \langle \xi^{''} \rangle = u_c,\ \langle \phi_E \rangle = (u_1, u_2, u_3)\,,\ \langle \phi_N \rangle = (v_1, v_2, v_3)\,,\ \langle \phi_h \rangle = v. \quad (7)$$

we obtain the mass matrix of the charged leptons

$$M_l = \begin{pmatrix} y_e u_1 & y_e u_2 & y_e u_3 \\ y_\mu u_1 & \omega y_\mu u_2 & \omega^2 y_\mu u_3 \\ y_\tau u_1 & \omega^2 y_\tau u_2 & \omega y_\tau u_3 \end{pmatrix} \quad (8)$$

where

$$y_e = \frac{\lambda_e}{\Lambda} v, \quad y_\mu = \frac{\lambda_\mu}{\Lambda} v, \quad y_e = \frac{\lambda_\tau}{\Lambda} v. \quad (9)$$

In the case $\langle \phi_E \rangle = (u, u, u)$, the charge lepton mass matrix can be written in the form

$$M_l = u U_0 \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}, \quad (10)$$

where

$$U_0 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}. \quad (11)$$

For the neutrino sector, we get a Majorana mass matrix

$$M_N = \begin{pmatrix} c_a + c_b + c_d & \epsilon_3 & \epsilon_2 \\ \epsilon_3 & c_a + \omega c_b + \omega^2 c_d & \epsilon_1 \\ \epsilon_2 & \epsilon_1 & c_a + \omega^2 c_b + \omega c_d \end{pmatrix} \equiv \begin{pmatrix} a & b & c \\ b & e & d \\ c & d & f \end{pmatrix}, \quad (12)$$

where $c_a = c_\xi u_a$, $c_b = c_{\xi^{'}} u_b$, $c_d = c_{\xi^{''}} u_c$, $\epsilon_1 = c_N v_1$, $\epsilon_2 = c_N v_2$, $\epsilon_3 = c_N v_3$. We also get a Dirac mass matrix

$$M_D = \begin{pmatrix} \lambda_{N_1} v & 0 & 0 \\ 0 & \lambda_{N_2} v & 0 \\ 0 & 0 & \lambda_{N_3} v \end{pmatrix} \equiv \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix}. \quad (13)$$

Following the seesaw mechanism we get a neutrino mass matrix,

$$M_\nu = -M_D^T M_N^{-1} M^D, \quad (14)$$

which, with (12) and (13) taken into account, has the form

$$M_\nu = \frac{-1}{\det(M)} \begin{pmatrix} \left(d^2 - ef\right) x^2 & (-cd + bf)xy & (-bd + ce)xz \\ (-cd + bf)xy & \left(c^2 - af\right) y^2 & (-bc + ad)yz \\ (-bd + ce)xz & (-bc + ad)yz & \left(b^2 - ae\right) z^2 \end{pmatrix} \equiv \begin{pmatrix} A & B & C \\ B & E & D \\ C & D & F \end{pmatrix}, \quad (15)$$

where $\det(M) = -2bcd + c^2 e + b^2 f + a\left(d^2 - ef\right)$ is the determinant of M_N.

One of the key problems of a neutrino mass model is to diagonalize the corresponding neutrino mass matrix, in this case, the matrix (15). To solve this problem, different methods and tricks have been used. Below we will follow a perturbation approach.

3. Diagonalization of the neutrino mass matrix

In the basis of the diagonalized charged lepton mass matrix (i.e., in the basis where this matrix has a diagonal form), where $M_\nu = U_0^T M_\nu^0 U_0$, and, as shown by the current neutrino oscillation experimental data, U_{PMNS} is a small deviation from the tribi-maximal form, one always has (see also [11])

$$M_\nu = M_0 + \lambda V, \quad (16)$$

with

$$M_0 = \begin{pmatrix} A & B & -B \\ B & E & -(A - E + B) \\ -B & -(A - E + B) & E \end{pmatrix}, \quad \lambda V = \begin{pmatrix} 0 & 0 & e_1 \\ 0 & 0 & e_3 \\ e_1 & e_3 & e_2 \end{pmatrix}, \quad (17)$$

where λ is a small (perturbation) parameter and M_0 can be diagonalized

$$\operatorname{diag}(m_1^0, m_2^0, m_3^0) = U_{TBM}^\dagger M_0 U_{TBM}, \quad (18)$$

by

$$U_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix} \sim \left(|1^0\rangle, |2^0\rangle, |3^0\rangle\right). \quad (19)$$

Using the perturbation expression

$$|n\rangle = |n^0\rangle + \lambda \sum_{k \neq n} |k^0\rangle \frac{V_{kn}}{m_n^0 - m_k^0}; \quad n, k = 1, 2, 3, \quad (20)$$

with

$$V_{kn} = \langle k|V|n\rangle, \quad (21)$$

one can diagonalize the matrix M_ν by

$$U_{PMNS} = \begin{pmatrix} \sqrt{\frac{2}{3}} + \sqrt{\frac{1}{3}}x^* & \sqrt{\frac{1}{3}} - \sqrt{\frac{2}{3}}x & -\sqrt{\frac{2}{3}}y - \sqrt{\frac{1}{3}}z \\ -\sqrt{\frac{1}{6}} + \sqrt{\frac{1}{3}}x^* + \sqrt{\frac{1}{2}}y^* & \sqrt{\frac{1}{3}} + \sqrt{\frac{1}{6}}x + \sqrt{\frac{1}{2}}z^* & \sqrt{\frac{1}{2}} + \sqrt{\frac{1}{6}}y - \sqrt{\frac{1}{3}}z \\ \sqrt{\frac{1}{6}} - \sqrt{\frac{1}{3}}x^* + \sqrt{\frac{1}{2}}y^* & -\sqrt{\frac{1}{3}} - \sqrt{\frac{1}{6}}x + \sqrt{\frac{1}{2}}z^* & \sqrt{\frac{1}{2}} - \sqrt{\frac{1}{6}}y + \sqrt{\frac{1}{3}}z \end{pmatrix}, \quad (22)$$

where

$$x = \frac{\sqrt{2}}{6}\left(\frac{2e_3 - e_1 - e_2}{m_1^0 - m_2^0}\right), \quad y = \frac{\sqrt{3}}{6}\left(\frac{2e_1 + e_2}{m_1^0 - m_3^0}\right), \quad z = \frac{1}{\sqrt{6}}\left(\frac{e_1 + e_2}{m_2^0 - m_3^0}\right). \quad (23)$$

From (22) a relation between the mixing angles and CP-violation Dirac phase δ_{CP} can be obtained and briefly discussed below. The mass spectrum derived by the present method of diagonalization of the mass matrix M_ν will be discussed in a separate work because it needs more analysis on the VEV's of the scalars and the Yukawa coupling coefficients as well as different phenomena and experimental results.

4. Relation between mixing angles and δ_{CP}

Denoting with U_{ij}, $i, j = 1, 2, 3$, a matrix element of (22), we obtain the relation

$$U_{21} + \sqrt{2}U_{22} - U_{31} - \sqrt{2}U_{32} = 2U_{11} - \sqrt{2}U_{12}. \tag{24}$$

The latter compared with the elements of the matrix U_{PMNS} of a general form

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{13}s_{23}c_{12}e^{i\delta} & c_{23}c_{12} - s_{13}s_{23}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta} & -s_{23}c_{12} - s_{13}c_{23}s_{12}e^{i\delta} & c_{23}c_{13}, \end{pmatrix}, \tag{25}$$

where $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$; $i, j = 1, 2, 3$, leads to a relation between $\delta_{CP} \equiv \delta$ and the mixing angles θ_{ij}:

$$\cos\delta \tan\theta_{13} = \frac{(\sqrt{2} - \tan\theta_{12})}{(1 + \sqrt{2}\tan\theta_{12})(1 - \tan\theta_{23})} \left(\frac{\sqrt{2}}{c_{23}} - \frac{1 + \tan\theta_{23}}{c_{13}} \right). \tag{26}$$

In general, it is not easy to determine δ_{CP} both theoretically and experimentally. Here, we obtain an explicit δ_{CP} as a function of the mixing angles which, thus, δ_{CP}, could be determined experimentally. Based on the relation (1) we can calculate δ_{CP} numerically using experimental inputs. The distributions of δ_{CP} are plotted in Fig. 1 and Fig. 2.

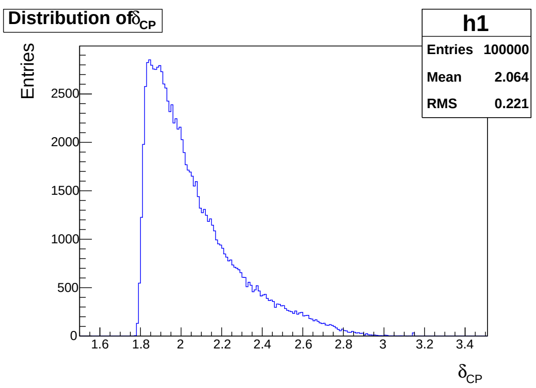

Figure 1: Distribution of δ_{CP}

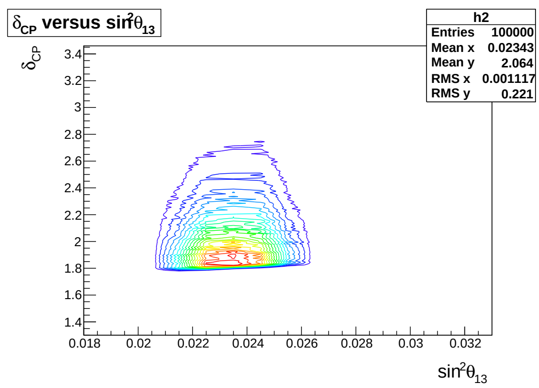

Figure 2: δ_{CP} versus $\sin^2\theta_{13}$.

From these figures we see that δ_{CP} distributes in the region $1.78 < \delta_{CP} < 3.2$. This distribution has a mean value at $\delta_{CP} = 2.064$ which is on the edge of the 1σ region from the best fit value (BFV) to the experimental data and reaches a maximum value around $\delta_{CP} = 1.9$ which is between 1σ from the BFV [5].

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Conclusions

A general neutrino mass matrix in an extended SM with an A4 flavour symmetry can be diagonalized by a pertubation method. Then, an explicit dependence between the CP-violation Dirac phase and the mixing angles leads to a quite good fit (near the 1σ region of the best fit) with recent experimental data at the normal hierarchy [5]. This result provides a more explicit form of neutrino mass and mixing matrices. The case with the inverse hierarchy is being investigated.

We are also considering pertubation of higher orders which may give a better fit (e.g., between the 1σ region) with the experimental data.

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