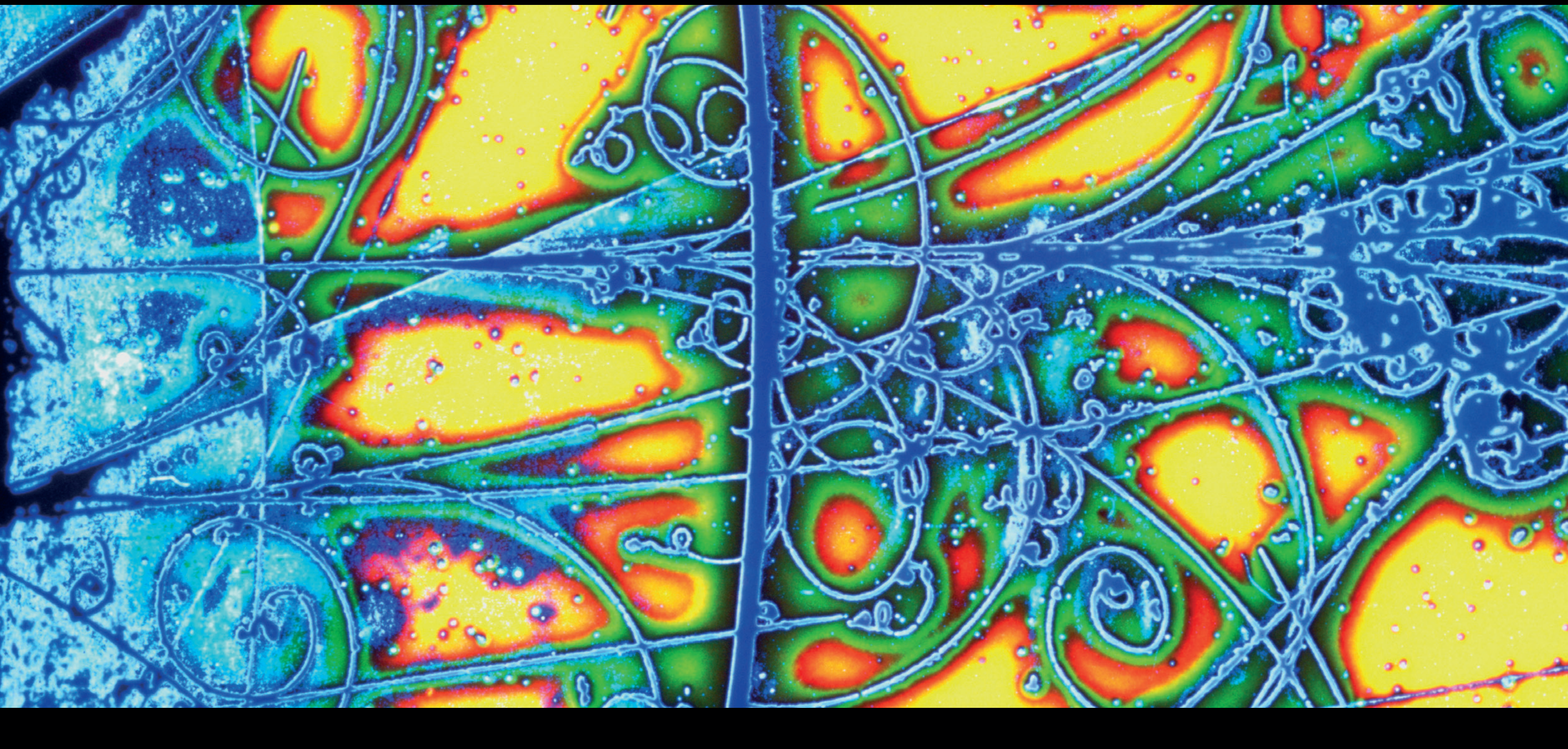
Advances in High Energy Physics

Non-Abelian Gauge Symmetries beyond the Standard Model

Guest Editors: Hoang Ngoc Long, Vicente Pleitez, Marc Sher, and Masaki Yasue



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Editorial Board

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Editorial

Non-Abelian Gauge Symmetries beyond the Standard Model

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The recent groundbreaking discovery of nonzero neutrino masses and oscillations is strong evidence of physics beyond the SM. The gauge symmetry of the SM and those of many extensions fix only the structure of gauge bosons. The fermions and Higgs representations have to be chosen somewhat arbitrarily. Thus, the models with the non-Abelian SM gauge symmetries are the main subject in this special issue. The authors have focused on models based on the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (3-3-1) gauge group, left-right symmetric model, and models with discrete symmetries. The gauge-Higgs unification based on the space-time extension has been also discussed in the issue.

The paper "*Mass mixing effect and oblique radiative corrections in extended $SU(2)_R \otimes SU(2)_L \otimes U(1)$ effective theory*" by Y. Zhang analyzes the properties of electroweak chiral effective Lagrangian with an extended $SU(2)_R$ gauge group. The non-Abelian $SU(2)_R$ contains sufficient complexity to incorporate interesting issues related to spontaneous parity violation and precise electroweak observables. The author discusses all possible mass-mixing terms and calculates the exact physical mass eigenvalues by diagonalization of mixing matrix without any approximations. The contributions to oblique radiative corrections parameters STU from $SU(2)_R$ fields are also presented.

In the paper "*Sources of FCNC in $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ models*" by J. M. Cabarcas et al., the authors explore the possible sources of flavor changing neutral currents and lepton flavor violation at tree level, in the 3-3-1 models. In the paper "*Non-standard neutrinos interactions in a 3-3-1 model with minimum Higgs sector*" by M. M. Jaime and P. C. de Holanda, the nonstandard neutrino interactions in the economical 3-3-1 model is presented. The limit on new gauge bosons is obtained.

The paper "*Gauge boson mixing in the 3-3-1 models with discrete symmetries*" by Dong Phung et al. deals with the mixing among gauge bosons in the 3-3-1 model with the discrete symmetries. The authors have shown that the neutrino tribimaximal mixing leads to the CPT violation. In the paper "*Radiatively generated leptogenesis in S_4 flavor symmetry models*" by T. P. Nguyen and D. Phung radiatively generated leptogenesis in the S_4 flavor symmetry models is presented. The authors have found a link between leptogenesis and amplitude of neutrinoless double beta decay $|m_{ee}|$ through a high-energy CP phase ϕ.

The paper "*Gauge-Higgs unification models in six dimensions with S_2/Z_2 extra space and GUT gauge symmetry*" by C.-W. Chiang et al. reviews gauge-Higgs unification models in six dimensions with S_2/Z_2 extra space and GUT gauge symmetry. It presents two scenarios for constructing a four-dimensional theory from the six-dimensional model, which leads to an SM-like gauge theory with the $SU(3) \otimes SU(2)_L \otimes U(1)_Y(\otimes U(1)_2)$ symmetry and the SM fermions in four dimensions. The gauge boson and Higgs boson masses are obtained.

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Review Article

Gauge-Higgs Unification Models in Six Dimensions with S^2/Z_2 Extra Space and GUT Gauge Symmetry

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We review gauge-Higgs unification models based on gauge theories defined on six-dimensional spacetime with S^2/Z_2 topology in the extra spatial dimensions. Nontrivial boundary conditions are imposed on the extra S^2/Z_2 space. This review considers two scenarios for constructing a four-dimensional theory from the six-dimensional model. One scheme utilizes the SO(12) gauge symmetry with a special symmetry condition imposed on the gauge field, whereas the other employs the E_6 gauge symmetry without requiring the additional symmetry condition. Both models lead to a standard model-like gauge theory with the $SU(3) \times SU(2)_L \times U(1)_Y (\times U(1)^2)$ symmetry and SM fermions in four dimensions. The Higgs sector of the model is also analyzed. The electroweak symmetry breaking can be realized, and the weak gauge boson and Higgs boson masses are obtained.

1. Introduction

The Higgs sector of the standard model (SM) plays an essential role in the spontaneous symmetry breaking (SSB) from the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group down to $SU(3)_C \times U(1)_{EM}$, thereby giving masses to the SM elementary particles. However, the SM does not address the most fundamental nature of the Higgs sector, such as the mass and self-coupling constant of the Higgs boson. Therefore, the Higgs sector is not only the last territory in the SM to be discovered, but will also provide key clues to new physics at higher energy scales.

Gauge-Higgs unification is one of many attractive approaches to physics beyond the SM in this regard [1–3] (for recent approaches, see [4–20]). In this approach, the Higgs particles originate from the extradimensional components of the gauge field defined on spacetime with the number of dimensions greater than four (for cases where). In other words, the Higgs sector is embraced into the gauge interactions in the higher-dimensional model, and many fundamental properties of Higgs boson are dictated by the gauge interactions.

In our recent studies, we have shown interesting properties of one type of gauge-Higgs unification models based on grand unified gauge theories defined on six-dimensional (6D) spacetime, with the extradimensional space having the topological structure of two-sphere orbifold S^2/Z_2 [21, 22].

In the usual coset space dimensional reduction (CSDR) approach [1, 23–26], one imposes on the gauge fields the symmetry condition which identifies the gauge transformation as the isometry transformation of S^2 due to its coset space structure $S^2 = SU(2)/U(1)$. The dimensional reduction is explicitly carried out by applying the solution of the symmetry condition. A background gauge field is introduced as part of the solution [1]. Such a background gauge field is also necessary for obtaining chiral fermions in four-dimensional (4D) spacetime, even without the symmetry condition. After the dimensional reduction, no Kaluza-Klein (KK) mode appears because of the imposed symmetry condition. The symmetry condition also restricts the gauge symmetry and the scalar contents originated from the extra gauge field components in the 4D spacetime. Moreover, a suitable potential for the scalar sector can be obtained to induce SSB at tree level.

In this paper, we consider two scenarios for constructing the 4D theory from a 6D model: one utilizing the symmetry condition for the gauge field with SO(12) symmetry [21], whereas the other without it for the gauge field with E_6 symmetry [22]. In the first scenario, however, we do not impose the condition on the fermions as used in other CSDR models. We then have massive KK modes for fermions but not the gauge and scalar fields in 4D. We can thus obtain a dark matter candidate under assumed KK parity. In the case without the symmetry condition, we find that the background gauge field is able to restrict the gauge symmetry and massless particle contents in 4D. Also, there are KK modes for each field, with the mass spectrum determined according to the model. Generically, massless modes do not appear in the KK mass spectrum because of the positive curvature of the S^2 space [27]. With the help of the background gauge field, however, we obtain massless KK modes for the gauge bosons and fermions.

In general, the gauge symmetry of a grand unified theory (GUT) tends to remain in 4D in these dimensional reduction approaches [24, 28–32]. Also, it is usually difficult to obtain an appropriate Higgs potential to break the GUT gauge symmetry to the SM-like one because of the gauge group structure. A GUT gauge symmetry can be broken to the SM-like gauge symmetry by imposing nontrivial boundary conditions (for cases with orbifold extra space, see, e.g., [4–8, 11, 12, 16–18, 33, 34]). Therefore, to solve the above-mentioned problems, we impose on the fields of the 6D model a set of nontrivial boundary conditions on the S^2/Z_2 space. Therefore, the gauge symmetry, scalar contents, and massless fermions are determined by these boundary conditions and the background gauge field. We find that in both scenarios, with or without the symmetry condition for the gauge field, the electroweak symmetry breaking (EWSB) can be realized spontaneously. The Higgs boson mass is predicted by analyzing the Higgs potential in the respective models.

This paper is organized as follows. In Section 2, we review two schemes for constructing a 4D theory from gauge models defined on 6D spacetime whose extra space has the S^2/Z_2 topology with a set of nontrivial boundary conditions. In Section 3, we show the models based on SO(12) and E_6 gauge symmetries, with the former being imposed with the symmetry condition on the gauge field and the latter without. We summarize our results in Section 4.

2. The 6D Gauge-Higgs Unification Model Construction Scheme with Extra S^2/Z_2 Space

There are two schemes for constructing a 4D theory from a 6D gauge theory, where the extra space is a two-sphere orbifold S^2/Z_2. Use of the symmetry condition is made on the first scheme but not the other. We apply nontrivial boundary condition in both schemes.

2.1. A Gauge Theory on 6D Spacetime with S^2/Z_2 Extraspace

2.1.1. The 6D Spacetime with S_2/Z_2 Extraspace

We begin by considering a 6D spacetime M^6 that is assumed to be a direct product of the 4D Minkowski spacetime M^4 and two-sphere orbifold S^2/Z_2, that is, $M^6 = M^4 \times S^2/Z_2$. The two-sphere S^2 is a unique two-dimensional coset space and can be written as $S^2 = \mathrm{SU}(2)_I/\mathrm{U}(1)_I$, where $\mathrm{U}(1)_I$ is a subgroup of $\mathrm{SU}(2)_I$. This coset space structure of S^2 requires that S^2 have the isometry group $\mathrm{SU}(2)_I$ and that the $\mathrm{U}(1)_I$ group be embedded in the group SO(2) which is in turn a subgroup of the full Lorentz group SO(1,5). We denote the coordinates of M^6 by $X^M = (x^\mu, y^\theta = \theta, y^\phi = \phi)$, where x^μ and $\{\theta, \phi\}$ are M^4 coordinates and S^2 spherical coordinates, respectively. The spacetime index M runs over $\mu \in \{0,1,2,3\}$ and $\alpha \in \{\theta,\phi\}$. The orbifold S^2/Z_2 is defined by the identification of (θ,ϕ) and $(\pi-\theta,-\phi)$ [35], leaving two fixed points: $(\pi/2, 0)$ and $(\pi/2,\pi)$. The metric g_{MN} of M^6 is written as

$$g_{MN} = \begin{pmatrix} \eta_{\mu\nu} & 0 \\ 0 & -g_{\alpha\beta} \end{pmatrix}, \tag{2.1}$$

where $\eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$ and $g_{\alpha\beta} = R^2\,\mathrm{diag}(1, \sin^2\theta)$ are the metrics for M^4 and S^2, respectively, and R is the radius of S^2. We define the vielbeins e_A^M that connect the metric of M^6 and that of the tangent space of M^6, denoted by h_{AB}, through the relation $g_{MN} = e_M^A e_N^B h_{AB}$. Here $A = (\mu, a)$, where $a \in \{4,5\}$, is the index for the coordinates of tangent space of M^6. The explicit forms of the vielbeins are

$$e_\theta^4 = R, \qquad e_\phi^5 = R\sin\theta, \qquad e_\phi^4 = e_\theta^5 = 0. \tag{2.2}$$

Also the nonzero components of the spin connection are

$$R_\phi^{45} = -R_\phi^{54} = -\cos\theta. \tag{2.3}$$

2.1.2. Lagrangian on 6D Spacetime with S^2/Z_2 Extra Space

We now discuss the general structure of a gauge theory on M^6. We first introduce a gauge field $A_M(x,y) = (A_\mu(x,y), A_\alpha(x,y))$, which belongs to the adjoint representation of a gauge group G, and fermions $\Psi(x,y)$, which lies in a representation F of G. The action of this theory is then given by

$$S = \int dx^4 R^2 \sin\theta\, d\theta\, d\phi \left(\overline{\Psi} i \Gamma^\mu D_\mu \Psi + \overline{\Psi} i \Gamma^a e_a^\alpha D_\alpha \Psi - \frac{1}{4g^2} g^{MN} g^{KL}\, \mathrm{Tr}[F_{MK}F_{NL}] \right), \tag{2.4}$$

where $F_{MN} = \partial_M A_N(X) - \partial_N A_M(X) - [A_M(X), A_N(X)]$ is the field strength, D_M is the covariant derivative including the spin connection, and Γ_A represents the Dirac matrices satisfying the 6D Clifford algebra. Here D_M and Γ_A can be written explicitly as

$$D_\mu = \partial_\mu - A_\mu, \qquad D_\theta = \partial_\theta - A_\theta, \qquad D_\phi = \partial_\phi - i\frac{\Sigma_3}{2}\cos\theta - A_\phi,$$
$$\Gamma_\mu = \gamma_\mu \otimes \mathbf{I}_2, \qquad \Gamma_4 = i\gamma_5 \otimes \sigma_1, \qquad \Gamma_5 = i\gamma_5 \otimes \sigma_2, \tag{2.5}$$

where $\{\gamma_\mu, \gamma_5\}$ are the 4D Dirac matrices, σ_i $(i = 1,2,3)$ are the Pauli matrices, $\mathbf{I}_d$ is the $d \times d$ identity matrix, and $\Sigma_3 = \mathbf{I}_4 \otimes \sigma_3$. The covariant derivative D_ϕ has the spin connection term $i(\Sigma_3/2)\cos\theta$ which is needed for space with a nonzero curvature- like S^2 and applied only to fermions. In 6D spacetime, one can define the chirality of fermions and the corresponding projection operators are

$$\Gamma_\pm = \frac{1 \pm \Gamma_7}{2}, \tag{2.6}$$

where $\Gamma_7 \equiv \gamma_5 \otimes \sigma_3$ is the chiral operator. The chiral fermions on 6D spacetime are thus

$$\Psi_\pm = \Gamma_\pm \Psi, \qquad \Gamma_7 \Psi_\pm = \pm\Psi_\pm. \tag{2.7}$$

The 6D chiral fermions can be also written in terms of 4D chiral fermions $\psi_{L(R)}$ as

$$\Psi_+ = \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix}, \tag{2.8}$$

$$\Psi_- = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}. \tag{2.9}$$

Here we note in passing that the mass dimensions of A_μ, A_α, Ψ and g in the 6D model are 1, 0, 5/2 and −1, respectively.

2.1.3. Nontrivial Boundary Conditions on the Two-Sphere Orbifold

On the two-sphere orbifold, one can consider parity operations $P: (\theta,\phi) \to (\pi-\theta,-\phi)$ and azimuthal translation $T_\phi: (\theta,\phi) \to (\theta,\phi+2\pi)$. Notice that here the periodicity $\phi \to \phi+2\pi$ is not associated with the orbifolding. We can impose the following two types of boundary conditions on both gauge and fermion fields under the two operations:

$$A_\mu(x,\pi-\theta,-\phi) = P_1 A_\mu(x,\theta,\phi) P_1, \qquad A_\mu(x,\pi-\theta,2\pi-\phi) = P_2 A_\mu(x,\theta,\phi) P_2, \tag{2.10}$$

$$A_\alpha(x,\pi-\theta,-\phi) = -P_1 A_\alpha(x,\theta,\phi) P_1, \qquad A_\alpha(x,\pi-\theta,2\pi-\phi) = -P_2 A_\alpha(x,\theta,\phi) P_2, \tag{2.11}$$

$$\Psi(x,\pi-\theta,-\phi) = \pm\gamma_5 P_1 \Psi(x,\theta,\phi), \qquad \Psi(x,\pi-\theta,2\pi-\phi) = \pm\gamma_5 P_2 \Psi(x,\theta,\phi), \tag{2.12}$$

or

$$A_\mu(x,\pi-\theta,-\phi)=P_1A_\mu(x,\theta,\phi)P_1,\qquad A_\mu(x,\theta,\phi+2\pi)=\widehat{P}_2A_\mu(x,\theta,\phi)\widehat{P}_2, \tag{2.13}$$

$$A_\alpha(x,\pi-\theta,-\phi)=-P_1A_\alpha(x,\theta,\phi)P_1,\qquad A_\alpha(x,\theta,\phi+2\pi)=\widehat{P}_2A_\alpha(x,\theta,\phi)\widehat{P}_2, \tag{2.14}$$

$$\Psi(x,\pi-\theta,-\phi)=\pm\gamma_5P_1\Psi(x,\theta,\phi),\qquad \Psi(x,\theta,\phi+2\pi)=\pm\widehat{P}_2\Psi(x,\theta,\phi), \tag{2.15}$$

where the former conditions are associated with P operation and combination of P and T_ϕ operations, while the latter conditions are associated with the P or T_ϕ operation individually. More explicitly, P_1, P_2, and $\widehat{P}_2$ correspond to operations P, PT_ϕ, and T_ϕ, respectively. These boundary conditions are determined by requiring invariance of the 6D action under the transformation $(\theta,\phi)\to(\pi-\theta,-\phi)$ and $\phi\to\phi+2\pi$. Note that at the poles ($\sin\theta=0$), the coordinate ϕ is not well-defined and the translation T_ϕ is irrelevant. Thus, only the components which are even under $\phi\to\phi+2\pi$ can exist without contradiction.

The projection matrices $P_{1,2}$ act on the gauge group representation space and have eigenvalues ±1. They assign different parities for different representation components. For fermion boundary conditions, the sign in front of γ_5 can be either + or − since the fermions always appear in bilinear forms in the action. The 4D action is then restricted by these parity assignments.

2.2. Dimensional Reduction Scheme with Symmetry Condition

Here we review the dimensional reduction scheme in which a symmetry condition is applied to the gauge field [21].

2.2.1. The Symmetry Condition

We impose on the gauge field $A_M(X)$ the symmetry which connects $SU(2)_I$ isometry transformation on S^2 and the gauge transformation of the field in order to carry out dimensional reduction. Moreover, the nontrivial boundary conditions of S^2/Z_2 are also utilized to restrict the 4D theory. The symmetry demands that the $SU(2)_I$ coordinate transformation should be compensated by a gauge transformation [1, 23]. It further leads to the following set of the symmetry condition on the gauge field:

$$\begin{aligned}\xi_i^\beta\partial_\beta A_\mu&=\partial_\mu W_i+\left[W_i,A_\mu\right],\\ \xi_i^\beta\partial_\beta A_\alpha+\partial_\alpha\xi_i^\beta A_\beta&=\partial_\alpha W_i+\left[W_i,A_\alpha\right],\end{aligned} \tag{2.16}$$

where ξ_i^α are the killing vectors that generate the $SU(2)_I$ symmetry, and W_i are some fields that generate an infinitesimal gauge transformation of G. Here the index $i=1,2,3$ corresponds to that of the SU(2) generators. The explicit forms of ξ_i^αs for S^2 are

$$\begin{aligned}&\xi_1^\theta=\sin\phi,\qquad \xi_1^\phi=\cot\theta\cos\phi,\\ &\xi_2^\theta=-\cos\phi,\qquad \xi_2^\phi=\cot\theta\sin\phi,\\ &\xi_3^\theta=0,\qquad \xi_3^\phi=-1.\end{aligned} \tag{2.17}$$

The LHS's and RHS's of (2.16) are infinitesimal isometry transformations and the corresponding infinitesimal gauge transformations, respectively.

2.2.2. Dimensional Reduction and Lagrangian in 4D Spacetime

The dimensional reduction of the gauge sector is explicitly carried out by applying the solutions of the symmetry condition equations (2.16). These solutions are given by Manton [1]

$$A_\mu = A_\mu(x), \qquad A_\theta = -\Phi_1(x), \qquad A_\phi = \Phi_2(x)\sin\theta - \Phi_3\cos\theta, \tag{2.18}$$

$$W_1 = -\Phi_3\frac{\cos\phi}{\sin\theta}, \qquad W_2 = -\Phi_3\frac{\sin\phi}{\sin\theta}, \qquad W_3 = 0, \tag{2.19}$$

where $\Phi_1(x)$ and $\Phi_2(x)$ are scalar fields and the Φ_3 term for A_ϕ corresponds to the background gauge field [36]. They satisfy the following constraints:

$$\left[\Phi_3, A_\mu\right] = 0, \tag{2.20}$$

$$[-i\Phi_3, \Phi_i(x)] = i\epsilon_{3ij}\Phi_j(x), \tag{2.21}$$

where the LHS shows the gauge transformation associated with Φ_3 and the RHS shows the $U(1)_I$ transformation embedded in Lorentz group SO(2). These constraints can be satisfied when $U(1)_I$ is embedded in the gauge group G and $-i\Phi_3$ should be chosen as the corresponding generator.

Substituting the solutions, (2.18), into $A_M(X)$ in the action, (2.4), one can easily obtain the 4D action by integrating out coordinates θ and ϕ in the gauge sector.

$$\begin{aligned} S_{4D}^{(\text{gauge})} = \int d^4x \Bigg(& -\frac{R^2}{4g^2}\,\text{Tr}\left[F_{\mu\nu}F^{\mu\nu}(x)\right] \\ & - \frac{1}{2g^2}\,\text{Tr}\Big[D'_\mu\Phi_1(x)D'^\mu\Phi_1(x) + D'_\mu\Phi_2(x)D'^\mu\Phi_2(x)\Big] \\ & - \frac{1}{2g^2R^2}\,\text{Tr}[(\Phi_3 + [\Phi_1(x),\Phi_2(x)])(\Phi_3 + [\Phi_1(x),\Phi_2(x)])]\Bigg), \end{aligned} \tag{2.22}$$

where $D'_\mu\Phi = \partial_\mu\Phi - [A_\mu, \Phi]$.

For fermions, we do not impose the symmetry condition. Then the gauge interaction term is not invariant under the coordinate transformation on S^2/Z_2. The fermion sector of the 4D action is thus obtained by expanding fermions in terms of the normal modes of S^2/Z_2 and then integrating out the S^2/Z_2 coordinates in the 6D action. As a result, the fermions have massive KK modes which can provide a dark matter candidate. Generally, the KK modes do not contain massless modes because of the positive curvature of S^2 [27]. Nevertheless, we can show that the fermion components satisfying the condition

$$-i\Phi_3\Psi = \frac{\Sigma_3}{2}\Psi \tag{2.23}$$

do have massless modes. The squared masses of the KK modes are eigenvalues of the square of the extradimensional Dirac-operator $-i\widehat{D}$. In the S^2 case,

$$\begin{aligned} -i\widehat{D} &= -ie^{\alpha a}\Gamma_a D_\alpha \\ &= -\frac{i}{R}\left[\Sigma_1\left(\partial_\theta + \frac{\cot\theta}{2}\right) + \Sigma_2\left(\frac{1}{\sin\theta}\partial_\phi + \Phi_3\cot\theta\right)\right], \end{aligned} \tag{2.24}$$

where $\Sigma_i = \mathbf{I}_4 \times \sigma_i$. Hence,

$$\begin{aligned} \left(-i\widehat{D}\right)^2 = -\frac{1}{R^2}\Bigg[&\frac{1}{\sin\theta}\partial_\theta(\sin\theta\partial_\theta) + \frac{1}{\sin^2\theta}\partial_\phi^2 + i(2(-i\Phi_3) - \Sigma_3)\frac{\cos\theta}{\sin^2\theta}\partial_\phi \\ &-\frac{1}{4} - \frac{1}{4\sin^2\theta} + \Sigma_3(-i\Phi_3)\frac{1}{\sin^2\theta} - (-i\Phi_3)^2\cot^2\theta\Bigg]. \end{aligned} \tag{2.25}$$

By acting the above operator on a fermion $\Psi(X)$ that satisfies (2.23), we obtain the relation

$$\left(-i\widehat{D}\right)^2\Psi = -\frac{1}{R^2}\left[\frac{1}{\sin\theta}\partial_\theta(\sin\theta\partial_\theta) + \frac{1}{\sin^2\theta}\partial_\phi^2\right]\Psi. \tag{2.26}$$

The eigenvalues of the operator on the RHS are less than or equal to zero. Hence, the fermion components satisfying (2.23) have massless modes, while other components have only massive KK modes. Note that the massless mode ψ_0 should be independent of S^2 coordinates θ and ϕ, that is,

$$\psi_0 = \psi(x). \tag{2.27}$$

The existence of massless fermions signifies the meaning and importance of the symmetry condition. Although the energy density of the gauge sector in the presence of the background field is higher than that with no background field, the massless fermions may help render a true ground state as a whole. In other words, the existence of the background field will give a positive contribution to the energy density of the gauge sector, indicating that the gauge sector with the background field alone is not at the ground state. Nevertheless, it gives rise to a negative contribution to the energy density of the fermion sector to induce massless fermions. We therefore expect that once both the gauge and fermion sectors are considered together, the existence of the background field renders the system at the ground state. We also note that one could impose symmetry condition on fermions [24, 37]. In that case, we obtain the massless condition equation (2.23) from the symmetry condition of fermion, and the solution of symmetry condition is independent of the S^2 coordinates: $\psi = \psi(x)$ with no massive KK mode. Therefore, the same discussion as before can be applied for this case if one only focuses on the massless mode in our scheme.

2.2.3. Gauge Symmetry and Particle Contents in 4D Spacetime

The symmetry condition and the nontrivial boundary conditions substantially constrain the 4D gauge group and the representations of the particle contents.

First, we show the prescriptions to identify gauge symmetry and field components which satisfy the constraint equations (2.20), (2.21), and (2.23). The gauge group H that satisfy the constraint equation (2.20) is identified as

$$H = C_G(U(1)_I), \tag{2.28}$$

where $C_G(U(1)_I)$ denotes the centralizer of $\mathrm{U}(1)_I$ in G [23]. Note that this implies $G \supset H = H' \times \mathrm{U}(1)_I$, where H' is some subgroup of G. In this way, the gauge group G is reduced to its subgroup $H = H' \times \mathrm{U}(1)_I$ by the symmetry condition.

Secondly, the scalar field components which satisfy the constraint equations (2.21) are specified by the following prescription. Suppose that the adjoint representations of $\mathrm{SU}(2)_I$ and G are decomposed according to the embeddings $\mathrm{SU}(2)_I \supset \mathrm{U}(1)_I$ and $G \supset H' \times \mathrm{U}(1)_I$ as

$$3(\text{adj SU}(2)) = \left(0(\text{adj U}(1))_I\right) + (2) + (-2), \tag{2.29}$$

$$\text{adj } G = (\text{adj } H)(0) + 1\left(0(\text{adj U}(1))_I\right) + \sum_g h_g(r_g), \tag{2.30}$$

where h_g's denote representations of H', and r_g's denote the $\mathrm{U}(1)_I$ charges. Then the scalar components satisfying the constraints belong to h_g's whose corresponding r_g's in (2.30) are ± 2.

Thirdly, the fermion components which satisfy the constraint equations (2.23) are determined as follows [37]. Let the group $\mathrm{U}(1)_I$ be embedded in the Lorentz group SO(2) in such a way that the vector representation 2 of SO(2) is decomposed according to $\mathrm{SO}(2) \supset \mathrm{U}(1)_I$ as

$$2 = (2) + (-2). \tag{2.31}$$

This embedding specifies a decomposition of the Weyl spinor representation $\sigma_6 = 4$ of $\mathrm{SO}(1,5)$ according to $\mathrm{SO}(1,5) \supset \mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)_I$ as

$$\sigma_6 = (2,1)(1) + (1,2)(-1), \tag{2.32}$$

where the $\mathrm{SU}(2) \times \mathrm{SU}(2)$ representations $(2,1)$ and $(1,2)$ correspond to left-handed and right-handed spinors, respectively. We note that this decomposition corresponds to (2.8) [or (2.9)]. We then decompose F according to $G \supset H' \times \mathrm{U}(1)_I$ as

$$F = \sum_f h_f(r_f). \tag{2.33}$$

Now the fermion components satisfying the constraints are identified as those h_f's whose corresponding r_f's in (2.33) are +1 for left-handed fermions and −1 for right-handed fermions.

Finally, we show which gauge symmetry and field components remain in 4D spacetime by surveying the consistency between the boundary conditions (2.13)–(2.15), the solutions in (2.18), and the massless fermion modes equation (2.27). By applying (2.18) and (2.27) to (2.13)–(2.15), we obtain the parity conditions

$$\begin{gathered}
A_\mu(x) = P_1\left(\widehat{P}_2\right) A_\mu(x) P_1\left(\widehat{P}_2\right), \\
-\Phi_1(x) = -P_1(-\Phi_1(x))P_1, \\
-\Phi_1(x) = \widehat{P}_2(-\Phi_1(x))\widehat{P}_2, \\
\Phi_2(x) + \Phi_3\cos\theta = -P_1\Phi_2(x)P_1 + P_1\Phi_3P_1\cos\theta, \\
\Phi_2(x) - \Phi_3\cos\theta = \widehat{P}_2\Phi_2(x)\widehat{P}_2 - \widehat{P}_2\Phi_3\widehat{P}_2\cos\theta, \\
\Psi(x) = \gamma_5 P_1\Psi(x), \\
\Psi(x) = \widehat{P}_2\Psi(x).
\end{gathered} \tag{2.34}$$

We find that the gauge fields, scalar fields, and massless fermions in 4D spacetime should be even for $P_1A_\mu P_1$ and $\widehat{P}_2A_\mu\widehat{P}_2$; $-P_1\Phi_{1,2}P_1$ and $\widehat{P}_2\Phi_{1,2}\widehat{P}_2$; $\gamma_5P_1\Psi$ and $\widehat{P}_2\Psi$, respectively. Φ_3 always remains in the spectrum because it is proportional to the $\mathrm{U}(1)_I$ generator and commutes with $P(P')$. Therefore, the particle spectrum contains those satisfying both the constraint equations (2.20), (2.21), and (2.23) and the parity conditions (2.34). The remaining 4D gauge symmetry can be readily identified by observing which components of the gauge field remain in the spectrum.

2.3. Dimensional Reduction Scheme without the Symmetry Condition

Here we review the dimensional reduction scheme which does not require the imposition of the symmetry condition on the gauge field [22].

2.3.1. Background Gauge Field and Gauge Group Reduction

Instead of utilizing the symmetry condition, we consider the background gauge field $A^B_\phi \equiv \widetilde{A}^B_\phi \sin\theta$ that corresponds to a Dirac monopole [36]

$$\widetilde{A}^B_\phi = -Q\frac{\cos\theta \mp 1}{\sin\theta}, \qquad \left(- : 0 \le \theta < \frac{\pi}{2},\ + : \frac{\pi}{2} \le \theta \le \pi\right), \tag{2.35}$$

where Q is proportional to the generator of a $\mathrm{U}(1)$ subgroup of the original gauge group. The background gauge field A^B_ϕ corresponds to $\Phi_3\cos\theta \subset A_\phi$ in (2.18).

Here we choose the background gauge field to belong to the $\mathrm{U}(1)_I$ group, which is a subgroup of original gauge group G:

$$G \supset G_{\mathrm{sub}} \otimes \mathrm{U}(1)_I. \tag{2.36}$$

We find that there is no massless mode for gauge field components with a nonzero $\mathrm{U}(1)_I$ charge. In fact, these components acquire masses due to the background field from the term proportional to $F_{\mu\phi}F^{\mu}_{\ \phi}$:

$$\begin{aligned}
&\mathrm{Tr}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2R^2\sin^2\theta}F_{\mu\phi}F^{\mu}_{\ \phi}\right] \\
&\quad\longrightarrow \mathrm{Tr}\left[-\frac{1}{4}(\partial_\mu A_\nu-\partial_\nu A_\mu)(\partial^\mu A^\nu-\partial^\nu A^\mu)-\frac{1}{2R^2\sin^2\theta}\left[A_\mu,A^B_\phi\right]\left[A^\mu,A^B_\phi\right]\right].
\end{aligned} \tag{2.37}$$

For the components of A_μ with nonzero $\mathrm{U}(1)_I$ charge, we have

$$A^i_\mu Q_i + A_{i\mu}Q^i \in A_\mu, \tag{2.38}$$

where Q_i ($Q^i = Q_i^\dagger$) are generators corresponding to distinct components in (3.30) that have nonzero $\mathrm{U}(1)_I$ charges, and $A_{i\mu}$ ($A^i_\mu = A^\dagger_{i\mu}$) are the corresponding components of A_μ. We find the term

$$\begin{aligned}
\frac{1}{\sin^2\theta}\mathrm{Tr}\left[\left[A_\mu,A^B_\phi\right]\left[A^\mu,A^B_\phi\right]\right] &= \frac{(\cos\theta\mp1)^2}{\sin^2\theta}\mathrm{Tr}\left[\left[A^i_\mu Q_i+A_{i\mu}Q^i,Q\right]\left[A^{i\mu}Q_i+A^\mu_i Q^i,Q\right]\right] \\
&= -2|q|^2\frac{(\cos\theta\mp1)^2}{\sin^2\theta}A^{i\mu}A_{i\mu},
\end{aligned} \tag{2.39}$$

where q is the Q charge of the relevant component. Use of the facts that A^B_ϕ belongs to $\mathrm{U}(1)_I$ and that $\mathrm{Tr}[Q_iQ^i]=2$ has been made in the above equation. A mass is thus associated with the lowest modes of those components of A_μ with nonzero $\mathrm{U}(1)_I$ charges:

$$\begin{aligned}
&\int d\Omega\,\mathrm{Tr}\left[-\frac{1}{4}(\partial_\mu A_\nu-\partial_\nu A_\mu)(\partial^\mu A^\nu-\partial^\nu A^\mu)-\frac{1}{2R^2\sin^2\theta}[A_\mu,A_B][A^\mu,A_B]\right]\Bigg|_{\mathrm{lowest}} \\
&\quad\longrightarrow -\frac{1}{2}\left[\partial_\mu A_{i\nu}(x)-\partial_\nu A_{i\mu}(x)\right]\left[\partial^\mu A^{i\nu}(x)-\partial^\nu A^{i\mu}(x)\right]+m_B^2A^i_\mu(x)A^{i\mu}(x),
\end{aligned} \tag{2.40}$$

where the subscript "lowest" means that only the lowest KK modes are kept. Here the lowest KK modes of A_μ correspond to the term $A_\mu(x)/\sqrt{4\pi}$ in the KK expansion. In short, any representation of A_μ carrying a nonzero $\mathrm{U}(1)_I$ charge acquires a mass m_B from the background field contribution after one integrates over the extra spatial coordinates. More explicitly,

$$m_B^2 = \frac{|q|^2}{4\pi R^2}\int d\Omega \frac{(\cos\theta \mp 1)^2}{\sin^2\theta} \simeq 0.39\frac{|q|^2}{R^2} \tag{2.41}$$

for the zero mode. Therefore the gauge group G is reduced to $G_{\text{sub}} \otimes \mathrm{U}(1)_I$ by the presence of the background gauge field. This condition is the same as the case with the symmetry condition.

2.3.2. Scalar Field Contents in 4D Spacetime

The scalar contents in 4D spacetime are obtained from the extradimensional components of the gauge field $\{A_\theta, A_\phi\}$ after integrating out the extra spatial coordinates. The kinetic term and potential term of $\{A_\theta, A_\phi\}$ are obtained from the gauge sector containing these components

$$\begin{aligned} S_{\text{scalar}} &= \int dx^4 d\Omega \left(\frac{1}{2g^2}\,\mathrm{Tr}\left[F_{\mu\theta}F^{\mu}{}_{\theta}\right] + \frac{1}{2g^2 \sin^2\theta}\,\mathrm{Tr}\left[F_{\mu\phi}F^{\mu}{}_{\phi}\right] - \frac{1}{2g^2R^2\sin^2\theta}\,\mathrm{Tr}\left[F_{\theta\phi}F_{\theta\phi}\right] \right) \\ &\longrightarrow \int dx^4 d\Omega \left(\frac{1}{2g^2} Tr\left[\left(\partial_\mu A_\theta - i\left[A_\mu, A_\theta\right]\right)^2 \right] + \frac{1}{2g^2} Tr\left[\left(\partial_\mu \widetilde{A}_\phi - i\left[A_\mu, \widetilde{A}_\phi\right]\right)^2 \right] \right) \\ &\quad - \frac{1}{2g^2R^2}\,\mathrm{Tr}\left[\left(\frac{1}{\sin\theta}\partial_\theta \left(\sin\theta \widetilde{A}_\phi + \sin\theta \widetilde{A}^B_\phi \right) - \frac{1}{\sin\theta}\partial_\phi A_\theta - i\left[A_\theta, \widetilde{A}_\phi + \widetilde{A}^B_\phi\right] \right)^2 \right], \end{aligned} \tag{2.42}$$

where we have taken $A_\phi = \widetilde{A}_\phi \sin\theta + \widetilde{A}^B_\phi \sin\theta$. In the second step indicated by the arrow in (2.42), we have omitted terms which do not involve A_θ and $\widetilde{A}_\phi$ from the right-hand side of the first equality. It is known that one generally cannot obtain massless modes for physical scalar components in 4D spacetime [14, 38]. One can see this by noting that the eigenfunction of the operator $(1/\sin\theta)\partial_\theta \sin\theta$ swith zero eigenvalue is not normalizable [14]. In other words, these fields have only KK modes. However, an interesting feature is that it is possible to obtain a negative squared mass when taking into account the interactions between the background gauge field $\widetilde{A}^B_\phi$ and $\{A_\theta, \widetilde{A}_\phi\}$. This happens when the component carries a nonzero $\mathrm{U}(1)_I$ charge, as the background gauge field belongs to $\mathrm{U}(1)_I$. In this case, the $(l = 1, m = 1)$ modes of these real scalar components are found to have a negative squared mass in 4D spacetime. They can be identified as the Higgs fields once they are shown to belong to the correct representation under the SM gauge group. Here the numbers (ℓ, m) are the angular momentum quantum number on S^2/Z_2, and each KK mode is characterized by these numbers. One can show that the $(l = 1, m = 0)$ mode has a positive squared mass and is not considered as the Higgs field. A discussion of the KK masses with general (ℓ, m) will be given in Section 3.2.5.

2.3.3. Chiral Fermions in 4D Spacetime

We introduce fermions as the Weyl spinor fields of the 6D Lorentz group SO(1,5). They can be written in terms of the SO(1,3) Weyl spinors as (2.8) and (2.9). In general, fermions on the two spheres do not have massless KK modes because of the positive curvature of the two spheres. The massless modes can be obtained by incorporating the background gauge field (2.35) though, for it can cancel the contribution from the positive curvature. In this case, the condition for obtaining a massless fermion mode is

$$Q\Psi = \pm\frac{1}{2}\Psi, \tag{2.43}$$

where Q comes from the background gauge field and is proportional to the $U(1)_I$ generator [35, 36, 38]. We observe that the upper [lower] component on the RHS of (2.8) [(2.9)] has a massless mode for the + [−] sign on the RHS of (2.43).

2.3.4. The Higgs Potential

The Lagrangian for the Higgs sector is derived from the gauge sector that contains extradimensional components of the gauge field $\{A_\theta, \tilde{A}_\phi\}$, as given in (2.42), by considering the lowest KK modes of them. The kinetic term and potential term are, respectively,

$$\begin{aligned} L_K &= \frac{1}{2g^2}\int d\Omega \left(\mathrm{Tr}\left[(\partial_\mu A_\theta - i[A_\mu, A_\theta])^2 \right] + \mathrm{Tr}\left[\left(\partial_\mu \tilde{A}_\phi - i\left[A_\mu, \tilde{A}_\phi\right] \right)^2 \right] \right)\Big|_{\text{lowest}}, \\ V &= \frac{1}{2g^2R^2}\int d\Omega\, \mathrm{Tr}\left[\left(\frac{1}{\sin\theta}\partial_\theta\left(\sin\theta \tilde{A}_\phi + \sin\theta \tilde{A}^B_\phi \right) - \frac{1}{\sin\theta}\partial_\phi A_\theta - i\left[A_\theta, \tilde{A}_\phi + \tilde{A}^B_\phi\right] \right)^2 \right]\Big|_{\text{lowest}}. \end{aligned} \tag{2.44}$$

In our model, scalar components other than the Higgs field have vanishing VEV because only the Higgs field has a negative mass-squared term, coming from the interaction with the background gauge field at tree level. Therefore, only the Higgs field contributes to the spontaneous symmetry breaking. Consider the (1,1) mode of the $\{(1,2)(3,-3,3) + \text{h.c.}\}$ representation in (3.31) as argued in the previous section. The gauge fields are given by the following KK expansions:

$$A_\theta = -\frac{1}{\sqrt{2}}\left[\Phi_1(x)\partial_\theta Y^-_{11}(\theta,\phi) + \Phi_2(x)\frac{1}{\sin\theta}\partial_\phi Y^-_{11}(\theta,\phi) \right] + \cdots, \tag{2.45}$$

$$\tilde{A}_\phi = \frac{1}{\sqrt{2}}\left[\Phi_2(x)\partial_\theta Y^-_{11}(\theta,\phi) - \Phi_1(x)\frac{1}{\sin\theta}\partial_\phi Y^-_{11}(\theta,\phi) \right] + \cdots, \tag{2.46}$$

where $\cdots$ represents higher KK mode terms [35]. The function $Y^-_{11} = -1/\sqrt{2}[Y_{11} + Y_{1-1}]$ is odd under $(\theta,\phi) \longrightarrow (\pi/2 - \theta, -\phi)$. We will discuss their higher KK modes and masses in the existence of the background gauge field in Section 3.2.5. With (2.45) and (2.46), the kinetic term becomes

$$L_K(x) = \frac{1}{2g^2}\left(\mathrm{Tr}\left[D_\mu\Phi_1(x)D^\mu\Phi_1(x)\right] + \mathrm{Tr}\left[D_\mu\Phi_2(x)D^\mu\Phi_2(x)\right]\right), \tag{2.47}$$

where $D_\mu\Phi_{1,2} = \partial_\mu\Phi_{1,2} - i[A_\mu, \Phi_{1,2}]$ is the covariant derivative acting on $\Phi_{1,2}$. The potential term, on the other hand, is

$$\begin{aligned} V = \frac{1}{2g^2R^2}\int d\Omega\,\mathrm{Tr}\Bigg[\Bigg(&-\sqrt{2}Y_{11}^-\Phi_2(x) + Q \\ &+ \frac{i}{2}[\Phi_1(x),\Phi_2(x)]\left\{\partial_\theta Y_{11}^-\partial_\theta Y_{11}^- + \frac{1}{\sin^2\theta}\partial_\phi Y_{11}^-\partial_\phi Y_{11}^-\right\} \\ &+ \frac{i}{\sqrt{2}}\left[\Phi_1(x),\tilde{A}_\phi^B\right]\partial_\theta Y_{11}^- + \frac{i}{\sqrt{2}}\left[\Phi_2(x),\tilde{A}_\phi^B\right]\frac{1}{\sin\theta}\partial_\phi Y_{11}^-\Bigg)^2\Bigg], \end{aligned} \tag{2.48}$$

where $\partial_\theta(\sin\theta\tilde{A}_\phi^B) = Q\sin\theta$ from (2.35) is used. Expanding the square in the trace, we get

$$\begin{aligned} V = \frac{1}{2g^2R^2}\int d\Omega\,\mathrm{Tr}\Bigg[&2\left(Y_{11}^+\right)^2\Phi_2^2(x) + Q^2 \\ &- \frac{1}{4}[\Phi_1(x),\Phi_2(x)]^2\left(\partial_\theta Y_{11}^-\partial_\theta Y_{11}^- + \frac{1}{\sin^2\theta}\partial_\phi Y_{11}^-\partial_\phi Y_{11}^-\right)^2 \\ &- \frac{1}{2}\left[\Phi_1(x),\tilde{A}_\phi^B\right]^2\left(\partial_\theta Y_{11}^-\right)^2 - \frac{1}{2}\left[\Phi_2(x),\tilde{A}_\phi^B\right]^2\left(\frac{1}{\sin\theta}\partial_\phi Y_{11}^-\right)^2 \\ &- 2i\Phi_2(x)\left[\Phi_1(x),\tilde{A}_\phi^B\right]Y_{11}^-\partial_\theta Y_{11}^- - \left[\Phi_1(x),\tilde{A}_\phi^B\right]\left[\Phi_2(x),\tilde{A}_\phi^B\right]\partial_\theta Y_{11}^-\frac{1}{\sin\theta}\partial_\phi Y_{11}^- \\ &+ iQ[\Phi_1(x),\Phi_2(x)]\left(\partial_\theta Y_{11}^-\partial_\theta Y_{11}^- + \frac{1}{\sin^2\theta}\partial_\phi Y_{11}^-\partial_\phi Y_{11}^-\right)\Bigg], \end{aligned} \tag{2.49}$$

where terms that vanish after the $d\Omega$ integration are directly omitted. In the end, the potential is simplified to

$$\begin{aligned} V = \frac{1}{2g^2R^2}\mathrm{Tr}\Bigg[&2\Phi_2^2(x) + 4\pi Q^2 - \frac{3}{10\pi}[\Phi_1(x),\Phi_2(x)]^2 + \frac{5i}{2}Q[\Phi_1(x),\Phi_2(x)] \\ &+ \mu_1[Q,\Phi_1(x)]^2 + \mu_2[Q,\Phi_2(x)]^2\Bigg], \end{aligned} \tag{2.50}$$

where use of $\tilde{A}_\phi^B = -Q(\cos\theta \mp 1)/\sin\theta$ has been made and $\mu_1 = 1 - (3/2)\ln 2$ and $\mu_2 = (3/4)(1 - 2\ln 2)$.

We now take the following linear combination of Φ_1 and Φ_2 to form a complex Higgs doublet,

$$\Phi(x) = \frac{1}{\sqrt{2}}(\Phi_1(x) + i\Phi_2(x)), \tag{2.51}$$

$$\Phi(x)^\dagger = \frac{1}{\sqrt{2}}(\Phi_1(x) - i\Phi_2(x)). \tag{2.52}$$

It is straightforward to see that

$$[\Phi_1(x), \Phi_2(x)] = i\left[\Phi(x), \Phi^\dagger(x)\right]. \tag{2.53}$$

The kinetic term and the Higgs potential now become

$$L_K = \frac{1}{g^2}\,\mathrm{Tr}\left[D_\mu \Phi^\dagger(x) D^\mu \Phi(x)\right], \tag{2.54}$$

$$V = \frac{1}{2g^2R^2}\,\mathrm{Tr}\Big[2\Phi_2^2(x) + 4\pi Q^2 + \frac{3}{10\pi}\left[\Phi(x), \Phi^\dagger(x)\right]^2 - \frac{5}{2}Q\left[\Phi(x), \Phi^\dagger(x)\right] + \mu_1[Q, \Phi_1(x)]^2 + \mu_2[Q, \Phi_2(x)]^2\Big]. \tag{2.55}$$

The last three terms in the potential are contributions to the squared mass term of the Higgs boson from the background gauge field and can lead to a negative value. This means that the existence of the background gauge field makes the minimum of Higgs potential lower.

3. The Models Based on Our Schemes

In this section, we show concrete models based on the scheme introduced in previous section. We review the model based on SO(12) gauge symmetry for the scheme with symmetry condition given in [21], and review the model based on E_6 gauge symmetry for the scheme without symmetry condition given in [22].

3.1. *The* $SO(12)$ *Model with Symmetry Condition*

Here we show a model based on a gauge group G = SO(12) and a representation F = 32 of SO(12) for fermions, under the scheme with symmetry condition [21]. The choice of G = SO(12) and F = 32 is motivated by the study based on CSDR which leads to an SO(10) × U(1) gauge theory with one generation of fermion in 4D spacetime [28] (for SO(12) GUT see also [39]).

3.1.1. A Gauge Symmetry and Particle Contents

First, we show the particle contents in 4D spacetime without parities equations (2.13)–(2.15). We assume that $U(1)_I$ is embedded into SO(12) such as

$$SO(12) \supset SO(10) \times U(1)_I. \tag{3.1}$$

Thus we identify $SO(10) \times U(1)_I$ as the gauge group which satisfy the constraint equations (2.20), using (2.28). The SO(12) gauge group is reduced to SO(10) × U(1) by the symmetry condition. We identify the scalar components which satisfy (2.21) by decomposing adjoint representation of SO(12):

$$SO(12) \supset SO(10) \times U(1)_I : 66 = 45(0) + 1(0) + 10(2) + 10(-2). \tag{3.2}$$

According to the prescription below (2.28) in Section 2, the scalar components 10(2) + 10(−2) remains in 4D spacetime. We also identify the fermion components which satisfy (2.23) by decomposing 32 representations of SO(12) as

$$SO(12) \supset SO(10) \times U(1)_I : 32 = 16(1) + \overline{16}(-1). \tag{3.3}$$

According to the prescription below (2.30) in Section 2, we have the fermion components as 16(1) for a left-handed fermion and $\overline{16}(-1)$ for a right-handed fermion, respectively, in 4D spacetime.

Next, we specify the parity assignment of $P_1(\widehat{P}_2)$ in order to identify the gauge symmetry and the particle contents that actually remain in 4D spacetime. We choose a parity assignment so as to break gauge symmetry as $SO(12) \supset SO(10) \times U(1)_I \supset SU(5) \times U(1)_X \times U(1)_I \supset SU(3) \times SU(2)_L \times U(1)_Y \times U(1)_X \times U(1)_I$ and to maintain Higgs-doublet in 4D spacetime. The parity assignment is written in 32 dimensional spinor basis of SO(12) such as

$$\begin{aligned}
&SO(12) \supset SU(3) \times SU(2)_L \times U(1)_Y \times U(1)_X \times U(1)_I, \\
&32 = (3,2)^{(+-)}(1,-1,1) + (\overline{3},2)^{(+-)}(-1,1,-1) \\
&\quad + (3,1)^{(--)}(4,1,-1) + \left(\overline{3},1\right)^{(--)}(-4,-1,1) \\
&\quad + (3,1)^{(-+)}(-2,-3,-1) + \left(\overline{3},1\right)^{(-+)}(2,3,1) \\
&\quad + (1,2)^{(++)}(3,-3,-1) + (1,2)^{(++)}(-3,3,1) \\
&\quad + (1,1)^{(--)}(6,-1,1) + (1,1)^{(--)}(-6,1,-1) \\
&\quad + (1,1)^{(-+)}(0,-5,1) + (1,1)^{(-+)}(0,5,-1),
\end{aligned} \tag{3.4}$$

where for example, (+,−) means that the parities $(P_1, \widehat{P}_2)$ of the associated components are (even, odd). We find the gauge symmetry in 4D spacetime by surveying parity assignment for the gauge field. The parity assignments of the gauge field under $A_\mu \rightarrow P_1 A_\mu P_1 (\widehat{P}_2 A_\mu \widehat{P}_2)$ are

$$
\begin{aligned}
66 = {} & (8,1)^{(++)}(0,0,0) + (1,3)^{(++)}(0,0,0) + (1,1)^{(++)}(0,0,0) \\
& + (1,1)^{(++)}(0,0,0) + (1,1)^{(++)}(0,0,0) \\
& + \Big[(3,2)^{(-+)}(-5,0,0) + \left(\overline{3},2\right)^{(-+)}(5,0,0) \\
& \quad + (3,2)^{(--)}(1,4,0) + \left(\overline{3},2\right)^{(--)}(-1,-4,0) \\
& \quad + (3,1)^{(+-)}(4,-4,0) + \left(\overline{3},1\right)^{(+-)}(-4,4,0) \\
& \quad + \underline{(3,1)^{(+-)}(-2,2,2) + \left(\overline{3},1\right)^{(+-)}(2,-2,-2)} \\
& \quad + \underline{(3,1)^{(++)}(-2,2,-2) + \left(\overline{3},1\right)^{(++)}(2,-2,2)} \\
& \quad + \underline{(1,2)^{(--)}(3,2,2) + (1,2)^{(--)}(-3,-2,-2)} \\
& \quad + \underline{(1,2)^{(-+)}(3,2,-2) + (1,2)^{(-+)}(-3,-2,2)} \\
& \quad + (1,1)^{(+-)}(6,4,0) + (1,1)^{(+-)}(-6,-4,0)\Big].
\end{aligned}
\tag{3.5}
$$

The components with an underline are originated from 10(2) and 10(−2) of $SO(10) \times U(1)_I$, which do not satisfy constraint equations (2.20), and hence these components do not remain in 4D spacetime. Thus we have the gauge fields with $(+,+)$ parity components without an underline in 4D spacetime, and the gauge symmetry is $SU(3) \times SU(2)_L \times U(1)_Y \times U(1)_X \times U(1)_I$.

The scalar particle contents in 4D spacetime are determined by the parity assignments, under $\Phi_{1,2} \rightarrow -P_1\, \Phi_{1,2} P_1$ and $\widehat{P}_2 \Phi_{1,2} \widehat{P}_2$:

$$
\begin{aligned}
66 = {} & (8,1)^{(-+)}(0,0,0) + (1,3)^{(-+)}(0,0,0) + (1,1)^{(-+)}(0,0,0) \\
& + (1,1)^{(-+)}(0,0,0) + (1,1)^{(-+)}(0,0,0) \\
& + \Big[(3,2)^{(++)}(-5,0,0) + \left(\overline{3},2\right)^{(++)}(5,0,0) \\
& \quad + (3,2)^{(+-)}(1,4,0) + \left(\overline{3},2\right)^{(+-)}(-1,-4,0) \\
& \quad + (3,1)^{(--)}(4,-4,0) + \left(\overline{3},1\right)^{(--)}(-4,4,0) \\
& \quad + \underline{(3,1)^{(--)}(-2,2,2) + \left(\overline{3},1\right)^{(--)}(2,-2,-2)} \\
& \quad + \underline{(3,1)^{(-+)}(-2,2,-2) + \left(\overline{3},1\right)^{(-+)}(2,-2,2)}
\end{aligned}
$$

$$
\begin{aligned}
&+ \underline{(1,2)^{(+-)}(3,2,2) + (1,2)^{(+-)}(-3,-2,-2)} \\
&+ \underline{(1,2)^{(++)}(3,2,-2) + (1,2)^{(++)}(-3,-2,2)} \\
&+(1,1)^{(--)}(6,4,0) + (1,1)^{(--)}(-6,-4,0)\Big].
\end{aligned}
\tag{3.6}
$$

Note that the relative sign for the parity assignment of P_1 is different from (3.5), and that the only underlined parts satisfy the constraint equations (2.21). Thus the scalar components in 4D spacetime are $(1,2)(3,2,-2)$ and $(1,2)(-3,\ -2,2)$.

We specify the massless fermion contents in 4D spacetime, by surveying the parity assignments for each components of fermion fields. We introduce two types of left-handed Weyl fermions that belong to 32 representation of SO(12), which have parity assignments $\psi^{(\widehat{P}_2)} \to \gamma_5 P_1 \psi^{(\widehat{P}_2)}(\widehat{P}_2\psi^{(\widehat{P}_2)})$ and $\psi^{(-\widehat{P}_2)} \to \gamma_5 P_1 \psi^{(-\widehat{P}_2)}(-\widehat{P}_2\psi^{(-\widehat{P}_2)})$, respectively. They have the parity assignments as

$$
\begin{aligned}
32_L^{(\widehat{P}_2)} = {} & \underline{(3,2)^{(--)}(1,-1,1)_L} + \left(\overline{3},2\right)^{(--)}(-1,1,-1)_L \\
& + \underline{\left(\overline{3},1\right)^{(+-)}(-4,-1,1)_L} + (3,1)^{(+-)}(4,1,-1)_L \\
& + \underline{\left(\overline{3},1\right)^{(++)}(2,3,1)_L} + (3,1)^{(++)}(-2,-3,-1)_L \\
& + \underline{(1,2)^{(-+)}(-3,3,1)_L} + (1,2)^{(-+)}(3,-3,-1)_L \\
& + \underline{(1,1)^{(+-)}(6,-1,1)_L} + (1,1)^{(+-)}(-6,1,-1)_L \\
& + \underline{(1,1)^{(++)}(0,-5,1)_L} + (1,1)^{(++)}(0,5,-1)_L, \\
32_R^{(\widehat{P}_2)} = {} & (3,2)^{(+-)}(1,-1,1)_R + \underline{\left(\overline{3},2\right)^{(+-)}(-1,1,-1)_R} \\
& + \left(\overline{3},1\right)^{(--)}(-4,-1,1)_R + \underline{(3,1)^{(--)}(4,1,-1)_R} \\
& + \left(\overline{3},1\right)^{(-+)}(2,3,1)_R + \underline{(3,1)^{(-+)}(-2,-3,-1)_R} \\
& + (1,2)^{(++)}(-3,3,1)_R + \underline{(1,2)^{(++)}(3,-3,-1)_R} \\
& + (1,1)^{(--)}(6,-1,1)_R + \underline{(1,1)^{(--)}(-6,1,-1)_R} \\
& + (1,1)^{(-+)}(0,-5,1)_R + \underline{(1,1)^{(-+)}(0,5,-1)_R}, \\
32_L^{(-\widehat{P}_2)} = {} & \underline{(3,2)^{(-+)}(1,-1,1)_L} + \left(\overline{3},2\right)^{(-+)}(-1,1,-1)_L \\
& + \underline{\left(\overline{3},1\right)^{(++)}(-4,-1,1)_L} + (3,1)^{(++)}(4,1,-1)_L
\end{aligned}
$$

$$
\begin{aligned}
&+ \underline{\left(\overline{3},1\right)^{(+-)}(2,3,1)_L} + (3,1)^{(+-)}(-2,-3,-1)_L \\
&+ \underline{(1,2)^{(--)}(-3,3,1)_L} + (1,2)^{(--)}(3,-3,-1)_L \\
&+ \underline{(1,1)^{(++)}(6,-1,1)_L} + (1,1)^{(++)}(-6,1,-1)_L \\
&+ \underline{(1,1)^{(+-)}(0,-5,1)_L} + (1,1)^{(+-)}(0,5,-1)_L, \\
32_R^{(-\widehat{P}_2)} = {} & (3,2)^{(++)}(1,-1,1)_R + \underline{\left(\overline{3},2\right)^{(++)}(-1,1,-1)_R} \\
&+ \left(\overline{3},1\right)^{(-+)}(-4,-1,1)_R + \underline{(3,1)^{(-+)}(4,1,-1)_R} \\
&+ \left(\overline{3},1\right)^{(-+)}(2,3,1)_R + \underline{(3,1)^{(-+)}(-2,-3,-1)_R} \\
&+ (1,2)^{(+-)}(-3,3,1)_R + \underline{(1,2)^{(+-)}(3,-3,-1)_R} \\
&+ (1,1)^{(-+)}(6,-1,1)_R + \underline{(1,1)^{(-+)}(-6,1,-1)_R} \\
&+ (1,1)^{(--)}(0,-5,1)_R + \underline{(1,1)^{(--)}(0,5,-1)_R},
\end{aligned}
\tag{3.7}
$$

where $L(R)$ means the left-handedness (right-handedness) of fermions in 4D spacetime, and the underlined parts correspond to the components which satisfy constraint equations (2.23). Note the relative sign for parity assignment of P_1 between left-handed fermion and right-handed fermion and that of $\widehat{P}_2$ between $32^{(\widehat{P}_2)}$ and $32^{(-\widehat{P}_2)}$. The difference between $32^{(\widehat{P}_2)}$ and $32^{(-\widehat{P}_2)}$ is allowed because of the bilinear form of the fermion sector. We thus find that the massless fermion components in 4D spacetime are one generation of SM-fermions with right-handed neutrino: $\{(3,2)(1,-1,1)_L,\ (3,1)(4,1,-1)_R,\ (3,1)(-2,-3,-1)_R,\ (1,2)(-3,3,1)_L,\ (1,1)(-6,1,-1)_R,\ (1,1)(0,5,-1)_R\}$.

3.1.2. The Higgs Sector of the Model

We analyze the Higgs-sector of our model. The Higgs-sector L_{Higgs} is the last two terms of(2.22)

$$
\begin{aligned}
L_{\text{Higgs}} = {} & -\frac{1}{2g^2}\operatorname{Tr}\left[D'_\mu\Phi_1(x)D'^\mu\Phi_1(x) + D'_\mu\Phi_2(x)D'^\mu\Phi_2(x)\right] \\
& - \frac{1}{2g^2R^2}\operatorname{Tr}\left[(\Phi_3 + [\Phi_1(x),\Phi_2(x)])(\Phi_3 + [\Phi_1(x),\Phi_2(x)])\right],
\end{aligned}
\tag{3.8}
$$

where the first term of RHS is the kinetic term of Higgs and the second term gives the Higgs potential. We rewrite the Higgs-sector in terms of genuine Higgs field in order to analyze it.

We first note that the Φ_is are written as

$$\Phi_i = i\phi_i = i\phi_i^a Q_a, \tag{3.9}$$

where Q_as are generators of gauge group SO(12), since Φ_is are originated from gauge fields $A_\alpha = iA_\alpha^a Q_a$; for the gauge group generator we assume the normalization $\mathrm{Tr}(Q_a Q_b) = -2\delta_{ab}$. Note that we assumed the $-i\Phi_3$ as the generator of $U(1)_I$ embedded in SO(12),

$$-i\Phi_3 = Q_I. \tag{3.10}$$

We change the notation of the scalar fields according to (2.29) such that,

$$\phi_+ = \frac{1}{2}(\phi_1 + i\phi_2), \qquad \phi_- = \frac{1}{2}(\phi_1 - i\phi_2), \tag{3.11}$$

in order to express solutions of the constraint equations (2.21) clearly. The constraint equations (2.21) then rewritten as

$$[Q_I, \phi_+] = \phi_+, \qquad [Q_I, \phi_-] = -\phi_-. \tag{3.12}$$

The kinetic term L_{KE} and potential $V(\phi)$ term are rewritten in terms of ϕ_+ and ϕ_-:

$$L_{KE} = -\frac{1}{g^2}\mathrm{Tr}\Big[D'_\mu\phi_+(x)D'^\mu\phi_-(x)\Big], \tag{3.13}$$

$$V = -\frac{1}{2g^2R^2}\mathrm{Tr}\Big[Q_I^2 - 4Q_I[\phi_+,\phi_-] + 4[\phi_+,\phi_-][\phi_+,\phi_-]\Big], \tag{3.14}$$

where covariant derivative D'_μ is $D'_\mu\phi_\pm = \partial_\mu\phi_\pm - [A_\mu, \phi_\pm]$.

Next, we change the notation of SO(12) generators Q_a according to decomposition (3.5) such that

$$\begin{aligned} Q_G = \Big\{ & Q_i, Q_\alpha, Q_Y, Q, Q_I, Q_{ax(-500)}, Q^{ax(500)} \\ & Q_{ax(140)}, Q^{ax(-1-40)}, Q_{a(4-40)}, Q^{a(-440)} \\ & Q_{a(-22-2)}, Q^{a(2-22)}, Q_{a(-222)}, Q^{a(2-2-2)} \\ & Q_{x(322)}, Q^{x(-3-2-2)}, Q_{x(32-2)}, Q^{x(-3-22)} \\ & Q_{(640)}, Q_{(-6-40)} \Big\}, \end{aligned} \tag{3.15}$$

Table 1: Commutation relations of $Q^{x(-3-22)}$, $Q_{x(32-2)}$, Q_α, Q_Y, Q, and Q_I.

$[Q^{x(-3-22)}, Q_{y(32-2)}] = -\sqrt{\frac{3}{10}}\delta^x_y Q_Y + -\sqrt{\frac{1}{5}}\delta^x_y Q + \delta^x_y Q_I + \frac{1}{\sqrt{2}}(\sigma^*_\alpha)^x_y Q_\alpha$	
$[Q_\alpha, Q_x] = -\frac{1}{\sqrt{2}}(\sigma_\alpha)^{y}_{x} Q_y$,	$[Q_\alpha, Q^x] = \frac{1}{\sqrt{2}}(\sigma^*_\alpha)^x_y Q^y$,
$[Q_x, Q_y] = 0$,	$[Q_Y, Q^x] = -\sqrt{\frac{3}{10}}Q^x$,
$[Q, Q^x] = -\sqrt{\frac{1}{5}}Q^x$,	$[Q_I, Q^x] = Q^x$,

where the order of generators corresponds to (3.5), index $i = 1$–8 corresponds to SU(3) adjoint rep, index $\alpha = 1$–3 corresponds to SU(2) adjoint rep, index $a = 1$–3 corresponds to SU(3)-triplet, and index $x = 1, 2$ corresponds to SU(2)-doublet. We write $\phi_\pm$ in terms of the genuine Higgs field ϕ_x which belongs to $(1,2)(3,2,-2)$, such that

$$\begin{aligned} \phi_+ &= \phi_x Q^{x(-3-22)} , \\ \phi_- &= \phi^x Q_{x(32-2)}, \end{aligned} \tag{3.16}$$

where $\phi^x = (\phi_x)^\dagger$. We also write gauge field $A_\mu(x)$ in terms of Qs in (3.38) as

$$A_\mu(x) = i\Big(A^i_\mu Q_i + A^\alpha_\mu Q_\alpha + B_\mu Q_Y + C_\mu Q + E_\mu Q_I\Big). \tag{3.17}$$

We need commutation relations of $Q^{x(-3-22)}$, $Q_{x(32-2)}$, Q_α, Q_Y, Q, and Q_I in order to analyze the Higgs sector; we summarized them in Table 1.

Finally, we obtain the Higgs sector with genuine Higgs field by substituting (3.16)–(3.17) into (3.13) and (3.14) and rescaling the fields $\phi \to (g/\sqrt{2})\phi$ and $A_\mu \to (g/R)A_\mu$, and the couplings $(\sqrt{2}/R)g = g_2$ and $\sqrt{6/(5R^2)}g = g_Y$,

$$L_{\text{Higgs}} = \left|D_\mu \phi_x\right|^2 - V(\phi), \tag{3.18}$$

where the covariant derivative $D_\mu\phi_x$ and potential $V(\phi)$ are

$$D_\mu\phi_x = \partial_\mu\phi_x + ig_2\frac{1}{2}(\sigma_\alpha)^y_x A_{\alpha\mu}\phi_y + ig_Y\frac{1}{2}B_\mu\phi_x + i\sqrt{\frac{1}{5}}gC_\mu\phi_x - igE_\mu\phi_x, \tag{3.19}$$

$$V = -\frac{2}{R^2}\phi^x\phi_x + \frac{3g^2}{2R^2}\left(\phi^x\phi_x\right)^2, \tag{3.20}$$

respectively. Notice that we omitted the constant term in the Higgs potential. We note that the $SU(2)_L \times U(1)_Y$ part of the Higgs sector has the same form as the SM Higgs sector. Therefore we obtain the electroweak symmetry breaking $SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$. The Higgs field ϕ^x acquires vacuum expectation value (VEV) as

$$\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}, \qquad v = \sqrt{\frac{4}{3}}\frac{1}{g}, \tag{3.21}$$

and W boson mass m_W and Higgs mass m_H are given in terms of radius R

$$m_W = g_2\frac{v}{2} = \sqrt{\frac{2}{3}}\frac{1}{R}, \qquad m_H = \sqrt{3}\frac{gv}{R} = \sqrt{4}\frac{1}{R}. \tag{3.22}$$

The ratio between m_W and m_H is predicted

$$\frac{m_H}{m_W} = \sqrt{6}. \tag{3.23}$$

We thus find m_H ~196 GeV in this model. The Weinberg angle is given by

$$\sin^2\theta_W = \frac{g_Y^2}{g_2^2 + g_Y^2} = \frac{3}{8}, \tag{3.24}$$

which is same as SU(5) GUT case. The prediction for the Weinberg angle at tree level is not consistent with the electroweak measurements. One should also take into account quantum corrections including contributions from the KK modes. It is, however, beyond the scope of this paper.

In principle, one-loop power divergences in the Higgs potential would reappear since the operator linear in F_{ab} is allowed, where $\{a, b\}$ denote extraspatial components [40]. Such an operator would have the form

$$F^{\alpha}_{\theta\phi}(x), \tag{3.25}$$

where α corresponds to the index of the U(1) generator remaining in 4D. This operator is potentially dangerous since its coefficient can be divergent. We can readily avoid this by requiring parity invariance on S^2/Z_2 as in the T^2/Z_2 case [41].

First, consider the parity transformation $\theta \to \pi-\theta$. The parity conditions for the fields are defined as

$$\begin{aligned} A_\mu(x,\theta,\phi) &\longrightarrow A_\mu(x,\pi-\theta,\phi), \\ A_\theta(x,\theta,\phi) &\longrightarrow -A_\theta(x,\pi-\theta,\phi), \\ A_\phi(x,\theta,\phi) &\longrightarrow A_\phi(x,\pi)(x,\pi-\theta,\phi), \\ \Psi(x,\theta,\phi) &\longrightarrow \pm\Gamma^4\Psi(x,\pi-\theta,\phi), \end{aligned} \tag{3.26}$$

where $\Gamma^4 = \gamma_5 \otimes \sigma_1$. It is easy to see that the action in 6D, (2.4), is invariant under such a parity transformation.

Secondly, we check the consistency between the orbifold boundary conditions on S^2/Z_2, (2.10)–(2.12), and the parity conditions, (3.26). By performing the parity transformation on both sides of the orbifold boundary conditions, (2.10)–(2.12), we obtain

$$\begin{aligned} A_\mu(x,\theta,-\phi) &= P_1 A_\mu(x,\pi-\theta,\phi)P_1, \\ -A_\theta(x,\theta,-\phi) &= P_1 A_\theta(x,\pi-\theta,\phi)P_1, \\ A_\phi(x,\theta,-\phi) &= -P_1 A_\phi(x,\pi-\theta,\phi)P_1, \\ \pm\Gamma^4\Psi(x,\theta,-\phi) &= \pm\gamma_5 P_1(\pm\Gamma^4)\Psi(x,\pi-\theta,\phi), \\ A_\mu(x,\pi-\theta,\phi+2\pi) &= \widehat{P}_2 A_\mu(x,\pi-\theta,\phi)P_2, \\ -A_\theta(x,\pi-\theta,\phi+2\pi) &= \widehat{P}_2 A_\theta(x,\pi-\theta,\phi)P_2, \\ A_\phi(x,\pi-\theta,\phi+2\pi) &= -\widehat{P}_2 A_\phi(x,\pi-\theta,\phi)P_2, \\ \pm\Gamma^4\Psi(x,\pi-\theta,\phi+2\pi) &= \pm\gamma_5\widehat{P}_2(\pm\Gamma^4)\Psi(x,\pi-\theta,\phi). \end{aligned} \tag{3.27}$$

Since (2.10)–(2.12) hold for any θ and ϕ and Γ^4 commutes with γ_5, we find that the orbifold boundary conditions still hold under the parity transformation with the identification of $\theta = \pi - \theta'$. In other words, the orbifold boundary conditions, (2.10)–(2.12), are parity invariant.

Finally, we find that under the parity, the operator $F^\alpha_{\theta\phi}$ transforms to $-F^\alpha_{\theta\phi}$. Therefore, this operator is forbidden by parity invariance of the action. An explicit calculation of one-loop corrections to the Higgs potential to show that this operator vanishes, however, is beyond the scope of this paper.

3.2. *The* E_6 *Model without Symmetry Condition*

Here we show a model based on a gauge group $G = \mathrm{E}_6$ with a representation **27** for a fermion, under the scheme without symmetry condition [22].

3.2.1. Gauge Group Reduction

We consider the following gauge group reduction

$$\begin{aligned}E_6 &\supset SO(10)\times U(1)_I\\ &\supset SU(5)\times U(1)_X\times U(1)_I\\ &\supset SU(3)\times SU(2)\times U(1)_Y\times U(1)_X\times U(1)_I.\end{aligned} \tag{3.28}$$

The background gauge field in (2.35) is chosen to belong to the $U(1)_I$ group. This choice is needed in order to obtain chiral SM fermions in 4D spacetime to be discussed later. There are two other symmetry reduction schemes. One can prove that the results in those two schemes are effectively the same as the one considered here once we require the correct U(1) combinations for the hypercharge and the background field.

We then impose the parity assignments with respect to the fixed points, (2.10)–(2.15). The parity assignments for the fundamental representation of E_6 is chosen to be

$$\begin{aligned}\mathbf{27} = {}&(1,2)(-3,-2,-2)^{(+,+)} + (1,2)(3,2,-2)^{(-,-)} + (1,2)(-3,3,1)^{(+,-)}\\ &+(1,1)(6,-1,1)^{(+,+)} + (1,1)(0,-5,1)^{(-,-)} + (1,1)(0,0,4)^{(-,+)}\\ &+(3,2)(1,-1,1)^{(-,+)} + (3,1)(-2,2,-2)^{(+,-)} + \left(\overline{3},1\right)(-4,-1,1)^{(+,+)}\\ &+\left(\overline{3},1\right)(2,3,1)^{(+,+)} + \left(\overline{3},1\right)(2,-2,-2)^{(-,+)},\end{aligned} \tag{3.29}$$

where, for example, $(+,-)$ means that the parities under P_1 and P_2 are (even, odd). By the requirement of consistency, we find that the components of A_μ in the adjoint representation have the parities under $A_\mu \to P_1 A_\mu P_1$ $(P_2 A_\mu P_2)$ as follows:

$$\begin{aligned}\mathbf{78}|_{A_\mu} = {}&\underline{(8,1)(0,0,0)^{(+,+)} + (1,3)(0,0,0)^{(+,+)}}\\ &+\underline{(1,1)(0,0,0)^{(+,+)} + (1,1)(0,0,0)^{(+,+)} + (1,1)(0,0,0)^{(+,+)}}\\ &+(3,2)(-5,0,0)^{(-,+)} + \left(\overline{3},2\right)(5,0,0)^{(-,+)}\\ &+(3,2)(1,4,0)^{(+,-)} + \left(\overline{3},2\right)(-1,-4,0)^{(+,-)}\\ &+(3,1)(4,-4,0)^{(-,-)} + \left(\overline{3},1\right)(-4,4,0)^{(-,-)}\\ &+(1,1)(-6,-4,0)^{(-,-)} + (1,1)(6,4,0)^{(-,-)}\\ &+(3,2)(1,-1,-3)^{(+,+)} + \left(\overline{3},2\right)(-1,1,3)^{(+,+)}\\ &+(3,1)(4,1,3)^{(-,+)} + \left(\overline{3},1\right)(-4,-1,-3)^{(-,+)}\\ &+(3,1)(-2,-3,3)^{(+,-)} + \left(\overline{3},1\right)(2,3,-3)^{(+,-)}\end{aligned}$$

$$
\begin{aligned}
&+ (1,2)(-3,3,-3)^{(-,-)} + (1,2)(3,-3,3)^{(-,-)} \\
&+ (1,1)(-6,1,3)^{(-,+)} + (1,1)(6,-1,-3)^{(-,+)} \\
&+ (1,1)(0,-5,-3)^{(+,-)} + (1,1)(0,5,3)^{(+,-)},
\end{aligned}
\tag{3.30}
$$

where the underlined components correspond to the adjoint representations of SU(3) × SU(2) × $U(1)_Y$ × $U(1)_X$ × $U(1)_I$, respectively. We note that the components with parity $(+,+)$ can have massless zero modes in 4D spacetime. Such components include the adjoint representations of SU(3) × SU(2) × $U(1)^3$, $(3,2)(1,-1,-3)$ and its conjugate. The latter components seem problematic. Yet they do not appear in the low-energy spectrum due to nonzero $U(1)_I$ charge. The zero modes of these components will get masses from the background field as in (2.41).

3.2.2. Scalar Field Contents in 4D Spacetime

With the parity assignments with respect to the fixed points, (2.11) and (2.14), we have for the A_θ and A_ϕ fields

$$
\begin{aligned}
\mathbf{78}|_{A_{\theta,\phi}} = {}& (8,1)(0,0,0)^{(-,-)} + (1,3)(0,0,0)^{(-,-)} \\
&+ (1,1)(0,0,0)^{(-,-)} + (1,1)(0,0,0)^{(-,-)} + (1,1)(0,0,0)^{(-,-)} \\
&+ (3,2)(-5,0,0)^{(+,-)} + \left(\overline{3},2\right)(5,0,0)^{(+,-)} \\
&+ (3,2)(1,4,0)^{(-,+)} + \left(\overline{3},2\right)(-1,-4,0)^{(-,+)} \\
&+ (3,1)(4,-4,0)^{(+,+)} + \left(\overline{3},1\right)(-4,4,0)^{(+,+)} \\
&+ (1,1)(-6,-4,0)^{(+,+)} + (1,1)(6,4,0)^{(+,+)} \\
&+ (3,2)(1,-1,-3)^{(-,-)} + \left(\overline{3},2\right)(-1,1,3)^{(-,-)} \\
&+ (3,1)(4,1,3)^{(+,-)} + \left(\overline{3},1\right)(-4,-1,-3)^{(+,-)} \\
&+ (3,1)(-2,-3,3)^{(-,+)} + \left(\overline{3},1\right)(2,3,-3)^{(-,+)} \\
&+ (1,2)(-3,3,-3)^{(+,+)} + (1,2)(3,-3,3)^{(+,+)} \\
&+ (1,1)(-6,1,3)^{(+,-)} + (1,1)(6,-1,-3)^{(+,-)} \\
&+ (1,1)(0,-5,-3)^{(-,+)} + (1,1)(0,5,3)^{(-,+)}.
\end{aligned}
\tag{3.31}
$$

Components with $(+,-)$ or $(-,+)$ parity do not have KK modes since they are odd under $\phi \to \phi{+}2\pi$ and the KK modes of gauge field are specified by integer angular momentum quantum numbers ℓ and m on the two spheres. In the S^1/Z_2 case, the translation group on S^1 is U(1) and any quantum number is allowed. After orbifolding, we obtain the quantum numbers allowed by parity and they can be nonintegers. On the other hand, the translation group on S^2 is SU(2) and only integer quantum numbers are allowed because they correspond to quantized angular momenta. We then concentrate on the components which have either $(+,+)$ or $(-,-)$ parity and nonzero $\mathrm{U}(1)_I$ charges as the candidate for the Higgs field. These include $\{(1,2)(3,-3,3)$ + h.c.$\}$ and $\{(3,2)(1,-1,-3)$ + h.c.$\}$ with parities $(+,+)$ and $(-,-)$, respectively. The representations $(1,2)(-3,3,-3)$ and $(1,2)(3,-3,3)$ have the correct quantum numbers for the SM Higgs doublet. Therefore, we identify the $(1,1)$ mode of these components as the SM Higgs fields in 4D spacetime.

3.2.3. Chiral Fermion Contents in 4D Spacetime

In our model, we choose the fermions as the Weyl fermions Ψ_- belonging to the **27** representation of E_6. The **27** representation is decomposed as in (3.29) under the group reduction, (3.28). In this decomposition, we find that our choice of the background gauge field of $\mathrm{U}(1)_I$ is suitable for obtaining massless fermions since all such components have $\mathrm{U}(1)_I$ charge 1. In the fundemantal representation, the $\mathrm{U}(1)_I$ generator is

$$Q_I = \frac{1}{6}\,\mathrm{diag}(-2,-2,-2,-2,1,1,1,1,4,1,1,1,1,1,1,-2,-2,-2,1,1,1,1,1,1,-2,-2,-2), \tag{3.32}$$

according to the decomposition equation (3.29). By identifying $Q = 3Q_I$, we readily obtain the condition

$$Q\Psi_- = \frac{1}{2}\Psi_-. \tag{3.33}$$

Therefore, the chiral fermions ψ_L in 4D spacetime have zero modes.

Next, we consider the parity assignments for the fermions with respect to the fixed points of S^2/Z_2. The boundary conditions are given by (2.12) and (2.15). It turns out that four **27** fermion copies with different boundary conditions are needed in order to obtain an entire generation of massless SM fermions. They are denoted by $\Psi^{(1,2,3,4)}$ with the following parity assignments:

$$\begin{aligned}\Psi^{(i)}_{\pm}(x,\pi-\theta,-\phi) &= \xi\gamma_5 P_1 \Psi^{(i)}_{\pm}(x,\theta,\phi),\\ \Psi^{(i)}_{\pm}(x,\pi-\theta,2\pi-\phi) &= \eta\gamma_5 P_2 \Psi^{(i)}_{\pm}(x,\theta,\phi),\end{aligned} \tag{3.34}$$

where γ_5 is the chirality operator, and $(\xi,\eta) = (+,+)$, $(-,-)$, $(+,-)$ and $(-,+)$ for $i = 1,2,3,4$, respectively. From these fermions we find that $\psi_{1,2,3,4}$ have the parity assignments

$$\begin{aligned}
\mathbf{27}_{\psi_L^{(1)}} = {} & (1,2)(-3,-2,-2)^{(-,-)} + (1,2)(3,2,-2)^{(+,+)} + (1,2)(-3,3,1)^{(-,+)} \\
& + (1,1)(6,-1,1)^{(-,-)} + \underline{(1,1)(0,-5,1)^{(+,+)}} + (1,1)(0,0,4)^{(+,-)} \\
& + (3,2)(1,-1,1)^{(+,-)} + (3,1)(-2,2,-2)^{(-,+)} + \left(\overline{3},1\right)(-4,-1,1)^{(-,-)} \\
& + \left(\overline{3},1\right)(2,3,1)^{(-,-)} + \left(\overline{3},1\right)(2,-2,-2)^{(+,-)}, \\
\mathbf{27}_{\psi_L^{(2)}} = {} & (1,2)(-3,-2,-2)^{(+,+)} + (1,2)(3,2,-2)^{(-,-)} + (1,2)(-3,3,1)^{(+,-)} \\
& + \underline{(1,1)(6,-1,1)^{(+,+)}} + (1,1)(0,-5,1)^{(-,-)} + (1,1)(0,0,4)^{(-,+)} \\
& + (3,2)(1,-1,1)^{(-,+)} + (3,1)(-2,2,-2)^{(+,-)} + \underline{\left(\overline{3},1\right)(-4,-1,1)^{(+,+)}} \\
& + \underline{\left(\overline{3},1\right)(2,3,1)^{(+,+)}} + \left(\overline{3},1\right)(2,-2,-2)^{(-,+)}, \\
\mathbf{27}_{\psi_L^{(3)}} = {} & (1,2)(-3,-2,-2)^{(-,+)} + (1,2)(3,2,-2)^{(+,-)} + (1,2)(-3,3,1)^{(-,-)} \\
& + (1,1)(6,-1,1)^{(-,+)} + (1,1)(0,-5,1)^{(+,-)} + (1,1)(0,0,4)^{(+,+)} \\
& + \underline{(3,2)(1,-1,1)^{(+,+)}} + (3,1)(-2,2,-2)^{(-,-)} + \left(\overline{3},1\right)(-4,-1,1)^{(-,+)} \\
& + \left(\overline{3},1\right)(2,3,1)^{(-,+)} + \left(\overline{3},1\right)(2,-2,-2)^{(+,+)}, \\
\mathbf{27}_{\psi_L^{(4)}} = {} & (1,2)(-3,-2,-2)^{(+,-)} + (1,2)(3,2,-2)^{(-,+)} + \underline{(1,2)(-3,3,1)^{(+,+)}} \\
& + (1,1)(6,-1,1)^{(+,-)} + (1,1)(0,-5,1)^{(-,+)} + (1,1)(0,0,4)^{(-,-)} \\
& + (3,2)(1,-1,1)^{(-,-)} + (3,1)(-2,2,-2)^{(+,+)} + \left(\overline{3},1\right)(-4,-1,1)^{(+,-)} \\
& + \left(\overline{3},1\right)(2,3,1)^{(+,-)} + \left(\overline{3},1\right)(2,-2,-2)^{(-,-)},
\end{aligned} \tag{3.35}$$

where the underlined components have even parities and $U(1)_I$ charge 1. One can readily identify one generation of SM fermions, including a right-handed neutrino, as the zero modes of these components.

A long-standing problem in the gauge-Higgs unification framework is the Yukawa couplings of the Higgs boson to the matter fields. Here we discuss about the Yukawa couplings in our model. As mentioned before, the SM Higgs is the ($\ell = 1$, $|m| = 1$) KK mode of the extraspatial component of the gauge field, the Yukawa term at tree level has the following form:

$$L_{\text{Yukawa}} \supset \overline{\psi}_L^{00}\Phi^{11}\psi_R^{\ell 1} + \overline{\psi}_L^{\ell 1}\Phi^{11}\psi_R^{00} + \text{h.c.}, \tag{3.36}$$

where $\psi^{\ell m}$s are the fermionic KK modes with the $(l = 0, m = 0)$ modes appearing as the chiral fermions and Φ^{11} denotes the SM Higgs field. We here identify the left-handed fermionic zero modes as SU(2) doublets and the right-handed fermionic zero modes as SU(2) singlets, as in the SM. Therefore, the $(\ell, |m| = 1)$ modes and the $(\ell = 0, |m| = 0)$ modes mix after spontaneous symmetry breaking. One needs to diagonalize the mass terms to obtain physical eigenstates. The Yukawa couplings in our model are thus more complicated than other gauge-Higgs unification models in the sense that there is mixing between KK modes including the zero modes without a bulk mass term or fixed point localized term. However, similar mixing occurs in models on warped 5D spacetime or even in models with a flat metric if one takes into account the bulk mass term or fixed point localized term. In such cases, diagonalization is necessary.

The difficulty of obtaining a realistic fermion mass spectrum comes from the fact that the Yukawa couplings arise from gauge interactions. However, one can overcome the difficulty by introducing SM fermions localized at an orbifold fixed point and additional massive bulk fermions. The realistic Yukawa couplings would be obtained from nonlocal interactions of the fixed point localized fermions involving Wilson lines after integrating out the massive bulk fermions [41–43]. Another possible solution is to consider fermions in 6D spacetime belonging to a higher dimensional representation of the original E_6 gauge group, rendering more than one generation of SM fermions. In that case, mixing among generations will be obtained from gauge interactions and is given by Clebsch-Gordan coefficients. We expect that realistic Yukawa couplings could be obtained using these methods. A detailed analysis of this issue is beyond the scope of the paper and left for a future work.

3.2.4. Higgs Potential of the Model

Here we analyze the Higgs potential for the E_6 model. To further simplify the Higgs potential, we need to find out the algebra of the gauge group generators. Note that the E_6 generators are chosen according to the decomposition of the adjoint representation given in (3.30)

$$\begin{aligned}
&\{Q_i, Q_\alpha, Q_Y, Q_X, Q_I, \\
&\quad Q_{ax(-5,0,0)}, Q^{ax(5,0,0)}, Q_{ax(1,4,0)}, Q^{ax(-1,-4,0)}, \\
&\quad Q_{a(4,-4,0)}, Q^{a(-4,4,0)}, Q_{(-6,-4,0)}, Q_{(6,4,0)}, \\
&\quad Q_{ax(1,-1,-3)}, Q^{ax(-1,1,3)}, Q_{a(4,1,3)}, Q^{a(-4,-1,-3)}, \\
&\quad Q_{a(-2,-3,3)}, Q^{a(2,3,-3)}, Q_{x(3,-3,3)}, Q^{x(-3,3,-3)}, \\
&\quad Q_{(-6,1,3)}, Q_{(6,-1,-3)}, Q_{(0,-5,-3)}, Q_{(0,5,3)}\},
\end{aligned} \tag{3.37}$$

where the generators are listed in the corresponding order of the terms in (3.30) and the indices

$$i = 1, \ldots, 8 \;:\; \text{SU(3) adj rep index} \Longrightarrow Q_i \;:\; \text{SU(3) generators}, \tag{3.38}$$

$$\alpha = 1, 2, 3 \;:\; \text{SU(2) adj rep index} \Longrightarrow Q_\alpha \;:\; \text{SU(2) generators}, \tag{3.39}$$

$$Q_{X,Y,I} \;:\; \text{U(1)}_{X,Y,I} \text{ generators}, \tag{3.40}$$

Table 2: Commutation relations of Q_α, $Q_{X,Y,I}$, $Q_{x(3,-3,3)}$ and $Q^{x(-3,3,-3)}$, where σ_i are the Pauli matrices.

$[Q_{x(3,-3,3)}, Q^{y(-3,3,-3)}] = \frac{1}{2}\delta_x^y Q_I - \frac{1}{2}\sqrt{\frac{3}{5}}\,\delta_x^y Q_X + \frac{1}{\sqrt{10}}\delta_x^y Q_Y + \frac{1}{\sqrt{6}}(\sigma_\alpha)_x^y Q_\alpha$	
$[Q_\alpha, Q_{x(3,-3,3)}] = \frac{1}{\sqrt{6}}(\sigma_\alpha)_x^y Q_{y(3,-3,3)}$,	$[Q_\alpha, Q^{x(-3,3,-3)}] = -\frac{1}{\sqrt{6}}(\sigma_\alpha^*)_x^y Q^{y(-3,3,-3)}$,
$[Q_{x(3,-3,3)}, Q_{y(3,-3,3)}] = 0$,	$[Q_I, Q_{x(3,-3,3)}] = \frac{1}{2}Q_{x(3,-3,3)}$,
$[Q_X, Q_{x(3,-3,3)}] = -\frac{1}{2}\sqrt{\frac{3}{5}}Q_{x(3,-3,3)}$,	$[Q_Y, Q_{x(3,-3,3)}] = \frac{1}{\sqrt{10}}Q_{x(3,-3,3)}$,

$$x = 1,2 \;:\; \mathrm{SU}(2) \text{ doublet index}, \tag{3.41}$$

$$a = 1,2,3 \;:\; \mathrm{SU}(3) \text{ color index}. \tag{3.42}$$

Here we take the normalization for generators, $\mathrm{Tr}[QQ^\dagger] = 2$ which is taken from [24]. The Higgs fields are in the representations of $(1,2)(3,-3,3)$ and $(1,2)(-3,3,-3)$. We write

$$\Phi(x) = \phi^x Q_{x(3,-3,3)} \qquad \left(\Phi^\dagger(x) = \phi_x Q^{x(-3,3,-3)}\right). \tag{3.43}$$

Likewise, the gauge field $A_\mu(x)$ in terms of the Q's in (3.38) is

$$A_\mu(x) = A_\mu^i Q_i + A_\mu^\alpha Q_\alpha + B_\mu Q_Y + C_\mu Q_X + E_\mu Q_I. \tag{3.44}$$

The commutation relations between the generators Q_α, $Q_{X,Y,I}$, $Q_{x(3,-3,3)}$, and $Q^{x(-3,3,-3)}$ are summarized in Table 2.

Finally, we obtain the Lagrangian associated with the Higgs field by applying (3.43) and (3.44) to (2.54) and (2.55) and carrying out the trace. Furthermore, to obtain the canonical form of kinetic terms, the Higgs field, the gauge field, and the gauge coupling need to be rescaled in the following way:

$$\begin{aligned} \phi &\longrightarrow \frac{g}{\sqrt{2}}\phi, \\ A_\mu &\longrightarrow \frac{g}{R}A_\mu, \\ \frac{g}{\sqrt{6\pi R^2}} &= g_2, \end{aligned} \tag{3.45}$$

where g_2 denotes the SU(2) gauge coupling. The Higgs sector is then given by

$$\mathcal{L}_{\text{Higgs}} = \left|D_\mu\phi\right|^2 - V(\phi), \tag{3.46}$$

where

$$D_\mu\phi = \left[\partial_\mu + ig_2\frac{\sigma_\alpha}{2}A_{\alpha\mu} + ig\frac{1}{\sqrt{40\pi R^2}}B_\mu - ig\frac{1}{2}\sqrt{\frac{3}{20\pi R^2}}C_\mu + ig\frac{1}{2\sqrt{4\pi R^2}}E_\mu\right]\phi, \tag{3.47}$$

$$V = -\frac{\chi}{8R^2}\phi^\dagger\phi + \frac{3g^2}{40\pi R^2}\left(\phi^\dagger\phi\right)^2, \tag{3.48}$$

where $\chi = 7 + 9\mu_1 + 9\mu_2$. The numerical values $\mu_{1,2}$ are given by $\mu_1 = 1 - (3/2)\ln 2$ and $\mu_2 = (3/4)(1 - 2\ln 2)$ as in Section 2.3.4. We have omitted the constant term in the Higgs potential. Comparing the potential derived above with the standard form $\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$ in the SM, we see that the model has a tree-level μ^2 term that is negative and proportional to R^{-2}. The negative contribution to the squared mass term comes from the interaction between background gauge field and ϕ as seen in Section 2.3.4. Moreover, the quartic coupling $\lambda = 3g^2/(40\pi R^2)$ is related to the 6D gauge coupling g and grants perturbative calculations because it is about 0.16, using the value of R to be extracted in the next section. Therefore, the order parameter in this model is controlled by a single parameter R, the compactification scale.

In fact, the $(1,1)$ mode of the $\{(3,2)(1,-1,-3) + \text{h.c.}\}$ representation also has a negative squared mass term because it has the same Q_I charge as the $\{(1,2)(3,-3,3) + \text{h.c.}\}$ representation. Therefore, it would induce not only electroweak symmetry breaking but also color symmetry breaking. This undesirable feature can be cured by adding brane terms

$$\frac{\alpha}{R^2\sin^2\theta}F^a_{\theta\phi}F^{a\theta\phi}\delta\left(\theta - \frac{\pi}{2}\right)\left[\delta(\phi) + \delta(\phi - \pi)\right], \tag{3.49}$$

where a denotes the group index of the $\{(3,2)(1,-1,-3) + \text{h.c.}\}$ representation. These brane terms preserve the Z_2' symmetry which corresponds to the symmetry under the transformation $(\phi \rightarrow \phi + \pi)$. With an appropriate choice of the dimensionless constant α, the squared mass of the $(1,1)$ can be lifted to become positive and sufficiently large. We need to forbid a similar brane term for the SU(2) doublet component, and it can be achieved by imposing some additional discrete symmetry. However, here we simply assume that such a brane term for the SU(2) doublet component does not exist.

Due to a negative mass term, the Higgs potential in (3.48) can induce the spontaneous symmetry breakdown: $SU(2) \times U(1)_Y \rightarrow U(1)_{EM}$ in the SM. The Higgs field acquires a vacuum expectation value (VEV):

$$\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix} \quad \text{with } v = \sqrt{\frac{5\pi\chi}{3}}\frac{1}{g} \simeq \frac{4.6}{g}. \tag{3.50}$$

One immediately finds that the W boson mass:

$$m_W = \frac{g_2}{2}v = \frac{1}{6}\sqrt{\frac{5\chi}{2}}\frac{1}{R} \simeq \frac{0.53}{R}, \tag{3.51}$$

from which the compactification scale $R^{-1} \simeq 152\,\text{GeV}$ is inferred. Moreover, the Higgs boson mass at the tree level is

$$m_H = \sqrt{\frac{3}{20\pi}\frac{gv}{R}} = 3\sqrt{\frac{2}{5}}m_W = \frac{\sqrt{\chi}}{2}\frac{1}{R}, \tag{3.52}$$

which is about 152 GeV, numerically very close to the compactification scale. Since the hypercharge of the Higgs field is 1/2, the $U(1)_Y$ gauge coupling is derived from (3.47) as

$$g_Y = \frac{g}{\sqrt{10\pi R^2}}. \tag{3.53}$$

The Weinberg angle is thus given by

$$\sin^2\theta_W = \frac{g_Y^2}{g_2^2 + g_Y^2} = \frac{3}{8}, \tag{3.54}$$

and the Z boson mass

$$m_Z = \frac{m_W}{\cos\theta_W} = m_W\sqrt{\frac{8}{5}}, \tag{3.55}$$

both at the tree level. These relations are the same as the SU(5) GUT at the unification scale. This is not surprising because this part only depends on the group structure. Again, this Weinberg angle is not consistent with experimental measurements, and we need to take into account quantum corrections.

We can repeat the discussion in Section 3.1.2 about the one-loop power divergence in the Higgs potential associated with the linear operator F_{ab}. The operator $F^{\alpha}_{\theta\phi}$ transform to $-F^{\alpha}_{\theta\phi}$ under the parity transformation $\theta \to \pi - \theta$. Hence, this operator is forbidden by parity invariance of the action. In this case, we check the consistency between the orbifold boundary conditions on S^2/Z_2, (2.13)–(2.15), and the parity conditions, (3.26). By performing the parity transformation on both sides of the orbifold boundary conditions, (2.13)–(2.15), we obtain

$$\begin{aligned}
A_\mu(x,\theta,-\phi) &= P_1 A_\mu(x,\pi-\theta,\phi)P_1,\\
-A_\theta(x,\theta,-\phi) &= P_1 A_\theta(x,\pi-\theta,\phi)P_1,\\
A_\phi(x,\theta,-\phi) &= -P_1 A_\phi(x,\pi-\theta,\phi)P_1,\\
\pm\Gamma^4\Psi(x,\theta,-\phi) &= \pm\gamma_5 P_1\left(\pm\Gamma^4\right)\Psi(x,\pi-\theta,\phi),\\
A_\mu(x,\theta,2\pi-\phi) &= P_2 A_\mu(x,\pi-\theta,\phi)P_2,\\
-A_\theta(x,\theta,2\pi-\phi) &= P_2 A_\theta(x,\pi-\theta,\phi)P_2,\\
A_\phi(x,\theta,2\pi-\phi) &= -P_2 A_\phi(x,\pi-\theta,\phi)P_2,\\
\pm\Gamma^4\Psi(x,\theta,2\pi-\phi) &= \pm\gamma_5 P_2\left(\pm\Gamma^4\right)\Psi(x,\pi-\theta,\phi).
\end{aligned} \tag{3.56}$$

Since (2.13)–(2.15) hold for any θ and ϕ and Γ^4 commutes with γ_5, we find that the orbifold boundary conditions still hold under the parity transformation with the identification of $\theta = \pi - \theta'$. In other words, the orbifold boundary conditions, (2.13)–(2.15), are parity invariant.

3.2.5. KK Mode Spectrum of Each Field

Since we did not impose symmetry condition, we have KK modes for each field in this model. Here we show KK mass spectrum under the existence of background field for our E_6 model. The masses are basically controlled by the compactification radius R of the two spheres. They receive two kinds of contributions: one arising from the angular momentum in the S^2 space and the other coming from the interactions with the background field.

The KK masses for fermions have been given in [35, 36, 38]. We give them in terms of our notation here:

$$M^{\mathrm{KK}}_{\ell m}(\psi_L) = \frac{1}{R}\sqrt{\ell(\ell+1) - \frac{4q^2-1}{4}}, \tag{3.57}$$

where q is proportional to the $U(1)_I$ charge of a fermion and determined by the action of $Q = 3Q_I$ on fermions as $Q\Psi = q\Psi = 3q_I\Psi$. Note that the mass does not depend on the quantum number m. The lightest KK mass, corresponding to $\ell = 1$ and $q_I = 1/6$, is about 214 GeV at the tree level. The range of ℓ is

$$\frac{2q\pm 1}{2} \le \ell \quad \left(+:\ \text{for } \psi_{R(L)} \text{ in } \Psi_{+(-)},\ -:\ \text{for } \psi_{L(R)} \text{ in } \Psi_{-(+)}\right). \tag{3.58}$$

We thus can have zero mode for $Q\Psi = \pm(1/2)\Psi$, where this condition is given in (2.43).

For the 4D gauge field A_μ, its kinetic term, and KK mass term are obtained from the terms:

$$L = \int d\Omega \,\mathrm{Tr}\left[-\frac{1}{4}F_{\mu\nu} + \frac{1}{2R^2}F_{\mu\theta}F^{\mu}{}_{\theta} + \frac{1}{2R^2\sin^2\theta}F_{\mu\phi}F^{\mu}{}_{\phi}\right]. \tag{3.59}$$

Taking terms quadratic in A_μ, we get

$$\begin{aligned} L_{\text{quad}} = \int d\Omega \,\mathrm{Tr}\Big[&-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) + \frac{1}{2R^2}\partial_\theta A_\mu \partial_\theta A^\mu \\ &+ \frac{1}{2R^2\sin^2\theta}\partial_\phi A_\mu \partial_\phi A^\mu - \frac{1}{2R^2}\left[A_\mu, \tilde{A}_\phi^B\right]\left[A^\mu, \tilde{A}_\phi^B\right]\Big], \end{aligned} \tag{3.60}$$

where $\tilde{A}_\phi^B$ is the background gauge field given in (2.35). The KK expansion of A_μ is

$$A_\mu = \sum_{\ell m} A_\mu^{\ell m}(x) Y_{\ell m}^{\pm}(\theta, \phi), \tag{3.61}$$

where $Y_{\ell m}^{\pm}(\theta,\phi)$ are the linear combinations of spherical harmonics satisfying the boundary condition $Y_{\ell m}^{\pm}(\pi-\theta,-\phi) = \pm Y_{\ell m}^{\pm}(\theta,\phi)$. Their explicit forms are [35]

$$\begin{aligned} Y_{\ell m}^{+}(\theta,\phi) &\equiv \frac{(i)^{\ell+m}}{\sqrt{2}}\left[Y_{\ell m}(\theta,\phi) + (-1)^{\ell} Y_{\ell -m}(\theta,\phi)\right] \quad \text{for } m \neq 0, \\ Y_{\ell m}^{-}(\theta,\phi) &\equiv \frac{(i)^{\ell+m+1}}{\sqrt{2}}\left[Y_{\ell m}(\theta,\phi) - (-1)^{\ell} Y_{\ell -m}(\theta,\phi)\right] \quad \text{for } m \neq 0, \\ Y_{\ell 0}^{+(-)}(\theta) &\equiv \begin{cases} Y_{\ell 0}(\theta) & \text{for } m = 0, \quad \ell = \text{even (odd)} \\ 0 & \text{for } m = 0, \quad \ell = \text{odd (even)}. \end{cases} \end{aligned} \tag{3.62}$$

Note that we do not have KK mode functions that are odd under $\phi \to \phi + 2\pi$ since the KK modes are specified by the integer angular momentum quantum numbers ℓ and m of gauge field A_M on the two spheres. Thus, the components of A_μ and $A_{\theta,\phi}$ with $(+,-)$ or $(-,+)$ parities do not have corresponding KK modes. Applying the KK expansion and integrating about $d\Omega$, we obtain the kinetic and KK mass terms for the KK modes of A_μ

$$\begin{aligned} L_M = &-\frac{1}{2}\left[\partial_\mu A_\nu^{\ell m}(x) - \partial_\nu A_\mu^{\ell m}(x)\right]\left[\partial^\mu A^{\ell m\nu}(x) - \partial^\nu A^{\ell m\mu}(x)\right] \\ &+ \frac{\ell(\ell+1)}{R^2} A_\mu^{\ell m}(x) A^{\ell m\mu}(x) \\ &+ \frac{9q_I^2}{R^2}\left[\int d\Omega \frac{(\cos\theta \pm 1)^2}{\sin^2\theta}(Y_{\ell m}^{\mp})^2\right] A_\mu^{\ell m}(x) A^{\ell m\mu}(x), \end{aligned} \tag{3.63}$$

where we have used $\mathrm{Tr}[Q_i Q^i] = 2$ and $[A_\mu(x), Q_I] = q_I(A^i_\mu(x)Q_i - A_{i\mu}(x)Q^i)$. Therefore, the KK masses of A_μ are

$$M^{\mathrm{KK}}_{\ell m}(A_\mu) = \frac{1}{R}\sqrt{\ell(\ell+1) + \left(m^B_{\ell m}\right)^2}, \tag{3.64}$$

$$\left(m^B_{\ell m}\right)^2 = 9q_I^2 \int d\Omega \frac{(\cos\theta \pm 1)^2}{\sin^2\theta}\left(Y^{\mp}_{\ell m}\right)^2, \tag{3.65}$$

where $m^B_{\ell m}$ corresponds to the contribution from the background gauge field. Note that (3.64) agrees with (2.41) when $\ell = 0$. Also, since the SM gauge bosons have $q_I = 0$, their KK masses are simply $\sqrt{\ell(\ell+1)}/R$ at the tree level.

The kinetic and KK mass terms of A_θ and A_ϕ are obtained from the terms in the higher dimensional gauge sector

$$\begin{aligned} L = \frac{1}{2g^2}\int d\Omega \Bigg\{ &\left(\mathrm{Tr}\left[(\partial_\mu A_\theta - i[A_\mu, A_\theta])^2\right] + \mathrm{Tr}\left[\left(\partial_\mu \tilde{A}_\phi - i\left[A_\mu, \tilde{A}_\phi\right]\right)^2\right]\right) \\ &-\frac{1}{R^2}\mathrm{Tr}\left[\left(\frac{1}{\sin\theta}\partial_\theta\left(\sin\theta \tilde{A}_\phi + \sin\theta \tilde{A}^B_\phi\right) - \frac{1}{\sin\theta}\partial_\phi A_\theta - i\left[A_\theta, \tilde{A}_\phi + \tilde{A}^B_\phi\right]\right)^2\right]\Bigg\}. \end{aligned} \tag{3.66}$$

The first line on the right-hand side of (3.66) corresponds to the kinetic terms, and the second line corresponds to the potential term. Applying the background gauge field (2.35), the potential becomes

$$L_V = -\frac{1}{2g^2R^2}\int d\Omega\, \mathrm{Tr}\left[\left(\frac{1}{\sin\theta}\partial_\theta\left(\sin\theta \tilde{A}_\phi\right) + Q - \frac{1}{\sin\theta}\partial_\phi A_\theta - i\left[A_\theta, \tilde{A}_\phi + \tilde{A}^B_\phi\right]\right)^2\right]. \tag{3.67}$$

For A_θ and A_ϕ, we use the following KK expansions to obtain the KK mass terms,

$$\begin{aligned} A_\theta(x,\theta,\phi) &= \sum_{\ell m(\neq 0)} \frac{-1}{\sqrt{\ell(\ell+1)}}\left[\Phi^{\ell m}_1(x)\partial_\theta Y^{\pm}_{\ell m}(\theta,\phi) + \Phi^{\ell m}_2(x)\frac{1}{\sin\theta}\partial_\phi Y^{\pm}_{\ell m}(\theta,\phi)\right], \\ A_\phi(x,\theta,\phi) &= \sum_{\ell m(\neq 0)} \frac{1}{\sqrt{\ell(\ell+1)}}\left[\Phi^{\ell m}_2(x)\partial_\theta Y^{\pm}_{\ell m}(\theta,\phi) - \Phi^{\ell m}_1(x)\frac{1}{\sin\theta}\partial_\phi Y^{\pm}_{\ell m}(\theta,\phi)\right], \end{aligned} \tag{3.68}$$

where the factor of $1/\sqrt{\ell(\ell+1)}$ is needed for normalization. These particular forms are convenient in giving diagonalized KK mass terms [35]. Applying the KK expansions equations (3.68), we obtain the kinetic term

$$L_K = \frac{1}{2g^2}\sum_{\ell m(\neq 0)} \mathrm{Tr}\left[\partial_\mu \Phi^{\ell m}_1(x)\partial^\mu \Phi^{\ell m}_1(x) + \partial_\mu \Phi^{\ell m}_2(x)\partial^\mu \Phi^{\ell m}_2(x)\right], \tag{3.69}$$

where only terms quadratic in $\partial_\mu \Phi$ are retained. The potential term is

$$
\begin{aligned}
L_V = -\frac{1}{2g^2R^2}\sum_{\{\ell m\}(\neq 0)}\int d\Omega\,\mathrm{Tr}\Bigg[\Bigg(&\frac{\Phi_2^{\ell m}}{\sqrt{\ell(\ell+1)}}\frac{1}{\sin\theta}\partial_\theta\left(\sin\theta\partial_\theta Y^{\pm}_{\ell m}\right) + Q + \frac{\Phi_2^{\ell m}}{\sqrt{\ell(\ell+1)}}\frac{1}{\sin^2\theta}\partial_\phi^2 Y^{\pm}_{\ell m} \\
&- \frac{i}{\sqrt{\ell(\ell+1)\ell'(\ell'+1)}}\left[-\Phi_1^{\ell m}\partial_\theta Y^{\pm}_{\ell m} - \Phi_2^{\ell m}\frac{1}{\sin\theta}\partial_\phi Y^{\pm}_{\ell m},\right. \\
&\Phi_2^{\ell' m'}\partial_\theta Y^{\pm}_{\ell' m'} - \Phi_1^{\ell' m'}\frac{1}{\sin\theta}\partial_\phi Y^{\pm}_{\ell' m'} \\
&\left. + \sqrt{\ell'(\ell'+1)}A_\phi^B\right]\Bigg)^2\Bigg].
\end{aligned}
\tag{3.70}
$$

Note that these terms are not diagonal in (ℓ, m) in general. Using the relation $(1/\sin\theta)\partial_\theta(\sin\theta\partial_\theta Y_{\ell m}) + (1/\sin^2\theta)\partial_\phi^2 Y_{\ell m} = -\ell(\ell+1)Y_{\ell m}$, the potential term is simplified as

$$
\begin{aligned}
L_V = &-\frac{1}{2g^2R^2} \\
&\times \sum_{\ell m(\neq 0)}\int d\Omega\,\mathrm{Tr}\Bigg[\Bigg(-\sqrt{\ell(\ell+1)}\Phi_2^{\ell m}Y^{\pm}_{\ell m} + Q + \frac{i}{\sqrt{\ell(\ell+1)\ell'(\ell'+1)}}\left[\Phi_1^{\ell m}, \Phi_2^{\ell' m'}\right] \\
&\times\left(\partial_\theta Y^{\pm}_{\ell m}\partial_\theta Y^{\pm}_{\ell' m'} + \frac{1}{\sin^2\theta}\partial_\phi Y^{\pm}_{\ell m}\partial_\phi Y^{\pm}_{\ell' m'}\right) \\
&+\frac{i}{\sqrt{\ell(\ell+1)}}\left[\Phi_1^{\ell m}, \tilde{A}_\phi^B\right]\partial_\theta Y^{\pm}_{\ell m} + \frac{i}{\sqrt{\ell(\ell+1)}}\left[\Phi_2^{\ell m}, \tilde{A}_\phi^B\right]\frac{\partial_\phi Y^{\pm}_{\ell m}}{\sin\theta}\Bigg)^2\Bigg].
\end{aligned}
\tag{3.71}
$$

To obtain the mass term, we focus on terms quadratic in $\Phi_{1,2}$:

$$
\begin{aligned}
L_M = -\frac{1}{2g^2R^2}\int d\Omega\,\mathrm{Tr}\Bigg[&\ell(\ell+1)\left(\Phi_2^{\ell m}\right)^2\left(Y^{\pm}_{\ell m}\right)^2 \\
&+\frac{2iQ}{\ell(\ell+1)}\left[\Phi_1^{\ell m}, \Phi_2^{\ell m}\right]\left(\partial_\theta Y^{\pm}_{\ell m}\partial_\theta Y^{\pm}_{\ell m} + \frac{1}{\sin^2\theta}\partial_\phi Y^{\pm}_{\ell m}\partial_\phi Y^{\pm}_{\ell m}\right) \\
&+2i\tilde{A}_\phi^B\left[\Phi_1^{\ell m}, \Phi_2^{\ell m}\right]Y^{\pm}_{\ell m}\partial_\theta Y^{\pm}_{\ell m} - \frac{1}{\ell(\ell+1)}\left[\Phi_1^{\ell m}, \tilde{A}_\phi^B\right]^2\left(\partial_\theta Y^{\pm}_{\ell m}\right)^2 \\
&-\frac{1}{\ell(\ell+1)}\left[\Phi_2^{\ell m}, \tilde{A}_\phi^B\right]^2\frac{\left(\partial_\phi Y^{\pm}_{\ell m}\right)^2}{\sin^2\theta}\Bigg].
\end{aligned}
\tag{3.72}
$$

Here we take terms which are diagonal in (ℓ, m) for simplicity. Note that we have dropped the term proportional to $[\Phi_1, \widetilde{A}^B_\phi][\Phi_2, \widetilde{A}^B_\phi]$ because this term vanishes after turning the field into the linear combinations of Φ and $\Phi^\dagger$, (2.51) and (2.52):

$$\begin{aligned}
\mathrm{Tr}\left[\left[\Phi_1, \widetilde{A}^B_\phi\right]\left[\Phi_1, \widetilde{A}^B_\phi\right]\right] &\longrightarrow \mathrm{Tr}\left[\left[\left(\Phi + \Phi^\dagger\right), Q\right]\left[\left(\Phi - \Phi^\dagger\right), Q\right]\right] \\
&\propto \mathrm{Tr}\left[\left(\Phi - \Phi^\dagger\right)\left(\Phi + \Phi^\dagger\right)\right] \\
&\propto \mathrm{Tr}\left[\Phi\Phi^\dagger\right] - \mathrm{Tr}\left[\Phi^\dagger\Phi\right] = 0.
\end{aligned} \tag{3.73}$$

Integrating the second term of (3.72) by part, we obtain

$$\begin{aligned}
L_M = -\frac{1}{2g^2R^2}\Bigg(&\ell(\ell+1)\,\mathrm{Tr}\left[\left(\Phi_2^{\ell m}\right)^2\right] + 2i\,\mathrm{Tr}\left[Q\left[\Phi_1^{\ell m}, \Phi_2^{\ell m}\right]\right] \\
&- 2i\,\mathrm{Tr}\left[Q\left[\Phi_1^{\ell m}, \Phi_2^{\ell m}\right]\right]\int d\Omega \frac{\cos\theta \mp 1}{\sin\theta} Y^{\pm}_{\ell m}\partial_\theta Y^{\pm}_{\ell m} \\
&- \frac{1}{\ell(\ell+1)}\left[\Phi_1^{\ell m}, Q\right]^2 \int d\Omega \frac{(\cos\theta \mp 1)^2}{\sin^2\theta}\left(\partial_\theta Y^{\pm}_{\ell m}\right)^2 \\
&- \frac{1}{\ell(\ell+1)}\left[\Phi_2^{\ell m}, Q\right]^2 \int d\Omega \frac{(\cos\theta \mp 1)}{\sin^2\theta}\frac{\left(\partial_\phi Y^{\pm}_{\ell m}\right)^2}{\sin^2\theta}\Bigg).
\end{aligned} \tag{3.74}$$

Therefore, the KK masses depend on the $U(1)_I$ charges of the scalar fields. Note that terms in the second line to the last line of (3.74) are not diagonal in (ℓ, m) in general.

For components with zero $U(1)_I$ charge, we write $\Phi_{1(2)}(x)$ as $\phi_{1(2)}(x)Q$ where Q is the corresponding generator of E_6 in (3.30) with zero $U(1)_I$ charge. Taking the trace, we have the following kinetic and KK mass terms instead:

$$L = \sum_{\ell m(\neq 0)}\left(\partial_\mu\phi_1^{\ell m}(x)\partial^\mu\phi_1^{\ell m}(x) + \partial_\mu\phi_2^{\ell m}(x)\partial^\mu\phi_2^{\ell m}(x) + \ell(\ell+1)\phi_2^{\ell m}(x)\phi_2^{\ell m}(x)\right), \tag{3.75}$$

where we have made the substitution $\phi_i \to g\phi_i$. Note that ϕ_1 is considered as a massless Nambu-Goldstone (NG) boson in this case. For components with nonzero $U(1)_I$ charge, mass terms are not diagonal for $\Phi_{1(2)}$, and Φ_1 does not correspond to the NG boson. In this case, we need to diagonalize the mass terms and some linear combination of $\Phi_{1(2)}$ becomes the NG boson mode.

For components with nonzero $U(1)_I$ charge, we use (2.51) and (2.52) and write $\Phi(x)$ as $\phi^i(x)Q_i$ where Q_i is the corresponding generator of E_6 in (3.30) with nonzero $U(1)_I$ charge. The commutator between Q and Φ is

$$[Q, \Phi] = 3[Q_I, Q_i]\phi^i = 3q_I\phi^i, \tag{3.76}$$

where we have used $Q = 3Q_I$ as required to obtain chiral fermions in Section 3.2.3, and that q_I is a constant determined by the $\mathrm{U}(1)_I$ charge of the corresponding component. Finally, the Lagrangian becomes

$$\begin{aligned} L = \sum_{\ell m(\neq 0)} \Bigg\{ & \partial_\mu \phi^\dagger_{\ell m} \partial^\mu \phi_{\ell m} \\ & - \frac{1}{4R^2} \Bigg[2\ell(\ell+1)\phi^\dagger_{\ell m}\phi_{\ell m} - 12 q_I \phi^\dagger_{\ell m}\phi_{\ell m} + 12 q_I \phi^\dagger_{\ell m}\phi_{\ell m} \int d\Omega \frac{\cos\theta \mp 1}{\sin\theta} Y^\pm_{\ell m} \partial_\theta Y^\pm_{\ell m} \\ & + \frac{18 q_I^2}{\ell(\ell+1)} \phi^\dagger_{\ell m}\phi_{\ell m} \int d\Omega \frac{(\cos\theta \mp 1)^2}{\sin^2\theta} \left(\left(\partial_\theta Y^\pm_{\ell m}\right)^2 + \frac{\left(\partial_\phi Y^\pm_{\ell m}\right)^2}{\sin^2\theta} \right) \Bigg] \Bigg\}, \end{aligned} \tag{3.77}$$

where the subscript i is omitted for simplicity. The KK masses of the complex scalar field ϕ are then

$$\begin{aligned} M^{\mathrm{KK}}_{\ell m}(\phi) &= \frac{1}{R}\sqrt{\frac{\ell(\ell+1)}{2} + \left(m^B_{\ell m}\right)^2}, \\ \left(m^B_{\ell m}\right)^2 &= -3q_I + 3q_I \int d\Omega \frac{\cos\theta \mp 1}{\sin\theta} Y^\pm_{\ell m}\partial_\theta Y^\pm_{\ell m} \\ &\quad + \frac{9q_I^2}{2\ell(\ell+1)} \int d\Omega \frac{(\cos\theta \mp 1)^2}{\sin^2\theta} \left(\partial_\theta Y^\pm_{\ell m}\right)^2 \\ &\quad + \frac{9q_I^2}{2\ell(\ell+1)} \int d\Omega \frac{(\cos\theta \mp 1)^2}{\sin^2\theta} \frac{\left(\partial_\phi Y^\pm_{\ell m}\right)^2}{\sin^2\theta}. \end{aligned} \tag{3.78}$$

The squared KK mass $(M^{\mathrm{KK}}_{\ell m})^2$ is always positive except for the lowest mode $(\ell = 1, m = 1)$. In fact, the squared KK mass of the $(1,1)$ mode agrees with the coefficient of quadratic term in the Higgs potential (3.48).

4. Summary and Discussions

We have reviewed a gauge theory defined on 6D spacetime with the S^2/Z_2 topology on the extra space. Two scenarios are considered to construct a 4D theory from the 6D model. One scenario based on the SO(12) gauge group requires a symmetry condition for the gauge field. The other involves the E_6 gauge group, but does not need the symmetry condition. Nontrivial boundary conditions on the extra space are imposed in both scenarios.

We explicitly give the prescriptions to identify the gauge field and the scalar field remaining in 4D spacetime after the dimensional reduction. We show that the $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_X \times \mathrm{U}(1)_I$ gauge symmetry remains in 4D spacetime, and that the SM Higgs doublet with a suitable potential for electroweak symmetry breaking can be derived from the gauge sector in both models. The Higgs boson mass is also predicted in such models. Our tree-level prediction of the Higgs boson mass is 196 GeV for the SO(12) model and 152 GeV for the E_6 model. These mass values are in the range of 127–600 GeV already excluded at 95% CL by recent LHC data [44, 45]. However, the mass value will become different once quantum corrections to the Higgs potential are taken into account. We expect that the Higgs boson mass in our model will become smaller than the lower limit of the exclusion region by quantum corrections. In particular, the E_6 case gives a 152 GeV Higgs boson mass at tree level that is not far from the lower limit of the exclusion region at 95%CL. However, a full analysis of the quantum corrections is beyond the scope of this paper and left as a future work. Massless fermion modes are also successfully obtained as the SM fermions by introducing appropriate field contents in 6D spacetime, with suitable parity assignments on the S^2/Z_2 extra dimension and incorporating the background gauge field. We also discuss about the massive KK modes of fermions for the scenario with the symmetry condition and the KK modes of all fields for the one without the symmetry condition. The lightest fermonic KK mode can serve as a dark matter candidate. In general, they may give rise to rich phenomena in collider experiments and implications in cosmological studies.

To make our models more realistic, there are several challenges such as eliminating the extra U(1) symmetries and constructing the realistic Yukawa couplings, which are the same as other gauge-Higgs unification models. We, however, can get Kaluza-Klein modes in our models. This suggests that we obtain the dark matter candidate in our model. Thus, it is very important to study these models further such as dark matter physics and collider physics.

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Research Article

Gauge Boson Mixing in the 3-3-1 Models with Discrete Symmetries

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The mixing among gauge bosons in the 3-3-1 models with the discrete symmetries is investigated. To get tribimaximal neutrino mixing, we have to introduce sextets containing neutral scalar components with lepton number $L = 1, 2$. Assignation of VEVs to these fields leads to the mixing of the new gauge bosons and those in the standard model. The mixing in the charged gauge bosons leads to the lepton number violating interactions of the W boson. The same situation happens in the neutral gauge boson sector.

1. Introduction

The experimental evidences of nonzero neutrino masses and mixing [1] have shown that the standard model of fundamental particles and interactions must be extended. Among many extensions of the standard model known today, the models based on gauge symmetry $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (called 3-3-1 models) [2–9] have interesting features. First, $[SU(3)_L]^3$ anomaly cancelation requires that the number of $SU(3)_L$ fermion triplets must be equal to that of antitriplets. If these multiplets are respectively enlarged from those of the standard model, the fermion family number is deduced to be a multiple of the fundamental color number, which is three, coinciding with the observation (see Frampton in [2]). In addition, one family of quarks has to transform under $SU(3)_L$ differently from the other two. This can lead to an explanation why the top quark is characteristically heavy (see, e.g., [10]). To complete the fundamental representations for leptons, the right-handed neutrinos or neutral fermions can be imposed which imply natural seesaw mechanisms for the neutrino small masses [11]. The 3-3-1 models can also provide a solution of electric charge quantization observed in the nature [12–16].

Table 1: Character table of S_3, where χ stands for character of representation and C for class.

Class	n	h	$\chi_{\underline{1}}$	$\chi_{\underline{1}'}$	$\chi_{\underline{2}}$
C_1	1	1	1	1	2
C_2	2	3	1	1	−1
C_3	3	2	1	−1	0

There are two typical versions of the 3-3-1 models concerning respective lepton contents. In the minimal 3-3-1 model [2–4] the lepton triplets include ordinary leptons of the standard model such as (ν_L, l_L, l_R^c). The 3-3-1 model with right-handed neutrinos [5–9] introduces right-handed neutrinos into the lepton sector, that is, (ν_L, l_L, ν_R^c) and l_R. In the framework of 3-3-1 models, to explain the smallness of neutrino masses and the tribimaximal mixing [17–20]

$$U^{\mathrm{HPS}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}, \tag{1.1}$$

we should propose another variant of the lepton sector such as (ν_L, l_L, N_R^c) and l_R where N_R are neutral chiral fermions carrying no lepton number (called 3-3-1 model with neutral fermions), and including discrete symmetries either A_4 or S_4 [21, 22]. The 3-3-1 model with neutral fermions based on S_3 flavor symmetry instead of A_4, S_4 has been studied in [23].

One of the most important ingredients is the sextets in which neutral scalar fields carrying lepton number $L = 1$ or 2. Assignation of VEVs to these fields leads to the mixing among new gauge bosons and that of the SM similarly in the economical 3-3-1 model [24–26], and such mixing leads to the lepton violating interactions. In this work we will pay attention to gauge bosons in the mentioned 3-3-1 models and give some phenomenological consequences.

The rest of this work is follows. In Section 2 we give a review of the 3-3-1 model with neutral fermions-based S_3 flavor symmetry. The other models with A_4 and S_4 can be done similarly, thus should be skip. Section 3 identifies gauge bosons and obtained the mixings among the standard model gauge bosons and the new ones. Section 4 is devoted to charged currents and give a constraint on the charged gauge boson mixing-angle. Finally we make conclusions in Section 5.

2. Brief Review of the Model

Before looking into the model, we provide a sketch of S_3 group theory [27, 28]. The S_3 that is a permutation group of three objects has six elements divided into three conjugacy classes. It possesses three nonequivalent irreducible representations $\underline{1}$, $\underline{1}'$ of one dimension, and $\underline{2}$ of two dimensions. Denoting n and h as the order of class and the order of elements within each class, respectively, the character table is given by Table 1.

We will work in the basis that $\underline{2}$ is complex (see, e.g. [27]). Decomposition rules are

$$\begin{aligned}
&\underline{1}\otimes\underline{1}=\underline{1}(11), \qquad \underline{1}'\otimes\underline{1}'=\underline{1}(11), \qquad \underline{1}\otimes\underline{1}'=\underline{1}'(11),\\
&\underline{1}\otimes\underline{2}=\underline{2}(11,12), \qquad \underline{1}'\otimes\underline{2}=\underline{2}(11,-12),\\
&\underline{2}\otimes\underline{2}=\underline{1}(12+21)\oplus\underline{1}'(12-21)\oplus\underline{2}(22,11).
\end{aligned} \tag{2.1}$$

Here the first and second factors of the terms appearing in the parentheses indicate to the multiplet components of the first and second representations given in l.h.s, respectively. In this basis, the conjugation rules are given by

$$\underline{2}^*(1^*,2^*)=\underline{2}(2^*,1^*), \qquad \underline{1}^*(1^*)=\underline{1}(1^*), \qquad \underline{1}'^*(1^*)=\underline{1}'(1^*). \tag{2.2}$$

The lepton number in the 3-3-1 model with S_3 symmetry [23] does not commute with the gauge symmetry. It is thus better to work with a new lepton charge $\mathcal{L}$ related to the lepton number L by diagonal matrices $L = xT_3 + yT_8 + \mathcal{L}$. Applying L to the lepton triplet with the notation that $L(N_R) = 0$, the coefficients are defined as $x = 0$, $y = 2/\sqrt{3}$, and thus $L = (2/\sqrt{3})T_8 + \mathcal{L}$ [29]. The leptons and quarks under $[\mathrm{SU}(3)_L, U(1)_X, U(1)_{\mathcal{L}}, \underline{S}_3]$ symmetries correspondingly transform as follows:

$$\begin{aligned}
&\psi_{1L}=\left(\nu_{1L},l_{1L},N^c_{1R}\right)^T\sim\left[3,-\frac{1}{3},\frac{2}{3},\underline{1}\right], \qquad l_{1R}\sim[1,-1,1,\underline{1}],\\
&\psi_{\alpha L}=\left(\nu_{\alpha L},l_{\alpha L},N^c_{\alpha R}\right)^T\sim\left[3,-\frac{1}{3},\frac{2}{3},\underline{2}\right], \qquad l_{\alpha R}\sim[1,-1,1,\underline{2}],\\
&Q_{1L}=(u_{1L},\ d_{1L},U_L)^T\sim\left[3,\frac{1}{3},-\frac{1}{3},\underline{1}\right],\\
&u_{1R}\sim\left[1,\frac{2}{3},0,\underline{1}\right], \qquad d_{1R}\sim\left[1,-\frac{1}{3},0,\underline{1}\right], \qquad U_R\sim\left[1,\frac{2}{3},-1,\underline{1}\right],\\
&Q_{\alpha L}=(d_{\alpha L},-u_{\alpha L},D_{\alpha L})^T\sim\left[3^*,0,\frac{1}{3},\underline{2}\right],\\
&u_{\alpha R}\sim\left[1,\frac{2}{3},0,\underline{2}\right], \qquad d_{\alpha R}\sim\left[1,-\frac{1}{3},0,\underline{2}\right], \qquad D_{\alpha R}\sim\left[1,-\frac{1}{3},1,\underline{2}\right],
\end{aligned} \tag{2.3}$$

where $\alpha = 2, 3$ is a family index of the last two lepton and quark families, which are in order defined as the components of $\underline{2}$ representations.

To generate masses for the charged leptons, we need two scalar multiplets:

$$\phi=\begin{pmatrix}\phi_1^+\\ \phi_2^0\\ \phi_3^+\end{pmatrix}\sim\left[3,\frac{2}{3},-\frac{1}{3},\underline{1}\right], \quad \phi'=\begin{pmatrix}\phi_1'^+\\ \phi_2'^0\\ \phi_3'^+\end{pmatrix}\sim\left[3,\frac{2}{3},-\frac{1}{3},\underline{1}'\right], \tag{2.4}$$

with VEVs $\langle \phi \rangle = (0, v, 0)^T$ and $\langle \phi' \rangle = (0, v', 0)^T$. To generate masses for quarks, we additionally acquire the following scalar multiplets:

$$\begin{aligned}
\chi &= \left(\chi_1^0, \chi_2^-, \chi_3^0\right)^T \sim \left[3, -\frac{1}{3}, \frac{2}{3}, \underline{1}\right], \\
\eta &= \left(\eta_1^0, \eta_2^-, \eta_3^0\right)^T \sim \left[3, -\frac{1}{3}, -\frac{1}{3}, \underline{1}\right], \\
\eta' &= \left(\eta_1^{'0}, \eta_2^{'-}, \eta_3^{'0}\right)^T \sim \left[3, -\frac{1}{3}, -\frac{1}{3}, \underline{1}'\right].
\end{aligned} \tag{2.5}$$

Suppose that the VEVs of η, η', and χ are u, u' and w, where $u = \langle \eta_1^0 \rangle$, $u' = \langle \eta_1^{'0} \rangle$, $w = \langle \chi_3^0 \rangle$, and $\langle \eta_3^0 \rangle$, $\langle \eta_3^{'0} \rangle$, and $\langle \chi_1^0 \rangle$ vanish. The exotic quarks get masses $m_U = f_1 w$ and $m_{D_{1,2}} = f w$. In addition, w has to be much larger than those of ϕ and η. Notice that the numbered subscripts are the indices of $SU(3)_L$.

Because of the $\mathcal{L}$-symmetry, the couplings $\overline{\psi}_L^c \psi_L \phi$ and $\overline{\psi}_L^c \psi_L \phi'$ are suppressed. We therefore propose a new $SU(3)_L$ antisextet instead coupling to $\overline{\psi}_L^c \psi_L$ responsible for neutrino masses. The antisextet transforms as

$$s = \begin{pmatrix} s_{11}^0 & s_{12}^+ & s_{13}^0 \\ s_{12}^+ & s_{22}^{++} & s_{23}^+ \\ s_{13}^0 & s_{23}^+ & s_{33}^0 \end{pmatrix} \sim \left[6^*, \frac{2}{3}, -\frac{4}{3}, \underline{2}\right], \tag{2.6}$$

where the numbered subscripts are the $SU(3)_L$ indices. Henceforth the indices of S_3 on scalar fields will be kept and should be understood. The VEVs of s is set as $(\langle s_1 \rangle, \langle s_2 \rangle)$ under S_3, where

$$\langle s_i \rangle = \begin{pmatrix} \lambda_i & 0 & v_i \\ 0 & 0 & 0 \\ v_i & 0 & \Lambda_i \end{pmatrix} \quad (i = 1, 2). \tag{2.7}$$

Due to the S_3 symmetry, all these VEVs are equal to each others, that is, $\lambda_1 = \lambda_2$, $v_1 = v_2$ and $\Lambda_1 = \Lambda_2$, which can be found from the potential minimization.

With the scalar multiplets as defined, the Yukawa lagrangian is given by

$$\begin{aligned}
\mathcal{L}_Y &= h_1 \overline{\psi}_{1L} \phi l_{1R} + h\left(\overline{\psi}_{2L} l_{2R} + \overline{\psi}_{3L} l_{3R}\right)\phi + h'\left(\overline{\psi}_{3L} l_{3R} - \overline{\psi}_{2L} l_{2R}\right)\phi' \\
&\quad + f_1 \overline{Q}_{1L} \chi U_R + f \overline{Q}_L \chi^* D_R + h_1^u \overline{Q}_{1L} \eta u_{1R} + h^d \overline{Q}_L \eta^* d_R \\
&\quad + h^{'d} \overline{Q}_L \eta^{'*} d_R + h_1^d \overline{Q}_{1L} \phi d_{1R} + h^u \overline{Q}_L \phi^* u_R + h^{'u} \overline{Q}_L \phi^{'*} u_R \\
&\quad + \frac{1}{2} x\left(\overline{\psi}_{2L}^c \psi_{2L} s_1 + \overline{\psi}_{3L}^c \psi_{3L} s_2\right) + \frac{1}{2} y \overline{\psi}_{1L}^c \left(\psi_{2L} s_2 + \psi_{3L} s_1\right) \\
&\quad + \text{H.c.}
\end{aligned} \tag{2.8}$$

It is easily shown that the charged leptons and ordinary quarks get consistent masses [23]. However, this case does not lead to neutrino masses and mixing consistent with the experimental data. The analysis in [21, 22] shows that (i) a "perturbation" is required:

$$\lambda_1 \simeq \lambda_2, \qquad v_1 \simeq v_2, \qquad \Lambda_1 \simeq \Lambda_2. \tag{2.9}$$

A possibility to derive this is to impose another antisextet s' but with the VEVs being very smaller than those of s, respectively. Thus, in the followings the s' should be skipped since it does not contribute at the first order. Otherwise, the s' contributions start from the second order in similarity to those of s which are easily included. (ii) A scalar triplet ρ similar to ϕ' must be imposed. The ρ is also skip for the same reason as s', that is, its contribution is similar to that of ϕ'. Let us emphasis that our conclusions remain unchanged if s' and ρ present.

The hierarchies in the VEVs were given in [23]:

$$\lambda_1, \lambda_2 < u_1,\ u_2 < v, v',\ u, u' < \omega,\ \Lambda_1,\ \Lambda_2. \tag{2.10}$$

In the following, the two limits are often taken into account: (i) the lepton-number violating parameters tend to zero, that is, $\lambda_{1,2}, u_{1,2} \to 0$, and (ii) the large scales of $\mathrm{SU}(3)_L$ symmetry break down to that of the standard model approx infinity, that is, $\omega, \Lambda_{1,2} \to \infty$. Let us note also that v, v', u, and u' are in the electroweak scale as well as the large scales all conserving the lepton number.

3. Gauge Bosons

The covariant derivative of a general triplet Φ is given by

$$\begin{aligned} D_\mu &= \partial_\mu + igT_a W_{a\mu} + ig_X T_9 X B_\mu \\ &\equiv \partial_\mu - i\mathcal{P}_\mu, \end{aligned} \tag{3.1}$$

where the gauge fields W_a and B transform as the adjoint representations of $\mathrm{SU}(3)_L$ and $\mathrm{U}(1)_X$, respectively, and the corresponding gauge coupling constants g and g_X. The $T_9 = \mathrm{diag}(1,1,1)/\sqrt{6}$ is chosen so that $\mathrm{Tr}(T_a T_b) = \delta_{ab}/2$ with $a, b = 1, 2, \ldots, 9$. The neutral gauge bosons of the theory get masses from the triplet as follows:

$$\mathcal{L}^{\Phi}_{\mathrm{mass}} = \left(D^H_\mu \langle\Phi\rangle\right)^+ \left(D^{H\mu}\langle\Phi\rangle\right), \tag{3.2}$$

where the subscript H denotes diagonal part of the covariant derivative:

$$D^H_\mu = \partial_\mu + igT_3 W^3_\mu + igT_8 W^8_\mu + ig_X T_9 X_\chi B_\mu. \tag{3.3}$$

The covariant derivative for an antisextet with the VEV part is [30]

$$D_\mu \langle s_i \rangle = -\frac{ig}{2}\left\{ A_\mu^a \lambda_a^* \langle s_i \rangle + \langle s_i \rangle A_\mu^a \lambda_a^{*T} \right\} + ig_X T_9 X B_\mu \langle s \rangle. \tag{3.4}$$

Let us denote the antisextet in term of the $SU(3)_L$ indices by Γ_{ij}. Then, the mass Lagrangian due to the antisextet's contribution is given by

$$\mathcal{L}_{\text{mass}}^{\Gamma} = \left(D_\mu^H \langle \Gamma \rangle_{ij} \right)^+ \left(D^{H\mu} \langle \Gamma \rangle_{ij} \right), \tag{3.5}$$

Let us denote the following combinations:

$$\begin{aligned} W_\mu^{'\pm} &\equiv \frac{W_{1\mu} \mp i W_{2\mu}}{\sqrt{2}}, \\ Y_\mu^{'\mp} &\equiv \frac{W_{6\mu} \mp i W_{7\mu}}{\sqrt{2}}, \\ X_\mu^{'0} &\equiv \frac{W_{4\mu} - i W_{5\mu}}{\sqrt{2}} \end{aligned} \tag{3.6}$$

having defined charges under the generators of the $SU(3)_L$ group. For the sake of convenience in further reading, we note that W_4 and W_5 are pure real and imaginary parts of $X_\mu^{'0}$ and $X_\mu^{'0*}$, respectively:

$$\begin{aligned} W_{4\mu} &= \frac{1}{\sqrt{2}}\left(X_\mu^{'0} + X_\mu^{'0*} \right), \\ W_{5\mu} &= \frac{i}{\sqrt{2}}\left(X_\mu^{'0} - X_\mu^{'0*} \right). \end{aligned} \tag{3.7}$$

Then $\not{D}_\mu$ is rewritten in a convenient form:

$$\frac{g}{2}\begin{pmatrix} W_{3\mu} + \frac{1}{\sqrt{3}} W_{8\mu} + t\sqrt{\frac{2}{3}} X B_\mu & \sqrt{2} W_\mu^{'+} & \sqrt{2} X_\mu^{'0} \\ \sqrt{2} W_\mu^{'-} & -W_{3\mu} + \frac{1}{\sqrt{3}} W_{8\mu} + t\sqrt{\frac{2}{3}} X B_\mu & \sqrt{2} Y_\mu^{'-} \\ \sqrt{2} X_\mu^{'0*} & \sqrt{2} Y_\mu^{'+} & -\frac{2}{\sqrt{3}} W_{8\mu} + t\sqrt{\frac{2}{3}} X B_\mu \end{pmatrix} \tag{3.8}$$

with $t \equiv g_X / g$.

The covariant derivative acting on the antisextet VEV is given by

$$D_\mu\langle s_i\rangle_{11} = -ig\left(\lambda_i W_{3\mu} + \lambda_i\frac{1}{\sqrt{3}}W_{8\mu} - t\lambda_i\sqrt{\frac{2}{3}}\frac{1}{3}B_\mu + \sqrt{2}\ u_i X^0_\mu\right),$$
$$D_\mu\langle s_i\rangle_{12} = -\frac{ig}{\sqrt{2}}\left(\lambda_i W^-_\mu + u_i Y^-_\mu\right),$$
$$D_\mu\langle s_i\rangle_{13} = -\frac{ig}{2}\left(u_i W_{3\mu} - u_i\frac{1}{\sqrt{3}}W_{8\mu} - tu_i\sqrt{\frac{2}{3}}\frac{2}{3}B_\mu + \sqrt{2}\Lambda_i\ X^0_\mu + \sqrt{2}\lambda_i\ X^{0*}_\mu\right),$$
$$D_\mu\langle s_i\rangle_{22} = 0,$$
$$D_\mu\langle s_i\rangle_{23} = -\frac{ig}{\sqrt{2}}\left(u_i W^-_\mu + \Lambda_i Y^-_\mu\right),\tag{3.9}$$
$$D_\mu\langle s_i\rangle_{33} = ig\left(2\Lambda_i\frac{1}{\sqrt{3}}W_{8\mu} + t\Lambda_i\sqrt{\frac{2}{3}}\frac{1}{3}B_\mu - \sqrt{2}u_i X^{*0}_\mu\right),$$
$$D_\mu\langle s_i\rangle_{12} = D_\mu\langle s_i\rangle_{21},$$
$$D_\mu\langle s_i\rangle_{13} = D_\mu\langle s_i\rangle_{31},$$
$$D_\mu\langle s_i\rangle_{23} = D_\mu\langle s_i\rangle_{32}.$$

The masses of gauge bosons in this model are followed from

$$\begin{aligned}\mathcal{L}^{\mathrm{GB}}_{\mathrm{mass}} &= (D_\mu\langle\phi\rangle)^+ D^\mu\langle\phi\rangle + (D_\mu\langle\phi'\rangle)^+ D^\mu\langle\phi'\rangle + (D_\mu\langle\chi\rangle)^+ D^\mu\langle\chi\rangle \\ &+ (D_\mu\langle\eta\rangle)^+ D^\mu\langle\eta\rangle\ + (D_\mu\langle\eta'\rangle)^+ D^\mu\langle\eta'\rangle + \mathrm{Tr}\left[(D_\mu\langle s_1\rangle)^+ D^\mu\langle s_1\rangle\right] \\ &+ \mathrm{Tr}\left[(D_\mu\langle s_2\rangle)^+ D^\mu\langle s_2\rangle\right]\ .\end{aligned}\tag{3.10}$$

In the following, we notice that $\langle s_1\rangle = \langle s_2\rangle$; namely, $u_1 = u_2$, $\lambda_1 = \lambda_2$, and $\Lambda_1 = \Lambda_2$ are taken into account.

From (3.10), the imaginary part W_5 is decoupled with mass given by

$$M^2_{W_5} = \frac{g^2}{2}\left(16u_1^2 + 4\lambda_1^2 - 8\Lambda_1\lambda_1 + 4\Lambda_1^2 + \omega^2 + u^2 + u'^2\right).\tag{3.11}$$

In the limit $\lambda_1, u_1 \to 0$,

$$M^2_{W_5} = \frac{g^2}{2}\left(u^2 + u'^2 + \omega^2 + 4\Lambda_1^2\right).\tag{3.12}$$

The charged gauge bosons W and Y mix via

$$\mathcal{L}^{\mathrm{CG}}_{\mathrm{mass}} = \frac{g^2}{4}\left(W^-_\mu, Y^-_\mu\right) M^2_{WY} \left(W^+_\mu, Y^+_\mu\right)^T, \tag{3.13}$$

where

$$M^2_{WY} = 2\begin{pmatrix} v^2 + v'^2 + u^2 + u'^2 + 4u_1^2 + 4\lambda_1^2 & 4(\Lambda_1 u_1 + \lambda_1 u_1) \\ 4(\Lambda_1 u_1 + \lambda_1 u_1) & v^2 + v'^2 + \omega^2 + 4\Lambda_1^2 + 4u_1^2 \end{pmatrix}. \tag{3.14}$$

Diagonalizing this mass matrix, we get *physical* charged gauge bosons

$$\begin{aligned} W^-_\mu &= \cos\theta W'^-_\mu + \sin\theta Y'^-_\mu, \\ Y^-_\mu &= -\sin\theta W'^-_\mu + \cos\theta Y'^-_\mu. \end{aligned} \tag{3.15}$$

The mixing angle is given by

$$\tan\theta = \frac{4(\Lambda_1 u_1 + \lambda_1 u_1)}{\omega^2 + 4\Lambda_1^2 - u^2 - u'^2 - 4\lambda_1^2} \sim \frac{u_1}{\Lambda_1}, \tag{3.16}$$

provided that $\omega^2 \sim \Lambda_1^2 \gg u^2, u'^2, u_1^2, \lambda_1^2$. The mass eigenvalues are

$$\begin{aligned} M^2_W = \frac{g^2}{4}\Big\{ & v^2 + v'^2 + 2u^2 + 2u'^2 + \omega^2 + 4\lambda_1^2 + 4\Lambda_1^2 \\ & - \Big[\left(v^2 + v'^2 - \omega^2\right)^2 + 16\lambda_1^4 + 16\Lambda_1^4 + 128\lambda_1\Lambda_1 u_1^2 \\ & +8\Lambda_1^2\left(\omega^2 - v^2 - v'^2 + 8u_1^2\right) - 8\lambda_1^2\left(4\Lambda_1^2\omega^2 - v^2 - v'^2 - 8u_1^2\right)\Big]^{1/2}\Big\}, \\ M^2_Y = \frac{g^2}{4}\Big\{ & v^2 + v'^2 + 2u^2 + 2u'^2 + \omega^2 + 4\lambda_1^2 + 4\Lambda_1^2 \\ & + \Big[\left(v^2 + v'^2 - \omega^2\right)^2 + 16\lambda_1^4 + 16\Lambda_1^4 + 128\lambda_1\Lambda_1 u_1^2 \\ & +8\Lambda_1^2\left(\omega^2 - v^2 - v'^2 + 8u_1^2\right) - 8\lambda_1^2\left(4\Lambda_1^2\omega^2 - v^2 - v'^2 - 8u_1^2\right)\Big]^{1/2}\Big\}. \end{aligned} \tag{3.17}$$

Note that, in the limit $\lambda_1, u_1 \to 0$, the mixing angle tends to zero and the mass eigenvalues are

$$\begin{aligned} M^2_W &= \frac{g^2}{2}\left(v^2 + v'^2 + u^2 + u'^2\right), \\ M^2_Y &= \frac{g^2}{2}\left(v'^2 + v^2 + \omega^2 + 4\Lambda_1^2\right). \end{aligned} \tag{3.18}$$

There is a mixing among the neutral gauge bosons W_3, W_8, B, and W_4. The mass Lagrangian in this case has the form

$$\mathcal{L}_{\text{mass}}^{\text{NG}} = \frac{1}{2} V^T M^2 V, \qquad V^T \equiv (W_3, W_8, B, W_4). \tag{3.19}$$

In the basis of these elements, the mass matrix is given by

$$M^2 = \frac{g^2}{4} \begin{pmatrix} M_{11}^2 & M_{12}^2 & M_{13}^2 & M_{14}^2 \\ M_{12}^2 & M_{22}^2 & M_{23}^2 & M_{24}^2 \\ M_{13}^2 & M_{23}^2 & M_{33}^2 & M_{34}^2 \\ M_{14}^2 & M_{24}^2 & M_{34}^2 & M_{44}^2 \end{pmatrix}, \tag{3.20}$$

where

$$\begin{aligned}
M_{11}^2 &= 2\left(4u_1^2 + 8\lambda_1^2 + u^2 + u'^2 + v^2 + v'^2\right), \\
M_{22}^2 &= \frac{2}{3}\left(4u_1^2 + 8\lambda_1^2 + u^2 + u'^2 + 32\Lambda_1^2 + v^2 + v'^2 + 4\omega^2\right), \\
M_{33}^2 &= \frac{4t^2}{27}\left(16u_1^2 + 8\lambda_1^2 + u^2 + u'^2 + 8\Lambda_1^2 + 4v^2 + 4v'^2 + \omega^2\right), \\
M_{44}^2 &= 2\left(\omega^2 + u^2 + u'^2 + 16u_1^2 + 4\Lambda_1^2 + 4\lambda_1^2 + 8\Lambda_1\lambda_1\right), \\
M_{12}^2 &= \frac{2}{\sqrt{3}}\left(-4u_1^2 + 8\lambda_1^2 + u^2 + u'^2 - v^2 - v'^2\right), \\
M_{13}^2 &= -\frac{2}{3}\sqrt{\frac{2}{3}}t\left(8u_1^2 + 8\lambda_1^2 + u^2 + u'^2 + 2v^2 + 2v'^2\right), \\
M_{14}^2 &= 8(3u_1\lambda_1 + u_1\Lambda_1), \\
M_{23}^2 &= \frac{2\sqrt{2}}{9}t\left(8u_1^2 + 8\lambda_1^2 - u^2 - u'^2 + 16\Lambda_1^2 + 2v^2 + 2v'^2 + 2\omega^2\right), \\
M_{24}^2 &= \frac{8}{\sqrt{3}}(u_1\lambda_1 - 5u_1\Lambda_1), \\
M_{34}^2 &= -\frac{32}{3}\sqrt{\frac{2}{3}}t(u_1\lambda_1 + u_1\Lambda_1).
\end{aligned} \tag{3.21}$$

This mass matrix contains one exact eigenvalue:

$$M_\gamma^2 = 0. \tag{3.22}$$

The associated eigenvector is

$$A_\mu = \frac{1}{\sqrt{18+4t^2}}\begin{pmatrix}\sqrt{3}t \\ -t \\ 3\sqrt{2} \\ 0\end{pmatrix}. \tag{3.23}$$

Using continuation of the gauge coupling constant g of the $\mathrm{SU}(3)_L$ at the spontaneous symmetry breaking point, we have [2–9]

$$t = \frac{3\sqrt{2}s_W}{\sqrt{3-4s_W^2}}. \tag{3.24}$$

In order to diagonalize the mass matrix, we choose the base of $(A_\mu, Z_\mu, Z'_\mu, W_{4\mu})$, with

$$\begin{aligned} Z_\mu &= c_W W_{3\mu} - s_W\left(-\frac{t_W}{\sqrt{3}}W_{8\mu} + \sqrt{1-\frac{t_W^2}{3}}B_\mu\right), \\ Z'_\mu &= \sqrt{1-\frac{t_W^2}{3}}W_{8\mu} + \frac{t_W}{\sqrt{3}}B_\mu. \end{aligned} \tag{3.25}$$

The new base is changed from the old by unitary matrix:

$$U = \begin{pmatrix} s_W & c_W & 0 & 0 \\ -\frac{c_W t_W}{\sqrt{3}} & \frac{s_W t_W}{\sqrt{3}} & \sqrt{1-\frac{t_W^2}{3}} & 0 \\ c_W\sqrt{1-\frac{t_W^2}{3}} & -s_W\sqrt{1-\frac{t_W^2}{3}} & \frac{t_W}{\sqrt{3}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{3.26}$$

In this basis, the mass matrix M^2 becomes

$$M'^2 = U^+ M^2 U = \frac{g^2}{4}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & M'^2_{22} & M'^2_{23} & M'^2_{24} \\ 0 & M'^2_{23} & M'^2_{33} & M'^2_{34} \\ 0 & M'^2_{24} & M'^2_{34} & M'^2_{44} \end{pmatrix}. \tag{3.27}$$

In the approximation $\lambda_1^2, u_1^2 \ll \Lambda_1^2$, we have

$$
\begin{aligned}
M_{22}'^2 &= 2\left(u^2 + u'^2 + v^2 + v'^2\right)\frac{1}{c_W^2}, \\
M_{23}'^2 &= \frac{-2\left[2s_W^2\left(u^2 + u'^2\right) - \left(v^2 + v'^2\right)\right]\sqrt{\alpha_0}}{c_W^2}, \\
M_{24}'^2 &= \frac{8u_1\Lambda_1}{c_W}, \\
M_{33}'^2 &= \frac{2\left(u^2 + u'^2\right)}{c_W^2\alpha_0} - \frac{2\left(v^2 + v'^2\right)\alpha_0}{c_W^2} + 8\omega^2 c_W^2\alpha_0 + 64\Lambda_1^2 c_W^2\alpha_0, \\
M_{34}'^2 &= \frac{-8x_0 u_1\Lambda_1}{c_W\sqrt{\alpha_0}}, \\
M_{44}'^2 &= 2\left(\omega^2 + u^2 + u'^2 + 4\Lambda_1\lambda_1 + 4\Lambda_1^2\right),
\end{aligned}
\tag{3.28}
$$

with

$$
\begin{aligned}
x_0 &= \left(4c_W^2 + 1\right), \\
\alpha_0 &= \frac{1}{\left(4c_W^2 - 1\right)}.
\end{aligned}
\tag{3.29}
$$

It is noteworthy that in the limit $u_1 = 0$, the elements M'_{24} and M'_{34} (or equivalently M_{14}, M_{24}, M_{34} in the old base) vanish. In this case, the mixing between W_4 and Z, Z' disappears.

Three bosons gain masses via seesaw mechanism:

$$
\begin{aligned}
M_Z^2 &= M_{22}'^2 - \left(M^{\text{off}}\right)^T\left(M_{2X2}'^2\right)^{-1}M^{\text{off}}, \\
M_{2X2}^2 &\approx M_{2X2}'^2,
\end{aligned}
\tag{3.30}
$$

where

$$
\begin{aligned}
M^{\text{off}} &= \begin{pmatrix} M_{23}'^2 \\ M_{24}'^2 \end{pmatrix}, \\
M_{2X2}'^2 &= \begin{pmatrix} M_{33}'^2 & M_{34}'^2 \\ M_{34}'^2 & M_{44}'^2 \end{pmatrix}.
\end{aligned}
\tag{3.31}
$$

We have then

$$M_Z^2 = \frac{g^2}{4}\left(M_{22}'^2 - \frac{\left(M_{23}'^2\right)^2 M_{44}'^2 - 2M_{23}'^2 M_{24}'^2 M_{34}'^2 + \left(M_{24}'^2\right)^2 M_{33}'^2}{M_{33}'^2 M_{44}'^2 - \left(M_{34}'^2\right)^2}\right)$$
$$= \frac{g^2}{2c_W^2}\left(u^2 + u'^2 + v^2 + v'^2\right) - \Delta_{M_{22}^2}, \tag{3.32}$$

where

$$\Delta_{M_{22}^2} = \frac{g^2}{\alpha_0^2}\frac{\left[32(2x_0x_1 + x_3)u_1^2\Lambda_1^2 + x_1^2 x_2\right]}{x_2x_3 - 32x_0^2u_1^2\Lambda_1^2},$$
$$x_1 = c_{2W}\left(u^2 + u'^2\right) - \left(v^2 + v'^2\right),$$
$$x_2 = \left(\omega^2 + u^2 + u'^2 + 4\Lambda_1\lambda_1 + 4\Lambda_1^2\right),$$
$$x_3 = c_{2W}\left(u^2 + u'^2\right) + v^2 + v'^2 + 4c_W^4\omega^2 + 32c_W^4\Lambda_1^2. \tag{3.33}$$

The ρ parameter in the our model is given by

$$\rho = 1 + \delta_{\text{tree}} + \delta_{\text{loop}}, \tag{3.34}$$

where δ_{loop} gets contribution from the oblique correction depending on the masses of top quark and standard model Higgs boson [1]. The tree level correction δ_{tree} describes the new physics as given by

$$\delta_{\text{tree}} = \frac{M_W^2}{c_W^2 M_Z^2} - 1 \simeq \frac{c_W^2\Delta_{M_{22}^2}}{M_W^2}. \tag{3.35}$$

It is noted that $\Delta_{M_{22}^2} \neq 0$ even if ω and Λ_1 go to infinity. This is because the 33 components of antisextets and the third components of scalar triplets can be integrated out. There leave the standard model scalar doublets and triplets (the submultiplets of the 3-3-1 model triplets and antisextets). Such standard model scalar triplets imply $\delta_{\text{tree}} \neq 0$ to be given by

$$\delta_{\text{tree}} \simeq \frac{8g^2c_W^2\left(4c_W^2 - 1\right)^2 u_1^2}{M_W^2} \neq 0. \tag{3.36}$$

The δ_{tree} parameter has already been given in [1] as $\rho_0 - 1$ from the global fit:

$$\delta_{\text{tree}} = 0.0008^{+0.0017}_{-0.0007}. \tag{3.37}$$

Hence

$$0.8403\,\mathrm{GeV}^2 \le \Delta_{M_{22}^2} \le 21.0065\,\mathrm{GeV}^2, \tag{3.38}$$

or

$$0.239\,\mathrm{GeV} \le u_1 \le 1.197\,\mathrm{GeV}, \tag{3.39}$$

where we have used $c_W^2 = 0.769$ and $M_W = 80.384$ GeV.

Diagonalizing the mass matrix M_{2X2}^2, we get new gauge bosons:

$$\begin{aligned} Z'_\mu &= \cos\phi Z''_\mu + \sin\phi W'_{4\mu}, \\ W_{4\mu} &= -\sin\phi Z''_\mu + \cos\phi W'_{4\mu}. \end{aligned} \tag{3.40}$$

The mixing angle is defined by

$$\tan\phi = \frac{2M_{34}'^2}{M_{44}'^2 - M_{33}'^2 + \sqrt{\left(M_{44}'^2 - M_{33}'^2\right)^2 + 4\left(M_{34}'^2\right)^2}}. \tag{3.41}$$

Substituting (3.28) into (3.41), we get

$$\begin{aligned} \tan\phi =& -2\sqrt{\alpha_1}u_1\Lambda_1 \Bigg\{ \alpha_2\left(u^2 + u'^2\right) + \alpha_3\left(v^2 + v'^2\right) - 2\alpha_0 w^2 + 8\lambda_1\Lambda_1 - 8x_0\alpha_0\Lambda_1^2 \\ & + \left[\left(\alpha_2\left(u^2+u'^2\right)+\alpha_3\left(v^2+v'^2\right) -2\alpha_0 w^2+8\lambda_1\Lambda_1-8x_0\alpha_0\Lambda_1^2\right)^2 +4\alpha_1 u_1^2\Lambda_1^2\right]^{1/2} \Bigg\}^{-1} \\ \simeq& \frac{\sqrt{4c_W^2 - 1}}{c_W}\frac{u_1}{\Lambda_1}, \end{aligned} \tag{3.42}$$

provided that $\omega^2 \sim \Lambda_1^2 \gg u^2, u'^2, v^2, v'^2, u_1^2, \lambda_1^2$, where

$$\begin{aligned} \alpha_1 &= \frac{64x_0^2\alpha_0}{c_W^2}, \\ \alpha_2 &= \frac{-2 + 6c_W^2}{c_W^2}\alpha_0, \\ \alpha_3 &= \frac{-2}{c_W^2}\alpha_0. \end{aligned} \tag{3.43}$$

The physical mass eigenvalues are defined by

$$
\begin{aligned}
M^2_{Z'',W'_{4\mu}} &= \frac{g^2}{4}\frac{M'^2_{44}+M'^2_{33}\mp\sqrt{\left(M'^2_{44}-M'^2_{33}\right)^2+4\left(M'^2_{34}\right)^2}}{2} \\
&= \frac{g^2}{8}\left(\alpha_4\left(u^2+u'^2\right)-\alpha_3\left(v^2+v'^2\right)+\alpha_5 w^2+8\lambda_1\Lambda_1+\alpha_6\Lambda_1^2\right. \\
&\quad \left.\pm\sqrt{\left(\alpha_2\left(u^2+u'^2\right)+\alpha_3(v^2+v'^2)-2\alpha_0 w^2+8\lambda_1\Lambda_1-8x_0\alpha_0\Lambda_1^2\right)^2+4\alpha_1 u_1^2\Lambda_1^2}\right)
\end{aligned}
\tag{3.44}
$$

with

$$
\begin{aligned}
\alpha_4 &= \frac{2-10c_W^2+16c_W^4}{c_W^2}\alpha_0, \\
\alpha_5 &= \left(16c_W^2-2\right)\alpha_0, \\
\alpha_6 &= 8\left(12c_W^2-1\right)\alpha_0.
\end{aligned}
\tag{3.45}
$$

In the limit $\lambda_1, u_1 \to 0$, we have

$$
\begin{aligned}
M^2_{Z''} &= \frac{g^2\left[c^2_{2W}\left(u^2+u'^2\right)+v^2+v'^2+4c_W^4\omega^2+32c_W^4\Lambda_1^2\right]}{2c_W^2}\alpha_0, \\
M^2_{W'_4} &= \frac{g^2}{2}\left(u^2+u'^2+\omega^2+4\Lambda_1^2\right).
\end{aligned}
\tag{3.46}
$$

Thus the W'_4 and W_5 components have the same mass. With this result, we should identify the combination of W'_4 and W_5

$$
\sqrt{2}X^0_\mu = W'_{4\mu} - iW_{5\mu} \tag{3.47}
$$

as *physical* neutral *non-Hermitian* gauge boson. The subscript 0 denotes neutrality of gauge boson X. However, to get tribimaximal mixing, the previous limit is not valid [21, 22]. This means that neutrino tribimaximal mixing leads to the masses of X^0 and $X^{0\dagger}$ to be different. Consequence of this fact is that there is CPT violation [1, 31] in the model under consideration. We will return to this problem in the future work.

In the limit $\omega^2 \sim \Lambda_1^2 \gg u^2, u'^2, v^2, v'^2, u_1^2, \lambda_1^2$ (or $\omega,\ \Lambda_1 \to \infty$), the mixings between the charged gauge bosons $W - Y$ and the neutral ones $W_4 - Z'$ are in the same order since from (3.16) and (3.42) they are proportional to u_1/Λ_1. In addition, from (3.46), $M^2_{Z''} \simeq 2g^2(\omega^2 + 16c_W^2\Lambda_1^1)$ is bigger than $M^2_{W_4} \simeq (g^2/2)(\omega^2+4\Lambda_1^2)$ (or $M^2_{X^0}$). It is also verified that $|M^2_Y - M^2_{X^0}| < M^2_W$. In that limit, the masses of X^0 and Y degenerate.

Note that the formulas for masses and mixing of gauge bosons previously presented, are *common* for the 3-3-1 models with more complicated Higgs sector such as with A_4 or S_4 discrete symmetries.

4. Charged Currents

The interaction among fermions with gauge bosons arises from part

$$i\overline{\psi}\gamma_\mu D^\mu \psi = \text{kinematic terms} + H^{\mathrm{CC}} + H^{\mathrm{NC}}. \tag{4.1}$$

Similarly in the economical 3-3-1 model, despite neutrality, the gauge bosons X^0 and X^{0*} belong to this section by their nature. Because of the mixing among the SM W boson and the charged bilepton Y as well as among $(X^0 + X^{0*})$ with (W_3, W_8, B), the new terms exist the same as the economical 3-3-1 model [25, 26]:

$$H^{\mathrm{CC}} = \frac{g}{\sqrt{2}}\left(J_W^{\mu-}W_\mu^+ + J_Y^{\mu-}Y_\mu^+ + J_X^{\mu0*}X_\mu^0 + \text{H.c.}\right), \tag{4.2}$$

where

$$\begin{aligned}
J_W^{\mu-} &= c_\theta\left(\overline{\nu}_{iL}\gamma^\mu e_{iL} + \overline{u}_{iL}\gamma^\mu d_{iL}\right) + s_\theta\left(\overline{\nu}_{iL}^c\gamma^\mu e_{iL} + \overline{U}_L\gamma^\mu d_{1L} + \overline{u}_{\alpha L}\gamma^\mu D_{\alpha L}\right),\\
J_Y^{\mu-} &= c_\theta\left(\overline{\nu}_{iL}^c\gamma^\mu e_{iL} + \overline{U}_L\gamma^\mu d_{1L} + \overline{u}_{\alpha L}\gamma^\mu D_{\alpha L}\right) - s_\theta\left(\overline{\nu}_{iL}\gamma^\mu e_{iL} + \overline{u}_{iL}\gamma^\mu d_{iL}\right),\\
J_X^{\mu0*} &= \left(1 - t_{2\theta}^2\right)\left(\overline{\nu}_{iL}\gamma^\mu \nu_{iL}^c + \overline{u}_{1L}\gamma^\mu U_L - \overline{D}_{\alpha L}\gamma^\mu d_{\alpha L}\right) - t_{2\theta}^2\left(\overline{\nu}_{iL}^c\gamma^\mu \nu_{iL} + \overline{U}_L\gamma^\mu u_{1L} - \overline{d}_{\alpha L}\gamma^\mu D_{\alpha L}\right)\\
&\quad + \frac{t_{2\theta}}{\sqrt{1 + 4t_{2\theta}^2}}\left(\overline{\nu}_i\gamma^\mu \nu_i + \overline{u}_{1L}\gamma^\mu u_{1L} - \overline{U}_L\gamma^\mu U_L - \overline{d}_{\alpha L}\gamma^\mu d_{\alpha L} + \overline{D}_{\alpha L}\gamma^\mu D_{\alpha L}\right).
\end{aligned} \tag{4.3}$$

All aforementioned interactions are lepton-number violating and weak (proportional to $\sin\theta$ or its square $\sin^2\theta$). However, these couplings lead to lepton-number violations only in the neutrino sector.

Let us consider some constraints on the parameters of the model; one of the ways to do that is the consideration for W decay. In our model, the W boson has the following *normal main* decay modes:

$$\begin{aligned}
W^- &\longrightarrow l\widetilde{\nu}_l \quad (l = e, \mu, \tau),\\
&\searrow u^c d, u^c s, u^c b, \quad (u \to c),
\end{aligned} \tag{4.4}$$

which are the same as in the SM and in the 331 with right-handed neutrinos. Beside the aforementioned modes, there are additional ones which are lepton-number violating ($\Delta L = 2$) the model's specific feature:

$$W^- \longrightarrow l\ \nu_l\ \ (l = e, \mu, \tau). \tag{4.5}$$

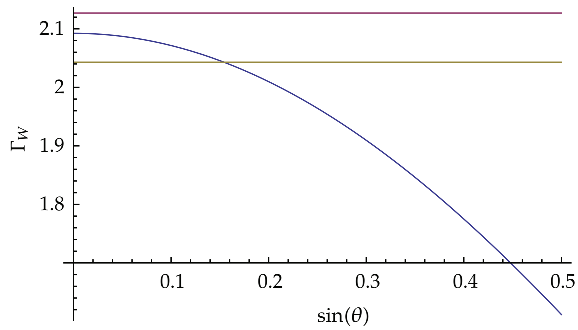

Figure 1: The total decay width of W [GeV] is depicted as a function of $\sin(\theta)$. The two horizontal lines are the upper and lower limits, respectively.

The interaction that provides these modes is as follows:

$$\mathcal{L} = \frac{g}{\sqrt{2}} s_\theta \overline{\nu}^c_{iL} \gamma^\mu e_{iL}, \tag{4.6}$$

where $\nu^c_L \equiv (N_R)^c$ is related to ν_L via the seesaw mechanism given by $\nu^c_L = M_D M_R^{-1} \nu_L$. Here M_R and M_D are right-handed Majorana and Dirac mass matrices (due to the contribution of s), respectively, which can be derived from the Yukawa Lagrangian above to yield $M_D M_R^{-1} = (u_1/\Lambda_1)\,\mathrm{diag}(1,1,1)$. On the other hand, from (3.16) we have $s_\theta \simeq \tan\theta \simeq u_1/\Lambda_1$ if Λ_1 is largest among the VEVs. It is therefore that $\nu^c_L \simeq s_\theta \nu_L$ and

$$\mathcal{L} = \frac{g}{\sqrt{2}} s^2_\theta \overline{\nu}_{iL} \gamma^\mu e_{iL}. \tag{4.7}$$

The total decay width of W is given by [25, 26]

$$\Gamma_W = 1.039 \frac{\alpha M_W}{2 s^2_W}\left(1 - s^2_\theta\right) + \frac{\alpha M_W}{4 s^2_W}\left(1 - s^2_\theta\right) + \frac{\alpha M_W}{4 s^2_W} s^4_\theta, \tag{4.8}$$

where the first term is due to the quark productions (with $\alpha_s = 0.1184$ chosen for the QCD radiative corrections), the second term comes from the normal modes with leptons, and the last one is for the unnormal modes. Let us choose $\alpha(M_Z) = 1/128$, $M_W = 80.399\,\mathrm{GeV}$, and $\Gamma^{\mathrm{ext}}_W = 2.085 \pm 0.042\,\mathrm{GeV}$ [1]. The total decay width is plotted in Figure 1. From the figure, we get an upper limit on the $\sin\theta$ in the model:

$$\sin\theta \leq 0.15, \tag{4.9}$$

which is bigger than that given in [25, 26].

There are lepton number violating interactions in the neutral Gauge boson sector, we refer interested reader to [25, 26].

5. Conclusions

In this paper, we have investigated Gauge boson sector: their mixing and masses. The vacuum expectation values u_i and λ_i are a source of lepton-number violations and a reason for the mixing between the charged Gauge bosons—the standard model W and the singly-charged bilepton Gauge bosons, as well as between neutral non-Hermitian X^0 and neutral Gauge bosons: the photon, the Z, and the new exotic Z'. The interesting new physics compared with 3-3-1 models is the neutrino physics. Due to lepton-number violating couplings, we have many interesting consequences. We have shown that the neutrino tribimaximal mixing leads to the CPT violation. This feature will be considered in the future publication.

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Research Article

Radiatively Generated Leptogenesis in S_4 Flavor Symmetry Models

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We study how leptogenesis can be implemented in the seesaw models with S_4 flavor symmetry, which lead to the tri-bimaximal neutrino mixing matrix. By considering renormalization group evolution from a high-energy scale of flavor symmetry breaking (the GUT scale is assumed) to the low-energy scale of relevant phenomena, the off-diagonal terms in a combination of Dirac Yukawa-coupling matrix can be generated and the degeneracy of heavy right-handed neutrino Majorana masses can be lifted. As a result, the flavored leptogenesis is successfully realized. We also investigate how the effective light neutrino mass $|\langle m_{ee}\rangle|$ associated with neutrinoless double beta decay can be predicted along with the neutrino mass hierarchies by imposing the experimental data on the low-energy observables. We find a link between the leptogenesis and the neutrinoless double beta decay characterized by $|\langle m_{ee}\rangle|$ through a high-energy CP phase ϕ, which is correlated with the low-energy Majorana CP phases. It is shown that the predictions of $|\langle m_{ee}\rangle|$ for some fixed parameters of the high-energy physics can be constrained by the current observation of baryon asymmetry.

1. Introduction

The neutrino experimental data can provide an important clue for elucidating the origin of observed hierarchies in the mass matrices of quarks and leptons. The recent experiments of neutrino oscillation have gone into a new phase of precise determination of the mixing angles and squared mass differences [1, 2], which indicate that the tri-bimaximal mixing (TBM) for the three flavors of leptons

$$U_{\rm TB} = \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{1.1}$$

can be regarded as the PMNS matrix $U_{\rm PMNS} \equiv U_{\rm TB} P_\nu$ [3–6], where P_ν is a diagonal matrix of CP phases. However, properties related to the leptonic CP violation have not been completely known yet. The large mixing angles, which may be suggestive of a flavor symmetry, are completely different from the quark mixing ones. Therefore, it is very important to find a model that naturally leads to those mixing patterns of quarks and leptons with a good accuracy. In recent years there have been a lot of efforts in searching for models which result in the TBM pattern naturally and a fascinating way seems to be the use of some discrete non-Abelian flavor groups added to the gauge groups of the standard model. There is a series of proposals based on groups A_4 [7–16], T' [17–21], and S_4 [22–36]. The common feature of these models is that they are naturally realized at a very-high-energy scale Λ and the groups are spontaneously broken due to a set of scalar multiplets, the flavons.

In addition to the explanation of smallness of observed neutrino masses, the seesaw mechanism [37–39] has another appearing feature so-called leptogenesis mechanism for generation of observed baryon asymmetry of the Universe (BAU), through the decay of heavy right-handed (RH) Majorana neutrinos [40–44]. If this BAU was made via the leptogenesis, then the CP violation in leptonic sector is required. For the Majorana neutrinos of three flavors there are one Dirac-type phase and two Majorana-type phases, one (or a combination) of which in principle can be measured through neutrinoless double beta ($0\nu2\beta$) decays [45–48]. The exact TBM pattern forbids at low energy the CP violation in neutrino oscillations, due to $U_{e3} = 0$. Therefore, any observation of the leptonic CP violation, for instance, in the $0\nu2\beta$ decay, can strengthen our belief in the leptogenesis by demonstrating that the CP is not a symmetry of leptons. It is interesting to explore this existence of the CP violation due to the Majorana CP-violating phases by measuring $|\langle m_{ee}\rangle|$ and examine a link between observable low-energy $0\nu2\beta$ decay and the BAU. The authors in [35, 36] have shown that the TBM pattern can be generated naturally in the framework of the seesaw mechanism with $SU(2)_L \times U(1)_Y \times S_4$ symmetry. The textures of mass matrices as given in [35, 36] also could not generate a lepton asymmetry which is essential for the baryogenesis. In this paper, we investigate possibility of radiative leptogenesis when renormalization group (RG) effects are taken into account. We will show that the leptogenesis can be linked to the $0\nu2\beta$ decay through the seesaw mechanism.

The rest of this work is organized as follows. In Section 2, we present the low-energy observables in two variants of supersymmetric seesaw model based on flavor symmetry S_4. We especially focus on the effective neutrino mass governing the $0\nu2\beta$ decay. In Section 3, we study RG effects on the Yukawa couplings and heavy Majorana neutrino mass matrices so that the leptogenesis becomes available. This leptogenesis in the two models due to the RG effects is studied in detail in Section 4. Finally, Section 5 is devoted to our conclusions.

Table 1: Transformation properties of the lepton sector and all the flavons of the BMM model where $\omega = e^{i2\pi/3}$.

Field	l	e^c	μ^c	τ^c	ν^c	$h_{u,d}$	θ	ψ	η	Δ	φ	ξ'
S_4	3_1	1_2	1_2	1_1	3_1	1_1	1_1	3_1	2	3_1	2	1_2
Z_5	ω^4	1	ω^2	ω^4	ω	1	1	ω^2	ω^2	ω^3	ω^3	1
$U(1)_{\text{FN}}$	0	1	0	0	0	0	–1	0	0	0	0	0

2. Two S_4 Models

In this section we give a review of the main features of Bazzocchi-Merlo-Morisi (BMM) model [35] and Ding model [36]. We simultaneously discuss the $0\nu2\beta$ decay, leptogenesis, and phenomenological difficulties associated with the models to be solved.

2.1. Bazzocchi-Merlo-Morisi Model

In this model the flavor symmetry is S_4 accompanied with cyclic group Z_5 and Froggatt-Nielsen symmetry $U(1)_{\text{FN}}$ [49], that is, $G_f = S_4 \times Z_5 \times U(1)_{\text{FN}}$. The matter fields and flavons are given in Table 1. The superpotential for the lepton sector reads

$$\begin{aligned} w_l &= \sum_{i=1}^{4} \frac{\theta}{\Lambda} \frac{y_{e,i}}{\Lambda^3} e^c (l X_i)_{1_2} h_d + \frac{y_\mu}{\Lambda^2} \mu^c (l\psi\eta)_{1_2} h_d + \frac{y_\tau}{\Lambda} \tau^c (l\psi)_{1_1} h_d + h.c. + \cdots, \\ w_\nu &= x(\nu^c l)_{1_1} h_u + x_d (\nu^c \nu^c \varphi)_{1_1} + x_t (\nu^c \nu^c \Delta)_{1_1} + h.c. + \cdots, \end{aligned} \tag{2.1}$$

where $X_i = \psi\psi\eta,\ \psi\eta\eta, \Delta\Delta\xi', \Delta\varphi\xi'$ and the dots denote higher-order contributions. The VEV alignment of flavons is

$$\begin{aligned} &\langle\psi\rangle = (0\ \ 1\ \ 0)^T v_\psi, \qquad \langle\Delta\rangle = (1\ \ 1\ \ 1)^T v_\Delta, \\ &\langle\eta\rangle = (0\ \ 1)^T v_\eta, \qquad \langle\varphi\rangle = (1\ \ 1)^T v_\varphi, \qquad \langle\xi'\rangle = v_{\xi'}, \end{aligned} \tag{2.2}$$

where all the VEVs are of the same order of magnitude and for this reason being parameterized as VEVs$/\Lambda = u$. The remaining VEV which originates from a different mechanism is v_θ, denoted by $v_\theta/\Lambda = t$. It is shown in [35] that u and t belong to a well-determined range $0.01 < u, t < 0.05$.

The mass matrix for the charged leptons is given by

$$m_l = \begin{pmatrix} y_e^{(1)} u^2 t & y_e^{(2)} u^2 t & y_e^{(2)} u^2 t \\ 0 & y_\mu u & 0 \\ 0 & 0 & y_\tau \end{pmatrix} u v_d. \tag{2.3}$$

The Dirac and RH-Majorana neutrino mass matrices are, respectively, obtained as

$$\begin{aligned} m_\nu^d &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} x\upsilon_u, \\ M_R &= Be^{i\alpha_1}\begin{pmatrix} 2re^{i\phi} & 1-re^{i\phi} & 1-re^{i\phi} \\ 1-re^{i\phi} & 1+2re^{i\phi} & -re^{i\phi} \\ 1-re^{i\phi} & -re^{i\phi} & 1+2re^{i\phi} \end{pmatrix}, \end{aligned} \tag{2.4}$$

where $B = 2|x_d|\upsilon_\varphi$, $C = 2|x_t|\upsilon_\Delta$, and $r = C/B$ are real and positive. The phases α_1 and α_2 are the arguments of $x_{d,t}$, and $\phi \equiv \alpha_2 - \alpha_1$ being the only physical phase remained in M_R. Notice that the M_R can be exactly diagonalized by the TBM matrix:

$$\begin{gathered} M_R^D = V_R^T M_R V_R = \text{Diag.}(M_1, M_2, M_3), \\ M_1 = B\left|3re^{i\phi} - 1\right|, \qquad M_2 = 2B, \qquad M_1 = B\left|3re^{i\phi} + 1\right|, \\ V_R = U_{TB}V_P, V_P = \text{Diag.}\left(e^{i\gamma_1/2}, 1, e^{i\gamma_2/2}\right), \\ \gamma_{1,2} = -\arg\left(3re^{i\phi} \mp 1\right). \end{gathered} \tag{2.5}$$

Integrating out the heavy degrees of freedom, we get the effective light neutrino mass matrix, which is given by the seesaw relation, $m_{\text{eff}} = -(m_\nu^d)^T M_R^{-1} m_\nu^d$ [37–39], and diagonalized by the TBM matrix:

$$\begin{aligned} U_\nu^T m_{\text{eff}} U_\nu &= \text{Diag.}(m_1, m_2, m_3) \\ &= -\text{Diag.}\left(\frac{x^2\upsilon_u^2}{M_1}, \frac{x^2\upsilon_u^2}{M_2}, \frac{x^2\upsilon_u^2}{M_3}\right), \\ U_\nu &= U_{\text{TB}}\text{Diag.}\left(e^{-i\gamma_1/2}, 1, e^{-i\gamma_2/2}\right). \end{aligned} \tag{2.6}$$

In order to find the lepton mixing matrix we need to diagonalize the charged-lepton mass matrix:

$$m_l^D = U_{l^c}^\dagger m_l U_l = \text{Diag.}\left(y_e u^2 t, y_\mu u, y_\tau\right) u\upsilon_d, \tag{2.7}$$

where U_l is unity matrix. Therefore we get

$$\begin{aligned} U_{\text{PMNS}} &= U_l^\dagger U_\nu \equiv U_\nu \\ &= e^{-i\gamma_1/2} U_{\text{TB}}\text{Diag.}\left(1, e^{i\beta_1}, e^{i\beta_2}\right), \end{aligned} \tag{2.8}$$

where $\beta_1 = \gamma_1/2$ and $\beta_2 = (\gamma_1 - \gamma_2)/2$ are Majorana CP violating phases. The phase factored out to the left has no physical meaning, since it can be eliminated by a redefinition of the charged lepton fields. The light neutrino mass eigenvalues are simply the inverse of the heavy

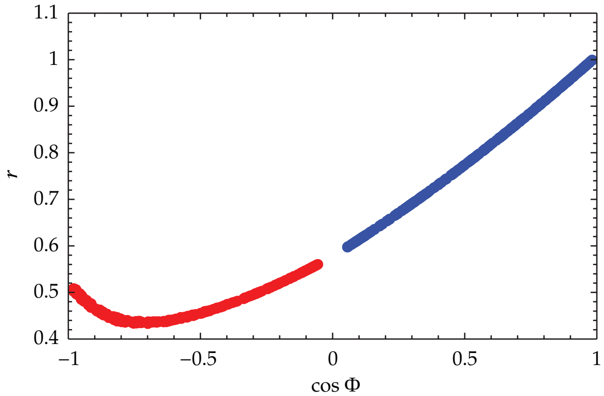

Figure 1: Allowed parameter region by the 1σ experimental constraints (2.9) for the ratio $r = C/B$ as a function of $\cos\phi$. The blue (dark) and red (light) curves correspond to the IH and NH spectra.

neutrino ones, a part from a minus sign and the global factor from m_ν^d, as can be seen in (2.6). There are nine physical parameters consisting of the three light neutrino masses, three mixing angles, and three CP-violating phases in general. The mixing angles are entirely fixed by the G_f symmetry group, predicting TBM and in turn no Dirac CP-violating phase. The remaining five physical parameters, β_1, β_2, m_1, m_2, and m_3, are determined by the five real parameters B, C, v_u, x, and ϕ.

The light neutrino mass spectrum can have both normal or inverted hierarchy depending on the sign of $\cos\phi$. If $\cos\phi < 0$, one has normal hierarchy (NH), whereas if $\cos\phi > 0$, one has inverted hierarchy (IH). In order to see how this correlation in the allowed parameter space is constrained, we consider the experimental data at 1σ [1, 2]:

$$\begin{aligned} \left|\Delta m_{31}^2\right| &= (2.29 - 2.52) \times 10^{-3}\,\mathrm{eV}^2, \\ \Delta m_{21}^2 &= (7.45 - 7.88) \times 10^{-5}\,\mathrm{eV}^2. \end{aligned} \tag{2.9}$$

(Hereafter, we always use the experimental data at 1σ for our numerical calculations of low-energy observables.) The correlations between r and $\cos\phi$ for the NH spectrum (red (light) plot) and IH one [blue (dark) plot) are, respectively, presented in Figure 1.

Because there is no Dirac CP-violating phase as mentioned, the only contribution from the Majorana phases to the $0\nu2\beta$ decay comes from β_1. The effective neutrino mass governing the $0\nu2\beta$ decay is given by

$$\begin{aligned} |\langle m_{ee}\rangle| &= \frac{1}{3}\left|2m_1 + m_2 e^{2i\beta_1}\right| \\ &= \frac{m_0}{3(1 - 6r\cos\phi + 9r^2)}\sqrt{8.5 + 13.5r^2 + 20.25r^4 - 3r(13 + 12r^2)\cos\phi + 9r^2\cos^2\phi}, \end{aligned} \tag{2.10}$$

where $m_0 = x^2 v_u^2/B$. The behavior of $|\langle m_{ee}\rangle|$ is plotted in Figure 2 as a function of ϕ. The horizontal line (0.2 eV) is the current lower bound sensitivity [50–53] while the dashed line (10^{-2} eV) is a future sensitivity [54, 55].

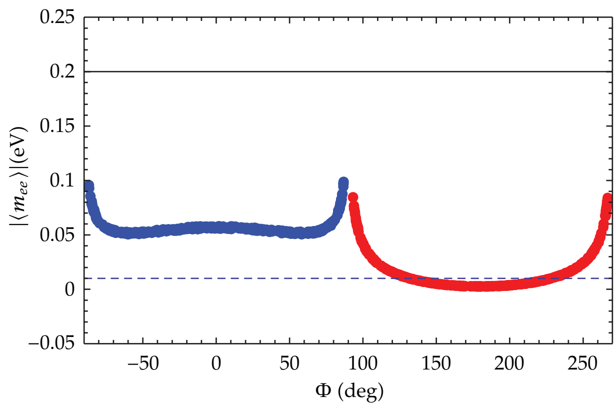

Figure 2: Prediction of the effective neutrino mass $|\langle m_{ee}\rangle|$ responsible for $0\nu 2\beta$ decay as a function of ϕ by the 1σ experimental constraints (2.9). The blue (dark) and red (light) curves correspond to the IH and NH spectra.

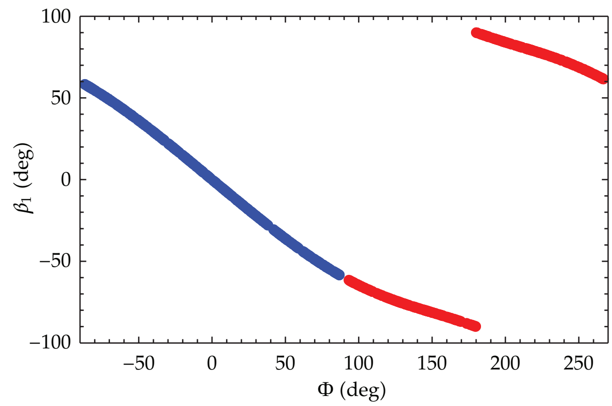

Figure 3: The Majorana CP phase β_1 as a function of ϕ plotted by the 1σ experimental constraints (2.9). The blue (dark) and red (light) curves correspond to the IH and NH spectra.

Using (10) we can obtain the explicit relation between ϕ and β_1:

$$\sin 2\beta_1 = \frac{-3r\sin\phi}{1 - 6r\cos\phi + 9r^2}. \tag{2.11}$$

Figure 3 represents this relation corresponding to the NH spectrum [red (light) plot] and IH one [blue (dark) plot].

In a basis where the charged current is flavor diagonal and the heavy neutrino mass matrix M_R is diagonal and real, the Dirac mass matrix m_ν^d gets modified to

$$m_\nu^d \longrightarrow Y_\nu v_u = V_R^T m_\nu^d, \tag{2.12}$$

Table 2: Representations of the matter fields of lepton sector and flavons under $S_4 \times Z_3 \times Z_4$.

Field	l	e^c	μ^c	τ^c	ν^c	$h_{u,d}$	φ	χ	ϑ	η	ϕ	Δ
S_4	$\mathbf{3}_1$	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{1}_1$	$\mathbf{3}_1$	$\mathbf{1}_1$	$\mathbf{3}_1$	$\mathbf{3}_2$	$\mathbf{1}_2$	$\mathbf{2}$	$\mathbf{3}_1$	$\mathbf{1}_2$
Z_3	ω	ω^2	ω^2	ω^2	1	1	1	1	1	ω^2	ω^2	ω^2
Z_4	1	i	-1	$-i$	1	1	i	i	1	1	1	-1

where $\upsilon_u = \upsilon \sin\beta$, $\upsilon = 176\,\text{GeV}$, and the coupling of N_i with leptons and scalar, Y_ν, is given by

$$Y_\nu = x e^{i\gamma_1/2} \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{6}} \\ \frac{e^{-i\beta_1}}{\sqrt{3}} & \frac{e^{-i\beta_1}}{\sqrt{3}} & \frac{e^{-i\beta_1}}{\sqrt{3}} \\ 0 & \frac{e^{-i\beta_2}}{\sqrt{2}} & \frac{-e^{-i\beta_2}}{\sqrt{2}} \end{pmatrix}. \tag{2.13}$$

Concerned with the CP violation, we notice that the CP phase ϕ originating from m_ν^d obviously takes part at the low-energy CP violation as the Majorana phases β_1 and β_2. On the other hand, the leptogenesis is associated with both the Yukawa coupling Y_ν and its combination:

$$H \equiv Y_\nu Y_\nu^\dagger = x^2 \cdot \text{Diag.}(1,1,1). \tag{2.14}$$

This directly indicates that all off-diagonal H_{ij} vanish, so the CP asymmetry could not be generated and neither leptogenesis. *For the leptogenesis to be viable, the off-diagonal H_{ij} have to be generated.*

2.2. Ding Model

Ding model, proposed in [36], possesses flavor symmetry group $G_f = S_4 \times Z_3 \times Z_4$, where the three factors play different roles. The S_4 controls the mixing angles, the Z_3 guarantees the misalignment in flavor space between neutrino and charged-lepton eigenstates, and the Z_4 is crucial to eliminating unwanted couplings and reproducing observed mass hierarchies. In this framework the mass hierarchies are controlled by spontaneously breaking of the flavor symmetry instead of the Froggatt-Nielsen mechanism [49]. The matter fields of lepton sector and flavons under G_f are assigned as in Table 2.

The superpotential for the lepton sector reads

$$\begin{aligned} w_l = {} & \frac{y_{e1}}{\Lambda^3} e^c (l\varphi)_{1_1} (\varphi\varphi)_{1_1} h_d + \frac{y_{e2}}{\Lambda^3} e^c ((l\varphi)_2 (\varphi\varphi)_2)_{1_1} h_d + \frac{y_{e3}}{\Lambda^3} e^c \left((l\varphi)_{3_1} (\varphi\varphi)_{3_1}\right)_{1_1} h_d \\ & + \frac{y_{e4}}{\Lambda^3} e^c ((l\chi)_2 (\chi\chi)_2)_{1_1} h_d \\ & + \frac{y_{e5}}{\Lambda^3} e^c \left((l\chi)_{3_1} (\chi\chi)_{3_1}\right)_{1_1} h_d + \frac{y_{e6}}{\Lambda^3} e^c (l\varphi)_{1_1} (\chi\chi)_{1_1} h_d + \frac{y_{e7}}{\Lambda^3} e^c ((l\varphi)_2 (\chi\chi)_2)_{1_1} h_d \end{aligned}$$

$$
\begin{aligned}
&+\frac{y_{e8}}{\Lambda^3}e^c\Big((l\varphi)_{3_1}(\chi\chi)_{3_1}\Big)_{1_1}h_d \\
&+\frac{y_{e9}}{\Lambda^3}e^c\big((l\chi)_2(\varphi\varphi)_2\big)_{1_1}h_d+\frac{y_{e10}}{\Lambda^3}e^c\Big((l\chi)_{3_1}(\varphi\varphi)_{3_1}\Big)_{1_1}h_d+\frac{y_\mu}{2\Lambda^2}\mu^c\Big(l(\varphi\chi)_{3_2}\Big)_{1_2}h_d \\
&+\frac{y_\tau}{\Lambda}\tau^c(l\varphi)_{1_1}h_d+\cdots, \\
w_\nu&=\frac{y_{\nu1}}{\Lambda}((\nu^c l)_2\eta)_{1_1}h_u+\frac{y_{\nu2}}{\Lambda}((\nu^c l)_{3_1}\phi)_{1_1}h_u+\frac{1}{2}M(\nu^c\nu^c)_{1_1}+\cdots,
\end{aligned}
\tag{2.15}
$$

where the dots denote higher-order contributions.

The VEV alignment of flavons are assumed as follows:

$$
\begin{aligned}
\langle\varphi\rangle&=(0,v_\varphi,0), \quad \langle\chi\rangle=(0,v_\chi,0), \qquad \langle\vartheta\rangle=v_\vartheta, \\
\langle\eta\rangle&=(v_\eta,v_\eta), \quad \langle\phi\rangle=(v_\phi,v_\phi,v_\phi), \qquad \langle\Delta\rangle=v_\Delta.
\end{aligned}
\tag{2.16}
$$

The charged-lepton mass matrix is obtained by

$$
m_l=\text{Diag.}\left(y_e\frac{v_\varphi^3}{\Lambda^3},y_\mu\frac{v_\varphi v_\chi}{\Lambda^2},y_\tau\frac{v_\varphi}{\Lambda}\right)v_d,
\tag{2.17}
$$

where all the components are assumed to be real. The neutrino sector gives rise to the following Dirac and RH-Majorana mass matrices

$$
\begin{aligned}
m_\nu^d&=e^{i\alpha_1}\begin{pmatrix}2be^{i\phi} & a-be^{i\phi} & a-be^{i\phi}\\ a-be^{i\phi} & a+2be^{i\phi} & -be^{i\phi}\\ a-be^{i\phi} & -be^{i\phi} & a+2be^{i\phi}\end{pmatrix}v_u, \\
M_R&=\begin{pmatrix}M&0&0\\0&0&M\\0&M&0\end{pmatrix},
\end{aligned}
\tag{2.18}
$$

where the quantity M is also supposed to be real and positive. The phase $\phi\equiv\alpha_2-\alpha_1$, where α_1,α_2 are denoted as the arguments of $y_{\nu1}$, $y_{\nu2}$, respectively, is the only physical phase survived because the global phase α_1 can be rotated away. The real and positive components a and b are defined as

$$
a=\left|y_{\nu1}\right|\frac{v_\eta}{\Lambda}, \qquad b=\left|y_{\nu2}\right|\frac{v_\phi}{\Lambda}.
\tag{2.19}
$$

After seesawing, the effective light neutrino mass matrix is obtained from $m_{\text{eff}}=-(m_\nu^d)^T M_R^{-1}m_\nu^d$, which can be diagonalized by the TBM matrix:

$$
U_\nu^T m_{\text{eff}}U_\nu=\text{Diag.}(m_1,m_2,m_3),
\tag{2.20}
$$

where

$$
\begin{aligned}
m_1 &= m_0\left(1 + 9r^2 - 6r\cos\phi\right),\\
m_2 &= 4m_0,\\
m_3 &= m_0\left(1 + 9r^2 + 6r\cos\phi\right),
\end{aligned}
\tag{2.21}
$$

with $m_0 = v_u^2 a^2/M$ and $r = b/a$. The lepton mixing matrix is given by

$$
U_{\mathrm{PMNS}} = U_\nu = e^{-i\gamma_1/2} U_{\mathrm{TB}} \mathrm{Diag.}\left(1, e^{i\beta_1}, e^{i\beta_2}\right),
\tag{2.22}
$$

where $\beta_1 = \gamma_1/2,\ \beta_2 = (\gamma_1 - \gamma_2)/2$ are Majorana CP-violating phases with

$$
\begin{aligned}
\gamma_1 &= \arg\left\{\left(a - 3be^{i\phi}\right)^2\right\},\\
\gamma_2 &= \arg\left\{-\left(a + 3be^{i\phi}\right)^2\right\}.
\end{aligned}
\tag{2.23}
$$

It is clear that the phase factored out to the left has no physical meaning. Moreover, the mixing angles are entirely fixed by the G_f symmetry, predicting TBM and in turn no Dirac CP-violating phase. There remain only five physical quantities, $\beta_1, \beta_2, m_1, m_2$, and m_3, completely determined by the five parameters M, v_u, a, b, and ϕ.

There are two possible orderings in the masses of effective light neutrinos depending on the sign of $\cos\phi$: the NH corresponding to $\cos\phi > 0$ while the IH to $\cos\phi < 0$, which contrast with the previous model. The relation between r and $\cos\phi$ for the NH spectrum (red plot) and IH one (blue plot) is included in Figure 4. Similarly to the previous model, the contribution to the $0\nu 2\beta$ decay entirely comes from the Majorana phase β_1. The relevant effective-neutrino mass is given by

$$
\begin{aligned}
|\langle m_{ee}\rangle| &= \frac{1}{3}\left|2m_1 + m_2 e^{2i\beta_1}\right|\\
&= m_0\sqrt{1 - 4r\cos\phi + 2r^2(2 + 3\cos 2\phi) - 12r^3\cos\phi + 9r^4},
\end{aligned}
\tag{2.24}
$$

where $m_0 = a^2 v_u^2/M$. The behavior of $|\langle m_{ee}\rangle|$ as a function of ϕ is plotted in Figure 5, where the horizontal line and dashed line are the current lower bound and the future one as mentioned. Moreover, the relation between ϕ and β_1 can be obtained from (2.23) as

$$
\sin 2\beta_1 = \frac{6r\sin\phi(1 - 3r\cos\phi)}{1 - 6r\cos\phi + 9r^2},
\tag{2.25}
$$

which is presented in Figure 6 corresponding to the NH ordering [red (light) plot] and IH one [blue (dark) plot].

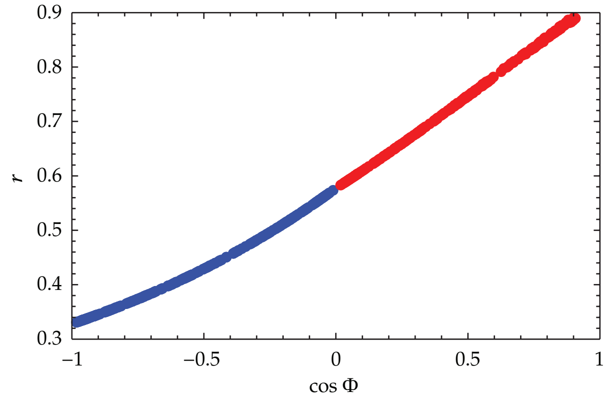

Figure 4: Allowed parameter region by the 1σ experimental constraints (2.9) for the ratio $r = b/a$ as a function of $\cos\phi$. The blue (dark) and red (light) curves correspond to the IH and NH ordering.

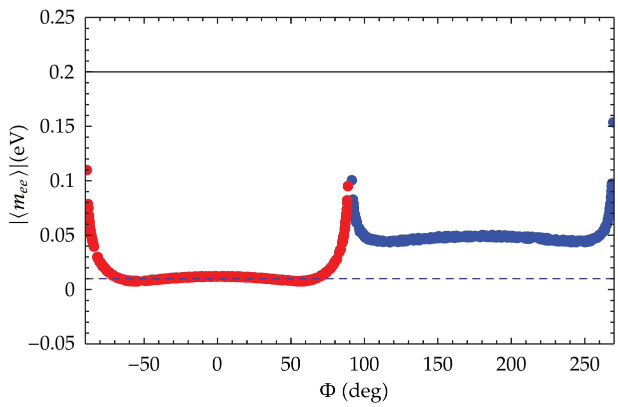

Figure 5: Prediction of the effective mass $|\langle m_{ee}\rangle|$ responsible for $0\nu 2\beta$ as a function of ϕ by the 1σ experimental constraints (2.9). The blue (dark) and red (light) curves correspond to the IH and NH ordering.

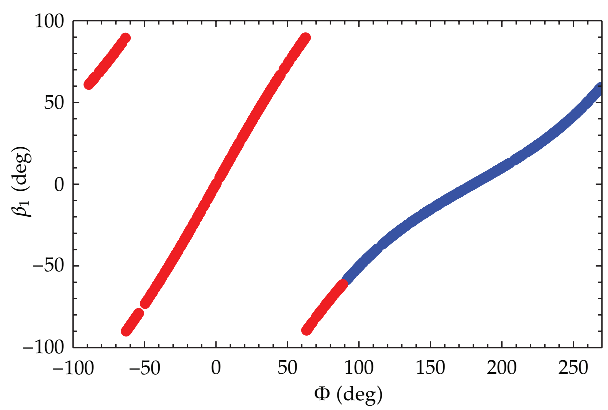

Figure 6: Relation between the phase β_1 and ϕ as given by the 1σ experimental constraints (2.9). The blue (dark) and red (light) curves correspond to the IH and NH ordering.

In a basis where the charged current is flavor diagonal, we diagonalize M_R in order to go into the physical mass basis of the RH neutrinos:

$$V_R^T M_R V_R = \text{Diag.}(M, M, -M), \tag{2.26}$$

where

$$V_R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \dfrac{1}{\sqrt{2}} & \dfrac{-1}{\sqrt{2}} \\ 0 & \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \end{pmatrix}. \tag{2.27}$$

In this basis, the Dirac mass matrix m_ν^d gets the form

$$m_\nu^d \longrightarrow Y_\nu \upsilon_u = V_R^T m_\nu^d, \tag{2.28}$$

where $\upsilon_u = \upsilon \cdot \sin\beta$, $\upsilon = 176\,\text{GeV}$ and the coupling of N_i with leptons and scalar, Y_ν, is given by

$$Y_\nu = \begin{pmatrix} 2be^{i\phi} & a - be^{i\phi} & a - be^{i\phi} \\ \sqrt{2}\left(a - be^{i\phi}\right) & \dfrac{a + be^{i\phi}}{\sqrt{2}} & \dfrac{a + be^{i\phi}}{\sqrt{2}} \\ 0 & \dfrac{-\left(a + 3be^{i\phi}\right)}{\sqrt{2}} & \dfrac{\left(a + 3be^{i\phi}\right)}{\sqrt{2}} \end{pmatrix}. \tag{2.29}$$

Again, the CP phase ϕ which comes from m_ν^d also takes part at the low-energy CP violation as the Majorana phases β_1 and β_2. On the other hand, the leptogenesis is associated with both the Yukawa coupling Y_ν and its combination:

$$H \equiv Y_\nu Y_\nu^\dagger = \begin{pmatrix} 2a^2 + 6b^2 - 4ab\cos\phi & \sqrt{2}\left(a^2 - 3b^2 + 2ab\cos\phi\right) & 0 \\ \sqrt{2}\left(a^2 - 3b^2 + 2ab\cos\phi\right) & 3a^2 + 3b^2 - 2ab\cos\phi & 0 \\ 0 & 0 & a^2 + 9b^2 + 6ab\cos\phi \end{pmatrix}, \tag{2.30}$$

which directly indicates that all $\text{Im}[H_{ij}]$ vanish and in turn unflavored leptogenesis could not take place. *However, flavored leptogenesis can work if the degeneracy of the heavy Majorana neutrino masses is lifted.*

3. Relevant RG Equations

In both models, the CP asymmetries due to the decay of heavy RH Majorana neutrinos at leading order vanish; therefore the leptogenesis could not take place. The radiative effects due to RG running from a high to low scale can naturally lead not only to a degenerate splitting of heavy Majorana masses (for Ding model), but also to an enhancement in vanished off-diagonal terms of $H = Y_\nu Y_\nu^\dagger$ (for BMM model), which are necessary ingredients for a successful leptogenesis mechanism.

The radiative behavior of heavy RH-Majorana mass matrix $\mathbf{M}_R$ is dictated by the following RG equation [56–60]:

$$\frac{d\mathbf{M}_R}{dt} = 2\left[\left(\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger\right)\mathbf{M}_R + \mathbf{M}_R\left(\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger\right)^T\right], \tag{3.1}$$

where $t = (1/16\pi^2)\ln(M/\Lambda')$ and M is an arbitrary renormalization scale. The cutoff scale Λ' can be regarded as the G_f breaking scale $\Lambda' = \Lambda$ and assumed to be in order of the GUT scale, $\Lambda' \sim 10^{16}$ GeV.

The RG equation for the Dirac neutrino Yukawa coupling can be written as

$$\frac{d\mathbf{Y}_\nu}{dt} = \mathbf{Y}_\nu\left[\left(T - 3g_2^2 - \frac{3}{5}g_1^2\right) + \mathbf{Y}_l^\dagger \mathbf{Y}_l + 3\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu\right], \tag{3.2}$$

where $T = \mathrm{Tr}(3Y_u^\dagger Y_u + \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)$, Y_u and $\mathbf{Y}_l$ are the Yukawa couplings of up-type quarks and charged leptons, and $g_{2,1}$ are the $SU(2)_L$ and $U(1)_Y$ gauge coupling constants, respectively.

Let us first reformulate (3.1) in the basis where $\mathbf{M}_R$ is diagonal. Since $\mathbf{M}_R$ is symmetric, it can be diagonalized by a unitary matrix V_R as mentioned:

$$V_R^T \mathbf{M}_R V_R = \mathrm{Diag.}(M_1, M_2, M_3). \tag{3.3}$$

As the structure of $\mathbf{M}_R$ changes with the evolution of the scale, the V_R depends on the scale too. The RG evolution of $V_R(t)$ can be written as

$$\frac{d}{dt}V_R = V_R A, \tag{3.4}$$

where A is an anti-Hermitian matrix $A^\dagger = -A$ due to the unitary of V_R. Differentiating (3.3) we obtain

$$\frac{dM_i\delta_{ij}}{dt} = A_{ij}^T M_j + M_i A_{ij} + 2\left\{V_R^T\left[\left(\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger\right)\mathbf{M}_R + \mathbf{M}_R\left(\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger\right)^T\right]V_R\right\}_{ij}. \tag{3.5}$$

Absorbing the unitary factor into the Dirac Yukawa coupling $Y_\nu \equiv V_R^T \mathbf{Y}_\nu$, the real diagonal part of (3.5) becomes

$$\frac{dM_i}{dt} = 4M_i\left(Y_\nu Y_\nu^\dagger\right)_{ii}. \tag{3.6}$$

The RG equation for Y_ν in the basis of diagonal $\mathbf{M}_R$ is given by

$$\frac{dY_\nu}{dt} = Y_\nu\left[\left(T - 3g_2^2 - \frac{3}{5}g_1^2\right) + \mathbf{Y}_l^\dagger \mathbf{Y}_l + 3Y_\nu^\dagger Y_\nu\right] + A^T Y_\nu. \tag{3.7}$$

Finally, we obtain the RG equation for H responsible for the leptogenesis:

$$\begin{aligned}\frac{dH}{dt} &= 2\left(T - 3g_2^2 - \frac{3}{5}g_1^2\right)H + 2Y_\nu\left(\mathbf{Y}_l^\dagger \mathbf{Y}_l\right)Y_\nu^\dagger \\ &\quad + 6H^2 + A^T H + HA^*.\end{aligned} \tag{3.8}$$

The heavy Majorana mass splitting generated through the relevant RG evolution is thus given by

$$\delta_N^{ij} = 4(H_{ii} - H_{jj})t, \tag{3.9}$$

where H is defined in (2.30). Neglecting the RG evolution of Y_ν and its combination $H = Y_\nu Y_\nu^\dagger$, all the necessary components for the flavored leptogenesis in Ding model are available. The flavored CP asymmetries ε_i^α can be obtained from (2.29), (2.30), (3.9), and (4.3).

Notice however that in BMM model a nonvanishing CP asymmetry requires $\mathrm{Im}[H_{ij}(Y_\nu)_{i\alpha}(Y_\nu)^*_{j\alpha}] \neq 0$ with Y_ν defined in (2.13). Therefore, to have a viable radiative leptogenesis we need to induce a nonvanishing $H_{ij}(i \neq j)$ at the leptogenesis scale. Indeed, this is possible since the RG effects due to the τ-Yukawa coupling contribution imply at the leading order yields [56–60]

$$H_{ij}(t) = 2y_\tau^2(Y_\nu)_{i3}(Y_\nu)^*_{j3} \times t. \tag{3.10}$$

The flavored CP asymmetries ε_i^α can then be obtained from (2.13), (2.14), (3.10), and (4.1).

4. Radiatively Induced Flavored Leptogenesis

As already noticed, the leptogenesis cannot be realized in the S_4 models at the leading order, so this section is devoted to study the flavored leptogenesis with the effects of RG evolution.

The lepton asymmetries, which are produced by out-of-equilibrium decays of heavy RH neutrinos in early Universe at temperatures above $T \sim (1 + \tan^2\beta) \times 10^{12}$ GeV, do not distinguish among lepton flavors, called conventional or unflavored leptogenesis. However, if the scale of heavy RH neutrino masses is about $M \leq (1 + \tan^2\beta) \times 10^{12}$ GeV, we need to take into account lepton flavor effects, called flavored leptogenesis.

In this case, the CP asymmetry as generated by the decay of ith heavy RH neutrino far from almost degenerate is given by [61–71]

$$\varepsilon_i^\alpha = \frac{1}{8\pi H_{ii}}\sum_{j\neq i}\mathrm{Im}\left[H_{ij}(Y_\nu)_{i\alpha}(Y_\nu)^*_{j\alpha}\right]g\left(\frac{M_j^2}{M_i^2}\right), \tag{4.1}$$

where Y_ν and $H = Y_\nu Y_\nu^\dagger$ are in the basis where M_R is real and diagonal. Here the loop function $g(M_j^2/M_i^2)$ is

$$g\left(\frac{M_j^2}{M_i^2}\right) \equiv g_{ij}(x) = \sqrt{x}\left[\frac{2}{1-x} - \ln\frac{1+x}{x}\right]. \tag{4.2}$$

This function depends strongly on the hierarchy of light neutrino masses.

For an almost degenerate heavy Majorana mass spectrum, the leptogenesis can be naturally implemented through the resonant leptogenesis [72, 73]. In this case, the CP asymmetry is generated by the ith heavy RH neutrino (N_i) when decaying into a lepton flavor α ($= e, \mu, \tau$) and dominated by the one-loop self-energy contributions [74],

$$\varepsilon_i^\alpha = \sum_{j\neq i}\frac{\mathrm{Im}\left[H_{ij}(Y_\nu)_{i\alpha}(Y_\nu)^*_{j\alpha}\right]}{16\pi H_{ii}\delta_N^{ij}}\left(1+\frac{\Gamma_j^2}{4M_j\delta_N^{ij2}}\right), \tag{4.3}$$

where $\Gamma_j = H_{jj}M_j/8\pi$ is the decay width of jth RH neutrino and δ_N^{ij} is mass splitting parameter defined as

$$\delta_N^{ij} = 1 - \frac{M_j}{M_i}. \tag{4.4}$$

As reminded in the previous section, by properly taking into account the RG effects, the nonzero flavored CP asymmetries ε_i^α as given above can be obtained.

Once the initial values of ε_i^α are fixed, the final result of BAU, η_B, can be given by solving a set of flavor-dependent Boltzmann equations including the decay, inverse decay, and scattering processes as well as the nonperturbative sphaleron interaction. In order to estimate the washout effects, we introduce parameters K_i^α which are the wash-out factors due to the inverse decay of Majorana neutrino N_i into the lepton flavor α. The explicit form of K_i^α is given by

$$K_i^\alpha = \frac{\Gamma_i^\alpha}{H(M_i)} = \left(Y_\nu^\dagger\right)_{\alpha i}(Y_\nu)_{i\alpha}\frac{v_u^2}{m_* M_i}, \tag{4.5}$$

where Γ_i^α is the partial decay width of N_i into the lepton flavors and Higgs scalars, $H(M_i) \simeq (4\pi^3 g_*/45)^{1/2}M_i^2/M_{\mathrm{Pl}}$ with the Planck mass $M_{\mathrm{Pl}} = 1.22 \times 10^{19}$ GeV and the effective number of degrees of freedom $g_* \simeq 228.75$ is the Hubble parameter at temperature $T = M_i$, and the equilibrium neutrino mass $m_* \simeq 10^{-3}$. From (2.13), (2.29), and (4.5) we can obtain the washout parameters corresponding to the two models.

Each lepton asymmetry for a single flavor ε_i^α is weighted differently by the corresponding washout parameter K_i^α, appearing with a different weight in the final formula for the baryon asymmetry [75, 76]:

$$\eta_B \simeq -10^{-2}\sum_{N_i}\left[\varepsilon_i^e\kappa\left(\frac{93}{110}K_i^e\right)+\varepsilon_i^\mu\kappa\left(\frac{19}{30}K_i^\mu\right)+\varepsilon_i^\tau\kappa\left(\frac{19}{30}K_i^\tau\right)\right], \tag{4.6}$$

provided that the scale of heavy RH neutrino masses is about $M \leq (1 + \tan^2\beta) \times 10^9$ GeV where the μ and τ Yukawa couplings are in equilibrium and all the flavors are to be treated separately. And

$$\eta_B \simeq -10^{-2} \sum_{N_i} \left[\varepsilon_i^2 \kappa\left(\frac{541}{761} K_i^2\right) + \varepsilon_i^\tau \kappa\left(\frac{494}{761} K_i^\tau\right)\right] \tag{4.7}$$

is given if $(1+\tan^2\beta)\cdot 10^9$ GeV $\leq M_i \leq (1+\tan^2\beta)\cdot 10^{12}$ GeV where only the τ Yukawa coupling is in equilibrium and treated separately while the e and μ flavors are indistinguishable. Here $\varepsilon_i^2 = \varepsilon_i^e + \varepsilon_i^\mu$, $K_i^2 = K_i^e + K_i^\mu$.

The wash-out factors are given by

$$\kappa_i^\alpha \simeq \left(\frac{8.25}{K_i^\alpha} + \left(\frac{K_i^\alpha}{0.2}\right)^{1.16}\right)^{-1}. \tag{4.8}$$

4.1. Bazzocchi-Merlo-Morisi Model

In this model, the RH neutrino masses are strongly hierarchical. For the NH case, the lightest RH neutrino mass is M_3, then the leptogenesis is governed by the decay of M_3 neutrino. The explicit form of flavored CP asymmetries ε_3^α is given from (2.13), (2.14), (3.10), and (4.1):

$$\begin{gathered}\varepsilon_3^e \simeq 0, \\ \varepsilon_3^\mu \simeq \varepsilon_3^\tau \simeq \frac{y_\tau^2 x^2}{24\pi}\left(\frac{1}{2}\sin 2\beta_2 \cdot g_{31} - \sin 2(\beta_1 - \beta_2)\cdot g_{32}\right)\cdot t.\end{gathered} \tag{4.9}$$

The corresponding washout parameters are

$$K_3^e = 0, \qquad K_3^{\mu,\tau} \simeq \frac{3}{4} K_1^e. \tag{4.10}$$

For the IH case, the lightest RH neutrino is of M_1, then the leptogenesis is governed by the decay of M_1 neutrino. The flavored CP asymmetries ε_1^α are obtained as

$$\begin{aligned}\varepsilon_1^e &\simeq \frac{-y_\tau^2 x^2}{36\pi}\sin 2\beta_1 \cdot g_{12}\cdot t, \\ \varepsilon_1^\mu &\simeq \frac{y_\tau^2 x^2}{24\pi}\left(\frac{1}{3}\sin 2\beta_1 \cdot g_{12} - \frac{1}{2}\sin 2\beta_2 \cdot g_{13}\right)\cdot t, \\ \varepsilon_1^\tau &\simeq \frac{y_\tau^2 x^2}{24\pi}\left(\frac{1}{3}\sin 2\beta_1 \cdot g_{12} + \frac{1}{2}\sin 2\beta_2 \cdot g_{13}\right)\cdot t,\end{aligned} \tag{4.11}$$

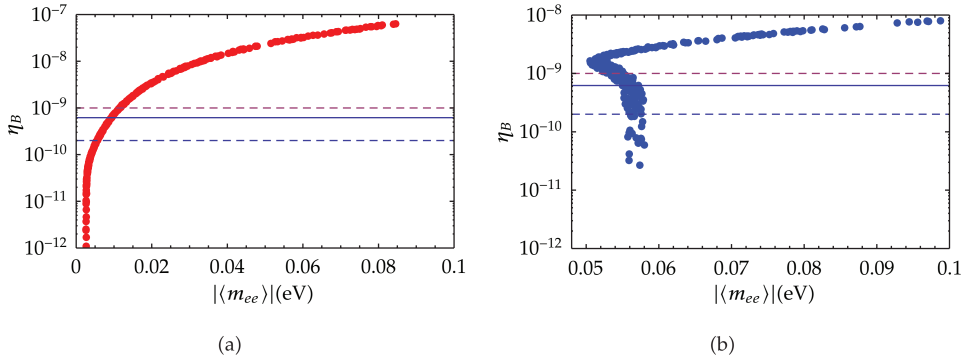

Figure 7: Prediction of η_B as a function of $|\langle m_{ee}\rangle|$ for the NH case (a) and IH case (b). The horizontal solid and dashed lines correspond to the experimental central value and phenomenologically allowed region.

with corresponding washout parameters

$$K_1^e \simeq \frac{2m_0}{3m_*(1-6r\cos\phi+9r^2)}, \qquad (4.12)$$
$$K_1^{\mu,\tau} \simeq \frac{1}{4}K_1^e.$$

Applying (4.6), (4.7), and (4.8), the BAU for two cases are then obtained. Notice also that in the NH case the leptogenesis has no contribution from the electron flavor decay channel which makes the scale of the heavy RH neutrino mass for a successful leptogenesis higher than that of the IH case.

The prediction for η_B as a function of $|\langle m_{ee}\rangle|$ is shown in Figure 7 where we have used $B = 10^{13}$ GeV for the NH case, $B = 10^{12}$ GeV for the IH case, and $\tan\beta = 30$ as inputs. The horizontal solid and dashed lines correspond to the central value of BAU experimental data $\eta_B^{\text{CMB}} = 6.1 \times 10^{-10}$ [77–79] and phenomenologically allowed region $2 \times 10^{-10} \leq \eta_B \leq 10^{-9}$, respectively. As shown in Figure 7, the current observation of η_B^{CMB} can narrowly constrain the value of $|\langle m_{ee}\rangle|$ for the NH and IH spectrum, respectively. Combining the results presented in Figures 2 and 3 with those from the leptogenesis, we can pin down the Majorana CP phase β_1 via the parameter ϕ.

4.2. Ding Model

In this model, all the heavy RH neutrinos are exactly degenerate. By considering the RG effects, their masses get a tiny splitting (almost degenerate), which lead to a resonant leptogenesis as contributed from all these heavy RH neutrinos. However, if we neglect the RG effects on the H matrix, the contribution of N_3 to lepton asymmetries ε_i^α can be negligible due to $H_{13(31)} = H_{23(32)} = 0$. (Actually, this is also correct if we take into account the RG effects on the H matrix since the radiative generation of $H_{13(31)}, H_{23(32)}$ is very small.) Combined with (2.29), (2.30), (3.9), and (4.3), the flavor-dependent CP asymmetries ε_i^{α} are obtained as

$$\varepsilon_1^e \simeq \frac{a^2 r \sin\phi}{32\pi(1-2r\cos\phi+3r^2)t}, \qquad \varepsilon_1^{\mu} \simeq \varepsilon_1^{\tau} \simeq \frac{-a^2 r\sin\phi}{64\pi(1-2r\cos\phi+3r^2)t},$$
$$\varepsilon_2^e \simeq \frac{-a^2 r \sin\phi}{16\pi(3-2r\cos\phi+3r^2)t}, \qquad \varepsilon_2^{\mu} \simeq \varepsilon_2^{\tau} \simeq \frac{a^2 r\sin\phi}{32\pi(3-2r\cos\phi+3r^2)t}. \tag{4.13}$$

Here the mass slitting parameter δ_N^{12} which can be calculated from (2.30) and (3.9),

$$\delta_N^{12} = -\delta_N^{21} = -4a^2\left(1-3r^2+2r\cos\phi\right), \tag{4.14}$$

has been used. The explicit form of K_i^{α} is found as

$$\begin{aligned} K_1^e &\simeq 4r^2\frac{m_0}{m_*}, \quad m_0 = \frac{a^2 v_u^2}{M}, \\ K_1^{\mu,\tau} &\simeq \frac{m_0}{m_*}(1-2r\cos\phi+r^2), \\ K_2^e &\simeq \frac{2m_0}{m_*}(1-2r\cos\phi+r^2), \\ K_2^{\mu,\tau} &\simeq \frac{m_0}{2m_*}(1+2r\cos\phi+r^2). \end{aligned} \tag{4.15}$$

With the help of (4.6), the BAU is obtained then.

The prediction for η_B as a function of $|\langle m_{ee}\rangle|$ is shown in Figure 8 where we have used $M = 10^3$ GeV and $\tan\beta = 1$. The horizontal solid and dashed lines correspond to the central value of the BAU experiment result $\eta_B^{\text{CMB}} = 6.1 \times 10^{-10}$ [77–79] and phenomenologically allowed region $2 \times 10^{-10} \le \eta_B \le 10^{-9}$, respectively. As seen in Figure 8, the current observation of η_B^{CMB} can narrowly constrain the value of $|\langle m_{ee}\rangle|$ for the NH and IH spectrum, respectively. Again, combining the results in Figures 5 and 6 with those from the leptogenesis, we can pin down the Majorana CP phase β_1 via the parameter ϕ.

5. Conclusions

We have studied the S_4 models in the context of a supersymmetric seesaw model which naturally lead to the TBM form for the lepton mixing matrix. In BMM model, the combination $Y_\nu Y_\nu^\dagger$ is proportional to unity whereas in Ding model the heavy RH Majorana masses are exactly degenerate. This would forbid the desirable leptogenesis to occur in each model. Therefore, for a viable leptogenesis the off-diagonal terms of $Y_\nu Y_\nu^\dagger$ in BMM model have to be generated, while in Ding model the degeneracy of heavy RH Majorana masses has to be lifted. We have shown that these can be easily achieved by the RG effects from a high-energy scale to the low-energy scale which result in the successful leptogenesis.

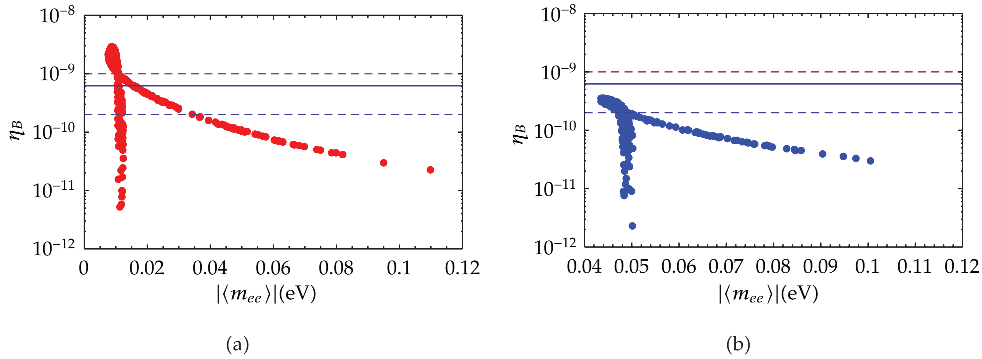

Figure 8: Prediction of η_B as a function of $|\langle m_{ee}\rangle|$ or the NH case (a) and IH case (b). The horizontal solid and dashed lines correspond to the experimental central value and phenomenologically allowed region.

We have also studied implications to the low-energy observables such as the $0\nu\beta\beta$ decay. It gives the definite predictions for $0\nu 2\beta$ decay parameter $|\langle m_{ee}\rangle|$. Interestingly we have found a link between the leptogenesis and amplitude of $0\nu 2\beta$ decay $|\langle m_{ee}\rangle|$ through a high-energy CP phase ϕ. We have shown how the high-energy CP phase ϕ is correlated to the low-energy Majorana CP phase and examined how the leptogenesis can be related with the $0\nu 2\beta$ decay. It is pointed out that the predictions of $|\langle m_{ee}\rangle|$ for the NH and IH spectra can be constrained by the current observation of the baryon asymmetry of the universe as 6.1×10^{-10}.

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Research Article

Nonstandard Neutrinos Interactions in a 331 Model with Minimum Higgs Sector

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We present a detailed analysis of a class of extensions to the SM Gauge chiral symmetry $SU(3)_C \times SU(3)_L \times U(1)_x$ (331 model), where the neutrino electroweak interaction with matter via charged and neutral current is modified through new gauge bosons of the model. We found the connections between the nonstandard contributions on 331 model with nonstandard interactions. Through limits of such interactions in cross-section experiments, we constrained the parameters of the model, obtaining that the new energy scale of this theory should obey $V > 1.3$ TeV and the new bosons of the model must have masses greater than 610 GeV.

1. Introduction

Although the standard model (SM) is a good phenomenological theory, describing very well all experimental results, it leaves several unanswered questions that suggest that the SM might be an effective model at low energies, originating from a more fundamental theory. Some of the unexplained aspects in the SM are the existence of three families and lepton flavour violation observed in solar [1–5], atmospheric [6–11], and reactor [12–17] neutrino experiments. These results demonstrate that new physics is required, being interpreted as a sign of physics beyond the SM.

In principle neutrinos new interactions not described by Standard Model can arise in extensions of the SM. We assume that the new physics which induces the nonstandard neutrino interactions (NSIs) [18–29] arises in some models enlarging the symmetry group where the SM is embedded. Models with larger symmetries that may allow us to understand the origin of the families have been proposed [30–34]. In some models, it is also possible to understand the number of families from the cancellation of chiral anomalies, necessary to preserve the renormalizability of the theory [35–37]. This is the case of the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$

or 331 models, which are an immediate extension of the SM [38–46]. There are a great variety of such models, which have generated new expectations and possibilities of solving several problems of the SM.

Our goal is to investigate how NSI with matter can be induced by new physics generated by 331 models. Through the constraints from neutrino elastic scattering experiments on this NSI parameters, we can constrain some values expected for 331 model parameters. We find that the constraints on vacuum expectation values of the model, as well as for the mass of the new bosons, are in full agreement with the limits found in the literature, which makes this class of models a viable theory for a higher energy level.

The paper is organized as follows. In Section 2 we briefly review NSI and present how new interactions can contribute to new matter effects, in addition to the SM electroweak ones. In Section 3 we introduce a specific 331 model and we give the fermion gauge-boson couplings. In Section 4 we calculate the interactions involving neutrinos and how these interactions can be interpreted as new terms beyond SM. Finally, in Section 5 we summarize our main results.

2. Nonstandard Neutrino Interactions

One convenient way to describe neutrino new interactions with matter in the electro-weak (EW) broken phase are the so-called nonstandard neutrino interactions (NSIs), which is a very widespread and convenient way of parameterizing the effects of new physics in neutrino oscillations [18–29]. NSIs with first generation of leptons and quarks for four-fermion operators are contained in the following Lagrangian density [18–22, 24, 25, 28]:

$$\mathcal{L}_{\text{eff}}^{\text{NSI}} = -2\sqrt{2}G_F \sum_{f,P} \varepsilon_{\alpha\beta}^{fP} \left[\overline{f}\gamma^{\mu} P f\right] \left[\overline{\nu}_{\alpha}\gamma^{\mu} L \nu_{\beta}\right], \tag{2.1}$$

where G_F is the Fermi constant, $f = u, d, e$, and $P = L, R$ with $2L = (1 - \gamma^5)$, $2R = (1 + \gamma^5)$, and the coefficients $\varepsilon_{\alpha\beta}^{fP}$ encode the deviation from standard interactions between neutrinos of flavor α with component P-handed of fermions f, resulting in a neutrinos of flavor β. Then, the neutrino oscillations in the presence of nonstandard matter effects can be described by an effective Hamiltonian, parameterized as

$$\widetilde{H} = \frac{1}{2E}\left[U\begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix}U^{\dagger} + a\begin{pmatrix} 1+\varepsilon_{ee} & \varepsilon_{e\mu} & \varepsilon_{e\tau} \\ \varepsilon_{e\mu}^{*} & \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau} \\ \varepsilon_{e\tau}^{*} & \varepsilon_{\mu\tau}^{*} & \varepsilon_{\tau\tau} \end{pmatrix}\right], \tag{2.2}$$

where $a = \sqrt{2}G_F n_f$, E is the neutrino energy and $\varepsilon_{\alpha\beta} = \sum_{f,P} \varepsilon_{\alpha\beta}^{fP} n_f / n_e$ with n_e and n_f the electrons and fermions f density in the medium, respectively. These parameters $\varepsilon_{\alpha\beta}$ can be found in solar [22, 47], atmospheric [20, 48], accelerator [18, 19, 22, 49], and cross-section [18, 19, 21, 50, 51] neutrino data experiment.

We focus on cross-section neutrino experiment, where at low energies the standard differential cross-section for $\nu_\alpha e \rightarrow \nu_\alpha e$ scattering processes has the well-know form:

$$\frac{d\sigma_\alpha}{dT} = \frac{2G_F m_e}{\pi}\left[(g_1^\alpha)^2 + (g_2^\alpha)^2\left(1 - \frac{T}{E_\nu}\right)^2 - g_1^\alpha g_2^\alpha \frac{m_e T}{E_\nu^2}\right], \tag{2.3}$$

where m_e is the electron mass, E_ν is the incident neutrino energy, and T_e is the electron recoil energy. The quantities g_1^α and g_2^α are related to the SM neutral current couplings of the electron $g_L^e = -1/2 + \sin^2\theta_W$ and $g_R^e = \sin^2\theta_W$, with $\sin^2\theta_W = 0,23119$. For $\nu_{\mu,\tau}$ neutrinos, which take part only in neutral current interactions, we have $g_1^{\mu,\tau} = g_L^e$ and $g_2^{\mu,\tau} = g_R^e$ while for electron neutrinos, which take part in both charge current (CC) and neutral current (NC) interactions, $g_1^e = 1 + g_L^e$, $g_2^e = g_R^e$. In the presence of nonuniversal standard interaction, the cross-section can be written in the same form of (2.3) but with $g_{1,2}^\alpha$ replaced by the effective nonstandard couplings $\tilde{g}_1^\alpha = g_1^\alpha + \varepsilon_{\alpha\alpha}^{eL}$ and $\tilde{g}_2^\alpha = g_2^\alpha + \varepsilon_{\alpha\alpha}^{eR}$, leading to the following differential scattering cross-section [19, 21, 50, 51]

$$\begin{aligned}\frac{d\sigma_\alpha}{dT} = \frac{2G_F m_e}{\pi}\Bigg\{&\left(g_1^\alpha + \varepsilon_{\alpha\alpha}^{eL}\right)^2 + \left(g_2^\alpha + \varepsilon_{\alpha\alpha}^{eR}\right)^2\left(1 - \frac{T_e}{E_\nu}\right)^2 \\ &-\left(g_1^\alpha + \varepsilon_{\alpha\alpha}^{eL}\right)\left(g_2^\alpha + \varepsilon_{\alpha\alpha}^{eR}\right)\frac{m_e T_e}{E_\nu}\Bigg\}.\end{aligned} \tag{2.4}$$

3. 331 Model

The success of the standard model (SM) implies that any new theory should contain the symmetry $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ (G_{321}) in a low energy limit. Then, it is natural that one possible modification of SM involves extensions of the representation content in matter and Higgs sector, leading to extension of symmetry group G_{321} to groups $SU(N_C)_C \otimes SU(m)_L \otimes U(1)_X$ with $SU(N_C)_C \otimes SU(m)_L \otimes U(1)_X \supset G_{321}$.

In early 90's, Pisano and Pleitez [38, 39] and Frampton [40] suggested an extension of the symmetry group $SU(2)_L \otimes U(1)_Y$ of electroweak sector to a group $SU(3)_L \otimes U(1)_X$, that is, with $N_C = m = 3$. The 331 models present some interesting features; for instance, they associate the number of families to internal consistence of the theory, preserving asymptotic freedom.

In these models, the SM doublets are part of triplets. In quark sector three new quarks are included to build the triplets, while in lepton sector we can use the right-handed neutrino to such role [38, 40]. Another option is to invoke three new heavy leptons, charged or not, depending on the choice of charge operator [41, 42]. In SM the electric charge operator is constructed as a combination of diagonal generators of $SU(2) \otimes U(1)_Y$. Then, it is natural to assume that this operator in $SU(3)_L \otimes U(1)_X$ is defined in the same way. The most general charge operator in $SU(3)_L \otimes U(1)_X$ is a linear superposition of diagonal generators of symmetry groups, given by

$$Q \equiv aT_{3L} + \frac{2}{\sqrt{3}}bT_{8L} + XI_3, \tag{3.1}$$

where the group generator is defined as $T_{iL} \equiv \lambda_{iL}/2$ with λ_{iL}, $i = 1, \ldots, 8$, being the Gell-Mann matrices for $SU(3)_L$, where the normalization chosen is $\mathrm{Tr}(\lambda_{iL}\lambda_{jL}) = 2\delta_{ij}$ and $I_3 = \mathrm{diag}(1,1,1)$ is the identity matrix, and a and b are two parameters to be determined. Then the charge operator in (3.1) acts on the representations 3 and 3^* of $SU(3)_L$ having the following form:

$$Q[3] = \mathrm{diag}\left[\frac{a}{2} + \frac{b}{3} + X,\ -\frac{a}{2} + \frac{b}{3} + X,\ -\frac{2b}{3} + X\right], \tag{3.2}$$

$$Q[3^*] = \mathrm{diag}\left[-\frac{a}{2} - \frac{b}{3} + X,\ +\frac{a}{2} - \frac{b}{3} + X,\ +\frac{2b}{3} + X\right], \tag{3.3}$$

where we have two free parameters to obtain the charge of fermions, a and b (X can be determined by anomalies cancellation). However, $a = 1$ is necessary to obtain doublets of isospins $SU(2) \otimes U(1)_Y$ correctly incorporated in the model $SU(3)_L \otimes U(1)_X$ [41, 42, 45]. Then we can vary b to create different models in 331 context, being a signature that differentiates such models. For $b = -3/2$, we have the original 331 model [38, 39].

To have local gauge invariance, we have the following covariant derivative: $D_\mu = \partial_\mu - i(g/2)\lambda_\alpha W_\mu^\alpha - ig_x X B_\mu$ and a total of 17 mediator bosons: one field B_μ associated with $U(1)_X$, eight fields associated with $SU(3)_C$, and another eight fields associated with $SU(3)_L$, written in the following form:

$$\mathbf{W}_\mu \equiv W_\mu^\alpha \lambda_\alpha = \begin{pmatrix} W_\mu^3 + \frac{1}{\sqrt{3}} W_\mu^8 & \sqrt{2} W_\mu^+ & \sqrt{2} K_\mu^{Q_1} \\ \sqrt{2} W_\mu^- & -W_\mu^3 + \frac{1}{\sqrt{3}} W_\mu^8 & \sqrt{2} K_\mu^{Q_2} \\ \sqrt{2} K_\mu^{-Q_1} & \sqrt{2} K_\mu^{-Q_2} & -\frac{2}{\sqrt{3}} W_\mu^8 \end{pmatrix}, \tag{3.4}$$

where

$$W_\mu^{\pm} = \frac{1}{\sqrt{2}}(W_{1\mu} \mp iW_{2\mu}), \qquad K_\mu^{\pm Q_1} = \frac{1}{\sqrt{2}}(W_{4\mu} \mp iW_{5\mu}), \qquad K_\mu^{\pm Q_2} = \frac{1}{\sqrt{2}}(W_{6\mu} \mp iW_{7\mu}). \tag{3.5}$$

Therefore, charge operator in (3.2) applied over (3.4) leads to $Q_1 = 1/2 + b$ and $Q_2 = (-1/2) + b$. Then the mediator bosons will have integer electric charge only if $b = \pm 1/2, \pm 3/2, \pm 5/2, \ldots, \pm (2n+1)/2, n = 0, 1, 2, 3, \ldots$. A detailed analysis shows that if a and b are associated with the fundamental representation 3, then $-a$ and $-b$ will be associated with antisymmetric representation 3^*.

3.1. The Representation Content

There are many representations for the matter content [46], for instance, $b = 3/2$ [38]. But we note that if we accommodate the doublets of $SU(2)_L$ in the superior components of triplets and antitriplets of $SU(3)_L$, and if we forbid exotic charges for the new fermions, we obtain from (3.2) the constrain $b = \pm 1/2$ (assuming $a = 1$). Since a negative value of b can be associated to the antitriplet, we obtain that $b = 1/2$ is a necessary and sufficient condition to exclude exotic electric charges in fermion and boson sector [41].

The fields left- and right-handed components transform under $SU(3)_L$ as triplets and singlets, respectively. Therefore the theory is chiral and can present anomalies of Alder-Bell-Jackiw [52, 53]. In a non-abelian theory, in the fermionic representation $\mathcal{R}$, the divergent anomaly is given by

$$\mathcal{A}^{abc} \propto \sum_{\mathcal{R}} \mathrm{Tr}\Big[\Big\{T_L^a(\mathcal{R}), T_L^b(\mathcal{R})\Big\}T_L^c(\mathcal{R}) - \Big\{T_R^a(\mathcal{R}), T_R^b(\mathcal{R})\Big\}T_R^c(\mathcal{R})\Big], \tag{3.6}$$

where $T^a(\mathcal{R})$ are the matrix representations for each group generator acting on the basis $\mathcal{R}$ with helicity left or right. Therefore, to eliminate the pure anomaly $[SU(3)_L]^3$, we should have that $\mathcal{A}^{abc} \propto \sum_{\mathcal{R}'} \mathrm{Tr}[\{T_L^a(\mathcal{R}'), T_L^b(\mathcal{R}')\}T_L^c(\mathcal{R}')] = 0$. We use the fact that $SU(3)_L$ has two fundamental representations, 3 and 3*, then its generators should be associated to T^a and T^{a*}, respectively, that is,

$$\begin{aligned}\sum_{\mathcal{R}'} \mathrm{Tr}\Big[\Big\{T_L^a(\mathcal{R}'), T_L^b(\mathcal{R}')\Big\}T_L^c(\mathcal{R}')\Big] &= \sum_{\mathcal{R}} \mathrm{Tr}\Big[\Big\{T_L^a(\mathcal{R}), T_L^b(\mathcal{R})\Big\}T_L^c(\mathcal{R})\Big] \\ &\quad - \sum_{\mathcal{R}^*} \mathrm{Tr}\Big[\Big\{T_L^{a*}(\mathcal{R}^*), T_L^{b*}(\mathcal{R}^*)\Big\}T_L^{c*}(\mathcal{R}^*)\Big],\end{aligned} \tag{3.7}$$

but we know that the matrix representations for each group generator satisfies that $T_L^{a*}(\mathcal{R}^*) = -T_L^a(\mathcal{R})$ [54]. So, we can see that for the anomalies to be canceled, the number of fields that transform as triplets (first term in equation above) and antitriplets under $SU(3)_L$ has to be the same; that is, two triplets quark families × 3 (color) = one antitriplet quark family × 3 (color) + 3 antitriplet lepton families. This implies that two families of quarks should transform differently than the third family, as will be discussed in next paragraph.

Usually the third quark family is chosen to transform in a different way than the first two families. But we will assume that the first family transform differently, to address the fact that $m_u < m_d$, $m_{\nu_\ell} < m_\ell$ while $m_c \gg m_s$ and $m_t \gg m_b$. To state this in a clearer way, we recall that in SM the $SU(2)_L$ doublets are $(\nu_\ell, \ell)^T$, $(u,d)^T$, $(c,s)^T$, $(t,b)^T$, with $\ell = e, \mu, \tau$. We can see that the first component of leptons doublets and first quark family is lighter than the second component. But for the second and third quark families, the opposite occurs. Then we use this idea to justify that first quark family transform as leptons.

3.2. Minimal 331 Model on Scalar Sector

Among the different possibilities of 331 models, we will present a detailed study on a minimal model on scalar sector without exotic electric charges for quarks and with three new leptons without charged [41] ($b = 1/2$), where the fermions present the following transformation structure under $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$:

$$\begin{gathered}
\psi_{\ell L} = \left(\ell^-,\ \nu_\ell,\ N_\ell^0\right)_L^T \sim \left(1, 3^*, -\frac{1}{3}\right), \\
\nu_{\ell R} \sim (1,1,0), \\
\ell_R^- \sim (1,1,-1), \\
N_{\ell R}^0 \sim (1,1,0), \\
Q_{1L} = (d,\ u, U_1)_L^T \sim \left(3, 3^*, \frac{1}{3}\right), \\
u_{iR} \sim \left(3, 1, \frac{2}{3}\right), \\
d_{iR} \sim \left(3, 1, -\frac{1}{3}\right), \\
U_{1R} \sim \left(3, 1, \frac{2}{3}\right), \\
Q_{aL} = (u_a,\ d_a,\ D_a)_L^T \sim (3,3,0), \\
D_{aR} \sim \left(3, 1, -\frac{1}{3}\right),
\end{gathered} \tag{3.8}$$

where $i = 1,2,3,\ \ell = e,\mu,\tau,\ a = 2,3$. We note that the leptons multiplets $\psi_{\ell L}$ consist of three fields $\ell = \{e,\mu,\tau\}$, the corresponding neutrinos $\nu_\ell = \{\nu_e,\nu_\mu,\nu_\tau\}$, and new neutral leptons $N_\ell^0 = \{N_e^0, N_\mu^0, N_\tau^0\}$. We can also see that the multiplet associated with the first quark family Q_{1L} consists of down and up quarks and a new quark with the same electric charge of quark up (named U_1), while the multiplet associated with second (third) family Q_{aL} consists of SM quarks of second (third) family and a new quark with the same electric charge of down quark (named D_2 (D_3)). The numbers on parenthesis refer to the transformation properties under $SU(3)_C,\ SU(3)_L$, and $U(1)_X$, respectively. With this choice, the anomalies are cancelled in a nontrivial way [55], and asymptotic freedom is guaranteed [56–59].

3.2.1. Scalar Sector and the Yukawa Couplings

The scalar fields have to be coupled to fermions by the Yukawa terms, invariants under $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$. In lepton sector, these couplings can be written as

$$\begin{gathered}
\overline{\psi}_{\ell L}\ell_R \sim \left(1, 3, \frac{1}{3}\right) \otimes (1,1,-1) = \underbrace{\left(1, 3, -\frac{2}{3}\right)}_{\rho^*}, \\
\overline{\psi}_{\ell L}\nu_{\ell R} \sim \left(1, 3, \frac{1}{3}\right) \otimes (1,1,0) = \underbrace{\left(1, 3, \frac{1}{3}\right)}_{\eta}, \\
\overline{\psi}_{\ell L}N_{\ell R}^0 \sim \left(1, 3, \frac{1}{3}\right) \otimes (1,1,0) = \underbrace{\left(1, 3, \frac{1}{3}\right)}_{\chi},
\end{gathered} \tag{3.9}$$

and writing only three terms in quarks sector, for example,

$$\begin{aligned}
\overline{Q}_{1L}u_{iR} &= \left(3^*,3,-\frac{1}{3}\right)\otimes\left(3,1,\frac{2}{3}\right) = \underbrace{\left(1,3,\frac{1}{3}\right)}_{\chi}\oplus\underbrace{\left(8,3,\frac{1}{3}\right)}_{\text{Color Higgs}},\\
\overline{Q}_{1L}d_{iR} &= \left(3^*,3,-\frac{1}{3}\right)\otimes\left(3,1,-\frac{1}{3}\right) = \underbrace{\left(1,3,-\frac{2}{3}\right)}_{\rho^*}\oplus\ldots,\\
\overline{Q}_{aL}d_{iR} &= \left(3^*,3^*,0\right)\otimes\left(3,1,-\frac{1}{3}\right) = \underbrace{\left(1,3^*,-\frac{1}{3}\right)}_{\eta^*}\oplus\ldots,\ \ldots.
\end{aligned} \tag{3.10}$$

As usual in these class of models, we impose colorless Higgs (i.e., selecting only the multiplets that transform as singlets under $SU(3)_C$). We note that we need only three Higgs multiplets, ρ, χ, and η, to couple the different fermionic fields and generate mass through spontaneous symmetry breaking. In (3.9) and (3.10) we note that quantum numbers of triplets χ and η are the same, which leads us to consider models with two or three Higgs triplets. We will adopt the first option, two Higgs triplets, due to the simpler scalar sector in comparison with the scenario with three triplets [41–44].

3.3. Model with Two Higgs Triplets

For the models with two Higgs triplets, we obtain (note that in this model we assumed $\Phi_1 = \chi, \eta$ e $\Phi_2 = \rho$)

$$\begin{aligned}
\Phi_1 &= \left(\phi_1^-,\phi_1'^0,\phi_1^0\right)^T \sim \left(1,3^*,-\frac{1}{3}\right),\\
\Phi_2 &= \left(\phi_2^0,\phi_2^+,\phi_2'^+\right)^T \sim \left(1,3^*,\frac{2}{3}\right).
\end{aligned} \tag{3.11}$$

Assuming the following choice to the Higgs triplets vacuum expectation value (VEV) [41] $\langle\Phi_1\rangle_0 = (0,\vartheta_1,V)^T$ and $\langle\Phi_2\rangle_0 = (\vartheta_2,0,0)^T$, we associate V with the mass of the new fermions, which lead us to assume $V \gg \vartheta_1, \vartheta_2$. We expand the scalar VEVs in the following way:

$$\phi_1^0 = V + \frac{H_{\phi_1}^0 + iA_{\phi_1}^0}{\sqrt{2}}, \qquad \phi_1'^0 = \vartheta_1 + \frac{H_{\phi_1}'^0 + iA_{\phi_1}'^0}{\sqrt{2}}, \qquad \phi_2^0 = \vartheta_2 + \frac{H_{\phi_2}^0 + iA_{\phi_2}^0}{\sqrt{2}}. \tag{3.12}$$

The real (imaginary) part $H_{\phi_i}(A_{\phi_i})$ is usually called CP-even (CP-odd) scalar field. The most general potential can be written as

$$\begin{aligned}
V(\Phi_1,\Phi_2) &= \mu_1^2\Phi_1^\dagger\Phi_1 + \mu_2^2\Phi_2^\dagger\Phi_2 + \lambda_1\left(\Phi_1^\dagger\Phi_1\right)^2 + \lambda_2\left(\Phi_2^\dagger\Phi_2\right)^2\\
&\quad + \lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right).
\end{aligned} \tag{3.13}$$

Demanding that in the displaced potential $V(\Phi_1, \Phi_2)$ the linear terms on the field should be absent, we have, in tree-level approximation, the following constraints:

$$\mu_1^2 + 2\lambda_1\left(\vartheta_1^2 + V^2\right) + \lambda_3\vartheta_2^2 = 0,$$
$$\mu_2^2 + \lambda_3\left(\vartheta_1^2 + V^2\right) + 2\lambda_2\vartheta_2^2 = 0. \tag{3.14}$$

The analysis of such equations shows that they are related to a minimum in scalar potential with the value $V_{\min} = -\vartheta_2^4\lambda_2 - (\vartheta_1^2 + V^2)[(\vartheta_1^2 + V^2)\lambda_1 + \vartheta_2^2\lambda_3]$. Then, replacing (3.12) and (3.14) in (3.13), we can calculate the mass matrix in $(H^0_{\phi_1}, H^0_{\phi_2}, H'^0_{\phi_1})$ basis through the relation $M^2_{ij} = 2(\partial^2 V(\Phi_1, \Phi_2)/\partial H^0_{\Phi_i}\partial H^0_{\Phi_j})$, obtaining

$$M_H^2 = 2\begin{pmatrix} 2\lambda_1 V^2 & \lambda_3\vartheta_2 V & 2\lambda_1\vartheta_1 V \\ \lambda_3\vartheta_2 V & 2\lambda_2\vartheta_2^2 & \lambda_3\vartheta_1\vartheta_2 \\ 2\lambda_1\vartheta_1 V & \lambda_3\vartheta_1\vartheta_2 & 2\lambda_1\vartheta_1^2 \end{pmatrix}. \tag{3.15}$$

Since (3.15) has vanishing determinant, we have one Goldstone boson G_1 and two massive neutral scalar fields H_1 and H_2 with masses (note that if $\lambda_3^2 = 4\lambda_1\lambda_2$, we obtain two Goldstone bosons, G_1 and H_2, and a massive scalar field H_1 with mass $M^2_{H_1} = 4[\lambda_1(\vartheta_1^2 + V^2) + \lambda_2\vartheta_2^2]$, where $\lambda_1\lambda_2 > 0$; then imposing $M^2_{H_1} > 0$ leads to $\lambda_1 > 0$ and $\lambda_2 > 0$)

$$M^2_{H_1,H_2} = 2\lambda_1\left(\vartheta_1^2 + V^2\right) + 2\lambda_2\vartheta_2^2$$
$$\pm 2\sqrt{\left[\lambda_1(\vartheta_1^2 + V^2) + \lambda_2\vartheta_2^2\right]^2 + \vartheta_2^2(\vartheta_1^2 + V^2)(\lambda_3^2 - 4\lambda_1\lambda_2)}, \tag{3.16}$$

where real values for λ's produce positive mass to neutral scalar fields only if $\lambda_1 > 0$ and $4\lambda_1\lambda_2 > \lambda_3^2$, which implies that $\lambda_2 > 0$. A detailed analysis shows that when $V(\Phi_1, \Phi_2)$ in (3.13) is expanded around the most general vacuum, given by (3.12) and using constrains in (3.14), we do not obtain pseudoscalar fields $A^0_{\Phi_i}$. This allows us do identify three more Goldstone bosons, $G_2 = A^0_{\Phi_1}$, $G_3 = A^0_{\Phi_2}$, and $G_4 = A'^0_{\Phi_1}$. For the mass spectrum in charged scalar sector on $(\phi_1^-, \phi_2^+, \phi_2'^+)$ basis, the mass matrix will be given by

$$M_+^2 = 2\lambda_4\begin{pmatrix} \vartheta_2^2 & \vartheta_1\vartheta_2 & \vartheta_2 V \\ \vartheta_1\vartheta_2 & \vartheta_1^2 & \vartheta_1 V \\ \vartheta_2 V & \vartheta_1 V & V^2 \end{pmatrix}, \tag{3.17}$$

with two eigenvalues equal to zero, equivalent to four Goldstone bosons $G_5^\pm$, $G_6^\pm$ and two physical charged scalar fields with large masses given by $\lambda_4(\vartheta_1^2 + \vartheta_2^2 + V^2)$, which leads to the constrain $\lambda_4 > 0$.

This analysis shows that, after symmetry breaking, the original twelve degrees of freedom in scalar sector leads to eight Goldstone bosons (four electrically neutral and four electrically charged), four physical scalar fields, two neutral (one of which being the SM Higgs scalar), and two charged. Eight Goldstone bosons should be absorbed by eight gauge fields as we will see in next section.

3.3.1. Gauge Sector with Two Higgs Triplets

The gauge bosons interaction with matter in electroweak sector appears with the covariant derivative for a matter field φ as

$$D_\mu^\varphi = \partial_\mu - \frac{i}{2} g W_\mu^a \lambda_{aL} - i g_X X_\varphi B_\mu = \partial_\mu - \frac{i}{2} g \mathcal{M}_\mu^\varphi, \tag{3.18}$$

where λ_{aL}, $a = 1, \ldots, 8$ are Gell-Mann matrices of $SU(3)_L$ algebra and X_φ is the charge of abelian factor $U(1)_X$ of the multiplet φ in which D_μ acts. The matrix $\mathcal{M}_\mu^\varphi$ contains the gauge bosons with electric charges q, defined by the generic charge operator in (3.1). For $b = 1/2$ the matrix $\mathcal{M}_\mu^\varphi$ will have the following form:

$$\mathcal{M}_\mu^\varphi = \begin{pmatrix} W_{3\mu} + \dfrac{W_{8\mu}}{\sqrt{3}} + 2tX_\varphi B_\mu & \sqrt{2} W_\mu^+ & \sqrt{2} K_\mu^+ \\ \sqrt{2} W_\mu^- & -W_{3\mu} + \dfrac{W_{8\mu}}{\sqrt{3}} + 2tX_\varphi B_\mu & \sqrt{2} K_\mu^0 \\ \sqrt{2} K_\mu^- & \sqrt{2} \overline{K}_\mu^0 & \dfrac{-2W_{8\mu}}{\sqrt{3}} + 2tX_\varphi B_\mu \end{pmatrix}, \tag{3.19}$$

where $t = g_x/g$ and nonphysical gauge bosons on nondiagonal entries, $W_\mu^\pm$ and $K_\mu^\pm$, are defined in (3.5) with $Q_1 = 1$, and

$$K_\mu^0 = \frac{1}{\sqrt{2}} \left(W_{6\mu} - i W_{7\mu} \right), \qquad \overline{K}_\mu^0 = \frac{1}{\sqrt{2}} \left(W_{6\mu} + i W_{7\mu} \right). \tag{3.20}$$

Then for the 331 model we are considering ($b = 1/2$), we have two neutral gauge bosons, K_μ^0 and $\overline{K}_\mu^0$, and four charged gauge bosons, $W_\mu^\pm$ and $K_\mu^\pm$. The three physical neutral eigenstates will be a linear combination of $W_{3\mu}$, $W_{8\mu}$, and B_μ. After breaking the symmetry with $\langle \Phi_i \rangle$, $i = 1, 2$, and using covariant derivative $D_\mu = \partial_\mu - (i/2) g \mathcal{M}_\mu^\varphi$ for the triplets Φ_i, we obtain the following masses for the charged physical fields:

$$M_{W'}^2 = \frac{1}{2} g^2 \vartheta_2^2, \qquad M_{K'}^2 = \frac{1}{2} g^2 \left(\vartheta_1^2 + \vartheta_2^2 + V^2 \right), \tag{3.21}$$

and the following physical eigenstates:

$$W_\mu'^{\pm} = \frac{1}{\sqrt{\vartheta_1^2 + V^2}} \left(-\vartheta_1 K_\mu^\pm + V W_\mu^\pm \right), \qquad K_\mu'^{\pm} = \frac{1}{\sqrt{\vartheta_1^2 + V^2}} \left(V K_\mu^\pm + \vartheta_1 W_\mu^\pm \right). \tag{3.22}$$

The neutral sector in approximation $(\vartheta_i/V)^n \approx 0$ for $n > 2$ leads to the following masses for the neutral physical fields:

$$
\begin{gathered}
M^2_{\text{photon}} = 0, \\
M^2_{K^0_R} = \frac{1}{2}g^2\left(V^2 + \vartheta_1^2\right), \\
M^2_Z \approx \frac{1}{2}g^2\vartheta_2^2\left(\frac{3g^2 + 4g_x^2}{3g^2 + g_x^2}\right), \\
M^2_{Z'} \approx \frac{2}{9}\left(V^2 + \vartheta_1^2\right)\left(3g^2 + g_x^2\right) + \frac{\vartheta_2^2\left(3g^2 + 4g_x^2\right)^2}{18\left(3g^2 + g_x^2\right)}, \\
M^2_{K^0_I} = \frac{1}{2}g^2\left(V^2 + \vartheta_1^2\right).
\end{gathered}
\tag{3.23}
$$

We can see from (3.21) and (3.23) that we have one nonmassive boson, which we associate with the photon, and four massive neutral fields, where the mass of one of them is proportional to ϑ_2 while the other three have masses proportional to V (new energy scale). Therefore we can associate the field Z with SM Z_μ and the fields Z', K^0_I, and K'^0_R with three new neutral bosons. We note that (3.23) contains two same of the eigenvalues; thus, the K^0_I and K'^0_R components have the same mass, and this conclusion contradicts the previous analysis in [41], but this is in agreement with [43, 44]. We also have four massive charged fields, where two of them have masses proportional to ϑ_2. Thus we can associate the fields $W'^{\pm}_\mu$ to the SM fields $W^{\pm}_\mu$, while the fields $K'^{\pm}_\mu$ are new bosons. The eigenstates $B_\mu,\ W_{3\mu},\ W_{8\mu}$, and $K^o_{R\mu}$ can be related to the physical eigenstates $A_\mu,\ K'^0_{R\mu},\ Z^0_\mu$, and Z'^0_μ by

$$
\begin{pmatrix} B_\mu \\ W_{3\mu} \\ W_{8\mu} \\ K^o_{R\mu} \end{pmatrix} = \mathbf{M}^{-1}\begin{pmatrix} A_\mu \\ K'^0_{R\mu} \\ Z^0_\mu \\ Z'^0_\mu \end{pmatrix}.
\tag{3.24}
$$

Assuming $(\vartheta_i/V)^n \sim 0$ for $n > 2$, we obtain

$$
\mathbf{M}^{-1} = \begin{pmatrix}
-\frac{1}{t}S_W & 0 & \frac{1}{t}T_W^2 C_W + \beta_1 & -\frac{1}{\sqrt{3}}T_W + \beta_2 \\
S_W & \frac{-\vartheta_1}{V} & C_W + \beta_3 & \beta_4 \\
\frac{1}{\sqrt{3}}S_W & \frac{\sqrt{3}\vartheta_1}{V} & -\frac{1}{\sqrt{3}}T_W S_W + \beta_5 & -\frac{1}{t}T_W + \beta_6 \\
0 & 1 - \beta_7 & \frac{\vartheta_1}{V}C_W^{-1} & \frac{\sqrt{3}\vartheta_1}{tV}T_W
\end{pmatrix},
\tag{3.25}
$$

where, again, $t = g_x/g$ and

$$S_W = \frac{\sqrt{3}g_x}{\sqrt{3g^2 + 4g_x^2}}, \qquad C_W = \sqrt{1 - S_W^2}, \qquad T_W = \frac{S_W}{C_W},$$
$$\beta_1 = -\frac{\vartheta_2^2}{4tV^2} T_W^2 C_W^{-3}, \qquad \beta_2 = -\frac{\sqrt{3}\vartheta_2^2}{4t^2V^2} T_W^3 C_W^{-2},$$
$$\beta_3 = -\frac{\vartheta_1^2}{2V^2} C_W^{-1}, \qquad \beta_4 = -\frac{\sqrt{3}\left(2C_W^2\vartheta_1^2 + \vartheta_2^2\right)}{4tV^2} T_W C_W^{-2}, \tag{3.26}$$
$$\beta_5 = \frac{6C_W^4\vartheta_1^2 - \left(3 - 4S_W^2\right)\vartheta_2^2}{4\sqrt{3}V^2C_W^5}, \qquad \beta_6 = \frac{\left(6C_W^4\vartheta_1^2 + S_W^2\vartheta_2^2\right)}{4tV^2C_W^4} T_W,$$
$$\beta_7 = -\frac{2t\vartheta_2^2}{V^2}.$$

We note that all β_i are of order $\mathcal{O}((\vartheta_i/V)^2)$. So, assuming $\vartheta_i \sim \mathcal{O}(10^{-1})$ TeV, for a new energy scale of order $V \sim 10$ TeV, all the β_i's are negligible.

3.3.2. Charged and Neutral Currents

The interaction between gauge bosons and fermions in flavor basis is given by the following Lagrangian density:

$$\mathcal{L}_f = \overline{R} i\gamma^\mu \left(\partial_\mu + i g_x B_\mu X_R\right) R + \overline{L} i\gamma^\mu \left(\partial_\mu + i g_x B_\mu X_L + \frac{ig}{2}\lambda_a W_\mu^a\right) L, \tag{3.27}$$

where R represents any right-handed singlet and L any left-handed triplet. We can write $\mathcal{L}_f = \mathcal{L}_{\text{lep}} + \mathcal{L}_{Q_1} + \mathcal{L}_{Q_a}$, and in lepton sector, we obtain

$$\mathcal{L}_{\text{lep}} = \mathcal{L}_{\text{lep}}^{\text{kin}} + \mathcal{L}_{\text{lep}}^{\text{CC}} + \mathcal{L}_{\text{lep}}^{\text{NC}}, \tag{3.28}$$

where

$$\mathcal{L}_{\text{lep}}^{\text{kin}} = \overline{R} i\gamma^\mu \partial_\mu R + \overline{L} i\gamma^\mu \partial_\mu L, \tag{3.29}$$

$$\mathcal{L}_{\text{lep}}^{\text{CC}} = -\frac{g}{\sqrt{2}} \overline{\ell_L} \gamma^\mu \nu_{\ell L} W_\mu^+ - \frac{g}{\sqrt{2}} \overline{\ell_L} \gamma^\mu N_{\ell L}^0 K_\mu^+ + \text{ h.c.}, \tag{3.30}$$

$$\begin{aligned}
\mathcal{L}_{\text{lep}}^{\text{NC}} = {} & \frac{g_x}{3}\left[\overline{\ell_L}\gamma^\mu \ell + \overline{\nu_{\ell L}}\gamma^\mu \nu_{\ell L} + \overline{N_{\ell L}^0}\gamma^\mu N_{\ell L}^0\right] B_\mu + g_x \overline{\ell_R}\gamma^\mu \ell_R B_\mu \\
& - \frac{g}{2\sqrt{3}}\left[\overline{\ell_L}\gamma^\mu \ell_L + \overline{\nu_{\ell L}}\gamma^\mu \nu_{\ell L} - 2t\overline{N_{\ell L}^0}\gamma^\mu N_{\ell L}^0\right] W_{8\mu} - \frac{g}{\sqrt{2}}\overline{\nu_{\ell L}}\gamma^\mu N_{\ell L}^0 K^{0\mu} \\
& - \frac{g}{2}\left[\overline{\ell_L}\gamma^\mu \ell_L - \overline{\nu_{\ell L}}\gamma^\mu \nu_{\ell L}\right] W_{3\mu} - \frac{g}{\sqrt{2}}\overline{N_{\ell L}^0}\gamma^\mu \nu_{\ell L} \overline{K}_\mu^0.
\end{aligned} \tag{3.31}$$

In quark sector we have that for the first family triplet $X = 1/3$, and for the singlets $d,\ u$, and U_1, we have $X = -1/3,\ 2/3$ and $2/3$, respectively. Then we have

$$\mathcal{L}^{\text{kin}}_{Q_1} = \overline{Q}_{1R} i\gamma^{\mu}\partial_{\mu}Q_{1R} + \overline{Q}_{1L} i\gamma^{\mu}\partial_{\mu}Q_{1L},$$
$$\mathcal{L}^{\text{CC}}_{Q_1} = -\frac{g}{\sqrt{2}}\overline{d_L}\gamma^{\mu}u_L W^{+}_{\mu} - \frac{g}{\sqrt{2}}\overline{d_L}\gamma^{\mu}U_{1L}K^{+}_{\mu} + \text{ h.c.}, \tag{3.32}$$

$$\begin{aligned}\mathcal{L}^{\text{NC}}_{Q_1} = {} & \frac{g_x}{3}\left(\overline{d_R}\gamma^{\mu}d_R - 2\overline{u_R}\gamma^{\mu}u_R - 2\overline{U_{1R}}\gamma^{\mu}U_{1R}\right)B_{\mu} + \frac{g}{2}\overline{u_L}\gamma^{\mu}u_L W_{3\mu} \\ & - \frac{g_x}{3}\left(\overline{d_L}\gamma^{\mu}d_L + \overline{u_L}\gamma^{\mu}u_L + \overline{U_{1L}}\gamma^{\mu}U_{1L}\right)B_{\mu} - \frac{g}{2}\overline{d_L}\gamma^{\mu}d_L W_{3\mu} - \frac{g}{\sqrt{2}}\overline{U_{1L}}\gamma^{\mu}u_L\overline{K}^{0}_{\mu} \\ & - \frac{g}{2\sqrt{3}}\left(\overline{d_L}\gamma^{\mu}d_L + \overline{u_L}\gamma^{\mu}u_L - 2\overline{U_{1L}}\gamma^{\mu}U_{1L}\right)W_{8\mu} - \frac{g}{\sqrt{2}}\overline{u_L}\gamma^{\mu}U_{1L}K^{0}_{\mu}.\end{aligned} \tag{3.33}$$

For second and third families we know that $X = 0$ for the triplets and $X = 2/3,\ -1/3$ and $-1/3$, for the singlets $u_{2,3},\ d_{2,3},\ D_{2,3}$, respectively, where $u_2 = c,\ u_3 = t,\ d_2 = s, d_3 = b$. Then we obtain for $a = 2, 3$

$$\begin{aligned}\mathcal{L}^{\text{kin}}_{Q_a} &= \overline{Q}_{aR} i\gamma^{\mu}\partial_{\mu}Q_{aR} + \overline{Q}_{aL} i\gamma^{\mu}\partial_{\mu}Q_{aL}, \\ \mathcal{L}^{\text{CC}}_{Q_a} &= -\frac{g}{\sqrt{2}}\overline{u_{aL}}\gamma^{\mu}d_{aL}W^{+}_{\mu} - \frac{g}{\sqrt{2}}\overline{u_{aL}}\gamma^{\mu}D_{aL}K^{+}_{\mu} + \text{ h.c.}, \\ \mathcal{L}^{\text{NC}}_{Q_a} &= \frac{g_x}{3}\left[-2\overline{u_{aR}}\gamma^{\mu}u_{aR} + \overline{d_{aR}}\gamma^{\mu}d_{aR} + \overline{D_{aR}}\gamma^{\mu}D_{aR}\right]B_{\mu} \\ & \quad - \frac{g}{2\sqrt{3}}\left[\overline{u_{aL}}\gamma^{\mu}u_{aL} + \overline{d_{aL}}\gamma^{\mu}d_{aL} - 4\overline{D_{aL}}\gamma^{\mu}D_{aL}\right]W_{8\mu} - \frac{g}{\sqrt{2}}\overline{d_{aL}}\gamma^{\mu}D_{aL}K^{0}_{\mu} \\ & \quad - \frac{g}{2}\left[\overline{u_{aL}}\gamma^{\mu}u_{aL} - \overline{d_{aL}}\gamma^{\mu}d_{aL}\right]W_{3\mu} - \frac{g}{\sqrt{2}}\overline{D_{aL}}\gamma^{\mu}d_{aL}\overline{K}^{0}_{\mu}.\end{aligned} \tag{3.34}$$

4. Neutrinos Interactions with Matter in 331 Model

It is well known that neutrino oscillation phenomenon in a material medium, as the sun, earth, or in a supernova, can be quite different from the oscillation that occurs in vacuum, since the interactions in the medium modify the dispersion relations of the particles traveling through it [60]. From the macroscopic point of view, the modifications of neutrino dispersion relations can be represented in terms of a refractive index or an effective potential. And according to [60, 61], the effective potential can be calculated from the amplitudes of coherent elastic scattering in relativistic limit.

In the present 331 model, the coherent scattering will be induced by neutral currents, NC, mediated by bosons $Z'^{0}_{\mu},\ Z^{0}_{\mu}$, and $K'^{0}_{R\mu}$ and by charged currents, CC, mediated by bosons $W'^{\pm}_{\mu}$ and $K'^{\pm}_{\mu}$. Following [61], we calculate in next sections the neutrino effective potentials in coherent scattering.

4.1. Charged Currents

The first term of (3.30) shows that the interaction of charged leptons with neutrinos occurs only through the gauge bosons $W_\mu^\pm$; then, by (3.22) we obtain that the interaction through charged bosons is given by

$$-\frac{g}{\sqrt{2}}\overline{\ell_L}\gamma^\mu\nu_{\ell L}W_\mu^+ = -\frac{Vg}{\sqrt{2}\sqrt{\vartheta_1^2+V^2}}\overline{\ell_L}\gamma^\mu\nu_{\ell L}W_\mu^{'\pm} - \frac{g\vartheta_1}{\sqrt{2}\sqrt{\vartheta_1^2+V^2}}\overline{\ell_L}\gamma^\mu\nu_{\ell L}K_\mu^{'\pm}. \tag{4.1}$$

The amplitude for the neutrino elastic scattering with charged leptons in tree level through CC is given by (note from (4.1) that only left-handed leptons interact with neutrinos, as in SM)

$$\begin{aligned}\mathcal{L}_{\text{int}}^{\text{cc}} = &-\left(-\frac{Vg}{\sqrt{2}\sqrt{\vartheta_1^2+V^2}}\right)^2\overline{\ell_L}(p_1)\gamma^\mu\nu_{\ell L}(p_2)\frac{-ig_{\mu\lambda}}{(p_2-p_1)^2-M_W^2}\overline{\nu}_{\ell L}(p_3)\gamma^\lambda\ell_L(p_4)\\ &-\left(-\frac{g\vartheta_1}{\sqrt{2}\sqrt{\vartheta_1^2+V^2}}\right)^2\overline{\ell_L}(p_1)\gamma^\mu\nu_{\ell L}(p_2)\frac{-ig_{\mu\lambda}}{(p_2-p_1)^2-M_K^2}\overline{\nu}_{\ell L}(p_3)\gamma^\lambda\ell_L(p_4).\end{aligned} \tag{4.2}$$

For low energies $M_{W'}^2,\ M_{K'}^2 \gg (p_2-p_1)^2$, the effective Lagrangian is given by

$$\mathcal{L}_{\text{eff}}^{\text{cc}} \approx -\frac{g^2}{2(\vartheta_1^2+V^2)}\left(\frac{V^2}{M_{W'}^2}+\frac{\vartheta_1^2}{M_{K'}^2}\right)\left[\overline{\ell}_L(p_1)\gamma^\mu\ell_L(p_4)\right]\left[\overline{\nu}_{\ell L}(p_3)\gamma_\mu\nu_{\ell L}(p_2)\right], \tag{4.3}$$

where we used the Fierz transformation [62] to go from (4.2) to (4.3). Replacing (3.21) in (4.3), we obtain

$$-\mathcal{L}_{\text{eff}}^{\text{cc}} \approx \left[\frac{1}{\vartheta_2^2}-\frac{\vartheta_1^2}{V^2\vartheta_2^2}+\left(\frac{\vartheta_1^2}{V^4}\right)_{K'}+\mathcal{O}\left(\frac{1}{V^4}\right)\right]\left\langle\overline{\ell}\gamma^\mu\frac{(1-\gamma_5)}{2}\ell\right\rangle\{\overline{\nu}_{\ell L}(p)\gamma^\mu\nu_{\ell L}(p)\}, \tag{4.4}$$

where we used $(\,)_{K'}$ to denote the term that appears from the new charged boson. We can see that for a new energy scale $V \gg \vartheta_1$ the term that comes from the new boson does not contribute to the process, as expected, since the new charged boson $K_\mu^{'\pm}$ has a mass of the order of the new energy scale of the theory (see (3.21)).

Now, since usual matter has only leptons from first family, we will restrain our calculations to the neutrino interactions with first family standard model particles. The term $\langle\ \rangle$ in (4.4) can be calculated following [61], where we have the correspondence $\langle\overline{e}\gamma^\mu\gamma_5 e\rangle \sim$ spin, $\langle\overline{e}\gamma^i e\rangle \sim$ velocity, and $\langle\overline{e}\gamma^0 e\rangle \sim n_e$, where n_e is the electronic density. Assuming nonpolarized medium and vanishing average velocity, we obtain that (4.4) can be written as

$$\mathcal{L}_{\text{eff}}^{\text{cc}} \approx -\left[\frac{1}{2\vartheta_2^2}-\frac{\vartheta_1^2}{2V^2\vartheta_2^2}+\left(\frac{\vartheta_1^2}{2V^4}\right)_{K'}+\mathcal{O}\left(V^{-4}\right)\right]n_e\overline{\nu}_{eL}\gamma^0\nu_{eL}. \tag{4.5}$$

The modifications on electronic neutrino dispersion relations can be represented by the following effective potential:

$$V_{CC}^{e} \approx \frac{1}{2\vartheta_2^2} n_e - \frac{\vartheta_1^2}{2V^2\vartheta_2^2} n_e + \left(\frac{\vartheta_1^2}{2V^4}\right)_{K'} n_e + \mathcal{O}\left(V^{-4}\right). \tag{4.6}$$

Disregarding the term $(\,)_{K'}$ since we are assuming $V \gg \vartheta_i$, and remembering that in Section 3.3.1 we associated boson W' with SM boson W, we can easily associate

$$\sqrt{2}G_F \approx \frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2}. \tag{4.7}$$

We note that (4.7) gives limits for the VEV of one of the Higgs triplets. Under assumption $\vartheta_1,\ \vartheta_2 \ll V$, we can write $G_F \approx (1/2\sqrt{2}\vartheta_2^2)(1 - \vartheta_1^2/V^2)$, from which we can see that the maximum value of ϑ_2^2 is achieved when we consider $(\vartheta_1^2/V^2) \to 0$, in which replacing $G_F = 1.16637(1) \times 10^{-5}\,\text{Gev}^{-2}$ leads to

$$\vartheta_2 \lesssim 174.105\,\text{GeV}. \tag{4.8}$$

4.2. Neutral Current

The Lagrangian for neutrino elastic scattering with fermions $f = e, u, d$ through NC is given by

$$\begin{aligned}
-\mathcal{L}_{\text{int}}^{\text{NC}} = {} & \overline{f}(p_1)\gamma^{\mu}\left(g_{z'L}^{f} + g_{z'R}^{f}\right) f(p_2) \frac{-i g_{\mu\lambda}}{(p_2 - p_1)^2 - M_{z'}^2} \overline{\nu}_{eL}(p_3)\gamma^{\lambda} g_{\nu z'} \nu_{eL}(p_4) \\
& + \overline{f}(p_1)\gamma^{\mu}\left(g_{zL}^{f} + g_{zR}^{f}\right) f(p_2) \frac{-i g_{\mu\lambda}}{(p_2 - p_1)^2 - M_{z}^2} \overline{\nu}_{eL}(p_3)\gamma^{\lambda} g_{\nu z} \nu_{eL}(p_4) \\
& + \overline{f}(p_1)\gamma^{\mu}\left(g_{k'L}^{f} + g_{k'R}^{f}\right) f(p_2) \frac{-i g_{\mu\lambda}}{(p_2 - p_1)^2 - M_{k'}^2} \overline{\nu}_{eL}(p_3)\gamma^{\lambda} g_{\nu k'} \nu_{eL}(p_2).
\end{aligned} \tag{4.9}$$

For low energies, we have that $M_{k'}^2, M_z^2, M_{z'}^2 \gg (p_2 - p_1)^2$ with $p_3 = p_4 = p$ and (4.9), and following the same procedure of Section 4.1, we obtain

$$\mathcal{L}_{\text{eff}}^{\text{NC}} \approx -\sum_{P=L,R}\left(g_{z'P}^{f}\frac{G_{\nu z'}}{M_{z'}^2} + g_{zP}^{f}\frac{G_{\nu z}}{M_{z}^2} + g_{k'P}^{f}\frac{G_{\nu k'}}{M_{k'}^2}\right)\frac{1}{2} n_f \overline{\nu}_{eL}\gamma_0 \nu_{eL}. \tag{4.10}$$

4.2.1. Leptons Sector

From (3.31) and (3.24), we obtain that for the known neutral leptons

$$\begin{aligned}\frac{g_x}{3}\overline{\nu}_{eL}\gamma^{\mu}\nu_{eL}B_{\mu} = \overline{\nu}_{eL}\gamma^{\mu}\nu_{eL}\Bigg[-\frac{g}{3}S_W A_{\mu} + \left(\frac{g}{3}T_W^2 C_W + \frac{g_x}{3}\beta_1\right)Z_{\mu}^0 \\ -\frac{g_x}{3}\left(\frac{1}{\sqrt{3}}T_W - \beta_2\right)Z_{\mu}^{\prime 0}\Bigg],\end{aligned} \tag{4.11}$$

$$\frac{g}{2}\overline{\nu}_{eL}\gamma^{\mu}\nu_{eL}W_3^{\mu} = \overline{\nu}_{eL}\gamma^{\mu}\nu_{eL}\left[\frac{g}{2}S_W A_{\mu} - \frac{g\vartheta_1}{2V}K_{R\mu}^{\prime 0} + \frac{g\left(C_W + \beta_3\right)}{2}Z_{\mu}^0 + \frac{g\beta_4}{2}Z_{\mu}^{\prime 0}\right], \tag{4.12}$$

$$\begin{aligned}\frac{-g}{2\sqrt{3}}\overline{\nu}_{eL}\gamma^{\mu}\nu_{eL}W_8^{\mu} = \overline{\nu}_{eL}\gamma^{\mu}\nu_{eL}\Bigg[\frac{-g}{6}S_W A_{\mu} - \frac{g\vartheta_1}{2V}K_{R\mu}^{\prime 0} + \left(\frac{g}{6}\frac{S_W^2}{C_W} - \frac{g\beta_5}{2\sqrt{3}}\right)Z_{\mu}^0 \\ +\frac{g}{2\sqrt{3}}\left(\frac{1}{t}T_W - \beta_6\right)Z_{\mu}^{\prime 0}\Bigg].\end{aligned} \tag{4.13}$$

By (4.11), (4.12), and (4.13), we obtain that vertex interactions with neutrinos can be written as

$$\overline{\nu}_{eL}\gamma^{\mu}\nu_{eL}A_{\mu} \propto 0, \tag{4.14}$$

$$\overline{\nu}_{eL}\gamma^{\mu}\nu_{eL}K_{R\mu}^{\prime 0} \propto -\frac{g\vartheta_1}{V} \equiv G_{\nu K'}, \tag{4.15}$$

$$\overline{\nu}_{eL}\gamma^{\mu}\nu_{eL}Z_{\mu}^{0} \propto \frac{1}{2}gC_W^{-1} + \eta_1 \equiv G_{\nu Z}, \tag{4.16}$$

$$\overline{\nu}_{eL}\gamma^{\mu}\nu_{eL}Z_{\mu}^{\prime 0} \propto \left(\frac{3g - 2g_x t}{6\sqrt{3}t}\right)T_W + \eta_2 \equiv G_{\nu Z'}, \tag{4.17}$$

where

$$\begin{aligned}\eta_1 &= \frac{-4gtC_W^2\vartheta_1^2 + g_x\left(1 - 2S_W^2\right)\vartheta_2^2}{8tV^2C_W^5}, \\ \eta_2 &= \frac{gt\left(1 - 4C_W^2\right)\vartheta_1^2}{2\sqrt{3}V^2C_W S_W} - \frac{\left(-gt^3 + 2gt^3C_W^2 + 8gt^3C_W^4 + 6g_x S_W^4\right)\vartheta_2^2}{24\sqrt{3}t^2V^2C_W^5 S_W}.\end{aligned} \tag{4.18}$$

We note from (4.14) that neutrinos do not interact electrically, as expected. For charged leptons, from (3.31) and (3.24), we obtain

$$\begin{aligned}
\frac{g_x}{3}\overline{\ell_L}\gamma^\mu \ell_L B_\mu &= \overline{\ell_L}\gamma^\mu \ell_L\left[\frac{-g}{3}S_W A_\mu + \left(\frac{g}{3}T_W^2 C_W + \frac{g_x}{3}\beta_1\right)Z_\mu^0 \right.\\
&\qquad \left. -\frac{g_x}{3}\left(\frac{1}{\sqrt{3}}T_W - \beta_2\right)Z_\mu^{'0}\right],\\
-\frac{g}{2}\overline{\ell}_L\gamma^\mu \ell_L W_3^\mu &= \overline{\ell}_L\gamma^\mu \ell_L\left[\frac{-g}{2}S_W A_\mu + \frac{g\vartheta_1}{2V}K_{R\mu}^{'0} - \frac{g\left(C_W+\beta_3\right)}{2}Z_\mu^0 - \frac{g\beta_4}{2}Z_\mu^{'0}\right],\\
\frac{-g}{2\sqrt{3}}\overline{\ell}_L\gamma^\mu \ell_L W_8^\mu &= \overline{\ell}_L\gamma^\mu \ell_L\left[\frac{-g}{6}S_W A_\mu - \frac{g\vartheta_1}{2V}K_{R\mu}^{'0} + \left(\frac{g}{6}\frac{S_W^2}{C_W} - \frac{g\beta_5}{2\sqrt{3}}\right)Z_\mu^0 \right.\\
&\qquad \left. +\frac{g}{2\sqrt{3}}\left(\frac{1}{t}T_W - \beta_6\right)Z_\mu^{'0}\right],\\
g_x\overline{\ell_R}\gamma^\mu \ell_R B_\mu &= \overline{\ell_R}\gamma^\mu \ell_R\left[-gS_W A_\mu + \left(gT_W^2 C_W + g_x\beta_1\right)Z_\mu^0 \right.\\
&\qquad \left. -g_x\left(\frac{1}{\sqrt{3}}T_W - \beta_2\right)Z_\mu^{'0}\right],
\end{aligned} \tag{4.19}$$

and therefore

$$\overline{\ell}\gamma^\mu \ell A_\mu \propto -gS_W, \tag{4.20}$$

$$\overline{\ell_L}\gamma^\mu \ell_L K_{R\mu}^{'0} \propto 0 \equiv g_{k'L}^\ell = g_{k'R}^\ell, \tag{4.21}$$

$$\begin{aligned}
\overline{\ell_L}\gamma^\mu \ell_L Z_\mu^0 &\propto \frac{1}{2}g\left(-1+T_W^2\right)C_W + \eta_3 \equiv g_{zL}^\ell,\\
\overline{\ell_R}\gamma^\mu \ell_R Z_\mu^0 &\propto gT_W^2 C_W + \eta_5 \equiv g_{zR}^\ell,
\end{aligned} \tag{4.22}$$

$$\begin{aligned}
\overline{\ell_L}\gamma^\mu \ell_L Z_\mu^{'0} &\propto \frac{1}{6\sqrt{3}t}(3g-2tg_x)T_W + \eta_4 \equiv g_{z'L}^\ell,\\
\overline{\ell_R}\gamma^\mu \ell_R Z_\mu^{'0} &\propto -\frac{g_x}{\sqrt{3}}T_W + \eta_6 \equiv g_{z'R}^\ell,
\end{aligned} \tag{4.23}$$

where

$$\begin{aligned}
\eta_3 &= \frac{\left(-1+2C_W^2\right)g_x\vartheta_2^2}{8tV^2C_W^5},\\
\eta_4 &= \frac{\left(gt^3\left(1+2C_W^2\right)^2 - 12gt^3S_W^2C_W^2 - 6g_xS_W^4\right)}{24\sqrt{3}t^2V^2C_W^5S_W},\\
\eta_5 &= -\frac{g_x\vartheta_2^2}{4tV^2C_W^3}T_W^2,\\
\eta_6 &= -\frac{\sqrt{3}g_x\vartheta_2^2}{4t^2V^2C_W^2}T_W^3,
\end{aligned} \tag{4.24}$$

and, again, $t = g_x/g$. We note that by (4.20) we can make the association $gS_W = |e|$. Then for $f = e$, (4.15)–(4.17) and (4.21)–(4.23) lead to

$$\begin{aligned}
\mathcal{L}^{\mathrm{NC}}_{\mathrm{eff}-e} &\approx -\sum_{P=L,R}\frac{1}{2}\left(g^e_{z'P}\frac{G_{\nu z'}}{M^2_{z'}} + g^e_{zP}\frac{G_{\nu z}}{M^2_z} + g^e_{k'P}\frac{G_{\nu k'}}{M^2_{k'}}\right)n_e\overline{\nu}_{\ell L}\gamma_0\nu_{\ell L} \\
&\approx -\left\{\left[\frac{T^4_W}{144t^2g^2_xV^2}(3g-2tg_x)^2 + \frac{T^2_W}{8V^2}\left(1-T^2_W\right)\right.\right. \\
&\qquad\left.+\frac{1}{2}\left(\frac{1}{2\vartheta^2_2} - \frac{\vartheta^2_1}{2V^2\vartheta^2_2}\right)\left(1-2C^2_W\right)\right]_L \\
&\qquad\left.+\left[\frac{T^4_W(2tg_x-3g)}{24tg_xV^2} - \frac{T^4_W}{4V^2} + \left(\frac{1}{2\vartheta^2_2} - \frac{\vartheta^2_1}{2V^2\vartheta^2_2}\right)S^2_W\right]_R\right\}n_e\overline{\nu}_{\ell L}\gamma_0\nu_{\ell L}.
\end{aligned} \tag{4.25}$$

Since intermediate neutral bosons in (4.9) do not distinguish between different lepton flavors, the interaction through NC with electron is described by the following effective potential:

$$\begin{aligned}
V^e_{\mathrm{NC}} &= V^\mu_{\mathrm{NC}} = V^\tau_{\mathrm{NC}} = V^\ell_{\mathrm{NC}}, \\
&= V^{\ell L}_{\mathrm{NC}} + V^{\ell R}_{\mathrm{NC}},
\end{aligned} \tag{4.26}$$

where

$$\begin{aligned}
V^{\ell L}_{\mathrm{NC}} &= \left[\frac{T^4_W}{144t^2g^2_xV^2}(3g-2tg_x)^2 + \frac{T^2_W}{8V^2}\left(1-T^2_W\right)\right. \\
&\qquad\left.+\frac{1}{2}\left(\frac{1}{2\vartheta^2_2} - \frac{\vartheta^2_1}{V^2\vartheta^2_2}\right)\left(1-2C^2_W\right)\right]n_e, \\
V^{\ell R}_{\mathrm{NC}} &= \left[\frac{T^4_W(2tg_x-3g)}{24tg_xV^2} - \frac{T^4_W}{4V^2} + \left(\frac{1}{2\vartheta^2_2} - \frac{\vartheta^2_1}{2V^2\vartheta^2_2}\right)S^2_W\right]n_e,
\end{aligned} \tag{4.27}$$

and index ℓ refers to neutrino flavor. We note that the potential through CC comes from interactions of electron neutrinos with left-handed electrons, while the effective potential through NC comes from left- and right-handed electrons.

Considering both NC and CC, we can write the effective potential felt by neutrinos as $V^\ell = V^{\ell L} + V^{\ell R}$, where

$$\begin{aligned}
V^{\ell L} &= \left(\frac{1}{2\vartheta^2_2} - \frac{\vartheta^2_1}{2V^2\vartheta^2_2}\right)\delta_{e\ell}n_e + V^{\ell L}_{\mathrm{NC}}, \\
V^{\ell R} &= V^{\ell R}_{\mathrm{NC}}.
\end{aligned} \tag{4.28}$$

Comparing with SM expression for such potential:

$$V^\ell_{\mathrm{NC}} = -\sqrt{2}G_F\left(\frac{1}{2} - 2S^2_W\right)n_e, \qquad V^e_{\mathrm{CC}} = \sqrt{2}G_F n_e, \tag{4.29}$$

we can find that

$$
\begin{aligned}
V^{\ell L} &= V^{\ell L} + \left[\frac{T_W^4}{144t^2 g_x^2 V^2}(3g - 2tg_x)^2 + \frac{T_W^2}{8V^2}\left(1 - T_W^2\right)\right] n_e, \\
V^{\ell R} &= V_{\rm NC}^{\ell R} + \left[\frac{T_W^4 (2tg_x - 3g)}{24tg_x V^2} - \frac{T_W^4}{4V^2}\right] n_e,
\end{aligned}
\tag{4.30}
$$

where we adopt in what follow, the convention that V denotes SM-like part of the model; thus, the new terms beyond SM [ ] can be associated with the parameters ε's in NSI [63]. So, in the approximation $(\vartheta_i/V)^n \approx 0$, for $n > 2$, we obtain

$$
\varepsilon_{\ell\ell}^{eL} \approx \frac{(1 - 2S_W^2)\vartheta_2^2}{8V^2 C_W^4}, \tag{4.31}
$$

$$
\varepsilon_{\ell\ell}^{eR} \approx -\frac{S_W^2 (1 + 2S_W^2)\vartheta_2^2}{4V^2 C_W^4}. \tag{4.32}
$$

We note that on limit $V \to \infty$, we recover SM. The NSIs are a subleading interaction, as expected. By (4.31) and (4.32), we obtain $\varepsilon_{\ell\ell}^{eR} \approx -2S_W^2 \varepsilon_{\ell\ell}^{eL} - (\vartheta_2^2/V^2)T_W^4$.

4.2.2. Quarks Sector

For the quarks of the first family, the Lagrangian density in (3.33) describes the interactions with gauge bosons $W_{3\mu}$, $W_{8\mu}$, and B_μ; then, by (3.24) and (3.25) we obtain the following interactions for up quarks:

$$
\begin{aligned}
-\frac{g_x}{3}\overline{u_L}\gamma^\mu u_L B_\mu &= \overline{u_L}\gamma^\mu u_L \left[\frac{g}{3} S_W A_\mu - \frac{g_x}{3}\left(\frac{1}{t}T_W^2 C_W + \beta_1\right) Z_\mu^0 \right. \\
&\qquad \left. + \frac{g_x}{3}\left(\frac{1}{\sqrt{3}}T_W - \beta_2\right) Z_\mu^{\prime 0}\right], \\
\frac{g}{2}\overline{u_L}\gamma^\mu u_L W_3^\mu &= \overline{u_L}\gamma^\mu u_L \left[\frac{g}{2} S_W A_\mu - \frac{g\vartheta_1}{2V} K_{R\mu}^{\prime 0} + \frac{g(C_W + \beta_3)}{2} Z_\mu^0 + \frac{g\beta_4}{2} Z_\mu^{\prime 0}\right], \\
\frac{-g}{2\sqrt{3}}\overline{u_L}\gamma^\mu u_L W_8^\mu &= \overline{u_L}\gamma^\mu u_L \left[\frac{-g}{6} S_W A_\mu - \frac{g\vartheta_1}{2V} K_{R\mu}^{\prime 0} \right. \\
&\qquad \left. + \frac{g}{2\sqrt{3}}\left(\frac{1}{\sqrt{3}}T_W S_W - \beta_5\right) Z_\mu^0 + \frac{g}{2\sqrt{3}}\left(\frac{1}{t}T_W - \beta_6\right) Z_\mu^{\prime 0}\right], \\
-\frac{2g_x}{3}\overline{u_R}\gamma^\mu u_R B_\mu &= \overline{u_R}\gamma^\mu u_R \left[\frac{2g}{3} S_W A_\mu - \frac{2g_x}{3}\left(\frac{1}{t}T_W^2 C_W + \beta_1\right) Z_\mu^0 \right. \\
&\qquad \left. + \frac{2g_x}{3}\left(\frac{1}{t}T_W - \beta_6\right) Z_\mu^{\prime 0}\right].
\end{aligned}
\tag{4.33}
$$

The couplings quark-quark-boson for the first family are given by

$$\overline{u_L}\gamma^{\mu}u_L A_{\mu} \propto \frac{2}{3}gS_W, \tag{4.34}$$

$$\overline{u_R}\gamma^{\mu}u_R A_{\mu} \propto \frac{2}{3}gS_W, \tag{4.35}$$

$$\begin{aligned}
\overline{u_L}\gamma^{\mu}u_L K'^{0}_{R\mu} &\propto -\frac{g\vartheta_1}{V} \equiv g^u_{k'L},\\
\overline{u_R}\gamma^{\mu}u_R K'^{0}_{R\mu} &\propto 0 \equiv g^u_{k'R},\\
\overline{u_L}\gamma^{\mu}u_L Z^0_{\mu} &\propto \frac{1}{6}g\left(3 - T_W^2\right)C_W + \zeta_1 \equiv g^u_{zL},\\
\overline{u_R}\gamma^{\mu}u_R Z^0_{\mu} &\propto -\frac{2}{3}gT_W^2 C_W + \zeta_3 \equiv g^u_{zR},\\
\overline{u_L}\gamma^{\mu}u_L Z'^{0}_{\mu} &\propto \frac{1}{6\sqrt{3}t}\left(3g + 2tg_x\right)T_W \equiv g^u_{z'L},\\
\overline{u_R}\gamma^{\mu}u_R Z'^{0}_{\mu} &\propto \frac{2}{3\sqrt{3}}g_x T_W + \zeta_4 \equiv g^u_{z'R},
\end{aligned} \tag{4.36}$$

where

$$\begin{aligned}
\zeta_1 &= \frac{g_x\left(-12C_W^4\vartheta_1^2 + \left(1 + 2C_W^2\right)\vartheta_2^2\right)}{24tV^2C_W^5},\\
\zeta_2 &= \frac{12gt^3C_W^4\left(1 - 4C_W^2\right)\vartheta_1^2 + \left(gt^3\left(1 - 2C_W^2 - 8C_W^4\right) + 6g_xS_W^4\right)\vartheta_2^2}{24\sqrt{3}t^2V^2C_W^5S_W},\\
\zeta_3 &= \frac{g}{6}\frac{S_W^2\vartheta_2^2}{C_W^5V^2},\\
\zeta_4 &= \frac{g_xS_W^3\vartheta_2^2}{2\sqrt{3}t^2V^2C_W^5}.
\end{aligned} \tag{4.37}$$

We note that (4.34) and (4.35) reflect the fact that quarks interact electrically through photons with coupling constant $Q_f \sin\theta_W$, as in SM. The effective Lagrangian at low energies for neutrino interaction with quarks up through neutral currents are given by (4.10) with $f = u$:

$$\begin{aligned}
\mathcal{L}^{\mathrm{NC}}_{\mathrm{quark},\,u} &\approx -\frac{1}{2}\sum_{P=L,R}\left(g^u_{z'P}\frac{G_{\nu z'}}{M_{z'}^2} + g^u_{zP}\frac{G_{\nu z}}{M_z^2} + g^u_{k'P}\frac{G_{\nu k'}}{M_{k'}^2}\right)n_u\overline{\nu}_{eL}\gamma_0\nu_{eL}\\
&\approx -\Bigg\{\Bigg[\frac{1}{24V^2}\left(3 + T_W^4\right) + \frac{T_W^4}{144t^4V^2}\left(9 - 4t^4\right)\\
&\qquad + \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2}\right)\left(\frac{1}{2} - \frac{2}{3}S_W^2\right) - \frac{\vartheta_1^2}{4V^2\vartheta_2^2}\Bigg]_L\\
&\qquad + \left[\frac{T_W^4}{6V^2} + \frac{T_W^4\left(3g - 2tg_x\right)}{36tg_xV^2} - \frac{2}{3}\left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2}\right)S_W^2\right]_R\Bigg\}n_u\overline{\nu}_{eL}\gamma_0\nu_{eL},
\end{aligned} \tag{4.38}$$

where n_u is the up quarks average density.

SM predictions, using result of (4.7), can be written as

$$\mathrm{V}_{\mathrm{NC}}^{u} = \mathrm{V}_{\mathrm{NC}}^{uL} + \mathrm{V}_{\mathrm{NC}}^{uR} = \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2}\right)\left(\frac{1}{2} - \frac{4}{3}S_W^2\right)n_u, \tag{4.39}$$

where

$$\begin{gathered} \mathrm{V}_{\mathrm{NC}}^{uL} = \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2}\right)\left(\frac{1}{2} - \frac{2}{3}S_W^2\right)n_u, \\ \mathrm{V}_{\mathrm{NC}}^{uR} = -\frac{2}{3}\left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2}\right)S_W^2 n_u. \end{gathered} \tag{4.40}$$

By comparison, we obtain

$$\begin{gathered} V_{\mathrm{NC}}^{uL} \approx V_{\mathrm{NC}}^{uL} + \left[\frac{1}{24V^2}\left(3 + T_W^4\right) + \frac{T_W^4}{144t^4V^2}\left(9 - 4t^4\right) - \frac{\vartheta_1^2}{4V^2\vartheta_2^2}\right]n_u, \\ V_{\mathrm{NC}}^{uR} \approx V_{\mathrm{NC}}^{uR} + \left[\frac{T_W^4}{6V^2} + \frac{T_W^4(3g - 2tg_x)}{36tg_xV^2}\right]n_u. \end{gathered} \tag{4.41}$$

Then we can say that $\varepsilon_{\ell\ell}^{u} = \varepsilon_{\ell\ell}^{uL} + \varepsilon_{\ell\ell}^{uR}$, where

$$\begin{gathered} \varepsilon_{\ell\ell}^{uL} \approx -\frac{\vartheta_1^2}{2V^2} + \frac{\vartheta_2^2}{24V^2C_W^4}\left(9 - 8S_W^2\right), \\ \varepsilon_{\ell\ell}^{uR} \approx \frac{\vartheta_2^2}{6V^2}\frac{S_W^2}{C_W^4}. \end{gathered} \tag{4.42}$$

Again, we obtain universal NSI, as for the electrons. We note that $\varepsilon_{\ell\ell}^{uL} = -(\vartheta_1^2/2V^2) + (3\vartheta_2^2/8V^2C_W^4) - 2\varepsilon_{\ell\ell}^{uR}$ and in the limit $V \to \infty$ we recover SM.

For down quarks by (3.33) and (3.24), we obtain that

$$\begin{aligned} -\frac{g_x}{3}\overline{d_L}\gamma^{\mu}d_L B_{\mu} &= \overline{d_L}\gamma^{\mu}d_L\left[\frac{g}{3}S_W A_{\mu} - \frac{g_x}{3}\left(\frac{1}{t}T_W^2 C_W + \beta_1\right)Z_{\mu}^0 \right. \\ &\qquad \left. + \frac{g_x}{3}\left(\frac{1}{\sqrt{3}}T_W - \beta_2\right)Z_{\mu}^{\prime 0}\right], \\ -\frac{g}{2}\overline{d_L}\gamma^{\mu}d_L W_3^{\mu} &= \overline{d_L}\gamma^{\mu}d_L\left[-\frac{gS_W}{2}A_{\mu} + \frac{g\vartheta_1}{2V}K_{R\mu}^{\prime 0} - \frac{g(C_W + \beta_3)}{2}Z_{\mu}^0 - \frac{g\beta_4}{2}Z_{\mu}^{\prime 0}\right], \\ \frac{-g}{2\sqrt{3}}\overline{d_L}\gamma^{\mu}d_L W_8^{\mu} &= \overline{d_L}\gamma^{\mu}d_L\left[\frac{-gS_W}{6}A_{\mu} + \frac{g}{2\sqrt{3}}\left(\frac{1}{\sqrt{3}}T_W S_W - \beta_5\right)Z_{\mu}^0 \right. \\ &\qquad \left. + -\frac{g\vartheta_1}{2V}K_{R\mu}^{\prime 0} + \frac{g}{2\sqrt{3}}\left(\frac{1}{t}T_W - \beta_6\right)Z_{\mu}^{\prime 0}\right], \end{aligned}$$

$$\frac{g_x}{3}\overline{d_R}\gamma^\mu d_R B_\mu = \overline{d_L}\gamma^\mu d_L\left[-\frac{gS_W}{3}A_\mu + \frac{g_x}{3}\left(\frac{1}{t}T_W^2 C_W + \beta_1\right)Z_\mu^0 + \frac{g_x}{3}\left(-\frac{1}{\sqrt{3}}T_W + \beta_2\right)Z_\mu^{\prime 0}\right]. \tag{4.43}$$

$$\begin{aligned}
\overline{d_L}\gamma^\mu d_L A_\mu &\propto -\frac{1}{3}gS_W, \\
\overline{d_R}\gamma^\mu d_R A_\mu &\propto -\frac{1}{3}gS_W, \\
\overline{d_L}\gamma^\mu d_L K_{R\mu}^{\prime 0} &\propto 0 \equiv g_{k'L}^d, \\
\overline{d_R}\gamma^\mu d_R K_{R\mu}^{\prime 0} &\propto 0 \equiv g_{k'R}^d, \\
\overline{d_L}\gamma^\mu d_L Z_\mu^0 &\propto -\frac{1}{6}g\left(3+T_W^2\right)C_W + \zeta_5 \equiv g_{zL}^d, \\
\overline{d_R}\gamma^\mu d_R Z_\mu^0 &\propto \frac{g}{3}T_W^2 C_W + \zeta_7 \equiv g_{zR}^d, \\
\overline{d_L}\gamma^\mu d_L Z_\mu^{\prime 0} &\propto \frac{1}{6\sqrt{3}t}\left(3g + 2tg_x\right)T_W + \zeta_6 \equiv g_{z'L}^d, \\
\overline{u_R}\gamma^\mu u_R Z_\mu^{\prime 0} &\propto -\frac{1}{3\sqrt{3}}g_x T_W + \zeta_8 \equiv g_{z'R}^d,
\end{aligned} \tag{4.44}$$

where

$$\begin{aligned}
\zeta_5 &= \frac{g\vartheta_2^2}{24V^2C_W^5}\left(3-2S_W^2\right), \\
\zeta_6 &= \frac{\left(-1+3C_W^2+6C_W^4-8C_W^6\right)}{24\sqrt{3}V^2C_W^5S_W^3}, \\
\zeta_7 &= -\frac{gS_W^2\vartheta_2^2}{12V^2C_W^5}, \\
\zeta_8 &= -\frac{g_xS_W^3\vartheta_2^2}{4\sqrt{3}t^2V^2C_W^5}.
\end{aligned} \tag{4.45}$$

Then by (4.10) for $f = d$, we obtain the following effective Lagrangian for NC:

$$\begin{aligned}
\mathcal{L}_{\text{quark},\, d}^{\text{NC}} &\approx -\left(g_{z'V}^d\frac{G_{\nu z'}}{M_{z'}^2} + g_{zV}^d\frac{G_{\nu z}}{M_z^2} + g_{k'V}^d\frac{G_{\nu k'}}{M_{k'}^2}\right)n_d\overline{\nu}_{eL}\gamma_0\nu_{eL} \\
&\approx -\left\{\left[\frac{\left(3S_W^2-2S_W^4\right)}{24V^2C_W^4} + \frac{\left(9-4t^4\right)}{144t^4V^2}T_W^4 + \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2}\right)\left(-\frac{1}{2}+\frac{1}{3}S_W^2\right)\right]_L \right. \\
&\quad \left. + \left[-\frac{S_W^2}{24V^2C_W^4} + \frac{1}{3}\left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2}\right)S_W^2\right]_R\right\}n_d\overline{\nu}_{eL}\gamma_0\nu_{eL},
\end{aligned} \tag{4.46}$$

and the effective potential felt by neutrinos when crossing a medium composed by a density n_d of *down* quarks is $V_{\rm NC}^{d} = V_{\rm NC}^{dL} + V_{\rm NC}^{dR}$, where

$$V_{\rm NC}^{dL} \approx \left[\frac{(3S_W^2 - 2S_W^4)}{24V^2C_W^4} + \frac{(9-4t^4)}{144t^4V^2}T_W^4 + \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2}\right)\left(-\frac{1}{2} + \frac{1}{3}S_W^2\right)\right]n_d, \tag{4.47}$$

$$V_{\rm NC}^{dR} \approx \left[-\frac{S_W^2}{24V^2C_W^4} + \frac{1}{3}\left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2}\right)S_W^2\right]n_d. \tag{4.48}$$

Then we can easily see that in SM the NC effective potential for neutrinos in a d-quark medium, using result of (4.7), will be given by

$$\begin{aligned} V_{\rm NC}^{d} = V_{\rm NC}^{dL} + V_{\rm NC}^{dR} &\approx -\left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2}\right)\left(\frac{1}{2} - \frac{2}{3}S_W^2\right)n_d, \\ V_{\rm NC}^{dL} &= \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2}\right)\left(-\frac{1}{2} + \frac{1}{3}S_W^2\right)n_d, \end{aligned} \tag{4.49}$$

$$V_{\rm NC}^{dR} = \frac{1}{3}\left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2}\right)S_W^2 n_d. \tag{4.50}$$

Then from (4.47)–(4.50), we obtain

$$\begin{aligned} V_{\rm NC}^{dL} &\approx V_{\rm NC}^{dL} + \left[\frac{(3S_W^2 - 2S_W^4)}{24V^2C_W^4} + \frac{(9-4t^4)}{144t^4V^2}T_W^4\right]n_d, \\ V_{\rm NC}^{dR} &\approx V_{\rm NC}^{dR} - \frac{S_W^2}{24V^2C_W^4}n_d, \end{aligned} \tag{4.51}$$

and neglecting terms of order $(\vartheta_i/V)^n$, for $n > 2$, we obtain that $\varepsilon_{\ell\ell}^{d} = \varepsilon_{\ell\ell}^{dL} + \varepsilon_{\ell\ell}^{dR}$, where

$$\varepsilon_{\ell\ell}^{dL} \approx \frac{\vartheta_2^2}{24V^2C_W^4}\left(3 - 2S_W^2\right), \tag{4.52}$$

$$\varepsilon_{\ell\ell}^{dR} \approx -\frac{S_W^2\vartheta_2^2}{12V^2C_W^4}. \tag{4.53}$$

Then we obtain $\varepsilon_{\ell\ell}^{dL} \approx (\vartheta_2^2/8V^2\, C_W^4) + \varepsilon_{\ell\ell}^{dR}$. Note that again in limit $V \to \infty$ we recover the SM.

5. Results

In last sections we saw that in 331 model we chose, all NSI parameters are universal and diagonal and will not affect oscillation experiments. However, measurements of cross-section will be sensitive to such parameters, through modifications on g_i^α [51]. We will now compare

Table 1: Values for NSI in 331 model and experimental limits taken of the strongest constraints on these parameters are given in [18, 19, 21, 22].

331 Model		Exp. 90% C.L.
$\varepsilon_{\ell\ell}^{eL} \approx \frac{(1-2S_W^2)\vartheta_2^2}{8V^2C_W^4}$	$0.114\left(\frac{\vartheta_2^2}{V^2}\right)$	$-0.14 < \varepsilon_{ee}^{eL} < 0.09$ $-0.033 < \varepsilon_{\mu\mu}^{eL} < 0.055$ $-0.6 < \varepsilon_{\tau\tau}^{eL} < 0.4$
$\varepsilon_{\ell\ell}^{eR} \approx -2S_W^2\varepsilon_{\ell\ell}^{eL} - \frac{\vartheta_2^2}{V^2}T_W^4$	$-0.143\left(\frac{\vartheta_2^2}{V^2}\right)$	$-0.03 < \varepsilon_{ee}^{eR} < 0.18$ $-0.040 < \varepsilon_{\mu\mu}^{eR} < 0.053$ $-0.4 < \varepsilon_{\tau\tau}^{eR} < 0.6$
$\varepsilon_{\ell\ell}^{uL} \approx -\frac{\vartheta_1^2}{2V^2} + \frac{\vartheta_2^2}{24V^2C_W^4}(9-8S_W^2)$	$0.50\left(\frac{\vartheta_2^2-\vartheta_1^2}{V^2}\right)$	$-1 < \varepsilon_{ee}^{uL} < 0.3$ $\lvert\varepsilon_{\mu\mu}^{uL}\rvert < 0.003$ $\lvert\varepsilon_{\tau\tau}^{uL}\rvert < 1.4$
$\varepsilon_{\ell\ell}^{uR} \approx \frac{\vartheta_2^2}{6V^2}\frac{S_W^2}{C_W^4}$	$0.065\left(\frac{\vartheta_2^2}{V^2}\right)$	$-0.4 < \varepsilon_{ee}^{uR} < 0.7$ $-0.008 < \varepsilon_{\mu\mu}^{uR} < 0.003$ $\lvert\varepsilon_{\tau\tau}^{uR}\rvert < 3$
$\varepsilon_{\ell\ell}^{dL} \approx \frac{\vartheta_2^2}{24V^2C_W^4}(3-2S_W^2)$	$0.179\left(\frac{\vartheta_2^2}{V^2}\right)$	$-0.3 < \varepsilon_{ee}^{dL} < 0.3$ $\lvert\varepsilon_{\mu\mu}^{dL}\rvert < 0.003$ $\lvert\varepsilon_{\tau\tau}^{dL}\rvert < 1.1$
$\varepsilon_{\ell\ell}^{dR} \approx -\frac{S_W^2\vartheta_2^2}{12V^2C_W^4}$	$-0.033\left(\frac{\vartheta_2^2}{V^2}\right)$	$-0.6 < \varepsilon_{ee}^{dR} < 0.5$ $-0.008 < \varepsilon_{\mu\mu}^{dR} < 0.015$ $\lvert\varepsilon_{\tau\tau}^{dR}\rvert < 6$

our results with those obtained in cross-section measurements. We will assume $\sin^2\theta_W = 0.23149(13)$.

In Table 1 we can see that constrains in $\varepsilon_{\ell\ell}^{eP}$ lead to $V^2 > 4.7\vartheta_2^2$., while the constrains in $\varepsilon_{\ell\ell}^{uR}$ lead to $V^2 > 21.7\vartheta_2^2$, and the constrains in $\varepsilon_{\ell\ell}^{dP}$ ($|\varepsilon_{\mu\mu}^{dL}| < 0.003$) lead to $V^2 > 60\vartheta_2^2$. If ϑ_2 has its maximum value of 174.105 GeV, then $V \gtrsim 1.3$ TeV. We note also that by $|\varepsilon_{\mu\mu}^{uL}| < 0.003$ we obtain $|\vartheta_2^2-\vartheta_1^2| < 0.006\,V^2$; then, for $V \sim 1.3$ TeV and $\vartheta_2 = 174$ GeV, we obtain 142 GeV< $\vartheta_1 < 201$ GeV. We therefore cannot predict any hierarchy to the VEV's ϑ_1 and ϑ_2. Based on those results, we obtain the following inferior limits for the new gauge bosons masses:

$$
\begin{gathered}
M_{K_I} = M_{Z'} > 610\,\text{GeV}, \\
M_{K'} > 613\,\text{GeV}, \\
M_{K_R} > 740\,\text{GeV}.
\end{gathered}
\tag{5.1}
$$

6. Conclusion

We presented in this work a procedure to show that models with extended gauge symmetries $SU(3)_C \times SU(3)_L \times U(1)_X$ can lead to neutrino nonstandard interactions, respecting the Standard Model Gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$, without spoiling the available experimental data and reproducing the known phenomenology at low energies. We also have shown that with an assumption about a mass hierarchy for the Higgs triplets VEV's we could qualitatively address the mass hierarchy problem in standard model. Finally we obtained limits for the triplets VEV's based on limits for NSI in cross-section experiments.

We believe that the class of model presented here is an interesting theoretical possibility to look for new physics beyond SM. We restrained our work to a simple scenario, but flavor-changing interactions can be naturally introduced in the model, leading to new constraints on NSI.

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Research Article

Sources of FCNC in $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ Models

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There are different models which are based on the gauge symmetry $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (331), and some of them include exotic particles, and others are constructed without any exotic charges assigned to the fermionic spectrum. Each model build-up on 331 symmetry has its own interesting properties according to the representations of the gauge group used for the fermionic spectrum, that is, the main reason to explore and identify the possible sources of flavor changing neutral currents and lepton flavor violation at tree level.

1. Introduction

The standard model (SM) [1–3] has been successful to describe leptons, quarks, and their interactions. But in any case, the SM leaves open questions concerning to the electroweak symmetry breaking sector of the model, as well as the particle content of the model: why there are three generations of quarks and leptons? These questions, among others, are the motivation to consider the SM as one important attempt to understand the elementary particles of nature and their interactions but not to consider the SM as the ultimate theory of nature. A common alternative to look for new physics beyond the SM is enlarging the gauge symmetry group, one of these alternatives is the gauge symmetry $SU(3)_C \times SU(3)_L \times U(1)_X$ (331) [4–11]. There are many motivations for this new gauge symmetry group, one of them is that there are some of models are based on 331 symmetry that explain why the family number must be three. This result is obtained from the anomaly-free condition which is satisfied when equal number of triplets and antitriplets (taking into account the $SU(3)_C$) are present and requiring the sum of all fermion charges to vanish, but even that each generation is anomalous and the anomaly cancellation is given for three generations or multiply of three. Other motivation is concerned with the feature that $\sin^2\theta_W$ in this model should be less than

1/4, it is related to the ratio of the coupling constants g' and g of $U(1)_X$ and $SU(3)_L$,

$$\left(\frac{g'}{g}\right)^2 = \frac{\sin^2\theta_W}{1 - 4\sin^2\theta_W}, \tag{1.1}$$

in this model, there is an energy scale at which the perturbative character is lost, and the energy scale is found using the condition $\sin^2\theta_W = 1/4$ and it is order of ~ 4 TeV [12].

On the other hand, in the breaking symmetry of the 331 gauge symmetry to the gauge group of the SM and then to the $U(1)_Q$, some new bosons appear such as a new neutral Z' boson which is heavier than the SM gauge bosons and in all the 331 models it can mediate flavor changing process at tree level. In contrast, in the framework of the SM it is well known that flavor changing neutral currents (FCNC) are strongly suppressed, because they appear only at one loop level. Therefore, these FCNC processes can help to put stringent bounds on the parameter space of these kind of models [13–20]. Our aim in this work is to review the possible models that can be built on the basis of extended gauge symmetry 331 and identify the different sources of FCNC in the quark sector as well as the lepton sector.

2. 331 Models

The gauge group to be consider is $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$. Left-handed particles are into $SU(3)_L$ triplets, there are the usual quarks and leptons plus new exotic particles, and the anomaly-free condition constrains the allowed fermion representations (3 or 3^*) and the quantum numbers. To describe the particle content of the model and to identify specific types of 331 models is important and so is how defined the electric charge operator, which can be written as a linear combination of the diagonal generators of the group

$$Q = T_3 + \beta T_8 + X, \tag{2.1}$$

where β is a parameter that characterizes the specific particle structure. The parameter β can be chosen $\beta = \pm\sqrt{3}$ or $\beta = \pm 1/\sqrt{3}$, obtaining 331 models with exotic electric charges or 331 models without exotic electric charges, and by exotic charges we mean charges different from those that appear in the SM framework.

Since each lepton family has three states, taking massless neutrinos, they can be arranged into $SU(3)_L$ antitriplets $\psi_i^T = (l_i^-, -\nu_i, l_i^+)$, where i is a family index. The first two components corresponds to the ordinary electroweak doublet. This model corresponds to $\beta = \sqrt{3}$ [4–11] for the charge operator in (2.1). Therefore each lepton family will be in the $(1, 3^*)_0$ representation of $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$. With these assumptions, there are no new leptons in the 331 model, and all three lepton families are treated identically. In contrast, one of the three quark families transform differently from the other two which is required to anomaly cancellation. Anomaly cancellation requires that two families of quarks transform as triplets $(3,3)_{-1/3}$, and the third one transforms as an antitriplet $(3,3^*)_{2/3}$. The right-handed spectrum is put in singlets in the usual way $(3^*,1)_{-2/3,1/3,4/3}$ for the first two families and $(3^*,1)_{-5/3,-2/3,1/3}$ for the third one. It is worth to notice that in general the assumption that one quark family is transforming differently to the other two families is a general condition in the framework of 331 models, and it is generally assumed that the unique generation corresponds to the third generation, and then, it could explain the heavy top quark mass.

In the gauge sector, five new gauge bosons beyond the SM are found. The new gauge bosons form a complex $SU(2)_L$ doublet of dileptons (Y^{++}, Y^{+}) with hypercharge 3 and a singlet W_μ^8. The gauge boson W_μ^8 mixes with the gauge boson X from the $U(1)_X$ to form the hypercharge B_μ boson and a new neutral Z'_μ boson.

In order to break the symmetry spontaneously, four Higgs multiplets are necessary. Three triplets in representations $(1,3)_1$, $(1,3)_0$, and $(1,3)_{-1}$ for the breaking of $SU(2)\times U(1)$ in order to give masses to all quarks, and a sextet $(1,6)_0$ is required for the lepton masses [4–11].

In this first model [4–11], there are new sources of FCNC processes at tree level coming from the new Z' boson in the quark sector, because the families are treated differently. Also, at one loop level appears new contributions coming from the charged bileptons and the charged scalar sector [21–26]. In this model, there are FCNC in the lepton sector, and they are mediated by the charged bileptons [21–26].

A possible variation of this original model is to consider a new lepton assignment using a heavy lepton E^+ instead of the e^c and adding e^c and E^- as singlets [27]. With this model, it is easy to generate small neutrino masses and lepton number violation can occur and one property of this model version is that bileptons only couple standard to exotic leptons [27].

On the other hand, it is possible to obtain models based on the gauge 331 symmetry but without new exotic charges for the fermions. One version of that is the model proposed by Özer [28], where it is introduced a right-handed neutrino. A systematic study of these kind of models was done in [29, 30]. According to the β value in (2.1), it is possible to get six different set of fermions and the fermion structure in order to avoid the quiral anomalies producing different 331 models. The fermion sets are four lepton sets and two quark sets.

The first set of leptons is

$$L_1 = \underset{(1,3)_{-2/3}}{\begin{pmatrix} \nu_i \\ e_i^- \\ E_i^- \end{pmatrix}} \Bigg| \underset{(1,1)_1}{e_i^+} \Bigg| \underset{(1,1)_1}{E_i^+} \tag{2.2}$$

using $i = 1,2,3$ as the family index and e_i, d_i, and u_i are the SM fermions and E_i, D_i, and U_i are the exotic ones.

The second set is

$$L_2 = \underset{(1,3^*)_{-1/3}}{\begin{pmatrix} e_i^- \\ \nu_i \\ N_i^0 \end{pmatrix}} \Bigg| \underset{(1,1)_1,}{e_i^+} \tag{2.3}$$

where there is a neutral exotic particle. For the third leptonic set,

$$L_3 = \underset{(1,3^*)_{-1/3}}{\begin{pmatrix} e_i^- \\ \nu_i \\ N_1^0 \end{pmatrix}} \Bigg| \underset{(1,3^*)_{-1/3}}{\begin{pmatrix} E_i^- \\ N_2^0 \\ N_3^0 \end{pmatrix}} \Bigg| \underset{(1,3^*)_{2/3},}{\begin{pmatrix} N_4^0 \\ E_i^+ \\ e_i^+ \end{pmatrix}} \tag{2.4}$$

Table 1: Anomalies for the six fermion sets.

Anomalies	L_1	L_2	L_3	L_4	Q_1	Q_2
$SU(3)_c^2U(1)_X$	0	0	0	0	0	0
$SU(3)_L^2U(1)_X$	−2/3	−1/3	0	−1	1	0
$\text{grav}^2U(1)_X$	0	0	0	0	0	0
$U(1)_X^3$	10/9	8/9	6/9	12/9	−12/9	−6/9

where there is a charged exotic particle and four new exotic neutral ones. Finally, for the forth set

$$L_4 = \underset{(1,3)_{-2/3}}{\begin{pmatrix} \nu_i \\ e_i^- \\ E_{1i}^- \end{pmatrix}} \Bigg| \underset{(1,3)_{1/3}}{\begin{pmatrix} E_{2i}^- \\ N_1^0 \\ N_2^0 \end{pmatrix}} \Bigg| \underset{(1,3)_{-2/3}}{\begin{pmatrix} N_3^0 \\ E_{2i}^- \\ E_{3i}^- \end{pmatrix}} \Bigg| \underset{(1,1)_1}{e_i^+} \Bigg| \underset{(1,1)_1}{E_{1i}^+} \Bigg| \underset{(1,1)_1,}{E_{3i}^+} \tag{2.5}$$

with three exotic charged particles and three neutral.

Now, the quark sets are

$$Q_1 = \underset{(3,3^*)_{1/3}}{\begin{pmatrix} d_i \\ u_i \\ U_i \end{pmatrix}} \Bigg| \underset{(3,1)_{1/3}}{d_i} \Bigg| \underset{(3,1)_{-2/3}}{u_i} \Bigg| \underset{(3,1)_{-2/3},}{U_i} \tag{2.6}$$

$$Q_2 = \underset{(3,3)_0}{\begin{pmatrix} u_i \\ d_i \\ D_i \end{pmatrix}} \Bigg| \underset{(3,1)_{-2/3}}{u_i} \Bigg| \underset{(3,1)_{1/3}}{d_i} \Bigg| \underset{(3,1)_{1/3}.}{D_i} \tag{2.7}$$

The anomaly contribution for each set is presented in Table 1.

On the basis of Table 1, it is possible to build up many models asking for the anomaly free condition. There are two one family models and eight three family models, referring to how cancel out the anomalies if it is needed one family or the three families. There are two one family models composed by the sets $Q_2 + L_3$ and $Q_1 + L_4$. These models were studied in [29–31] and their relation with the grand unified theories established. For the three family models, there are the combinations $3L_2 + Q_1 + 2Q_2$, $3L_1 + 2Q_1 + Q_2$, $2(Q2 + L3) + (Q1 + L4)$ and $2(Q1 + L4) + (Q2 + L3)$, and there are other two models particularly interesting, because they treat the three family leptons completely different, they are the combinations $L_1 + L_2 + L_3 + Q_1 + 2Q_2$ and $L_1 + L_2 + L_4 + 2Q_1 + Q_2$ [31, 32].

In the gauge sector, there are 17 gauge bosons, one gauge boson B^μ associated to $U(1)_X$, eight gluons associated to $S(3)_c$ and eight gauge fields from the $SU(3)_L$. The gauge bosons associated with $SU(3)_L$ transform according to the adjoint representation of the group, and they can be written as

$$\mathbf{W}_\mu = W_\mu^a \frac{\lambda^a}{2} = \frac{1}{2}\begin{pmatrix} W_\mu^3 + \frac{1}{\sqrt{3}}W_\mu^8 & \sqrt{2}W_\mu^+ & \sqrt{2}K_{1\mu} \\ \sqrt{2}W_\mu^- & -W_\mu^3 + \frac{1}{\sqrt{3}}W_\mu^8 & \sqrt{2}K_{2\mu} \\ \sqrt{2}\overline{K}_{1\mu} & \sqrt{2}\overline{K}_{2\mu} & -\frac{2}{\sqrt{3}}W_\mu^8 \end{pmatrix}, \tag{2.8}$$

where λ^a are the Gell-Mann matrices, and the electric charges of K_1 and K_2 are given by $Q_1 = 1/2 + \sqrt{3}\beta/2$ and $Q_2 = 1/2 - \sqrt{3}\beta/2$, respectively.

In general, it is convenient to rotate the neutral gauge bosons W_μ^3, W_μ^8 and B_μ into new states A_μ, Z_μ, and Z'_μ, given by

$$\begin{pmatrix} A_\mu \\ Z_\mu \\ Z'_\mu \end{pmatrix} = \begin{pmatrix} S_W & \beta S_W & C_W\sqrt{1-\beta^2 T_W^2} \\ C_W & -\beta S_W T_W & -S_W\sqrt{1-\beta^2 T_W^2} \\ 0 & -\sqrt{1-\beta^2 T_W^2} & \beta T_W \end{pmatrix}\begin{pmatrix} W_\mu^3 \\ W_\mu^8 \\ B_\mu \end{pmatrix}, \tag{2.9}$$

where the angle θ_W is defined by $T_W = \tan\theta_W = g'/\sqrt{g^2+\beta^2 g'^2}$, g, g' being the coupling constants associated to the groups $SU(3)_L$ and $U(1)_X$, respectively, ($S_W = \sin\theta_W$, etc.). In the new basis, A_μ (the photon) is the gauge boson corresponding to the generator Q, while Z_μ can be identified with the SM Z boson. As in the SM, the extended electroweak symmetry is spontaneously broken in 331 models by the presence of elementary scalars having nonzero vacuum expectation values [33–37]. The symmetry breakdown follows a hierarchy

$$SU(3)_L \otimes U(1)_X \xrightarrow{V} SU(2)_L \otimes U(1)_Y \xrightarrow{v} U(1)_Q, \tag{2.10}$$

in which two VEV scales V and v, with $V \gg v$, are introduced. The photon is kept as the only massless gauge boson, while the remaining neutral gauge bosons get mixed. In this way, Z and Z' turn out to be only approximate mass eigenstates.

3. FCNC in 331 Models

First of all, the extension of the gauge group which embedded the SM group implies a new neutral gauge Z' boson, which in general in all the 331 models presented generates FCNC at tree-level. This fact is that because in 331 models it is not possible to accommodate all the SM spectrum in multiplets with the same quantum numbers; therefore, the Z' couplings are not universal for all the fermions, and that is the origin of a new source of FCNC. Particularly, to treat in a different manner the third generation, as is usually assumed, to the other two generations produces FCNC contributions. This property is common to all the 331 models in the quark sector. It is worth to mention that even in the left-handed couplings of the standard fermions to the Z neutral boson appear FCNC at tree level through the mixing of $Z - Z'$ and also coming from the mixing between the standard quarks and the exotic ones included in each case. Moreover, the mixing between neutral gauge bosons should take into account the gauge bosons which transform according to the adjoint representation of the $SU(3)_L$, some noted $K^{\pm}$ and K^0 gauge bosons for the charged sector and neutral sector (they are related to $K_{1,2}$). In order to notice these effects clearly, the Lagrangian for the new Z' boson with a β arbitrary is the following:

$$\begin{aligned}
\mathcal{L}^{Z'} = -\frac{g'}{2T_W}Z^{\mu'}\Bigg[&\sum_{m=1}^{2}\overline{D_m^0}\gamma_\mu\left(\frac{P_L}{\sqrt{3}}+\frac{T_W^2\beta}{3}(P_L-2P_R)\right)D_m^0 \\
&+\overline{D_3^0}\gamma_\mu\left(-\frac{P_L}{\sqrt{3}}+\frac{T_W^2\beta}{3}(P_L-2P_R)\right)D_3^0+\sum_{m=1}^{2}\overline{U_m^0}\gamma_\mu\left(\frac{P_L}{\sqrt{3}}+\frac{T_W^2\beta}{3}(P_L+4P_R)\right) \\
&\times U_m^0+\overline{U_3^0}\gamma_\mu\left(-\frac{P_L}{\sqrt{3}}+\frac{T_W^2\beta}{3}(P_L+4P_R)\right)U_3^0+\overline{L^0}\gamma_\mu\left(-\frac{P_L}{\sqrt{3}}-T_W^2\beta(P_L+2P_R)\right) \\
&\times L^0+\overline{\nu^0}\gamma_\mu\left(-\frac{1}{\sqrt{3}}-T_W^2\beta\right)P_L\nu^0+\sum_{m=1}^{2}\overline{J^0}_m\gamma_\mu\left(-\frac{2P_L}{\sqrt{3}}+T_W^2\left(\frac{1}{3}+\frac{3\beta}{\sqrt{3}}\right)\right)J_m^0 \\
&+\overline{J_3^0}\gamma_\mu\left(-\frac{2P_L}{\sqrt{3}}+T_W^2\left(\frac{1}{3}-\frac{3\beta}{\sqrt{3}}\right)(P_L-P_R)\right)J_3^0 \\
&+\overline{E^0}\gamma_\mu\left(\frac{2P_L}{\sqrt{3}}+T_W^2\left(-\frac{1}{3}-\frac{3\beta}{\sqrt{3}}\right)\times(-P_L+P_R)\right)E^0\Bigg],
\end{aligned} \tag{3.1}$$

where $D^0 = (d_1^0\ d_2^0\ d_3^0)^T$, $U^0 = (u_1^0\ u_2^0\ u_3^0)^T$, $L^0 = (e_1^0\ e_2^0\ e_3^0)^T$, $E^0 = (E_1^0\ E_2^0\ E_3^0)^T$, and the exotic quarks j_i^0 with electric charges are given by $q_{J_1} = Q_{J_2} = 1/6 + \sqrt{3}\beta/2$ and $q_{J_3} = 1/6 - \sqrt{3}\beta/2$. There is explicitly shown the no universal couplings between the quarks D_i, U_i and the Z' boson, and it is because one family is in the 3 representation, while the other two are in the 3^* (or vice versa). As a consequence, the FCNCs arise once the fields U_i and D_i are rotated to the mass eigenstates. The number of extra fermions up-quark type or down-quark type depends on the parameter β, for $\beta = -1/\sqrt{3}$ will have $N_U = 1$ and $N_D = 2$ and for $\beta = 1/\sqrt{3}$ will have $N_U = 2$ and $N_D = 1$. Therefore, there is not only FCNC at tree level through the Z' boson, but also the usual Z boson due to the mix of these new exotic quarks with the ordinary ones. To notice this, for the case of $\beta = +1/\sqrt{3}$, the following definitions are useful $U_0^T = (u_1^0, u_2^0, u_3^0, T_1^0, T_2^0)$, $D_0^T = (d_1^0, d_2^0, d_3^0, B_1^0)$, $E_0^T = (e^0, \mu^0, \tau^0, E_1^0, E_2^0, E_3^0)$, and $N_0^T = (\nu_e^0, \nu_\mu^0, \nu_\tau^0)$. Meanwhile, for the case $\beta = -1/\sqrt{3}$, the definitions are $U_0^T = (u_1^0, u_2^0, u_3^0, T_1^0)$, $D_0^T = (d_1^0, d_2^0, d_3^0, B_1^0, B_2^0)$, $E_0^T = (e^0, \mu^0, \tau^0)$, and $N_0^T = (\nu_e^0, \nu_\mu^0, \nu_\tau^0, , N_1^0, N_2^0, N_3^0)$. With this vector notation, the Lagrangian for neutral currents is

$$\begin{aligned}
\mathcal{L}_{\mathrm{NC}} = \sum_{\Psi} &-\frac{gZ^\mu}{2C_W}\left\{\overline{\Psi^0}\gamma_\mu\epsilon_{\Psi_{(L)}}^{(1)}P_L\Psi^0+\overline{\Psi^0}\gamma_\mu\epsilon_{\Psi_{(R)}}^{(1)}P_R\Psi^0\right\} \\
&-\frac{g'Z'^\mu}{2\sqrt{3}S_WC_W}\left\{\overline{\Psi^0}\gamma_\mu\epsilon_{\Psi_{(L)}}^{(2)}P_L\Psi^0+\overline{\Psi^0}\gamma_\mu\epsilon_{\Psi_{(R)}}^{(2)}P_R\Psi^0\right\} \\
&-\frac{g}{\sqrt{2}}\left\{\overline{\Psi^0}\gamma_\mu\epsilon_{\Psi_{(L)}}^{(3)}P_L\Psi^0\,\mathrm{Re}\,K^\mu+i\overline{\Psi^0}\gamma_\mu\epsilon_{\Psi_{(L)}}^{(4)}P_L\Psi^0\,\mathrm{Im}\,K^\mu\right\},
\end{aligned} \tag{3.2}$$

where the sum is over U_0, D_0, E_0, and N_0. The couplings $\epsilon^{(1,2)}_{\Psi_{(L,R)}}$ depends on the parameter β. With $\beta = \pm 1/\sqrt{3}$, the Z^0 interaction is

$$\begin{aligned}
\epsilon^{(1)}_{\mathfrak{U}_{(L)}} &= \left(C_W^2 - \frac{S_W^2}{3}\right)\mathbf{1}_{(3+N_U^{\pm})\times(3+N_U^{\pm})} - \begin{pmatrix} 0_{(3\times3)} & \\ & \mathbf{1}_{(N_U^{\pm}\times N_U^{\pm})}\end{pmatrix}, \\
\epsilon^{(1)}_{\mathfrak{U}_{(R)}} &= -\left(\frac{4S_W^2}{3}\right)\mathbf{1}_{(3+N_U^{\pm})\times(3+N_U^{\pm})}, \\
\epsilon^{(1)}_{\mathfrak{D}_{(L)}} &= \left(-C_W^2 - \frac{S_W^2}{3}\right)\mathbf{1}_{(3+N_D^{\pm})\times(3+N_D^{\pm})} + \begin{pmatrix} 0_{(3\times3)} & \\ & \mathbf{1}_{(N_D^{\pm}\times N_D^{\pm})}\end{pmatrix}, \\
\epsilon^{(1)}_{\mathfrak{D}_{(R)}} &= +\left(\frac{2S_W^2}{3}\right)\mathbf{1}_{(3+N_D^{\pm})\times(3+N_D^{\pm})},
\end{aligned} \tag{3.3}$$

where the no universality of the left handed quarks is clear, while the right-handed couplings drive for $\epsilon^{(1)}_{\mathfrak{U},\mathfrak{D}_{(R)}}$ are universals.

In a similar way for the Z'^0 boson, the couplings are

$$\begin{aligned}
\epsilon^{(2)}_{\mathfrak{U}_{(L)}} &= \left(C_W^2 \pm \frac{S_W^2}{3}\right)\mathbf{1}_{(3+N_U^{\pm})\times(3+N_U^{\pm})} - 2C_W^2\begin{pmatrix} 0_{(2\times2)} & \\ & \mathbf{1}_{(N_T^{\pm}+1)\times(N_T^{\pm}+1)}\end{pmatrix} \\
&\quad + \left(C_W^2 \mp 2C_W^2 \pm S_W^2\right)\begin{pmatrix} 0_{(3\times3)} & \\ & \mathbf{1}_{(N_U^{\pm}\times N_U^{\pm})}\end{pmatrix}, \\
\epsilon^{(2)}_{\mathfrak{U}_{(R)}} &= \pm\frac{4S_W^2}{3}\mathbf{1}_{(3+N_U^{\pm})\times(3+N_U^{\pm})}, \\
\epsilon^{(2)}_{\mathfrak{D}_{(L)}} &= \left(C_W^2 \pm \frac{S_W^2}{3}\right)\mathbf{1}_{(3+N_D^{\pm})\times(3+N_D^{\pm})} - 2C_W^2\begin{pmatrix} 0_{(2\times2)} & \\ & \mathbf{1}_{(N_D^{\pm}+1)\times(N_D^{\pm}+1)}\end{pmatrix} \\
&\quad + \left(C_W^2 \pm 2C_W^2 \mp S_W^2\right)\begin{pmatrix} 0_{(3\times3)} & \\ & \mathbf{1}_{(N_D^{\pm}\times N_D^{\pm})}\end{pmatrix}, \\
\epsilon^{(2)}_{\mathfrak{D}_{(R)}} &= \mp\left(\frac{2S_W^2}{3}\right)\mathbf{1}_{(3+N_D^{\pm})\times(3+N_D^{\pm})}.
\end{aligned} \tag{3.4}$$

At this point, it is important to mention that the couplings in (3.2)–(3.4) are in the interaction basis, thus to obtain the mass eigenstates, it is necessary to get the rotation matrices which diagonalize the mass matrices in the Yukawa sector. Therefore, the mass eigenstates U and D are defined by

$$U_L^0 = V_L^u U_L, \qquad D_L^0 = V_L^d D_L, \tag{3.5}$$

with matrices V_L of dimensions $(3 + N_U^{\pm}) \times (3 + N_U^{\pm})$ and $(3 + N_D^{\pm}) \times (3 + N_D^{\pm})$, respectively. It is useful to write the matrices $V_L^{u,d}$ as

$$V_L^u = \begin{pmatrix} V^u_{0\,(3\times 3)} & V^u_{X\,(3\times N_U^{\pm})} \\ V^u_{Y\,(N_U^{\pm}\times 3)} & V_{U\,(N_U^{\pm}\times N_U^{\pm})} \end{pmatrix}, \qquad V_L^d = \begin{pmatrix} V^d_{0\,(3\times 3)} & V^d_{X\,(3\times N_D^{\pm})} \\ V^d_{Y\,(N_D^{\pm}\times 3)} & V_{D\,(N_D^{\pm}\times N_D^{\pm})} \end{pmatrix}, \tag{3.6}$$

using submatrices in such a way that $V_{\mathrm{CKM}} = V_0^{u\dagger} V_0^d$ and in general the CKM matrix is not unitary.

In addition, the models include new gauge bosons K_μ which coupled to the left handed fermions, the couplings in (3.2) for the K_2^μ boson when $\beta = 1/\sqrt{3}$ are

$$\begin{gathered} \epsilon^{(3)}_{U_{(L)}} = \begin{pmatrix} 0_{2\times 2} & & \mathbf{1}_{2\times 2} \\ & 0 & \\ \mathbf{1}_{2\times 2} & & 0_{2\times 2} \end{pmatrix}, \qquad \epsilon^{(3)}_{D_{(L)}} = \begin{pmatrix} 0_{2\times 2} & & \\ & 0 & 1 \\ & 1 & 0 \end{pmatrix}, \\ \epsilon^{(3)}_{E_{(L)}} = \begin{pmatrix} & \mathbf{1}_{3\times 3} \\ \mathbf{1}_{3\times 3} & \end{pmatrix}, \qquad \epsilon^{(4)}_{U_{(L)}} = \begin{pmatrix} 0_{2\times 2} & & \mathbf{1}_{2\times 2} \\ & 0 & \\ -\mathbf{1}_{2\times 2} & & 0_{2\times 2} \end{pmatrix}, \\ \epsilon^{(4)}_{D_{(L)}} = \begin{pmatrix} 0_{2\times 2} & & \\ & 0 & -1 \\ & 1 & 0 \end{pmatrix}, \qquad \epsilon^{(4)}_{E_{(L)}} = \begin{pmatrix} & \mathbf{1}_{3\times 3} \\ -\mathbf{1}_{3\times 3} & \end{pmatrix}. \end{gathered} \tag{3.7}$$

And when $\beta = -1/\sqrt{3}$ for the K_1^μ boson, they are

$$\begin{gathered} \epsilon^{(3)}_{U_{(L)}} = \begin{pmatrix} 0_{2\times 2} & & \\ & 0 & 1 \\ & 1 & 0 \end{pmatrix}, \qquad \epsilon^{(3)}_{D_{(L)}} = \begin{pmatrix} 0_{2\times 2} & & -\mathbf{1}_{2\times 2} \\ & 0 & \\ -\mathbf{1}_{2\times 2} & & 0_{2\times 2} \end{pmatrix}, \\ \epsilon^{(3)}_{N_{(L)}} = \begin{pmatrix} & \mathbf{1}_{3\times 3} \\ \mathbf{1}_{3\times 3} & \end{pmatrix}, \qquad \epsilon^{(4)}_{D_{(L)}} = \begin{pmatrix} 0_{2\times 2} & & \\ & 0 & 1 \\ & -1 & 0 \end{pmatrix}, \\ \epsilon^{(4)}_{U_{(L)}} = \begin{pmatrix} 0_{2\times 2} & & \mathbf{1}_{2\times 2} \\ & 0 & \\ -\mathbf{1}_{2\times 2} & & 0_{2\times 2} \end{pmatrix}, \qquad \epsilon^{(4)}_{N_{(L)}} = \begin{pmatrix} & -\mathbf{1}_{3\times 3} \\ \mathbf{1}_{3\times 3} & \end{pmatrix}. \end{gathered} \tag{3.8}$$

It is often assumed that the vacuum expectation values of the scalar fields are real, and this assumption implies that there is not any spontaneous CP symmetry breaking. In that case, the state $\mathrm{Im}\,K$ decouples and therefore turn into an exact mass eigenstate. However, the bosons Z^μ, $Z^{'\mu}$ and $\sqrt{2}\,\mathrm{Re}\,K$ mix, and it is possible to get the mass basis (Z_1, Z_2, and Z_3)

through an orthogonal matrix which depends on the vacuum expectation values of the Higgs bosons,

$$\begin{pmatrix} Z \\ Z' \\ \sqrt{2}\,\mathrm{Re}\,K \end{pmatrix} = R \begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \end{pmatrix}, \tag{3.9}$$

and therefore, the Lagrangian can be rewritten as

$$\begin{aligned} \mathcal{L}_{\mathrm{NC}} = -\sum_{\Psi=\mathcal{U},\mathcal{D}} \Bigg[& Q_\Psi \overline{\Psi}\gamma^\mu \Psi A_\mu + \sum_{j,k=1}^{3} g_j \overline{\Psi}\gamma^\mu \left(E^{(j)}_{\Psi_L} P_L + E^{(j)}_{\Psi_R} P_R \right) \Psi R_{jk} Z_{k\mu} \\ & + i\frac{g}{2} \overline{\Psi}\gamma^\mu \left(E^{(4)}_{\Psi_L} P_L + E^{(4)}_{\Psi_R} P_R \right) \Psi \sqrt{2}\,\mathrm{Im}\,K_\mu \Bigg], \end{aligned} \tag{3.10}$$

where Q_Ψ is the electric charge and the coupling constants g_j are

$$g_1 = \frac{g}{2C_W}, \qquad g_2 = \frac{g'}{2\sqrt{3}S_W C_W} = \frac{g}{2\sqrt{3}C_W\sqrt{C_W^2 - \beta^2 S_W^2}}, \qquad g_3 = \frac{g}{2}, \tag{3.11}$$

and the matrices $E^{(i)}_{\Psi_{L,R}}$ are given by

$$E^{(i)}_{\Psi_L} = V_L^{\Psi\dagger} \epsilon^{(i)}_{\Psi_L} V_L^{\Psi}, \qquad E^{(i)}_{\Psi_R} = V_R^{\Psi\dagger} \epsilon^{(i)}_{\Psi_R} V_R^{\Psi} = \epsilon^{(i)}_{\Psi_R}. \tag{3.12}$$

Finally, about the sources of FCNC in the framework of the 331 models, they are two models which are very interesting, because they have some special properties from the phenomenological point of view. They are the models build up with the fermionic sets $L_1 + L_2 + L_3 + Q_1 + 2Q_2$ and $L_1 + L_2 + L_4 + 2Q_1 + Q_2$. They not only differentiate the quark generations, doing one family specially different, but they also do in the leptonic sector. These models will have the usual FCNC at tree level in 331 models in the quark sector through the Z' boson but also they present FCNC in the leptonic sector through the scalar fields and through the Z' boson [32]. To notice the new sources of FCNC arising in these models, the neutral current Lagrangian is going to be obtained. First of all, the spectrum should be specified

$$\ell_{1L} = \begin{pmatrix} \nu_1 \\ e_1^- \\ E_1^- \end{pmatrix}_L, \qquad \ell_{mL} = \begin{pmatrix} e_m^- \\ \nu_m \\ N_k^0 \end{pmatrix}_L, \qquad \ell_{5L} = \begin{pmatrix} E_2^- \\ N_3^0 \\ N_4^0 \end{pmatrix}_L, \qquad \ell_{4L} = \begin{pmatrix} N_5^0 \\ E_2^+ \\ e_3^+ \end{pmatrix}_L, \tag{3.13}$$

where $m = 2,3$, $k = 1,2$ and note that one of the leptonic triplets is in the adjoint representation respect to the other two then FCNC at tree level will arise through the Z' boson. Using vector notation, the neutral current Lagrangian in this case is

$$\begin{aligned}\mathcal{L}_{\rm NC} = \sum_{\Psi}\Bigg[& A_\mu \left\{\overline{\Psi^0}\gamma_\mu \epsilon^{A}_{\Psi_{(L)}} P_L \Psi^0 + \overline{\Psi^0}\gamma_\mu \epsilon^{A}_{\Psi_{(R)}} P_R \Psi^0\right\} \\ & + \frac{gZ^\mu}{2C_W}\left\{\overline{\Psi^0}\gamma_\mu \epsilon^{Z}_{\Psi_{(L)}} P_L \Psi^0 + \overline{\Psi^0}\gamma_\mu \epsilon^{Z}_{\Psi_{(R)}} P_R \Psi^0\right\} \\ & + \frac{g' Z'^\mu}{2\sqrt{3}S_W C_W}\left\{\overline{\Psi^0}\gamma_\mu \epsilon^{Z'}_{\Psi_{(L)}} P_L \Psi^0 + \overline{\Psi^0}\gamma_\mu \epsilon^{Z'}_{\Psi_{(R)}} P_R \Psi^0\right\}\Bigg].\end{aligned} \tag{3.14}$$

Defining the vector $E^T = (e_1^-, e_2^-, e_3^-, E_1^-, E_2^-)$, the couplings are

$$\begin{aligned}\epsilon^{A}_{E_{(L)}} &= gS_W I_{5\times 5}, \qquad \epsilon^{A}_{\mathcal{E}_{(R)}} = gS_W I_{5\times 5}, \\ \epsilon^{Z}_{E_{(L)}} &= \frac{g}{2C_W}\mathrm{Diag}\left(C_{2W}, C_{2W}, C_{2W}, -2S_W^2, C_{2W}\right), \\ \epsilon^{Z}_{E_{(R)}} &= \frac{g}{2C_W}\mathrm{Diag}\left(-2S_W^2, -2S_W^2, -2S_W^2, -2S_W^2, C_{2W}\right), \\ \epsilon^{Z'}_{E_{(L)}} &= \frac{g'}{2\sqrt{3}S_W C_W}\mathrm{Diag}(1, -C_{2W}, -C_{2W}, -C_{2W}, -C_{2W}), \\ \epsilon^{Z'}_{E_{(R)}} &= \frac{g'}{2\sqrt{3}S_W C_W}\mathrm{Diag}\left(2S_W^2, 2S_W^2, -C_{2W}, 2S_W^2, 1\right),\end{aligned} \tag{3.15}$$

where $C_{2W} = \cos(2\theta_W)$, and it is worthwhile to point out that the right handed couplings are not universal, and it is a new feature of this model. Usually, in the framework of the 331 models, only the left-handed couplings are not universal, but the right-handed are universal as it was shown in (3.1).

For the neutral sector, $N^T = (\nu_1^0, \nu_2^0, \nu_3^0, N_1^0, N_2^0, N_3^0, N_4^0)$ is defined, and the left-handed couplings are

$$\begin{aligned}\epsilon^{A}_{N_{(L)}} &= 0, \qquad \epsilon^{Z}_{N_{(L)}} = \frac{g}{2C_W}\mathrm{Diag}(1,1,1,0,0,1,0,-1), \\ \epsilon^{Z}_{N_{(L)}} &= \frac{g'}{2\sqrt{3}S_W C_W}\mathrm{Diag}\left(1, -C_{2W}, -C_{2W}, 2C_W^2, 2C_W^2, -C_{2W}, 2C_W^2, -1\right).\end{aligned} \tag{3.16}$$

4. Summary

One of the most intriguing options to consider physics beyond the SM consists of extending the gauge symmetry group to $SU(3)_C \times SU(3)_L \times U(1)_X$. There are many models which are based on the 331 symmetry, and one intriguing feature of these models is the presence of FCNC at tree level, but the source of that new interactions is not unique and depends on how the model is built up. In the Pleitez-Frampton model, the first one proposed, it was established the presence of FCNC at tree level coming from the new Z' boson and due to the different assignment of the quark representation for one of the quark families; doing the left-handed couplings between quarks and the Z' boson not universal. On the other hand, it is possible to build up models on the basis of 331 symmetry contrary to the Pleitez-Frampton model without any exotic charges for the new particles in the spectrum. These kind of models correspond to a $\beta = \pm 1/\sqrt{3}$ in the electric charge operator (2.1). These models include new exotic up-quark type and down-quark type which are going to mix with the standard quarks. In one version appears five up quark type and four down quark type, and another version include four-up quark type and five-down quark type; also, these models include extra charged leptons in one case and neutral leptons in the other one. The mixing obtained is a source of FCNC at tree level when the quark fields are written in the mass basis. There are also a new source of FCNC which is coming from the mixing in the gauge sector between the bosons (A, Z, Z', K). The mixing in this sector is usually reduced to the mixing between Z and Z'. If we consider the mixing between the quarks and the mixing (Z, Z'), then the FCNC interactions appear through the Z and the Z' mediation. In the leptonic sector, something similar is going to happen. Finally, there are models which not only treat different the quark families but the leptonic families too. One of these models is presented and the neutral current Lagrangian obtained, and one interesting new and additional feature is the nonuniversal couplings in the right-handed sector through the Z and Z' bosons. This new contributions to the FCNC processes could help to relax the bounds obtained on the Z' Boson mass.

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Research Article

Mass Mixing Effect and Oblique Radiative Corrections in Extended $SU(2)_R \times SU(2)_L \times U(1)$ Effective Theory

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We analyze the properties of electroweak chiral effective Lagrangian with an extended $SU(2)_R$ gauge group. Right-handed $SU(2)_R$ gauge bosons affect electroweak observables by mixing with electroweak gauge bosons $W_{L,\mu}$ and B_μ. We discuss all possible mass mixing terms and calculate the exact physical mass eigenvalues by diagonalization of mixing matrix without any approximate assumptions. The contributions to oblique radiative corrections parameters STU from $SU(2)_R$ fields are also presented.

1. Introduction

Although the standard model (SM) has been checked very successfully by more and more high energy physics experiments, the as yet undiscovered Higgs, introduced as a basic scaler field in SM, remains as the only unknown component of the electroweak symmetry-breaking mechanism (EWSBM) unknown. That situation has prompted many extensions to SM [1–3]. A new $SU(1)_R$ group, associated with an additional triplet of gauge bosons $W'^{\pm}$ and Z', is often considered for different reasons as an extension to the gauge symmetry [4–6]. This extension often appears in superstring-inspired models as well as GUT models [7]. The non-Abelian $SU(2)_R$ contains sufficient complexity to incorporate interesting issues related to spontaneous parity violation (SPV) and precise electroweak observables, although remains simple enough that phenomenology can be subjected to analysis. $SU(2)_R$ gauge bosons can improve unitarity of not only WW but also WZ scattering processes and delay the breaking scale of unitarity.

Many left-right symmetry models with symmetry group $SU(2)_R \times SU(2)_L \times U(1)$ have been used in studying EWSBM. The common feature of these models is the existence of multi-Higgs bosons that then raises phenomenological issues related to multi-Higgs structure dependencies. To obtain an universal physical analysis, we adopt the nonlinear realization of the chiral Lagrangian to describe extended $SU(2)_R$ electroweak gauge models given the symmetry breaking pattern $SU(2)_R \times SU(2)_L \times U(1) \rightarrow U(1)_{\mathrm{em}}$. This chiral Lagrangian has already been written down in [8]. The model is a generalization of the conventional linearly realized models with multi-Higgs. Within the extended non-Abelian chiral effective Lagrangian, multi-Higgs effects are parameterized by a set of coefficients that describes all possible interactions among the gauge bosons and provides a model-independent platform to investigate interesting physics [8].

In the paper, we focus on mass mixing effects in left-right chiral effective Lagrangian. Mass mixings are main focus in the contribution of the right-handed gauge bosons to electroweak observables at low-energy scales. The $SU(2)_R$ gauge triplet can be regarded as a copy of the $SU(2)_L$ gauge triplet of SM, but with heavier masses. Right-handed charged gauge bosons $W_R^{\pm}$ can mix with left-handed $W_L^{\pm}$, and physical mass eigenstates of $W'^{\pm}$ and $W^{\pm}$ are eigenvalues of the charged mass matrix. Similarly, W_R^3 takes part in $W_R^3 - W_L^3 - B$ three-body mixing to form physical massive neutral bosons Z', Z, and a massless photon. The nonlinearly realized chiral effective Lagrangian provides us with all possible mass-mixing channels that are allowed by left-right symmetry. Calculating these mixings, we obtain a complete mass mixing contribution to the electroweak observables and a largest parameter space for new physics. Oblique radiative corrections of $SU(2)_R$ bosons can be obtained from the mass mixing rotation matrix, which indicates shifts to the SM with new physics.

The paper is organized as follows. Section 2 reviews $SU(2)_R \times SU(2)_L \times U(1)$ effective theory with all possible mass mixing terms in the gauge eigenstates basis. Section 3 presents calculations of the charged and neutral mass eigenvalues to obtain physical boson masses estimates. We improved our diagonalization calculation program for the neutral bosons sector in our paper [8] to yield a set of exact solutions for the rotation matrix and the mass eigenvalues without making any approximating assumptions. Oblique radiative corrections coming from the nonstandard mass mixing beyond SM are studied in Section 4. Furthermore, two kinds of special cases are considered corresponding to condition $M_{W_R} \gg M_{W_L}$ case and left-right symmetry. Finally, we give a short summary in Section 5.

2. Left-Right Symmetry Effective Lagrangian

Let $W^a_{R,\mu}$, $W^a_{L,\mu}$, B_μ be electroweak gauge fields ($a = 1,2,3$) corresponding to the gauge group $SU(2)_R$, $SU(2)_L$, and $U(1)$, respectively, and $U_{L,R}$ be the two by two unitary unimodular matrices corresponding to left- and right-handed Goldstone boson fields. Under $SU(2)_R \otimes SU(2)_L \otimes U(1)$ transformations, the gauge boson fields transform as

$$\begin{aligned}
\frac{\tau^a}{2} W^a_{i,\mu} &\longrightarrow R_i \frac{\tau^a}{2} W^a_{i,\mu}(x) R_i^{\dagger} - \frac{i}{g_i} R_i \partial_\mu R_i^{\dagger}, \\
B_\mu &\longrightarrow B_\mu - \frac{1}{g} \partial_\mu \theta^0, \\
U_i &\longrightarrow R_i U_i R_0^{\dagger}
\end{aligned} \tag{2.1}$$

with $R_0 = e^{(i/2)\tau^3\theta^0(x)}$ and $R_i = e^{(i/2)\tau^a\theta_i^a(x)}$ for $i = R, L$. The covariant derivative of the Goldstone fields takes the form

$$\begin{aligned} D_\mu U_R &= \partial_\mu U_R + i g_R \frac{\tau^a}{2} W^a_{R,\mu} U_R - i g U_L \frac{\tau_3}{2} B_\mu, \\ D_\mu U_L &= \partial_\mu U_L + i g_L \frac{\tau^a}{2} W^a_{L,\mu} U_L - i g U_L \frac{\tau_3}{2} B_\mu. \end{aligned} \tag{2.2}$$

For convenience in present discussion, we will discard conventional EWCL $SU(2)$ covariant building blocks [9–13] and introduce $U(1)$ invariant building blocks (for $i = L, R$)

$$\begin{aligned} X_i^\mu &= U_i^\dagger (D^\mu U_i), \\ \overline{W}_{i,\mu\nu} &= U_i^\dagger g_i W_{i,\mu\nu} U_i, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu. \end{aligned} \tag{2.3}$$

Here,

$$W_{i,\mu\nu} = W^a_{i,\mu\nu}\frac{\tau^a}{2} = \partial_\mu W^a_{i,\nu}\frac{\tau^a}{2} - \partial_\nu W^a_{i,\mu}\frac{\tau^a}{2} + i g_i \left[W^a_{i,\mu}\frac{\tau^a}{2}, W^b_{i,\nu}\frac{\tau^b}{2}\right]. \tag{2.4}$$

With the help of these building blocks, we can write a leading-order chiral Lagrangian as

$$\begin{aligned} \mathcal{L}_M = &-\frac{1}{4} f_L^2 \left\langle X_{L,\mu} X_L^\mu \right\rangle - \frac{1}{4} f_R^2 \left\langle X_{R,\mu} X_R^\mu \right\rangle + \frac{1}{2}\tilde{\kappa} f_L f_R \left\langle X_L^\mu X_R^\mu \right\rangle \\ &+ \frac{1}{4}\beta_{L,1} f_L^2 \left\langle \tau^3 X_{L,\mu} \right\rangle^2 + \frac{1}{4}\beta_{R,1} f_L^2 \left\langle \tau^3 X_{R,\mu} \right\rangle^2 + \frac{1}{4}\tilde{\beta}_1 f_L f_R \left\langle \tau^3 X_{L,\mu} \right\rangle \left\langle \tau^3 X_R^\mu \right\rangle. \end{aligned} \tag{2.5}$$

Here, $\langle\ \rangle$ stands for the trace in flavor space. f_L and f_R are the scales for spontaneous symmetry breaking in the electroweak sector and parity, respectively. The coefficient $\beta_{L,R,1}$ generates extra mass for the left-handed (right-handed) third component in breaking the $SU(2)_{L,R}$ isospin symmetry. The coefficient κ parameterizes the mixing between the left- and right-handed gauge bosons whereas the coefficient $\tilde{\beta}_1$ controls the mixing between left-handed W_L^3 and right-handed W_R^3.

The neutral current interactions are

$$-\mathcal{L}_{\rm NC} = W^3_{R\mu} J_R^\mu + W^3_{L\mu} J_L^\mu + B_\mu J_0^\mu \tag{2.6}$$

whereas the charged current interactions are

$$-\mathcal{L}_{\rm CC} = W^+_{R\mu} J_R^{-,\mu} + W^+_{L\mu} J_L^{-,\mu} + \text{h.c.} \tag{2.7}$$

Here,

$$J_{L,R}^{\pm,\mu} = \frac{g_{L,R}}{\sqrt{2}}\overline{\Psi}_{L,R}\tau^{\pm}\gamma^{\mu}\Psi_{L,R}. \tag{2.8}$$

The kinetic part has the simple form

$$\mathcal{L}_K = -\frac{1}{4}W^a_{L,\mu\nu}W_L^{\mu\nu,a} - \frac{1}{4}W^a_{R,\mu\nu}W_R^{\mu\nu,a} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + i\overline{\Psi}_i\gamma_\mu D_\mu \Psi_i. \tag{2.9}$$

Adding Yukawa terms

$$\mathcal{L}_Y = \overline{\Psi}_L U_L M U_R^{\dagger}\Psi_R + \text{h.c.}, \tag{2.10}$$

the total Lagrangian is the sum of all the above terms

$$\mathcal{L} = \mathcal{L}_M + \mathcal{L}_K + \mathcal{L}_{\text{NC}} + \mathcal{L}_{\text{CC}} + \mathcal{L}_Y. \tag{2.11}$$

3. Diagonalization and Mass Eigenstates

In this section, we calculate the mass eigenvalues of the left-right symmetry effective Lagrangian by rotating the mass mixing matrix from the gauge basis to the mass basis.

3.1. Charged Gauge Bosons

Taking the unitary gauge $U_L = U_R = 1$, the charged gauge boson mass terms can be expressed as

$$\begin{aligned}\mathcal{L}_{\text{CM}} = &\frac{1}{4}f_L^2 g_L^2 W^+_{L,\mu}W^-_{L,\mu} + \frac{1}{4}f_R^2 g_R^2 W^+_{R,\mu}W^-_{R,\mu} \\ &- \frac{1}{4}\tilde{\kappa} f_L f_R g_L g_R\left(W^+_{L,\mu}W^-_{R,\mu} + W^+_{R,\mu}W^-_{L,\mu}\right).\end{aligned} \tag{3.1}$$

Here, we have used charged boson definitions $W^1_{i,\mu} = (W^+_{i,\mu} + W^-_{i,\mu})/\sqrt{2}$ and $W^2_{i,\mu} = i(W^+_{i,\mu} - W^-_{i,\mu})/\sqrt{2}$ for $i = L, R$.

We make an orthogonal rotation V for $W_L^{\pm}$ and $W_R^{\pm}$

$$\begin{pmatrix} W_R^{\pm} \\ W_L^{\pm}\end{pmatrix} = \begin{pmatrix} \cos\xi & \sin\xi \\ -\sin\xi & \cos\xi\end{pmatrix}\begin{pmatrix} W'^{\pm} \\ W^{\pm}\end{pmatrix} \equiv V\begin{pmatrix} W'^{\pm} \\ W^{\pm}\end{pmatrix} \tag{3.2}$$

to eliminate the cross-terms involving W_L and W_R in (3.1) to keep the kinetic term diagonal. The mixing angle ξ is expressed as

$$\tan 2\xi = \frac{2\tilde{\kappa} f_L f_R g_L g_R}{f_L^2 g_L^2 - f_R^2 g_R^2}. \tag{3.3}$$

After this rotation, the charged boson mass-squared matrix for the charged bosons becomes

$$V^T \mathcal{M}_C V = \mathrm{diag}\left(M_{W'}^2, M_W^2\right), \tag{3.4}$$

and the heavy and light charged boson masses are

$$\begin{aligned}
M_{W'}^2 &= \frac{1}{8}\left[f_L^2 g_L^2 + f_R^2 g_R^2 + \sqrt{\left(f_L^2 g_L^2 - f_R^2 g_R^2\right)^2 + 4\tilde{\kappa}^2 f_L^2 f_R^2 g_L^2 g_R^2}\right] \\
&\simeq \frac{1}{4} f_R^2 g_R^2 \left\{1 + \tilde{\kappa}^2 \frac{f_L^2 g_L^2}{f_R^2 g_R^2 - f_L^2 g_L^2}\right\}, \\
M_W^2 &= \frac{1}{8}\left[f_L^2 g_L^2 + f_R^2 g_R^2 - \sqrt{\left(f_L^2 g_L^2 - f_R^2 g_R^2\right)^2 + 4\tilde{\kappa}^2 f_L^2 f_R^2 g_L^2 g_R^2}\right] \\
&\simeq \frac{1}{4} f_L^2 g_L^2 \left\{1 - \tilde{\kappa}^2 \frac{f_R^2 g_R^2}{f_R^2 g_R^2 - f_L^2 g_L^2}\right\}.
\end{aligned} \tag{3.5}$$

We notice that the charged boson mixing angle ξ is controlled by the coefficient $\tilde{\kappa}$. $W - W'$ mixing causes W couplings to the right-handed fermion with $g_R^W = g_L \sin\xi/\sqrt{2}$. g_R^W can yield the contributions to $b \rightarrow s\gamma$ (see paper [14]) and must be restrained so that $g_R^W/g_L^W < 4 \times 10^{-3}$, which requires $\xi < 4 \times 10^{-3}$.

3.2. *Neutral Gauge Bosons*

Now, let us discuss the neutral boson sector. The neutral mass terms in our chiral Lagrangian (2.5) can be readily separated out

$$\begin{aligned}
\mathcal{L}_{M_n} = {} & \frac{1}{8}(1 - 2\beta_{L,1}) f_L^2 \left(g_L W_{L,\mu}^3 - g B_\mu\right)^2 + \frac{1}{8}(1 - 2\beta_{R,1}) f_R^2 \left(g_R W_{R,\mu}^3 - g B_\mu\right)^2 \\
& - \frac{1}{4}\left(\tilde{\kappa} + \tilde{\beta}_1\right) f_L f_R \left(g_L W_{L,\mu}^3 - g B_\mu\right)\left(g_R W_R^{3,\mu} - g B_\mu\right).
\end{aligned} \tag{3.6}$$

It can be written in matrix form

$$\mathcal{L}_{M_n} = \frac{1}{2} \mathcal{G}_\mu^T \mathcal{M}_n \mathcal{G}^\mu \tag{3.7}$$

with neutral gauge bosons $\mathcal{G}_\mu \equiv (W_{R,\mu}, W_{\mathrm{L},\mu}, B_\mu)$ and mass-squared matrix

$$\mathcal{M}_n \equiv \begin{pmatrix}
\dfrac{f_R^2 g_R^2}{4} & -\dfrac{\kappa f_R f_L g_R g_L}{4} & \dfrac{f_R g_R g_0 (\kappa f_L - f_R)}{4} \\
-\dfrac{\kappa f_R f_L g_R g_L}{4} & \dfrac{f_L^2 g_L^2}{4} & \dfrac{f_L g_L g_0 (\kappa f_R - f_L)}{4} \\
\dfrac{f_R g_R g_0 (\kappa f_L - f_R)}{4} & \dfrac{f_L g_L g_0 (\kappa f_R - f_L)}{4} & \dfrac{(f_R^2 + f_L^2 - 2\kappa f_R f_L) g_0^2}{4}
\end{pmatrix}. \tag{3.8}$$

Note that the $\beta_{L,R,1}$ do not appear in the above mass-squared matrix because these can be absorbed by a redefinition of VEV $f_{L,R}$

$$f_{L,R} \longrightarrow \frac{f_{L,R}}{\sqrt{1-2\beta_{L,R,1}}}. \tag{3.9}$$

For the sake of convenience, we will retain using the same notation for the redefined VEV $f_{L,R}$ but keep in mind that this redefinition has been made. The new parameter κ in the above formula is a combination of $\tilde{\kappa}$ and $\tilde{\beta}_1$, namely, $\kappa = \tilde{\kappa} + \tilde{\beta}_1$. Taking into account the VEVs re-definition, we have

$$\kappa = \frac{\tilde{\kappa} + \tilde{\beta}_1}{\sqrt{1-2\beta_{L,1}}\sqrt{1-2\beta_{R,1}}}. \tag{3.10}$$

The physical masses of the neutral bosons are the eigenvalues of the matrix $\mathcal{M}_n$. To obtain the diagonalized eigenvalues, we define the mass eigenstates as $\overline{\mathcal{G}}_\mu = (Z', Z_\mu, A_\mu)^T$ which are related to $\mathcal{G}_\mu$ by a special rotation U^{-1}

$$\begin{aligned}
\overline{\mathcal{G}}_\mu &= \begin{pmatrix} \dfrac{G_4 g_R}{G_1G_4 - G_2G_3} & -\dfrac{G_2 g_L}{G_1G_4 - G_2G_3} & \dfrac{(G_2 - G_4)g_0}{G_1G_4 - G_2G_3} \\ -\dfrac{G_3 g_R}{G_1G_4 - G_2G_3} & \dfrac{G_1 g_L}{G_1G_4 - G_2G_3} & \dfrac{(G_3 - G_1)g_0}{G_1G_4 - G_2G_3} \\ \dfrac{g_R}{G_5} & \dfrac{\lambda_1 g_L}{G_5} & \dfrac{\lambda_2 g_0}{G_5} \end{pmatrix} \mathcal{G}_\mu && (3.11) \\
&\equiv U^{-1}\mathcal{G}_\mu && (3.12)
\end{aligned}$$

with undetermined couplings G_i ($i = 1, \ldots, 5$) and parameters λ_i ($i = 1, 2$). This complicated rotation is motivated by the following simple relations: the rotation U relates

$$\begin{aligned}
g_R W_{R,\mu} - g_0 B_\mu &= G_1 \overline{Z'}_\mu + G_2 \overline{Z}_\mu, \\
g_L W_{L,\mu} - g_0 B_\mu &= G_3 \overline{Z'}_\mu + G_4 \overline{Z}_\mu, \\
g_R W_{R,\mu} + \lambda_1 g_L W_{L,\mu} + \lambda_2 g_0 B_\mu &= G_5 \overline{A}_\mu
\end{aligned} \tag{3.13}$$

which diagonalizes the $B - W_L$ and $B - W_R$ mixings automatically while simultaneously keeping the photon massless. To maintain a diagonal kinetic energy matrix, U must satisfy six independent orthogonality conditions

$$UU^T = 1. \tag{3.14}$$

Adding one mass diagonalization condition for the $W_R - W_L$ mass mixing, there are seven independent equations that determine five G_i $(i = 1, \ldots, 5)$ and two λ_i $(i = 1, 2)$. Solving these equations, we obtain

$$
\begin{aligned}
\lambda_1 &= \frac{g_R^2}{g_L^2}, \\
\lambda_2 &= \frac{g_R^2}{g_0^2}, \\
G_1 &= \frac{(\kappa f_R C - f_L) f_L}{(f_R C - \kappa f_L) f_R} G_3, \\
G_2 &= C G_4, \\
G_3 &= \frac{(f_R C - \kappa f_L) f_R \sqrt{g_0^2 (1 - C)^2 + g_R^2 + C^2 g_L^2}}{f_R^2 C^2 + f_L^2 - 2C\kappa f_R f_L}, \\
G_4 &= \frac{1}{f_R^2 C^2 + f_L^2 - 2C\kappa f_R f_L} \Big(f_R^2 f_L^2 \left[g_R^2 + C^2 g_L^2 + g_0^2 (1 + C)^2 \right] \kappa^2 \\
&\quad - 2 f_R f_L \left[f_R^2 C \left(g_R^2 + g_0^2 \right) + f_L^2 C \left(g_L^2 + g_0^2 \right) + g_0^2 \left(f_R^2 C^2 + f_L^2 \right) \right] \kappa \\
&\quad + g_R^2 f_R^4 C^2 + g_L^2 f_L^4 + g_0^2 \left(f_R^2 C + f_L^2 \right)^2 \Big)^{1/2}, \\
G_5 &= g_R^2 \sqrt{\frac{1}{g_R^2} + \frac{1}{g_L^2} + \frac{1}{g_0^2}}
\end{aligned}
\tag{3.15}
$$

with a real C that satisfies the quadratic equation

$$
\left(\kappa f_R f_L \left(g_L^2 + g_0^2 \right) - f_R^2 g_0^2 \right) C^2 + \left[f_R^2 \left(g_R^2 + g_0^2 \right) - f_L^2 \left(g_L^2 + g_0^2 \right) \right] C + f_L^2 g_0^2 - \kappa f_R f_L \left(g_R^2 + g_0^2 \right) = 0. \tag{3.16}
$$

The mass eigenvalues of the physical Z' and Z then become

$$
\begin{aligned}
M_{Z'}^2 &= \left(U^T \mathcal{M} U \right)_{1,1}, \\
M_Z^2 &= \left(U^T \mathcal{M} U \right)_{2,2}.
\end{aligned}
\tag{3.17}
$$

Up to now, we have obtained the exact rotation matrix elements without any approximate assumption. The total rotation U in (3.12) can be expressed in terms of (3.11), (3.15), and (3.16).

4. Oblique Radiative Corrections

To clearly see the new physics correction, we can separate a standard electroweak rotation from the total rotation in (3.12)

$$U \equiv U' U_{\text{em}} \tag{4.1}$$

with the standard electroweak rotation

$$U_{\text{em}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\theta & s_\theta \\ 0 & -s_\theta & c_\theta \end{pmatrix}. \tag{4.2}$$

From (3.12) and (4.1), we can calculate the oblique radiative corrections coming from the right-handed gauge bosons in light of Holdom's work [15]

$$\begin{gathered} S = \frac{4 s_\theta c_\theta}{\alpha} \left\{ \left(s_\theta^2 - c_\theta^2 \right) U'_{32} - 2 c_\theta s_\theta \left(U'_{33} - 1 \right) + 2 c_\theta s_\theta \left(U'_{22} - 1 \right) \right\}, \\ T = \frac{2}{\alpha} \left\{ \left(U'_{22} - 1 \right) - \Delta M_Z \right\}, \\ U = -\frac{8 s_\theta^2}{\alpha} \left\{ c_\theta s_\theta U'_{32} + s_\theta^2 \left(U'_{33} - 1 \right) + c_\theta^2 \left(U'_{22} - 1 \right) \right\}, \end{gathered} \tag{4.3}$$

where s_θ and c_θ are the respective sine and cosine of the standard Weinberg angle from SM, and ΔM_Z is the new physical shift in the Z mass $\Delta M_Z = M_Z - M_Z|_{SM}$. Furthermore, we calculate to leading order the results for two special conditions.

4.1. Case 1: $f_R \gg f_L$ *and* $g_R \gg g_{L/0}$

This case corresponds to a $SU(2)_R$ breaking scale that is much higher than the electroweak breaking scale and $M_{W_R} \gg M_{W_L}$. It is easy to calculate the U' rotation from (4.1), (4.2), and (3.15). We only list leading-order terms

$$\begin{aligned} U'_{11} &\simeq 1, \\ U'_{12} &\simeq \frac{c_\theta s_\theta r^3}{2}, \\ U'_{13} &\simeq r, \end{aligned}$$

$$
\begin{aligned}
U'_{21} &\simeq -\frac{\kappa f_R}{f_L}\frac{s_\theta c_\theta}{r}\left(1+\frac{(3-c_\theta^2)r^2}{2}\right),\\
U'_{22} &\simeq 1,\\
U'_{23} &\simeq -\frac{c_\theta s_\theta r^2}{2},\\
U'_{31} &\simeq r+\frac{\kappa f_R}{r f_L}\left(1+\frac{(1-2c_\theta^2)r^2}{2}\right),\\
U'_{32} &\simeq \frac{c_\theta s_\theta r^2}{2},\\
U'_{33} &\simeq 1
\end{aligned}
\tag{4.4}
$$

with coupling ratio $r \equiv g_0/g_R$. Obviously, in the limit of heavy M_{W_R}, $g_R \gg g_{L,0}$, this new physics rotation matrix U' becomes a unitary matrix. Indeed, it is a requirement of the SM structure and a good self-checking condition of our calculation.

From (3.17), we can calculate the gauge boson mass eigenvalues

$$M_{Z'}^2 = \left(U^T \mathcal{M} U\right)_{1,1} \simeq \frac{f_R^2 g_R^2}{4}\left(1+r^2\right)\left(1-\kappa^2\right), \tag{4.5}$$

$$M_Z^2 = \left(U^T \mathcal{M} U\right)_{2,2} \simeq \frac{f_L^2(g_L^2+g_0^2)}{4}\left\{1+\left(2\frac{\kappa f_L}{f_R}-s_\theta^2\right)r^2\right\}. \tag{4.6}$$

From (4.5), the mass shift can be calculated

$$\Delta M_Z \simeq -\frac{s_\theta^2}{2} r^2. \tag{4.7}$$

Using (4.3), the leading-order terms to the oblique radiative correction parameters are

$$
\begin{aligned}
\alpha S &\simeq s_\theta^2 c_\theta^2\left(1+2s_\theta^2\right)r^2,\\
\alpha T &\simeq s_\theta^2 r^2,\\
\alpha U &\simeq 4 s_\theta^6 r^2.
\end{aligned}
\tag{4.8}
$$

Adopting the new physics constraints $S < 0.11$, $T < 0.14$, $U < 0.16$ [16] and taking $s_\theta^2 = 0.2311$, $\alpha = 1/137$, we can estimate the coupling ratio $r < 0.05$.

4.2. Case 2: $f_R = f_L$ *and* $g_R \gg g_{L/0}$

The conditions correspond to left-right symmetry. $M_{W_R} \gg M_{W_L}$ requires $g_R \gg g_{L/0}$. Hence, the leading-order terms to the matrix elements of U' are

$$
\begin{aligned}
U'_{11} &\simeq 1,\\
U'_{12} &\simeq -\frac{r^3 c_\theta \left(c_\theta^2 + c_\theta^2 s_\theta^2 + 1\right)}{s_\theta} + \frac{r c_\theta \left(2 + s_\theta^4\right)}{s_\theta},\\
U'_{13} &\simeq r - \frac{\kappa r \left(2 + s_\theta^4\right)}{2},\\
U'_{21} &\simeq -r^3 s_\theta c_\theta \left(\frac{3}{2} - c_\theta^2\right) + s_\theta c_\theta r^3 \kappa,\\
U'_{22} &\simeq 1\\
U'_{23} &\simeq -s_\theta c_\theta r^2 + \left\{\frac{s_\theta^3 c_\theta}{2} - \frac{r^2 s_\theta^3 c_\theta \left(1 + 3c_\theta^2\right)}{4}\right\}\kappa,\\
U'_{31} &\simeq r + c_\theta^2 s_\theta^3 r^3 \kappa,\\
U'_{32} &\simeq \frac{s_\theta c_\theta r^2}{2} + \left\{\frac{s_\theta^3 c_\theta}{2} - \frac{r^2 c_\theta}{4 s_\theta}\left[4 + 3 s_\theta^4 \left(1 + c_\theta^2\right)\right]\right\}\kappa,\\
U'_{33} &\simeq 1.
\end{aligned}
\tag{4.9}
$$

When taking $r \to 0$ and $\kappa \to 0$, matrix U' becomes unitary. The leading order terms for the gauge boson masses are

$$
\begin{aligned}
M_{Z'}^2 &= \left(U^T \mathcal{M} U\right)_{1,1} \simeq \frac{f^2 g_R^2}{4}\left(1 + r^2\right),\\
M_Z^2 &= \left(U^T \mathcal{M} U\right)_{2,2} \simeq \frac{f^2 \left(g_L^2 + g_0^2\right)}{4}\left(1 - \frac{s_\theta^2 r^2}{2}\right).
\end{aligned}
\tag{4.10}
$$

The shift in mass of Z is

$$
\Delta M_Z \simeq -\frac{s_\theta^2}{4} r^2. \tag{4.11}
$$

In this case, the leading-order terms of the oblique radiative correction parameters are

$$\alpha S \simeq 2r^2 s_\theta^2\left(1-2s_\theta^2\right)c_\theta^2+\left\{6\left(1-2c_\theta^2\right)s_\theta^4+r^2\left(4c_\theta^2-3\right)\left(3s_\theta^2+4\right)\right\}c_\theta^2\kappa,$$
$$\alpha T \simeq \frac{s_\theta^2 r^2}{2}, \tag{4.12}$$
$$\alpha U \simeq 4s_\theta^6 r^2+2\left\{2s_\theta^2\left(1-3c_\theta^2 s_\theta^2\right)+r^2\left(2c_\theta^2-1\right)\left(3s_\theta^4+4\right)\right\}s_\theta^2\kappa.$$

From $T < 0.10$, we can estimate coupling ratio $r < 0.09$ implying a lower limit for the Z' mass of about 0.8 TeV.

5. A Short Summary

To summarize, we have reviewed nonlinearly realized electroweak chiral Lagrangian for the gauge group $SU(2)_R \times SU(2)_L \times U(1)$ and diagonalized gauge eigenstates using all possible mass mixing terms to obtain exact mass eigenstates and the rotation matrix. The oblique radiative corrections from right-handed gauge bosons have been estimated to leading order.

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