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Inflation and leptogenesis in the 3-3-1-1 model

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We consider the $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X \otimes \mathrm{U}(1)_N$ (3-3-1-1) model at the grand unified theory scale with implication for inflation and leptogenesis. The mass spectra of the neutral Higgs bosons and neutral gauge bosons are reconsidered when the scale of the 3-3-1-1 breaking is much larger than that of the ordinary $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X$ (3-3-1) breaking. We investigate how the 3-3-1-1 model generates an inflation by identifying the scalar field that spontaneously breaks the $\mathrm{U}(1)_N$ symmetry to inflaton as well as including radiative corrections for the inflaton potential. We figure out the parameter spaces appeared in the inflaton potential that satisfy the conditions for an inflation model and obtain the inflaton mass an order of 10^{13} GeV. The inflaton can dominantly decay into a pair of light Higgs bosons or a pair of heavy Majorana neutrinos which lead, respectively, to a reheating temperature of 10^9 GeV order appropriate to a thermal leptogenesis scenario or to a reduced reheating temperature corresponding to a nonthermal leptogenesis scenario. We calculate the lepton asymmetry which yields baryon asymmetry successfully for both the thermal and nonthermal cases.

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I. INTRODUCTION

Cosmological inflation is a popular postulate for the early universe. It can solve the difficulties of the hot big bang theory and provide the predictions for quantum fluctuations in the inflating background. In order to recover the conventions of the hot big bang theory and to know how the Universe is reheated, we must understand what is the inflaton field, and how it is connected to particle physics. These problems were first investigated with the chaotic inflation scenario by Linde [1]. According to this scenario, the inflation may begin even if there was no thermal equilibrium in the early universe. It can occur in a theory with a very simple potential such as $V(\phi) \propto \phi^2$. There is no limit to the theory with a polynomial potential: Chaotic inflation occurs in any theory where the potential has a sufficiently flat region [1]. On the other hand, the recent measurements of B modes by BICEP2 Collaboration [2] have yielded very interesting results, which could be the direct measurements of quantum gravitation excitations from the early universe. The ratio of the tensor and scalar is measured as $0.16^{+0.06}_{-0.05}$. Combined with the Planck and WMAP measurements suggests that the inflation model must be a larger field model. Hence, the inflationary scenario does not work on the framework of the Standard Model (SM) without a nonminimal coupling to gravity.

Furthermore, what is the origin of matter-antimatter asymmetry in the Universe? The neutrino experiments such as Super-Kamiokande [3], KamLAND [4] and SNO [5] have confirmed that the neutrinos have small masses and large flavor mixing. According to the Planck mission team, and based on the standard model of cosmology, there exits dark matter (DM) which lies beyond the SM. All the experiments call for extensions beyond the SM. One way to extend the SM is to expand the gauge symmetry group. There exists a simple extension of the SM gauge group to $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X$, the so-called 3-3-1 models. These models can explain the following issues [6]:

(i) Why the electric charges are quantized.
(ii) Why there are only three observed families of fermions.
(iii) Why top quark is oddly heavy.
(iv) Why the strong CP nonconservation is disappeared.
(v) 3-3-1 models can provide the neutrino small masses as well as candidates for the DM [7–9].

There have recently emerged an extension of the 3-3-1 models, based on the $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X \otimes \mathrm{U}(1)_N$ (3-3-1-1) gauge group, which not only contains all the good features of the 3-3-1 models as mentioned [8,9], but also has the following advantages:

(i) The $B-L$ number is naturally gauged by combination of the $\mathrm{SU}(3)_L$ and $\mathrm{U}(1)_N$ charges. It leads to an unification of the electroweak and $B-L$ interactions.
(ii) The right-handed neutrinos appear in the model as fundamental fermions that solve the small masses of neutrinos through a type I seesaw mechanism.

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(iii) There exists a W-parity symmetry as a (Z_2) remnant subgroup of the gauge symmetry. Almost all the new particles have wrong lepton numbers transforming as odd fields under W-parity. The lightest wrong lepton-number particle is identified to the DM. Because of W-parity conservation, the model can work better under experimental constraints than the 3-3-1 models.

Other highlights of the 3-3-1-1 model is that the energy scale of the symmetry breaking $U(1)_N$ can happen at a very high scale like the grand unified theory (GUT) one [8]. The inflationary scenario can be linked to $U(1)_N$ breaking and driven by the Higgs ϕ potential. Due to the local gauge $U(1)_N$ symmetry, a radiative correction to the inflaton potential can arise from the coupling of inflaton with the $U(1)_N$ gauge boson (Z_2). There exist the couplings of inflaton ϕ with right-handed neutrinos and Higgs triplets, which also contribute to the inflation potential. We would like to stress that the well-known advantages of a spontaneously broken gauge $U(1)_N$ symmetry include a seesaw mechanism for the neutrino physics [8]. The presence of the right-handed neutrinos that directly interact to the inflaton may be compatible with the leptogenesis scenario. The aim of this work is to show that the chaotic inflationary scenario can be driven by the singlet Higgs ϕ potential. We also focus on how the leptogenesis happened after the inflation through the couplings of the right-handed neutrinos.

Our paper is organized as follows: In Sec. II, we briefly review the 3-3-1-1 model and especially concentrate on the Higgs and gauge boson spectra in the large Λ limit. In Sec. III, we present the inflation model by assuming the singlet Higgs ϕ as an inflation field. The leptogenesis related to the matter-antimatter asymmetry of the Universe and neutrino properties is studied in Sec IV. Finally we summarize our works in Sec. V.

II. BRIEF DESCRIPTION OF THE 3-3-1-1 MODEL

The fermion content of the 3-3-1-1 model is given as [8,9]

$$\psi_{aL} = \begin{pmatrix} \nu_{aL} \\ e_{aL} \\ (N_{aR})^c \end{pmatrix} \sim (1, 3, -1/3, -2/3), \tag{1}$$

$$\nu_{aR} \sim (1, 1, 0, -1), \qquad e_{aR} \sim (1, 1, -1, -1), \tag{2}$$

$$Q_{\alpha L} = \begin{pmatrix} d_{\alpha L} \\ -u_{\alpha L} \\ D_{\alpha L} \end{pmatrix} \sim (3, 3^*, 0, 0),$$
$$Q_{3L} = \begin{pmatrix} u_{3L} \\ d_{3L} \\ U_L \end{pmatrix} \sim (3, 3, 1/3, 2/3), \tag{3}$$

$$u_{aR} \sim (3, 1, 2/3, 1/3), \qquad d_{aR} \sim (3, 1, -1/3, 1/3), \tag{4}$$

$$U_R \sim (3, 1, 2/3, 4/3), \qquad D_{\alpha R} \sim (3, 1, -1/3, -2/3), \tag{5}$$

where the quantum numbers in the parentheses are defined upon the gauge symmetries $[SU(3)_C,\ SU(3)_L,\ U(1)_X,\ U(1)_N]$, respectively. The family indices are $a = 1, 2, 3$ and $\alpha = 1, 2$. N_R, U and D are the exotic fermions, which have incorrect lepton numbers. The other fermions have ordinary lepton numbers. Note that the neutral fermions N_R are truly sterile since they do not have any gauge interaction, which contradicts to the ν_R ones as usually considered.

To break the gauge symmetry, one uses the following scalar multiplets [8]:

$$\rho = \begin{pmatrix} \rho_1^+ \\ \rho_2^0 \\ \rho_3^+ \end{pmatrix} \sim (1, 3, 2/3, 1/3),$$
$$\eta = \begin{pmatrix} \eta_1^0 \\ \eta_2^- \\ \eta_3^0 \end{pmatrix} \sim (1, 3, -1/3, 1/3), \tag{6}$$

$$\chi = \begin{pmatrix} \chi_1^0 \\ \chi_2^- \\ \chi_3^0 \end{pmatrix} \sim (1, 3, -1/3, -2/3), \qquad \phi \sim (1, 1, 0, 2), \tag{7}$$

with the vacuum expectation values (VEVs) that conserve electric charge and R-parity being respectively given by

$$\langle\rho\rangle = \frac{1}{\sqrt{2}}(0, v, 0)^T, \qquad \langle\eta\rangle = \frac{1}{\sqrt{2}}(u, 0, 0)^T,$$
$$\langle\chi\rangle = \frac{1}{\sqrt{2}}(0, 0, \omega)^T, \qquad \langle\phi\rangle = \frac{1}{\sqrt{2}}\Lambda. \tag{8}$$

The pattern of the symmetry breaking of the model is given by the following scheme:

$$\text{3-3-1-1} \overset{\langle\chi\rangle\langle\rho\rangle\langle\eta\rangle}{\longmapsto} SU(3)_C \otimes U(1)_Q \otimes U(1)_{B-L}$$
$$\overset{\langle\phi\rangle}{\longmapsto} SU(3)_C \otimes U(1)_Q \otimes P, \tag{9}$$

where the electric charge Q, $B - L$ and matter parity P take the forms,

$$Q = T_3 - \frac{1}{\sqrt{3}}T_8 + X, \qquad B - L = -\frac{2}{\sqrt{3}}T_8 + N,$$
$$P = (-1)^{3(B-L)}. \tag{10}$$

Here, $T_i (i = 1, 2, 3, \ldots, 8)$, X and N are the $\mathrm{SU}(3)_L$, $\mathrm{U}(1)_X$ and $\mathrm{U}(1)_N$ charges, respectively.

The Lagrangian of the 3-3-1-1 model is given by [8]

$$\mathcal{L} = \sum_{\text{fermion multiplets}} \bar{\Psi} i\gamma^\mu D_\mu \Psi + \sum_{\text{scalar multiplets}} (D^\mu \Phi)^\dagger (D_\mu \Phi) - \frac{1}{4} G_{i\mu\nu} G_i^{\mu\nu} - \frac{1}{4} A_{i\mu\nu} A_i^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} C_{\mu\nu} C^{\mu\nu} - V(\rho, \eta, \chi, \phi) + \mathcal{L}_{\text{Yukawa}}, \tag{11}$$

where the Yukawa Lagrangian and scalar potential are obtained [8,9] as follows:

$$\mathcal{L}_{\text{Yukawa}} = h^e_{ab} \bar{\psi}_{aL} \rho e_{bR} + h^\nu_{ab} \bar{\psi}_{aL} \eta \nu_{bR} + h'^\nu_{ab} \bar{\nu}^c_{aR} \nu_{bR} \phi + h^U \bar{Q}_{3L} \chi U_R + h^D_{\alpha\beta} \bar{Q}_{\alpha L} \chi^* D_{\beta R} + h^u_a \bar{Q}_{3L} \eta u_{aR} + h^d_a \bar{Q}_{3L} \rho d_{aR} + h^d_{\alpha a} \bar{Q}_{\alpha L} \eta^* d_{aR} + h^u_{\alpha a} \bar{Q}_{\alpha L} \rho^* u_{aR} + \text{H.c.}, \tag{12}$$

$$\begin{aligned} V(\rho, \eta, \chi, \phi) = {} & \mu_1^2 \rho^\dagger \rho + \mu_2^2 \chi^\dagger \chi + \mu_3^2 \eta^\dagger \eta + \lambda_1 (\rho^\dagger \rho)^2 + \lambda_2 (\chi^\dagger \chi)^2 + \lambda_3 (\eta^\dagger \eta)^2 \\ & + \lambda_4 (\rho^\dagger \rho)(\chi^\dagger \chi) + \lambda_5 (\rho^\dagger \rho)(\eta^\dagger \eta) + \lambda_6 (\chi^\dagger \chi)(\eta^\dagger \eta) \\ & + \lambda_7 (\rho^\dagger \chi)(\chi^\dagger \rho) + \lambda_8 (\rho^\dagger \eta)(\eta^\dagger \rho) + \lambda_9 (\chi^\dagger \eta)(\eta^\dagger \chi) + (f \epsilon^{mnp} \eta_m \rho_n \chi_p + \text{H.c.}) \\ & + \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 + \lambda_{10} (\phi^\dagger \phi)(\rho^\dagger \rho) + \lambda_{11} (\phi^\dagger \phi)(\chi^\dagger \chi) + \lambda_{12} (\phi^\dagger \phi)(\eta^\dagger \eta). \end{aligned} \tag{13}$$

Because of the 3-3-1-1 gauge symmetry, the Yukawa Lagangian and scalar potential as given take the standard forms which contain no lepton-number violating interactions.

The fermion masses that result from the Yukawa Lagrangian have been presented in [8]. The phenomenology of the 3-3-1-1 model with the Λ scale of the $\mathrm{U}(1)_N$ breaking comparable to the ω scale of the 3-3-1 symmetry breaking has been studied in [9]. Below, we will compute the physical states and masses for the scalar and gauge sectors in the limit $\Lambda \gg \omega$, which is needed for our further analysis.

A. Scalar sector

In this part, we identify the physical particles in the scalar sector. We expand the neutral scalars around their VEVs [8] such as

$$\rho = \begin{pmatrix} \rho_1^+ \\ \frac{1}{\sqrt{2}}(v + S_2 + iA_2) \\ \rho_3^+ \end{pmatrix}; \quad \eta = \begin{pmatrix} \frac{1}{\sqrt{2}}(u + S_1 + iA_1) \\ \eta_2^- \\ \frac{1}{\sqrt{2}}(S_3' + iA_3') \end{pmatrix}; \quad \chi = \begin{pmatrix} \frac{1}{\sqrt{2}}(S_1' + iA_1') \\ \chi_2^- \\ \frac{1}{\sqrt{2}}(\omega + S_3 + iA_3) \end{pmatrix}; \tag{14}$$

$$\phi \sim \frac{1}{\sqrt{2}}(\Lambda + S_4 + iA_4). \tag{15}$$

In the scalar sector, all scalar fields with W-parity even, S_1, S_2, S_3, S_4, mix via the mass matrix such as

$$M_S^2 = \begin{pmatrix} 2\lambda_3 u^2 - \frac{1}{\sqrt{2}} f \frac{v\omega}{u} & \lambda_5 uv + \frac{1}{\sqrt{2}} f\omega & \lambda_6 u\omega + \frac{1}{\sqrt{2}} fv & \lambda_{12} u\Lambda \\ \lambda_5 uv + \frac{1}{\sqrt{2}} f\omega & 2\lambda_1 v^2 - \frac{1}{\sqrt{2}} f \frac{u\omega}{v} & \lambda_4 \omega v + \frac{1}{\sqrt{2}} fu & \lambda_{10} v\Lambda \\ \lambda_6 u\omega + \frac{1}{\sqrt{2}} fv & \lambda_4 \omega v + \frac{1}{\sqrt{2}} fu & 2\lambda_2 \omega^2 - \frac{1}{\sqrt{2}} f \frac{vu}{\omega} & \lambda_{11} \omega\Lambda \\ \lambda_{12} u\Lambda & \lambda_{10} v\Lambda & \lambda_{11} \omega\Lambda & 2\lambda\Lambda^2 \end{pmatrix}. \tag{16}$$

We assume that $\Lambda \gg \omega \sim -f \gg u, v$ then the mass matrix given in Eq. (16) has form as

$$M_S^2 = \begin{pmatrix} C & B^T \\ B & A \end{pmatrix}, \tag{17}$$

where

$$A = 2\lambda\Lambda^2, \tag{18}$$

$$B = (\,\lambda_{12}u\Lambda \quad \lambda_{10}v\Lambda \quad \lambda_{11}\omega\Lambda\,), \tag{19}$$

$$C = \begin{pmatrix} 2\lambda_3 u^2 - \frac{fv\omega}{\sqrt{2}u} & \lambda_5 uv + \frac{f\omega}{\sqrt{2}} & \lambda_6 u\omega + \frac{fv}{\sqrt{2}} \\ \lambda_5 uv + \frac{f\omega}{\sqrt{2}} & 2\lambda_1 v^2 - \frac{fu\omega}{\sqrt{2}v} & \lambda_4 v\omega + \frac{fu}{\sqrt{2}} \\ \lambda_6 u\omega + \frac{fv}{\sqrt{2}} & \lambda_4 v\omega + \frac{fu}{\sqrt{2}} & 2\lambda_2\omega^2 - \frac{fuv}{\sqrt{2}\omega} \end{pmatrix}. \tag{20}$$

Since $(\Lambda \gg -f, \omega \gg u, v)$, we get $A \gg B, C$. The matrix given in (17) can be diagonalized by using block diagonalizing method. The approximately unitary matrix U,

$$U = \begin{pmatrix} 1 & B^\dagger A^{-1} \\ -A^{-1}B & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & \frac{u\lambda_{12}}{2\lambda\Lambda} \\ 0 & 1 & 0 & \frac{v\lambda_{10}}{2\lambda\Lambda} \\ 0 & 0 & 1 & \frac{\omega\lambda_{11}}{2\lambda\Lambda} \\ -\frac{u\lambda_{12}}{2\lambda\Lambda} & -\frac{v\lambda_{10}}{2\lambda\Lambda} & -\frac{\omega\lambda_{11}}{2\lambda\Lambda} & 1 \end{pmatrix}, \tag{21}$$

transform M_S^2 into approximately block-diagonal form:

$$U^\dagger M_S^2 U \approx \begin{pmatrix} C - B^\dagger A^{-1}B & 0 \\ 0 & A \end{pmatrix}. \tag{22}$$

In the limit $(\Lambda \gg -f, \omega \gg u, v)$, $U \simeq I$ then $H_3 \simeq S_4$ gets mass $m^2_{H_3} = 2\lambda\Lambda^2$. S_1, S_2, S_3 are mixing with the mixing mass matrix obtained as

$$C - B^\dagger A^{-1}B = \begin{pmatrix} 2\lambda_3 u^2 - \frac{fv\omega}{\sqrt{2}u} - \frac{u^2\lambda_{12}^2}{2\lambda} & \lambda_5 uv + \frac{f\omega}{\sqrt{2}} - \frac{uv\lambda_{10}\lambda_{12}}{2\lambda} & \lambda_6 u\omega + \frac{fv}{\sqrt{2}} - \frac{u\omega\lambda_{11}\lambda_{12}}{2\lambda} \\ \lambda_5 uv + \frac{f\omega}{\sqrt{2}} - \frac{uv\lambda_{10}\lambda_{12}}{2\lambda} & 2\lambda_1 v^2 - \frac{fu\omega}{\sqrt{2}v} - \frac{v^2\lambda_{10}^2}{2\lambda} & \lambda_4 v\omega + \frac{fu}{\sqrt{2}} - \frac{v\omega\lambda_{10}\lambda_{11}}{2\lambda} \\ \lambda_6 u\omega + \frac{fv}{\sqrt{2}} - \frac{u\omega\lambda_{11}\lambda_{12}}{2\lambda} & \lambda_4 v\omega + \frac{fu}{\sqrt{2}} - \frac{v\omega\lambda_{10}\lambda_{11}}{2\lambda} & 2\lambda_2\omega^2 - \frac{fuv}{\sqrt{2}\omega} - \frac{\omega^2\lambda_{11}^2}{2\lambda} \end{pmatrix}. \tag{23}$$

At the leading order $(-f, \omega \gg u, v)$, the mass matrix given in Eq. (23)) can be rewritten as

$$\begin{pmatrix} -\frac{fv\omega}{\sqrt{2}u} & \frac{f\omega}{\sqrt{2}} & 0 \\ \frac{f\omega}{\sqrt{2}} & -\frac{fu\omega}{\sqrt{2}v} & 0 \\ 0 & 0 & 2\lambda_2\omega^2 - \frac{\omega^2\lambda_{11}^2}{2\lambda} \end{pmatrix}. \tag{24}$$

The physical fields with respective masses can be written as

$$H = \frac{uS_1 + vS_2}{\sqrt{u^2+v^2}}, \qquad m_H^2 = 0, \qquad H_1 = \frac{-vS_1 + uS_2}{\sqrt{u^2+v^2}},$$
$$m^2_{H_1} = -\frac{f(u^2+v^2)\omega}{\sqrt{2}uv}, \qquad H_2 = S_3, \qquad m^2_{H_2} = \frac{(4\lambda\lambda_2 - \lambda_{11}^2)\omega^2}{2\lambda}. \tag{25}$$

In the new basics, (H, H_1, H_2), the squared mass matrix given in (23) can be written as

$$\begin{pmatrix} C' & B'^T \\ B' & A' \end{pmatrix}, \tag{26}$$

where

$$C' = \frac{v^4(4\lambda\lambda_1 - \lambda_{10}^2) - u^4(\lambda_{12}^2 - 4\lambda\lambda_3) - 2u^2v^2(\lambda_{10}\lambda_{11} - 2\lambda\lambda_5)}{2(u^2+v^2)\lambda}, \tag{27}$$

$$B' = \begin{pmatrix} \frac{uv(v^2(\lambda_{10}(-\lambda_{10}+\lambda_{11}) + \lambda(4\lambda_1 - 2\lambda_5)) + u^2(-\lambda_{10}\lambda_{11} + \lambda_{11}^2 - 4\lambda\lambda_3 + 2\lambda\lambda_5))}{2(u^2+v^2)\lambda} \\ \frac{2\sqrt{2}fuv\lambda - \omega(v^2(\lambda_{10}\lambda_{11} - 2\lambda\lambda_4) + u^2(\lambda_{11}\lambda_{12} - 2\lambda\lambda_6))}{2\sqrt{u^2+v^2}\lambda} \end{pmatrix}, \tag{28}$$

$$A' = \begin{pmatrix} \frac{-\sqrt{2}f(u^2+v^2)^2\omega\lambda + u^3v^3(-(\lambda_{10}-\lambda_{12})^2 + 4\lambda(\lambda_1+\lambda_3-\lambda_5))}{2uv(u^2+v^2)\lambda} & \frac{\sqrt{2}f(u^2-v^2)\lambda + uv\omega(-\lambda_{10}\lambda_{11}+\lambda_{11}\lambda_{12}+2\lambda\lambda_4-2\lambda\lambda_6)}{2\sqrt{u^2+v^2}\lambda} \\ \frac{\sqrt{2}f(u^2-v^2)\lambda + uv\omega(-\lambda_{10}\lambda_{11}+\lambda_{11}\lambda_{12}+2\lambda\lambda_4-2\lambda\lambda_6)}{2\sqrt{u^2+v^2}\lambda} & -\frac{\sqrt{2}fuv\lambda+\omega^3(\lambda_{11}^2-4\lambda\lambda_2)}{2\omega\lambda} \end{pmatrix}. \quad (29)$$

Since $-f, \omega \gg u, v$, we get $A' \gg B', C'$. If we kept explicitly the $\mathcal{O}(\frac{u,v}{\omega})$, the H_1, H_2, H Higgs bosons can gain mass by using block diagonalizing method as

$$\begin{aligned} m'^2_{H_1} &= m^2_{H_1} + \mathcal{O}\left(\frac{u,v}{\omega}\right), \\ m'^2_{H_2} &= m^2_{H_2} + \mathcal{O}\left(\frac{u,v}{\omega}\right), \\ m'^2_{H} &= \frac{v^4(4\lambda\lambda_1 - \lambda_{10}^2) - u^4(\lambda_{12}^2 - 4\lambda\lambda_3) - 2u^2v^2(\lambda_{10}\lambda_{12} - 2\lambda\lambda_5)}{2(u^2+v^2)\lambda} \\ &\quad + \frac{1}{2\sqrt{2}(u^2+v^2)\lambda(\lambda_{11}^2 - 4\lambda\lambda_2)}\left(m_0 + m_1\frac{f}{\omega} + m_2\frac{f^2}{\omega^2}\right), \end{aligned} \quad (30)$$

where

$$\begin{aligned} m_0 &= \sqrt{2}(v^2(\lambda_{10}\lambda_{11} - 2\lambda\lambda_4) + u^2(\lambda_{11}\lambda_{12} - 2\lambda\lambda_6))^2, \\ m_1 &= 8uv\lambda(v^2(-\lambda_{10}\lambda_{11} + 2\lambda\lambda_4) + u^2(-\lambda_{11}\lambda_{12} + 2\lambda\lambda_6)), \\ m_2 &= 8\sqrt{2}u^2v^2\lambda^2. \end{aligned} \quad (31)$$

For the remaining fields in the pseudoscalar sector, the mass spectrum is similar to that of work given in [9]. Let us give a brief result.

(i) The pseudoscalar A_4 is massless and is identified to the Goldstone boson of Z_N.

(ii) Two other fields are massless that are identified to the Goldstone bosons of Z and Z'

$$\begin{aligned} G_Z &= \frac{-uA_1 + vA_2}{\sqrt{u^2+v^2}}; \\ G_{Z'} &= \frac{-\omega^{-1}(u^{-1}A_1 + v^{-1}A_2) + (u^{-2}+v^{-2})A_3}{\sqrt{(u^{-2}+v^{-2}+\omega^{-2})(u^{-2}+v^{-2})}}. \end{aligned} \quad (32)$$

(ii) One neutral complex Goldstone boson, $G_X = \frac{\omega\chi_1 - u\eta_3^*}{\sqrt{u^2+\omega^2}}$, that is eaten by X gauge boson.

(iii) One neutral complex Higgs , namely $H' = \frac{u\chi_1^* + \omega\eta_3}{\sqrt{u^2+\omega^2}}$ with the squared mass $m^2_{H'} = (\frac{1}{2}\lambda_9 - \frac{fv}{\sqrt{2}u\omega})(u^2+\omega^2)$.

(iv) One physical pseudoscalar (A) with mass

$$m_A^2 = -\frac{f}{\sqrt{2}}\frac{u^2v^2 + u^2\omega^2 + v^2\omega^2}{uv\omega}, \quad (33)$$

and the physical state respectively

$$A = \frac{u^{-1}A_1 + v^{-1}A_2 + \omega^{-1}A_3}{\sqrt{u^{-2}+v^{-2}+\omega^{-2}}}. \quad (34)$$

For charged scalars, the mass spectrum is seminar to that of work given in [8].

$$\begin{aligned} H_4^- &= \frac{v\chi_2^- + \omega\rho_3^-}{\sqrt{v^2+\omega^2}}, \\ H_5^- &= \frac{v\eta_2^- + u\rho_1^-}{\sqrt{u^2+v^2}}, \end{aligned} \quad (35)$$

with respective masses

$$\begin{aligned} m^2_{H_4} &= \left(\frac{1}{2}\lambda_7 - \frac{fu}{\sqrt{2}v\omega}\right)(v^2+\omega^2), \\ m^2_{H_5} &= \left(\frac{1}{2}\lambda_8 - \frac{f\omega}{\sqrt{2}uv}\right)(u^2+v^2). \end{aligned} \quad (36)$$

The model contains two massive charged Higgs and two massless Higgs that are identified to the Goldstone bosons of Y and W bosons.

$$\begin{aligned} G_Y^- &= \frac{\omega\chi_2^- - v\rho_3^-}{\sqrt{v^2+\omega^2}}, \\ G_W^- &= \frac{u\eta_2^- - v\rho_1^-}{\sqrt{u^2+v^2}}. \end{aligned} \quad (37)$$

B. Gauge sector

In this section, let us consider the gauge boson spectrum. The mass Lagrangian is given as

$$\begin{aligned}\mathcal{L}_{\mathrm{gaugemass}} &= \left(0,0,\frac{\omega}{\sqrt{2}}\right)\left(gA_{a\mu}T_a - \frac{1}{3}g_X B_\mu - \frac{2}{3}g_N C_\mu\right)^2\left(0,0,\frac{\omega}{\sqrt{2}}\right)^T \\ &+ \left(\frac{u}{\sqrt{2}},0,0\right)\left(gA_{a\mu}T_a - \frac{1}{3}g_X B_\mu + \frac{1}{3}g_N C_\mu\right)^2\left(\frac{u}{\sqrt{2}},0,0\right)^T \\ &+ \left(0,\frac{v}{\sqrt{2}},0\right)\left(gA_{a\mu}T_a + \frac{2}{3}g_X B_\mu + \frac{1}{3}g_N C_\mu\right)^2\left(0,\frac{v}{\sqrt{2}},0\right)^T \\ &+ 2(g_N C_\mu \Lambda)^2. \end{aligned} \tag{38}$$

Let us denote the following combinations:

$$W^\pm_\mu = \frac{A_{1\mu} \mp iA_{2\mu}}{\sqrt{2}}, \quad Y^\mp_\mu = \frac{A_{6\mu} \mp iA_{7\mu}}{\sqrt{2}}. \tag{39}$$

The non-Hermitian gauge bosons $W^\pm_\mu, Y^\mp_\mu$ have the following masses:

$$M^2_W = \frac{1}{4}g^2(u^2+v^2), \quad M^2_Y = \frac{1}{4}g^2(v^2+\omega^2). \tag{40}$$

It is worth noting that $A_{4\mu}$ and $A_{5\mu}$ gain the same mass. Therefore, these vectors can be combined in the following physical states:

$$X^0_\mu = \frac{A_{4\mu} - iA_{5\mu}}{\sqrt{2}}, \tag{41}$$

and its mass is given:

$$M^2_X = \frac{1}{4}g^2(u^2+\omega^2). \tag{42}$$

There is a mixing among $A_{3\mu}, A_{8\mu}, B_\mu, C_\mu$ components. In the basis of these elements, the mass matrix denoted by M^2 is given as follows:

$$\frac{g^2}{2}\begin{pmatrix} \frac{1}{2}(u^2+v^2) & \frac{u^2-v^2}{2\sqrt{3}} & -\frac{t_1(u^2+2v^2)}{3} & \frac{t_2(u^2-v^2)}{3} \\ \frac{u^2-v^2}{2\sqrt{3}} & \frac{1}{6}(u^2+v^2+4\omega^2) & -\frac{t_1(u^2-2(v^2+\omega^2))}{3\sqrt{3}} & \frac{t_2(u^2+v^2+4\omega^2)}{3\sqrt{3}} \\ -\frac{t_1(u^2+2v^2)}{3} & -\frac{t_1(u^2-2(v^2+\omega^2))}{3\sqrt{3}} & \frac{2}{9}t_1^2(u^2+4v^2+\omega^2) & -\frac{2}{9}t_1t_2(u^2-2(v^2+\omega^2)) \\ \frac{t_2(u^2-v^2)}{3} & \frac{t_2(u^2+v^2+4\omega^2)}{3\sqrt{3}} & -\frac{2}{9}t_1t_2(u^2-2(v^2+\omega^2)) & \frac{2}{9}t_2^2(u^2+v^2+4(\omega^2+9\Lambda)) \end{pmatrix}, \tag{43}$$

where $t_1 \equiv g_X/g$, $t_2 \equiv g_N/g$.

The mass matrix in (43) contains one exact zero eigenvalue with the corresponding eigenstate as follows:

$$A_\mu = \frac{\sqrt{3}}{\sqrt{3+4t_1^2}}\left(t_1 A_{3\mu} - \frac{t_1}{\sqrt{3}}A_{8\mu} + B_\mu\right). \tag{44}$$

It is worth to notice that A_μ is the combination of $A_{3\mu}, A_{8\mu}$, and B_μ without contribution of the new gauge boson C_μ. The factor t_1 can be expressed in terms of the sine of the weak mixing angle s_W by identifying the coefficient of the $\bar{e}e\gamma$ vertex with the electromagnetic coupling constant e, similarly as the analysis in [10]. We get

$$t_1 = \frac{\sqrt{3}s_W}{\sqrt{3-4s_W^2}}. \tag{45}$$

The diagonalization of the mass matrix is done via three steps. In the first step, in the base of $A_\mu, Z_\mu, Z'_\mu, C_\mu$, the two remaining Z_μ, Z'_μ gauge vectors are given by

$$Z_\mu = \frac{\sqrt{3+t_1^2}}{\sqrt{3+4t_1^2}}A_{3\mu} + \frac{t_1(\sqrt{3}t_1A_{8\mu} - 3B_\mu)}{\sqrt{3+t_1^2}\sqrt{3+4t_1^2}}, \quad Z'_\mu = \frac{\sqrt{3}}{\sqrt{3+t_1^2}}A_{8\mu} + \frac{t_1}{\sqrt{3+t_1^2}}B_\mu. \tag{46}$$

In this basis, the mass matrix M^2 becomes

$$\begin{pmatrix} 0 & 0 \\ 0 & M'^2 \end{pmatrix}, \tag{47}$$

where M'^2 is the 3×3 mixing mass matrix of Z_μ, Z'_μ, C_μ gauge bosons given as

$$\frac{g^2}{2}\begin{pmatrix} \frac{(3+4t_1^2)(u^2+v^2)}{2(3+t_1^2)} & -\frac{\sqrt{3+4t_1^2}((-3+2t_1^2)u^2+(3+4t_1^2)v^2)}{6(3+t_1^2)} & \frac{\sqrt{3+4t_1^2}t_2(u^2-v^2)}{3\sqrt{3+t_1^2}} \\ -\frac{\sqrt{3+4t_1^2}((-3+2t_1^2)u^2+(3+4t_1^2)v^2)}{6(3+t_1^2)} & \frac{(3-2t_1^2)^2u^2+(3+4t_1^2)^2v^2+4(3+t_1^2)^2\omega^2}{18(3+t_1^2)} & \frac{t_2((3-2t_1^2)u^2+(3+4t_1^2)v^2+4(3+t_1^2)\omega^2)}{9\sqrt{3+t_1^2}} \\ \frac{\sqrt{3+4t_1^2}t_2(u^2-v^2)}{3\sqrt{3+t_1^2}} & \frac{t_2((3-2t_1^2)u^2+(3+4t_1^2)v^2+4(3+t_1^2)\omega^2)}{9\sqrt{3+t_1^2}} & \frac{2}{9}t_2^2(u^2+v^2+4(\omega^2+9\Lambda^2)) \end{pmatrix}. \tag{48}$$

The matrix given in (48) can be diagonalized by using block diagonalizing method. In new basis $(\mathcal{Z}_\mu, \mathcal{Z}'_\mu, Z_\mu^N)$, the mass mixing matrix is given as

$$\begin{pmatrix} A & 0 \\ 0 & m_{Z^N}^2 \end{pmatrix}, \tag{49}$$

where A is the 2×2 matrix

$$A = \frac{g^2}{2}\begin{pmatrix} \frac{(3+4t_1^2)(u^2+v^2)}{2(3+t1^2)} + \mathcal{O}\left(\frac{v^4}{\Lambda^2}\right) & -\frac{\sqrt{3+4t_1^2}((-3+2t_1^2)u^2+(3+4t_1^2)v^2)}{6(3+t_1^2)} + \mathcal{O}\left(\frac{v^2\omega^2}{\Lambda^2}\right) \\ -\frac{\sqrt{3+4t_1^2}((-3+2t_1^2)u^2+(3+4t_1^2)v^2)}{6(3+t_1^2)} + \mathcal{O}\left(\frac{v^2\omega^2}{\Lambda^2}\right) & \frac{(3-2t_1^2)^2u^2+(3+4t_1^2)^2v^2+4(3+t_1^2)^2\omega^2}{18(3+t_1^2)} + \mathcal{O}\left(\frac{\omega^4}{\Lambda^2}\right) \end{pmatrix}, \tag{50}$$

$$m_{Z^N}^2 \simeq 4g^2t_2^2\Lambda^2. \tag{51}$$

The new basis $(\mathcal{Z}_\mu, \mathcal{Z}'_\mu, Z_\mu^N)$ is related to the basis (Z_μ, Z'_μ, C_μ) as following:

$$\begin{pmatrix} \mathcal{Z}_\mu \\ \mathcal{Z}'_\mu \\ Z_\mu^N \end{pmatrix} = \begin{pmatrix} 1 & 0 & -\epsilon_1 \\ 0 & 1 & -\epsilon_2 \\ \epsilon_1 & \epsilon_2 & 1 \end{pmatrix}\begin{pmatrix} Z_\mu \\ Z'_\mu \\ C_\mu \end{pmatrix}, \tag{52}$$

where

$$\begin{aligned} \epsilon_1 &= -\frac{3\sqrt{3+4t_1^2}(u^2-v^2)}{2\sqrt{3+t_1^2}t_2(u^2+v^2+4(\omega^2+9\Lambda^2))}, \\ \epsilon_2 &= \frac{(3-2t_1^2)u^2+(3+4t_1^2)v^2+4(3+t_1^2)\omega^2}{2\sqrt{3+t_1^2}t_2(u^2+v^2+4(\omega^2+9\Lambda^2))}. \end{aligned} \tag{53}$$

In the limit $\Lambda \gg \omega \gg u, v$,

$$\mathcal{Z}_\mu \sim Z_\mu, \qquad \mathcal{Z}'_\mu \sim Z'_\mu, \qquad Z_\mu^N \sim C_\mu. \tag{54}$$

The new heavy gauge boson Z_μ^N is imbedded to the gauge group $U(1)_N$. It approximately does not mix to other gauge bosons. $\mathcal{Z}_\mu$ and $\mathcal{Z}'_\mu$ are mixing of the two physical fields Z_μ^1, Z_μ^2.

$$\begin{aligned} &Z_\mu^1 = \cos\xi\mathcal{Z}_\mu - \sin\xi\mathcal{Z}'_\mu, \qquad Z_\mu^2 = \sin\xi\mathcal{Z}_\mu + \cos\xi\mathcal{Z}'_\mu, \\ &m_{Z^1}^2 \simeq \frac{g^2}{8}\left(u^2+\omega^2+\frac{u^2+4v^2+\omega^2}{3-4s_W^2} - 4\frac{\sqrt{c_W^4u^4+v^4-c_{2W}v^2\omega^2+c_W^4\omega^4+u^2(-c_{2W}v^2+(-1+2s_W^4)\omega^2)}}{(3-4s_W^2)}\right), \\ &m_{Z^2}^2 \simeq \frac{g^2}{8}\left(u^2+\omega^2+\frac{u^2+4v^2+\omega^2}{3-4s_W^2} + 4\frac{\sqrt{c_W^4u^4+v^4-c_{2W}v^2\omega^2+c_W^4\omega^4+u^2(-c_{2W}v^2+(-1+2s_W^4)\omega^2)}}{(3-4s_W^2)}\right), \end{aligned} \tag{55}$$

where $\tan 2\xi = \frac{\sqrt{3-4s_W^2}(c_{2W}u^2-v^2)}{((-1+2s_W^4)u^2-c_{2W}v^2+2c_W^4\omega^2)}$.

If we assume $\omega \gg u, v$, then $\tan 2\xi \to 0$. We get

$$Z^1_\mu \sim \mathcal{Z}_\mu, \qquad m^2_{Z^1} \simeq \frac{g^2(u^2+v^2)}{4c^2_W},$$
$$Z^2_\mu \sim \mathcal{Z}'_\mu, \qquad m^2_{Z^2} \simeq \frac{g^2 c^2_W \omega^2}{(3-4s^2_W)}. \tag{56}$$

The gauge boson Z^1_μ is identified as Z_μ in the standard model.

III. GENERATION OF INFLATION IN THE 3-3-1-1 MODEL

We would like to note that the scalar singlet ϕ is completely breaking $U(1)_N$. The VEV $\langle\phi\rangle$ can stay at the same scale as ω's scale and the interesting phenomenology of the model at TeV scale was studied in [9]. In a different situation, this VEV can be very high that can be integrated out from the low energy effective potential and a new gauge boson Z_N decoupling from the gauge boson spectrum. In this part, we expect that the VEV of ϕ is very high and consider the singlet scalar ϕ plays the role of inflaton field. The potential for ϕ at the tree level can be read off from Eq. (13) as

$$V_\phi = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 + \lambda_{10}(\phi^\dagger\phi)(\rho^\dagger\rho) + \lambda_{11}(\phi^\dagger\phi)(\chi^\dagger\chi) + \lambda_{12}(\phi^\dagger\phi)(\eta^\dagger\eta). \tag{57}$$

Due to the larger VEV of ϕ, the interaction terms of the singlet scalar Higgs and the ordinary 3-3-1 model Higgs triplets can be ignored. During inflation, we get

$$V_\phi = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2. \tag{58}$$

This potential is taken part in the chaotic inflation. However, the inflaton field has coupling to the matter fields which allow it to make the transition to hot big bang cosmology at the end of inflation, namely

$$\mathcal{L} \supset 4g^2_N C^\mu C_\mu \phi^2 + h'^\nu_{ab}\bar{\nu}^c_{aR}\nu_{bR}\phi. \tag{59}$$

We take into account quantum corrections to V_ϕ following the analysis of Coleman and Weinberg [11]

$$V_{\rm eff} = \frac{1}{64\pi^2}\sum_i\left[(-1)^{2J}(2J+1)m^4_i \ln\frac{m^2_i}{\Delta^2}\right], \tag{60}$$

where $i = \nu_{aR}, \phi, C_\mu, \chi, \rho, \eta$.

$$m_{\nu_{aR}} = -2h'^\nu_{ab}\Phi; \qquad m^2_\phi = 2(\mu^2+3\lambda\Phi^2);$$
$$m^2_{C_\mu} = 8g^2_N\Phi^2; \qquad m^2_\rho = 2\lambda_{10}\Phi^2;$$
$$m^2_\chi = 2\lambda_{11}\Phi^2; \qquad m^2_\eta = 2\lambda_{12}\Phi^2. \tag{61}$$

We get

$$V_{\rm eff} = \frac{1}{64\pi^2}\left\{\left[-32\sum_i(h'^\nu_{ii})^4 + 192g^4_N + 4(\lambda^2_{10}+\lambda^2_{11}+\lambda^2_{12})\right] \times \Phi^4\ln\frac{\Phi^2}{\Delta^2} + 4(\mu^2+3\lambda\Phi^2)^2\ln\frac{\mu^2+3\lambda\Phi^2}{\Delta^2}\right\}$$
$$= \frac{1}{64\pi^2}\left\{a\Phi^4\ln\frac{\Phi}{\Delta} + 4(\mu^2+3\lambda\Phi^2)^2\ln\frac{\mu^2+3\lambda\Phi^2}{\Delta^2}\right\}, \tag{62}$$

where

$$a = 2\left[-32\sum_i(h'^\nu_{ii})^4 + 192g^4_N + 4(\lambda^2_{10}+\lambda^2_{11}+\lambda^2_{12})\right]. \tag{63}$$

We identify the inflaton with the real part of the $B-L$ Higgs field, $\Phi = \sqrt{2}\mathcal{R}[\phi]$. In the leading- log approximation, we obtain

$$V(\Phi) = V_{\rm tree} + V_{\rm eff} \simeq \frac{\mu^2}{2}\Phi^2 + \frac{\lambda}{4}\Phi^4 + V_{\rm eff}. \tag{64}$$

We would like to remain that the inflation occurs as the inflaton slowly rolls to the minimal potential. The inflationary slow roll parameters are given [12] by

$$\epsilon(\Phi) = \frac{1}{2}m^2_P\left(\frac{V'}{V}\right)^2, \qquad \eta(\Phi) = m^2_P\left(\frac{V''}{V}\right),$$
$$\zeta^2(\Phi) = m^4_P\frac{V'V'''}{V^2}, \tag{65}$$

where $m_P = 2.4\times10^{18}$ GeV and a prime is denoted as a derivative of Φ. The slow roll condition means that $\epsilon(\Phi) \ll 1, |\eta(\Phi)| \ll 1, \zeta(\Phi) \ll 1$. In this limit, the spectral index n_s, the tensor to scalar ratio r (a canonical measure of gravity wave from inflation) and the running index α can be written as

$$n_s = 1-6\epsilon+2\eta, \qquad r = 16\epsilon,$$
$$\alpha = 16\epsilon\eta - 14\epsilon^2 - 2\zeta^2. \tag{66}$$

The spectrum index n_s is estimated by BICEP2 experiment [2], Planck [13] and WMAP9 [14] measurements. It is closed to 0.96. The tensor to scalar ratio is proven by BICEP2 [2], $r = 0.20^{+0.07}_{-0.05}$ while the Planck and WMAP9 experiments gave the bound $r < 0.11(0.12)$.

The number of e-folds is given by

$$N = \int^{\Phi_0}_{\Phi_e}\frac{Vd\Phi}{V'}, \tag{67}$$

where Φ_e is the inflaton value at the end of inflation and defined by $\max(\epsilon(\Phi), \eta(\Phi), \zeta(\Phi)) = 1$. Φ_0 is the inflation value at the horizon exit. The value of N is around 50–60 and depends on the energy scale during inflation.

The amplitude of the curvature perturbation is given as follows:

$$\triangle^2_{\mathcal{R}} = \frac{V}{24\pi^2 m_P^4 \epsilon(\Phi)}. \quad (68)$$

The value of curvature perturbation should satisfy the Planck measurement [15]: $\triangle^2_{\mathcal{R}} = 2.215 \times 10^{-9}$ at the scale $k_0 = 0.05\ \mathrm{Mpc}^{-1}$.

Let us study parameter space of μ, λ, Δ, a appeared in the potential $V(\Phi)$. If $\mu^2 \gg \lambda\Phi^2$ or $\mu^2 \sim \lambda\Phi^2$, both $\triangle^2_{\mathcal{R}}$ and r are not in agreement with the Planck and WMAP9 experimental results. For example, taking $\Delta = 30 m_P$, and random values of other parameters $10^{-10} m_P^2 < |\mu^2| < 10^4 m_P^2$, $10^{-15} < |a| < 10^3$, $10^{-10} < |\lambda| < 1$ we get $|\triangle^2_{\mathcal{R}}| > 10^3$. If we assume that $\mu^2 \ll \lambda\Phi^2$, the potential (64) can be rewritten in simple form

$$V(\Phi) = \lambda'\left(\Phi^4 + a'\Phi^4 \ln\frac{\Phi}{\Delta}\right), \quad (69)$$

where

$$\lambda' = \frac{\lambda}{4}, \qquad a' = \frac{a + 72\lambda^2}{16\pi^2\lambda}. \quad (70)$$

The coupling constant λ is determined to satisfy the constraint on $\triangle^2_{\mathcal{R}}$, while as the predictions for n_s, r, α are given for fixed values of a', Δ. Figure 1 shows the predicted values of n_s, r and α for $\Delta = 0.1 m_P$ (green), $\Delta = 30 m_P$ (red), $\Delta = 50 m_P$ (pink), and $\Delta = 500 m_P$ (blue) in the range of $-10^3 < a' < 10^3$ with the number of e-folds $N = 60$. We can see that for $\Delta = 0.1 m_P$ and $\Delta = 500 m_P$, r runs out of experimental region for almost values of a' in the range $-10^3 < a' < 10^3$. For $\Delta = 30 m_P$, we need to require $a' < -36$ or $a' > 6$ to make sure n_s and r are in agreement with experimental results [15], $n_s \in (0.94, 0.98)$, and $r \in (0.001, 0.15)$.

If we vary a', Δ in the parameter region satisfying experimental results, the order of $\langle\Phi\rangle$ and the inflaton mass mostly does not change. From now on we take $a' = -10^2, \Delta = 30 m_P$ for the below numerical calculations.

From the minimal potential condition, we get $\langle\Phi\rangle \simeq 23.6 m_P$. The inflaton mass is calculated by the second derivative of the effective potential at the minimum. For Z^N and ν_{kM}, the mass arises from (59) with notice that $\Phi = \sqrt{2}\mathcal{R}[\phi]$. We obtain

$$m_\Phi = \sqrt{V''(\Phi)}|_{\Phi=\langle\Phi\rangle} \simeq 2.67 \times 10^{13}\ \mathrm{GeV},$$
$$m_{Z^N} = 2g_N\langle\Phi\rangle, \qquad m_{\nu_{iR}} = -\sqrt{2}h'^{\nu}_{ii}\langle\Phi\rangle. \quad (71)$$

Now let us calculate the reheating temperature. In this model, the inflaton couples to pair of Higgs, pair of gauge boson Z^N and pair of Majorana neutrinos. We assume that $m_\Phi < m_{Z^N}$, hence the inflaton cannot decay into pair of Z^N. The inflaton can decay into pair of Higgs with the decay rate

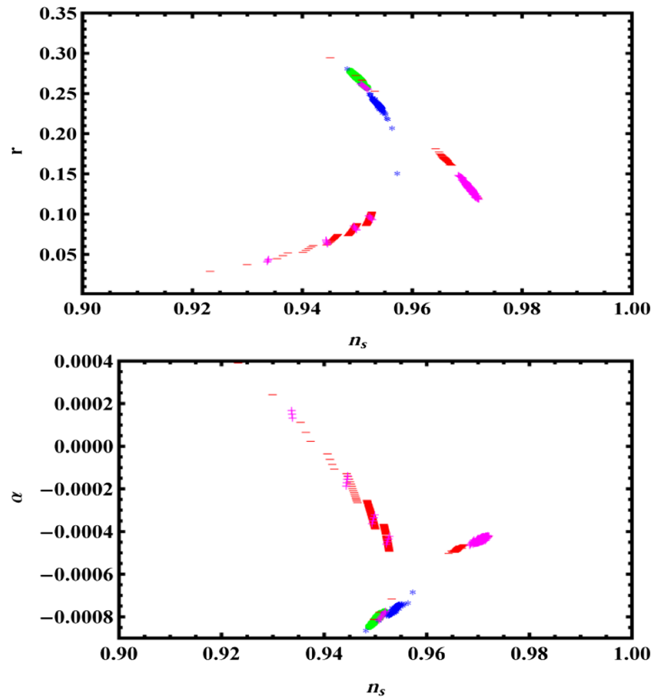

FIG. 1 (color online). r vs n_s (upper panel) and α vs n_s (lower panel) for $\Delta = 0.1 m_P$ (green dot), $\Delta = 30 m_P$ (red minus), $\Delta = 50 m_P$ (pink plus), and $\Delta = 500 m_P$ (blue asterisk) in the range of $-10^3 < a' < 10^3$, with the number of e-folds $N = 60$.

$$\Gamma(\Phi \to hh) = \frac{\lambda^2_{10;11;12}\langle\Phi\rangle^2}{32\pi m_\Phi}. \quad (72)$$

If the mass condition is allowed, the inflaton can decay into pair of ν_{iR}

$$\Gamma(\Phi \to \nu_{iR}\nu_{iR}) = \frac{(h'^{\nu}_{ii})^2 m_\Phi}{16\pi}. \quad (73)$$

If $|\lambda_{10;11;12}| \gg \frac{\sqrt{2}|h'^{\nu}_{ii}|m_\Phi}{\langle\Phi\rangle}$, we get $\Gamma(\Phi \to hh) \gg \Gamma(\Phi \to \nu_{iR}\nu_{iR})$. The inflaton dominantly decays into pair of Higgs, therefore the reheating temperature is estimated as

$$T_R = \left(\frac{90}{\pi^2 g^*}\right)^{\frac{1}{4}} (\Gamma_\Phi m_P)^{\frac{1}{2}} \sim 10^{20}\ \mathrm{GeV} \times |\lambda_{10;11;12}|, \quad (74)$$

where $g^* = 106.75$ is the number of degrees of the freedom active at the temperature of the asymmetry production. The constraint $\Gamma(\Phi \to hh) \ll m_\Phi$ requires $|\lambda_{10;11;12}| \ll \frac{\sqrt{32\pi}m_\Phi}{\langle\Phi\rangle} \sim 10^{-6}$. We find the limit $T_R(\max) < 10^{14}$ GeV. Taking $|\lambda_{10;11;12}| \sim 10^{-11}$ then $T_R \sim 10^9$ GeV satisfying the upper bound on reheating temperature to prevent gravitinos problem. In this case the thermal leptogenesis scenario may work to explain the baryon asymmetry.

In other case, we assume

$$|h'^{\nu}_{11}| \ll \frac{m_\Phi}{\sqrt{2}\langle\Phi\rangle} \sim 3.33\times 10^{-7} < |h'^{\nu}_{22}| \sim |h'^{\nu}_{33}|, \quad (75)$$

therefore,

$$m_{\nu_{1R}} \ll m_\Phi < m_{\nu_{2R}} \sim m_{\nu_{3R}}. \quad (76)$$

If $|\lambda_{10;11;12}| \ll \frac{|h'^{\nu}_{11}|m_\Phi}{\langle\Phi\rangle}$, we get $\Gamma(\Phi\to hh) \ll \Gamma(\Phi\to \nu_{1R}\nu_{1R})$. When $\lambda_{10;11;12}$ are negligibly small, the inflaton dominantly decays into pair of ν_{1R}. The produced reheating temperature is given as

$$T_R = \left(\frac{90}{\pi^2 g^*}\right)^{\frac{1}{4}}(\Gamma_\Phi m_P)^{\frac{1}{2}} \sim 10^{14}\times |h'^{\nu}_{11}|. \quad (77)$$

This temperature is much lower than the right-handed neutrino mass since $\langle\Phi\rangle$ is at Planck value. We can apply nonthermal leptogenesis scenario, in which the ν_{1R} is produced through the direct nonthermal decay of the inflaton Φ.

IV. LEPTOGENESIS IN THE 3-3-1-1 MODEL

First, we consider the scalar sector. The scalar mass spectrum is considered in [8,9], in which $-f\sim\omega\sim\Lambda\gg u\sim v$. In this work we assume $\Lambda\gg\omega\gg u\sim v$; the considered model contains the following

(i) There are 9 Goldstone bosons $A_4, G_Z, G'_Z, G_X, G^*_X, G^{\pm}_W, G^{\pm}_Y$; their interactions can be gauged away by a unitary transformation.

(ii) One Higgs gains mass at the electroweak breaking scale. This is the lightest massive Higgs boson H and is identified as SM Higgs.

(iii) There are 9 new Higgs bosons namely, $A, H_1, H_2, H^{\pm}_4, H^{\pm}_5, H', H'^*$, which are heavy at the ω scale, while the mass of H_3 is proportional to Λ.

In the gauge sector, let us collect the new gauge bosons beyond the SM. In the limit $\Lambda\gg\omega\gg u, v$, we get

(i) One super heavy gauge boson $Z^N_\mu \sim C_\mu$ with the mass $m^2_{Z^N} \simeq 4g^2_N\Lambda^2$.

(ii) All the other new gauge bosons, $Z^2_\mu, X^0_\mu, X^{0*}_\mu, Y^{\pm}_\mu$, have mass in order $\mathcal{O}(\omega)$.

The lepton number of particles are considered in [8,9]. In particular, the SM particles have a lepton number as usual. The new particles $(G_X, H'^*, H^-_4, G^-_Y, X^0, Y^-)$ have the lepton number equal to one; their complex conjugate have the lepton number equal to minus one while the remaining Higgs and gauge bosons have zero lepton number.

Now in order to account for leptogenesis, we have to verify the lepton-number violating interactions. Seeing that the lepton number L and baryon number B are conserved by VEVs of η,χ,ρ as mentioned in [8]. All interaction terms appeared after symmetry breaking in the considered model are conserved by the lepton number. Hence it is clear that B, L violating number interactions should be broken in other ways in order to explain neutrino mass and mixing as well as the matter-antimatter asymmetry of the Universe. The lepton number only can be violated in the interactions of Majorana neutrinos with nonzero lepton-number particles.

Let us remind the seesaw mechanism that explains the tiny neutrino mass and large mixing. The Lagrangian relevant to the neutrino mass has a form as

$$L_{\nu-\text{mass}} = (h^{\nu}_{ab}\bar{\psi}_{aL}\eta\nu_{bR} + h'^{\nu}_{ab}\bar{\nu}^c_{aR}\nu_{bR}\phi + \text{H.c.}). \quad (78)$$

The left-handed neutrinos couple to the right-handed neutrinos through the first term of the Eq. (78) and have a Dirac mass as

$$[m^D_\nu]_{ab} = -\frac{h^{\nu}_{ab}}{\sqrt{2}}u, \quad (79)$$

while the right-handed neutrinos couple to themselves through the second term given in the Eq. (78) and have a Majorana mass as

$$[m^M_\nu]_{ab} = -\sqrt{2}h'_{ab}\Lambda. \quad (80)$$

Hence, we can explain the smallness of the light neutrino masses via a type I seesaw mechanism [8] and predict six Majorana neutrinos as mass eigenstates, three heavy neutrinos ν_{iM} and three light neutrinos ν_{iE},

$$\nu_{iM} = \nu_{iR} + \nu^c_{iR}; \qquad m_{\nu_M} = m^M_\nu = -\sqrt{2}h'\Lambda, \quad (81)$$

$$\nu_{iE} = \nu_{iL} + \nu^c_{iL};$$

$$m^{\text{eff}}_{\nu_E} = -m^D_\nu(m^M_\nu)^{-1}(m^D_\nu)^T = \frac{u^2}{2\sqrt{2}\Lambda}h^\nu(h'^\nu)^{-1}(h^\nu)^T. \quad (82)$$

We note that the considered model also contains three new neutral fermions N_{aR}. They obtain the Majorana masses [8] via an effective interaction as

$$\frac{\lambda_{ab}}{M}(\bar{\psi}^c_{aL})_m(\psi_{bL})_n(\chi_m\chi_n)^* + \text{H.c.} \quad (83)$$

The Majorana masses of the neutral fermions N_{aR} are given

$$[m_{N_R}]_{ab} = -\frac{\lambda_{ab}\omega^2}{M} \quad (84)$$

and the Majorana fermion states are

$$N_i = N_{iR} + N^c_{iR}. \quad (85)$$

Based on the Majorana fermion states given in Eqs. (81), (82) and (85), we can rewrite the Lagrangian $\mathcal{L}_{\nu_R}$ including the Yukawa terms in Eq. (78) and the gauge-fermion interaction $\bar{\nu}_{iR}i\gamma^\mu D_\mu\nu_{iR}$ as follows:

TABLE I. Nonzero couplings of fermions appearing in loop diagram of $\nu_M \to e + H_5^+$ and $\nu_M \to N + H'^*$.

Vertex	Coupling	Vertex	Coupling
$\bar{\nu}_{aE}\nu_{bM}H$	$\frac{uh^\nu_{ab}}{\sqrt{2}\sqrt{u^2+v^2}}P_R$	$\bar{\nu}_{aE}\nu_{bM}H_1$	$-\frac{vh^\nu_{ab}}{\sqrt{2}\sqrt{u^2+v^2}}P_R$
$\bar{e}_a\nu_{bM}H_5^-$	$\frac{vh^\nu_{ab}}{\sqrt{u^2+v^2}}P_R$	$\bar{N}_a\nu_{bM}H'$	$\frac{\omega h^\nu_{ab}}{\sqrt{u^2+\omega^2}}P_R$
$\bar{\nu}_{aM}\nu_{bM}H_3$	$\frac{h'^\nu_{ab}}{\sqrt{2}}P_R + \frac{h'^{\nu *}_{ba}}{\sqrt{2}}P_L$	$\bar{\nu}_{aM}\nu_{bM}Z^N_\mu$	$g_N\gamma^\mu P_R$
$\bar{\nu}_{aE}e_bH_5^+$	$\frac{uh^e_{ab}}{\sqrt{u^2+v^2}}P_R$	$\bar{N}_ae_bH_4^+$	$\frac{\omega h^e_{ab}}{\sqrt{v^2+\omega^2}}P_R + \frac{v\omega\lambda_{ab}}{\sqrt{2}\sqrt{v^2+\omega^2}M}P_L$
$\bar{N}_a\nu_{bE}H'$	$\frac{u\omega\lambda_{ab}}{\sqrt{2}\sqrt{u^2+\omega^2}M}P_L$	$\bar{e}_a\nu_{bE}W^-_\mu$	$-\frac{g\gamma^\mu}{\sqrt{2}}P_L$
$\bar{e}_ae_bH$	$\frac{vh^e_{ab}}{\sqrt{2}\sqrt{u^2+v^2}}P_R + \frac{vh^{e*}_{ba}}{\sqrt{2}\sqrt{u^2+v^2}}P_L$	$\bar{e}_ae_bH_1$	$\frac{uh^e_{ab}}{\sqrt{2}\sqrt{u^2+v^2}}P_R + \frac{uh^{e*}_{ba}}{\sqrt{2}\sqrt{u^2+v^2}}P_L$
$\bar{e}_ae_bA_\mu$	$gs_W\gamma^\mu$	$\bar{e}_ae_bZ^k_\mu$	$\gamma^\mu(g_{kV} - g_{kA}\gamma^5), k = 1, 2, N$
$\bar{\nu}_{aE}N_bX_\mu$	$-\frac{g\gamma^\mu}{\sqrt{2}}P_L$	$\bar{e}_aN_bY^-_\mu$	$-\frac{g\gamma^\mu}{\sqrt{2}}P_L$
$\bar{N}_aN_bZ^2_\mu$	$\frac{gc_W}{\sqrt{3-4s_W^2}}\gamma^\mu P_L$	$\bar{N}_aN_bZ^N_\mu$	$\frac{2}{3}g_N\gamma^\mu P_L$
$\bar{N}_aN_bH_2$	$\frac{\omega\lambda_{ab}}{2M}P_L + \frac{\omega\lambda^*_{ba}}{2M}P_R$		

$$\mathcal{L}_{\nu_R} = \left(h^\nu_{ab}\bar{\psi}_{aL}\eta\nu_{bM} + h'^\nu_{ab}\bar{\nu}_{aM}P_R\nu_{bM}\phi - \frac{1}{2}[m^M_\nu]_{ab}\bar{\nu}_{aM}P_R\nu_{bM} + \text{H.c.}\right) + g_N\bar{\nu}_{iM}\gamma^\mu P_R\nu_{iM}Z^N_\mu + \bar{\nu}_{iM}i\gamma^\mu\partial_\mu P_R\nu_{iM} + \text{H.c.} \quad (86)$$

To rely on Higgs physical states mentioned above, we obtain the physical interaction terms that violate the lepton number. In particular the lepton violating interactions appeared in Eq. (86) are: $\bar{e}P_R\nu_M H_5^-, \bar{N}P_RH'\nu_M$.

We would like to emphasize that the lepton-number violating terms also appear via the interactions of the light Majorana neutrinos, namely, $\bar{e}\nu_E W^-, \bar{N}\nu_E X^{0*}$. However these interactions do not generate baryon asymmetry by [16]. In brief, this model contains the lepton-number violating interactions, which are $\bar{e}P_R\nu_M H_5^-, \bar{N}P_RH'\nu_M$, $\bar{e}\nu_E W^{-f}, \bar{N}\nu_E X^{0*}$. We consider leptogenesis scenario at the temperature T_Γ satisfying $T_\Gamma \gg 1$ TeV. It implies that only ν_M can generate lepton asymmetry.

Before calculating the CP asymmetry of ν_{aM}, for convenience, we list all nonzero couplings of fermions appearing in loop diagram of $\nu_M \to e + H_5^+$ and $\nu_M \to N + H'^*$, given in Table 1.

All possible one-loop diagrams, which can contribute to the CP asymmetry from the decay $\nu_M \to e + H_5^+$ are listed in Fig. 2. The interference of the tree level and one-loop level (2a), (2b), (3b) with the propagator ν_{jM}, (6) with the propagator (H_5^-, e_l) gives dominated contribution to the CP asymmetry. We obtain

$$\begin{aligned}\epsilon^{i(1)}_{\nu_{kM}} &= \frac{\Gamma(\nu_{kM} \to e_i + H_5^+) - \Gamma(\nu_{kM} \to \bar{e}_i + H_5^-)}{2\Gamma_{\nu_{kM}}} \\ &\simeq \frac{1}{8\pi C_0}\left[\frac{1}{2}g\lambda_{W^+H_1H_5^-} + e\lambda_{AH_5^+H_5^-} + (g_{1V} + g_{1A})\lambda_{Z_1H_5^+H_5^-} + (g_{2V} + g_{2A})\lambda_{Z_2H_5^+H_5^-}\right. \\ &\quad \left. + (g_{NV} + g_{NA})\lambda_{Z_NH_5^+H_5^-}(4 - 2g_z\log[1 + 1/g_z])\right]s^2_\beta\sum_l \text{Im}[h^\nu_{ik}h^\nu_{lk}] \\ &\quad + \frac{s^4_\beta}{8\pi C_0}\sum_j\sqrt{g_j}[1 - (1 + g_j)\log[1 + 1/g_j] + (1 - g_j)^{-1}]\text{Im}[(h^{\nu\dagger}h^\nu)_{kj}h^\nu_{ik}h^\nu_{ij}],\end{aligned} \quad (87)$$

where $\Gamma_{\nu_{kM}}$ is the total decay rate of ν_{kM} at tree level,

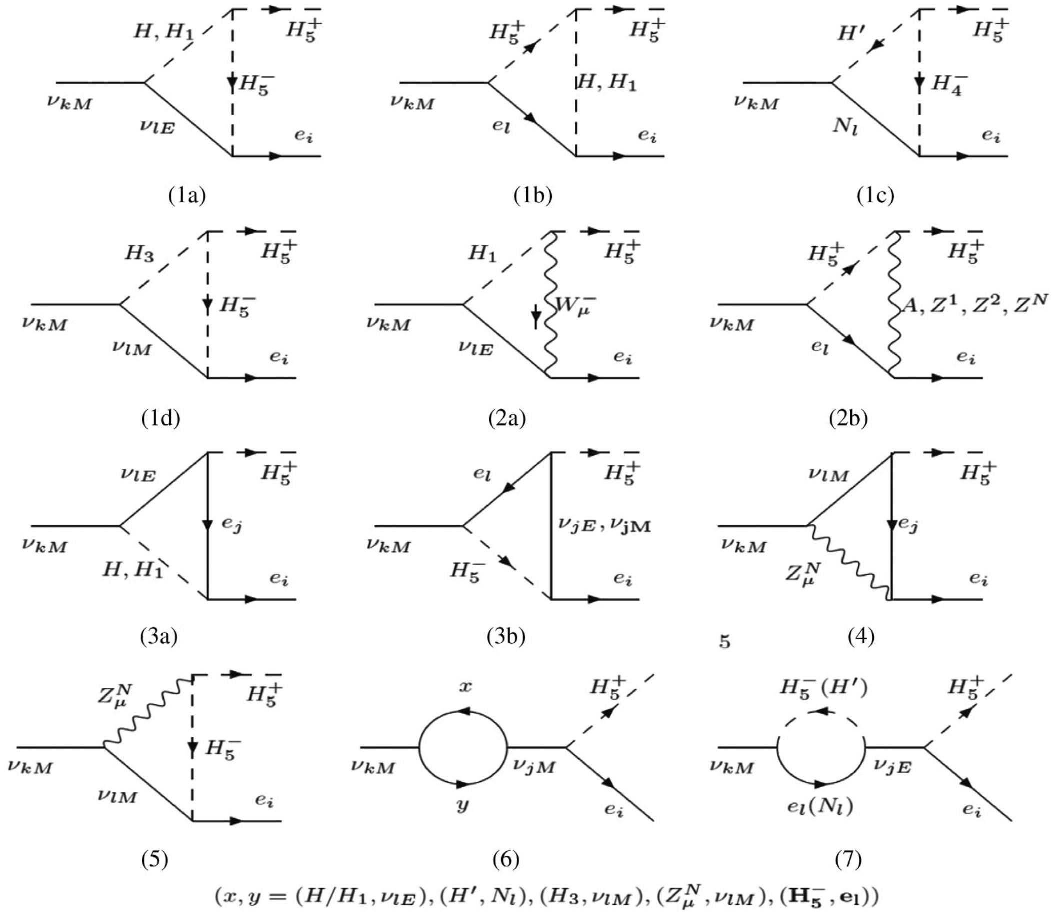

FIG. 2. One-loop diagram contributing to the asymmetry from the decay $\nu_M \to e + H_5^+$.

$$g_z = \frac{m_{Z_N}^2}{m_{\nu_{kM}}^2}, \qquad g_j = \frac{m_{\nu_{jM}}^2}{m_{\nu_{kM}}^2}, \qquad C_0 = (2 + s_\beta^2)\sum_i |h_{ik}^\nu|^2 = (2 + s_\beta^2)(h^{\nu\dagger} h^\nu)_{kk}, \qquad t_\beta = v/u,$$

$$\lambda_{W^+ H_1 H_5^-} = \frac{g}{2}, \qquad \lambda_{A H_5^+ H_5^-} = -e, \qquad \lambda_{Z_1 H_5^+ H_5^-} = -\frac{g c_{2W}}{2 c_W}, \qquad \lambda_{Z_2 H_5^+ H_5^-} = -\frac{g(c_\beta^2 - c_{2W} s_\beta^2)}{2 c_W \sqrt{3 - 4 s_W^2}},$$

$$\lambda_{Z_N H_5^+ H_5^-} = -\frac{g_N c_{2\beta}}{3}, \qquad (g_{1V} + g_{1A}) = -(g_{2V} + g_{2A}) = \frac{g c_{2W}}{2 c_W}, \qquad (g_{NV} + g_{NA}) = \frac{2 g_N}{3}. \tag{88}$$

Here we ignore the mixing between Z_μ^1 and Z_μ^2 since $\omega \gg u, v$.

Now we consider *CP* asymmetry of the decay $\nu_M \to N + H'^*$. All possible loop diagrams are listed in the Fig. 3. The interference of the tree level and one-loop level (2c), (3) with the propagator ν_{jM}, (6) with the propagator (H_5^-, e_l) gives dominated contribution to the *CP* asymmetry. We obtain

$$\begin{aligned}\varepsilon_{\nu_{kM}}^{i(2)} &= \frac{\Gamma(\nu_{kM} \to N_i + H'^*) - \Gamma(\nu_{kM} \to N_i + H')}{2\Gamma_{\nu_{kM}}} \\ &\simeq \frac{1}{8\pi C_0}\left[\frac{g c_W}{\sqrt{3 - 4 s_W^2}} \lambda_{Z_2 H'^* H'} + \frac{2}{3} g_N \lambda_{Z_N H'^* H'} (4 - 2 g_z \log[1 + 1/g_z])\right] \sum_l \mathrm{Im}[h_{ik}^{\nu *} h_{lk}^\nu] \\ &\quad + \frac{1}{8\pi C_0} \sum_j \sqrt{g_j}\,[1 - (1 + g_j)\log[1 + 1/g_j] + s_\beta^2 (1 - g_j)^{-1}]\,\mathrm{Im}[(h^{\nu\dagger} h^\nu)_{kj} h_{ik}^{\nu *} h_{ij}^\nu],\end{aligned} \tag{89}$$

where

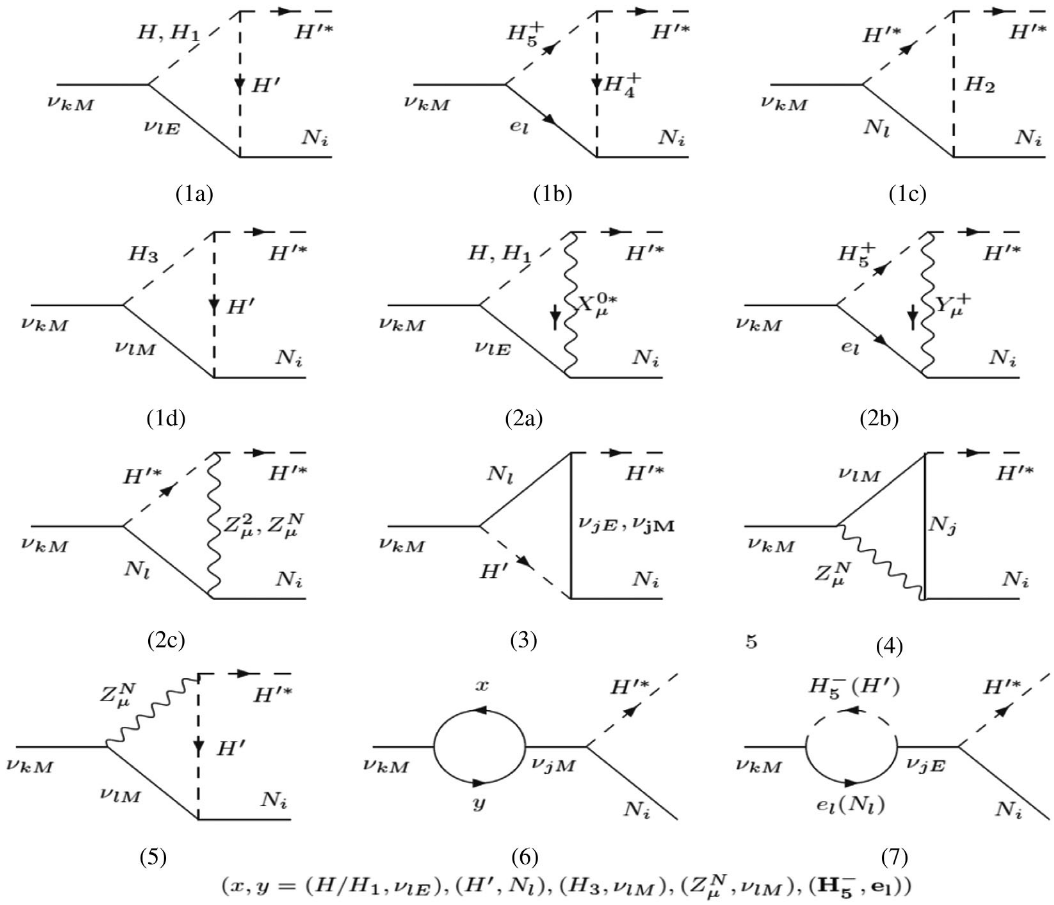

FIG. 3. One-loop diagram contributing to the asymmetry from the decay $\nu_M \to N + H'^*$.

$$\lambda_{Z_2 H'^* H'} = -\frac{g c_W}{\sqrt{3 - 4 s_W^2}}, \qquad \lambda_{Z_N H'^* H'} = \frac{g_N}{3}. \tag{90}$$

We would like to notice that since the coupling $\lambda_{\bar{e}_a \nu_{bM} H_5^-} = s_\beta h^\nu_{ab} P_R$ while $\lambda_{\bar{N}_a \nu_{bM} H'} \simeq h^\nu_{ab} P_R$, the factors s^2_β, s^4_β appear in (87) while s^0_β, s^2_β appear in (89). In this work we take $u \sim v$ and thus $s_\beta = 1/\sqrt{2}$.

Let us comment on neutrino mass and mixing. The light neutrino mass matrix is given Eq. (82). In order to diagonal this matrix, we have to use the U matrix. It is nice to note that the lepton mixing matrix was studied by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS). The standard form of this mixing matrix is given

$$U_{\mathrm{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{91}$$

where $c_{ij} = \cos\theta_{ij}, s_{ij} = \sin\theta_{ij}$ and the values of θ_{ij} are determined by the global analysis [17], namely

$$\sin^2\theta_{23} = 0.466^{+0.073,0178}_{-0.058,0.135};$$
$$\sin^2\theta_{12} = 0.312^{+0.019,0.063}_{-0.0018,0,049};$$
$$\sin^2\theta_{13} = 0.016 \pm 0.010 (\leq 0.046). \tag{92}$$

δ is unknown CP violating Dirac phase.

On the other hand, the square of charged lepton mass matrix and light neutrino mass matrix are diagonalized by two unitary transformations

$$U_l^\dagger M_l^+ M_l U_l = \mathrm{Diag}(m_e^2, m_\mu^2, m_\tau^2);$$
$$U_\nu^T m^{\mathrm{eff}}_{\nu E} U_\nu = \mathrm{Diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}). \tag{93}$$

The U_{PMNS} is defined as

$$U_{\rm PMNS}P = U_l^\dagger U_\nu, \tag{94}$$

where

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & e^{i\rho} \end{pmatrix}, \tag{95}$$

where ρ, σ are CP violating Majorana phases. If we ignore the mixing between the charged lepton, then we can get

$$U_\nu = U_{\rm PMNS}P. \tag{96}$$

We assume that $h'^\nu = {\rm Diag}(h'^\nu_{11}, h'^\nu_{22}, h'^\nu_{33})$ then $m_\nu^M = {\rm Diag}(m_{\nu_{1M}}, m_{\nu_{2M}}, m_{\nu_{3M}})$. Using the analysis in [18], the most general h^ν matrix is given by

$$h^\nu = \frac{\sqrt{2}}{u}{\rm Diag}(\sqrt{m_{\nu_{1M}}}, \sqrt{m_{\nu_{2M}}}, \sqrt{m_{\nu_{3M}}}) \\ .R.{\rm Diag}(\sqrt{m_{\nu_1}}, \sqrt{m_{\nu_2}}, \sqrt{m_{\nu_3}}).U_\nu{}^\dagger, \tag{97}$$

where R is orthogonal matrix expressed in terms of arbitrary complex angles $\hat\theta_1, \hat\theta_2, \hat\theta_3$ as following

$$R = \begin{pmatrix} \hat c_2\hat c_3 & -\hat c_1\hat s_3 - \hat s_1\hat s_2\hat c_3 & \hat s_1\hat s_3 - \hat c_1\hat s_2\hat c_3 \\ \hat c_2\hat s_3 & \hat c_1\hat c_3 - \hat s_1\hat s_2\hat s_3 & -\hat s_1\hat c_3 - \hat c_1\hat s_2\hat s_3 \\ \hat s_2 & \hat s_1\hat c_2 & \hat c_1\hat c_2 \end{pmatrix}, \tag{98}$$

where $\hat c_i = \cos\hat\theta_i, \hat s_i = \sin\hat\theta_i$, $i = 1, 2, 3$.

From the Eq. (97) $h^{\nu\dagger}h^\nu$ has the form

$$h^{\nu\dagger}h^\nu = \frac{2}{u^2}U_\nu.{\rm Diag}(\sqrt{m_{\nu_1}}, \sqrt{m_{\nu_2}}, \sqrt{m_{\nu_3}}) \\ .R^\dagger.{\rm Diag}(m_{\nu_{1M}}, m_{\nu_{2M}}, m_{\nu_{3M}}) \\ .R.{\rm Diag}(\sqrt{m_{\nu_1}}, \sqrt{m_{\nu_2}}, \sqrt{m_{\nu_3}}).U_\nu{}^\dagger. \tag{99}$$

For the light neutrinos masses, we fit the experimental results

$$\Delta m^2_{\nu_{12}} = m^2_{\nu_2} - m^2_{\nu_1} = 7.53\times 10^{-5}\ {\rm eV}^2, \\ \Delta m^2_{\nu_{23}} = m^2_{\nu_3} - m^2_{\nu_2} = 2.44\times 10^{-3}\ {\rm eV}^2. \tag{100}$$

The asymmetry $\varepsilon^{i(1)}_{\nu_{kM}}, \varepsilon^{i(2)}_{\nu_{kM}}$ now can be considered as function of the phase δ, ρ, σ, the heavy Majorana neutrinos masses and the complex angles $\hat\theta_1, \hat\theta_2, \hat\theta_3$. For simplicity, we assume $\hat\theta_1 = \hat\theta_2 = \hat\theta_3 \equiv \hat\theta$. In this work we consider the CP asymmetry due to the decays of the lightest heavy Majorana ν_{1M}. The detail will be presented in the subsections below.

The baryon asymmetry and lepton asymmetry are given as

$$\eta_B = \frac{n_B - n_{\bar B}}{s}, \\ \eta_L = \frac{n_l - n_{\bar l}}{s}, \tag{101}$$

where s is the entropy density. The lepton asymmetry can be transformed into a baryon asymmetry by nonperturbative $B + L$ violating (sphaleron) processes [19], giving

$$\eta_B = a(\eta_B - \eta_L) = \frac{a}{a-1}\eta_L, \tag{102}$$

where

$$a = \frac{8n_g + 4n_H}{22n_g + 13n_H}, \tag{103}$$

with n_H is the number of Higgs and n_g is the number of fermion generations. We get

$$\eta_B = -\frac{8n_g + 4n_H}{14n_g + 9n_H}\eta_L. \tag{104}$$

As the analysis in [20], taking $n_H = 2$ and $n_g = 3$, we get

$$\eta_B = -\frac{8}{15}\eta_L. \tag{105}$$

Now let us calculate η_L in thermal and nonthermal leptogenesis scenario.

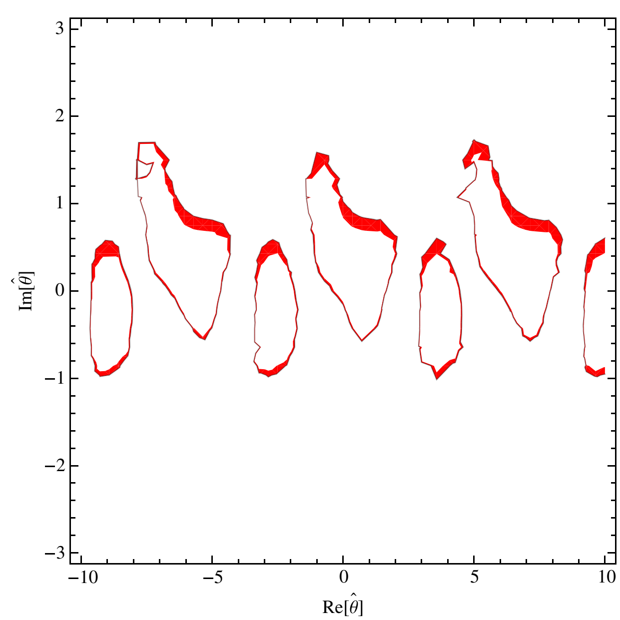

FIG. 4 (color online). Contour plot of η_B in the region $5\times 10^{-11} < \eta_B < 10^{-10}$ on the plane of the complex angle $\hat\theta$ for $\delta = 4.3$ rad, $\sigma = -1.5$ rad, $\rho = -1$ rad, $m_{\nu_{2M}} = m_{\nu_{3M}} = 10^3 m_{\nu_{1M}}$, $m_{\nu_{1M}} = 10^9$ GeV, $m_{\nu_1} = 0.01$ eV.

A. Thermal production

In the thermal scenario, the heavy Majorana neutrinos are produced in a thermal bath. At $T > m_{\nu_{1M}}$, the CP asymmetry generated by ν_{1M} decays can be washed out due to inverse decays and scattering processes. That why the CP asymmetry is weighted by the washout efficiency.

For the channel $\nu_{kM} \to e_i H_5^+, \bar{e}_i H_5^-$ the CP asymmetry depends on flavor because $L_i(e_i) = 1$. However, since $L(N_i) = 0, L(H') = -1$, the CP asymmetry due to the decay $\nu_{kM} \to N_i H'^*, N_i H'$ is considered flavor independent. The Boltzmann equations for the lepton asymmetry can be divided by two forms, one is the equation for the flavored lepton asymmetry corresponding to $\varepsilon^{i(1)}_{\nu_{1M}}$, and another is treated by the conventional computation for $\varepsilon^{(2)}_{\nu_{1M}} = \sum_i \varepsilon^{i(2)}_{\nu_{1M}}$.

The interference of the tree level with loop diagrams contained gauge propagator is vanished in summation of all the indexes $i, l = 1, 2, 3$ if the CP asymmetry has the same weight for all flavors,

$$\sum_{i,l} \mathrm{Im}[h^\nu_{ik} h^\nu_{lk}] = \mathrm{Im}\left[\sum_{i,l} h^\nu_{ik} h^\nu_{lk}\right] = 0. \tag{106}$$

Therefore, from Eq. (89)

$$\varepsilon^{(2)}_{\nu_{1M}} = \sum_i \varepsilon^{i(2)}_{\nu_{1M}} = \frac{1}{8\pi C_0} \sum_j \sqrt{g_j}\,[1 - (1 + g_j)\log[1 + 1/g_j] + s^2_\beta (1 - g_j)^{-1}]\mathrm{Im}[((h^{\nu\dagger} h^\nu)_{1j})^2]$$

$$\simeq -1.6 \times 10^{-2} \sum_j \frac{\mathrm{Im}[((h^{\nu\dagger} h^\nu)_{1j})^2]}{\sqrt{g_j}(h^{\nu\dagger} h^\nu)_{11}}. \tag{107}$$

Equation (87) can be reduced by taking $g = 0.65, s^2_W = 0.231$ as

$$\varepsilon^{i(1)}_{\nu_{1M}} = \frac{-1.6 \times 10^{-4} \sum_l \mathrm{Im}[h^{\nu *}_{i1} h^\nu_{l1}] - 0.6 \times 10^{-2} \sum_j \sqrt{g_j^{-1}}\,\mathrm{Im}[(h^{\nu\dagger} h^\nu)_{1j} h^\nu_{i1} h^\nu_{ij}]}{(h^{\nu\dagger} h^\nu)_{11}}. \tag{108}$$

In the thermal leptogenesis, the washout parameters are defined as

$$K_i = \frac{\Gamma(\nu_{1M} \to e_i H_5^+, \bar{e}_i H_5^-)}{H(T = m_{\nu_{1M}})} = \frac{s^2_\beta h^\nu_{i1} h^\nu_{i1} m_{\nu_{1M}}}{8\pi},$$
$$K = \frac{\Gamma(\nu_{1M})}{H(T = m_{\nu_{1M}})} = \frac{(2 + s^2_\beta)(h^{\nu\dagger} h^\nu)_{11} m_{\nu_{1M}}}{8\pi}. \tag{109}$$

By varying $\delta, \sigma, \rho, \hat{\theta}$ for $m_{\nu_{1M}} \sim 10^9$ GeV, $m_{\nu_{2M}} \sim m_{\nu_{3M}} \sim 10^3 m_{\nu_{1M}}$ we figure out $K \gg 1$ for all values of CP parameters δ, σ, ρ, and the complex angle $\hat{\theta}$. The lepton asymmetry can be approximated as given in [21]. In the strong washout regime, $K \gg 1$

$$\eta^0_L \simeq 0.3 \frac{\varepsilon^{(2)}_{\nu_{1M}}}{g_*} \left(\frac{0.55}{K}\right)^{1.16},$$
$$\eta^i_L \simeq \frac{\varepsilon^{i(1)}_{\nu_{1M}}}{g_*} \left(\frac{8.25}{|A_{ii}| K_i} + \left(\frac{|A_{ii}| K_i}{0.2}\right)^{1.16}\right)^{-1}, \tag{110}$$

where $A_{11} = -151/179, A_{22} = A_{33} = -344/537$.

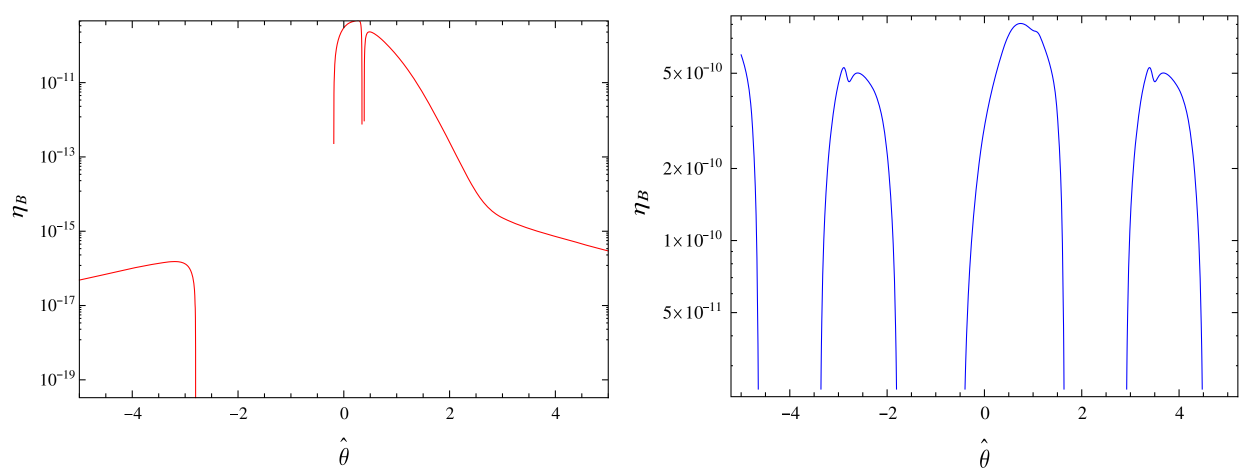

FIG. 5 (color online). η_B vs pure imaginary $\hat{\theta}$ (red) and pure real $\hat{\theta}$ (blue) for $\delta = 4.3$ rad, $\sigma = -1.5$ rad, $\rho = -1$ rad, $m_{\nu_{2M}} = m_{\nu_{3M}} = 10^3 m_{\nu_{1M}}$, $m_{\nu_{1M}} = 10^9$ GeV, $m_{\nu_1} = 0.01$ eV.

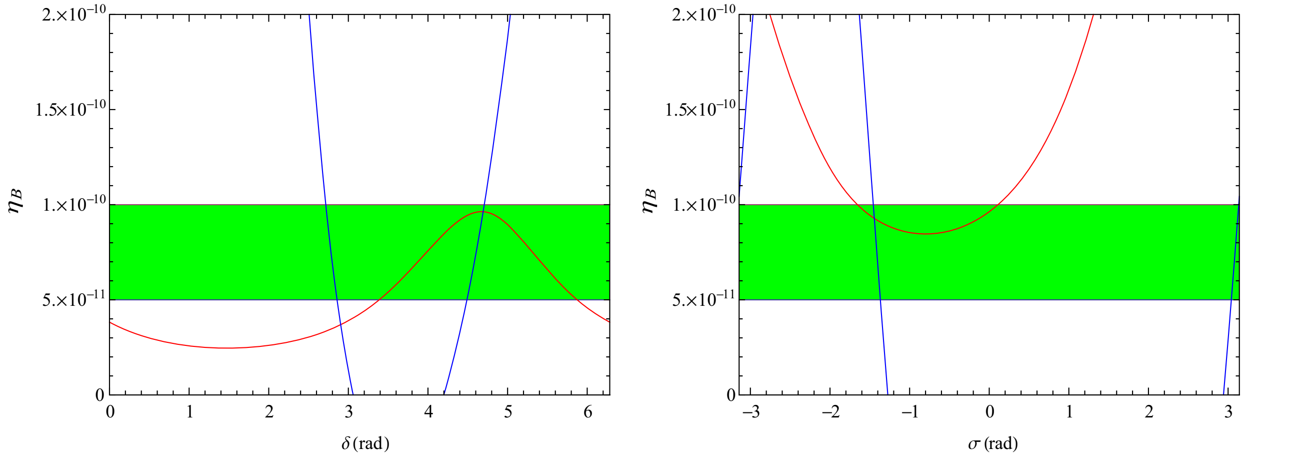

FIG. 6 (color online). η_B vs δ (left) and η_B vs $\sigma = \rho$ (right) for $\hat{\theta} = 0.87I$ (red) and $\hat{\theta} = -0.18I$ (blue), and other parameters given in (112).

The baryon asymmetry is related to the lepton asymmetry as

$$\eta_B = -\frac{8}{15}\left(\sum_{i=1,2,3}\eta_L^i + \eta_L^0\right). \tag{111}$$

From all expressions above, we see that η_B depends on δ, σ, ρ, and $\hat{\theta}$. The baryon asymmetry η_B in the region $(5\times 10^{-11}, 10^{-10})$ on the plane of the complex angle $\hat{\theta}$ is shown in Fig. 4 for

$$\begin{aligned} &\delta = 4.3\ \text{rad}, \qquad \sigma = -1.5\ \text{rad}, \qquad \rho = -1\ \text{rad}, \\ &m_{\nu_{2M}} = m_{\nu_{3M}} = 10^3 m_{\nu_{1M}}, \qquad m_{\nu_{1M}} = 10^9\ \text{GeV}, \\ &m_{\nu_1} = 0.01\ \text{eV}. \end{aligned} \tag{112}$$

The red regions indicate that in order to satisfy $5\times 10^{-11} < \eta_B < 10^{-10}$, we need to require $-1.01 < \text{Im}[\hat{\theta}] < 1.8$ when varying $\text{Re}[\hat{\theta}]$. The limit of $\text{Im}[\hat{\theta}]$ keeps the same if we extend the range of $\text{Re}[\hat{\theta}]$. The η_B is considered as function of pure imaginary $\hat{\theta}$ (red) and pure real $\hat{\theta}$ (blue) as shown in Fig. 5. We see that η_B changes a lot when varying pure $\text{Im}[\hat{\theta}]$ while it seems to keep the same order when the pure $\text{Re}[\hat{\theta}]$ alters. The baryon asymmetry varies little as function of the *CP* phases presented in Fig. 6.

If we study the case $m_{\nu_{1M}} = 10^9$ GeV, $m_{\nu_{2M}} = m_{\nu_{3M}} = 10^5 m_{\nu_{1M}}$, the constraint on the complex angle $\hat{\theta}$ is stricter in order to satisfy the experimental results on baryon asymmetry.

B. Nonthermal production

In the nonthermal scenario the reheating temperature can be lower than the lightest heavy Majorana. The total *CP* asymmetry is the summation of all flavor *CP* asymmetry,

$$\varepsilon_{\nu_{kM}} = \sum_i (\varepsilon^{i(1)}_{\nu_{kM}} + \varepsilon^{i(2)}_{\nu_{kM}}) = \frac{\sum_{j\neq k} B_j \text{Im}[[(h^{\nu\dagger}h^\nu)_{kj}]^2]}{(h^{\nu\dagger}h^\nu)_{kk}}, \tag{113}$$

where

$$\begin{aligned} B_j &= \frac{1}{8\pi(2+s_\beta^2)}\sqrt{g_j}[(s_\beta^4+1)(1-(1+g_j)\log[1+1/g_j]) \\ &\quad + s_\beta^2(s_\beta^2+1)(1-g_j)^{-1}] \\ &\simeq -\frac{11}{160\pi\sqrt{g_j}}. \end{aligned} \tag{114}$$

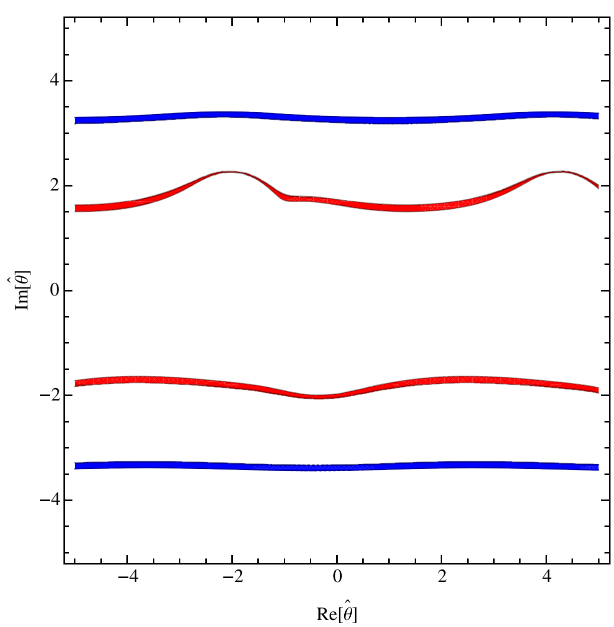

FIG. 7 (color online). Contour plot of η_B in the region $(5\times 10^{-11} < \eta_B < 10^{-10})$ on the plane of the complex angle $\hat{\theta}$ for $m_{\nu_{1M}} = 10^{11}$ GeV (red) and $m_{\nu_{1M}} = 10^9$ GeV (blue).

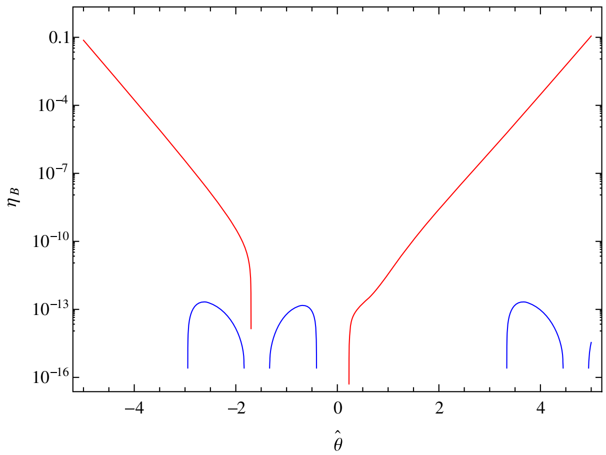

FIG. 8 (color online). η_B vs pure imaginary (red) and pure real (blue) $\hat{\theta}$.

The lepton asymmetry is related with the CP asymmetry through

$$\eta_L = \frac{3}{2}\varepsilon_{\nu_{kM}} \times Br_k \times \frac{T_R}{m_\Phi}, \qquad (115)$$

where Br_k denotes the branching ratio of the decay channel $\Phi \to \nu_{kM}\nu_{kM}$.

As analysis in the previous section, we assumed that $m_{\nu_{1M}} \ll m_\Phi < m_{\nu_{2M}} \sim m_{\nu_{3M}}, m_\Phi < m_{Z^N}$ and $\Gamma(\Phi \to hh) \ll \Gamma(\Phi \to \nu_{1R}\nu_{1R})$ when $\lambda_{10;11;12}$ are negligibly small, therefore,

$$\eta_L \simeq \frac{3}{2}\varepsilon_{\nu_{1M}} \times \frac{T_R}{m_\Phi}. \qquad (116)$$

Combining Eqs. (77), (105), (113) with $u \sim v \sim 174$ GeV we get

$$\eta_B \simeq 0.4 \times \frac{m_{\nu_{1M}}}{\sqrt{2}\langle\Phi\rangle} \times \frac{\sum_{j=2,3}\frac{m_{\nu_{1M}}}{m_{\nu_{jM}}}\mathrm{Im}[[(h^{\nu\dagger}h^\nu)_{1j}]^2]}{(h^{\nu\dagger}h^\nu)_{11}}, \qquad (117)$$

with notice that the formula $h^{\nu\dagger}h^\nu$ given in Eq. (99). Putting

$$\begin{aligned} &\delta = 4.3\ \mathrm{rad}, \qquad \sigma = -1.5\ \mathrm{rad}, \qquad \rho = -1\ \mathrm{rad}, \\ &\hat{\theta} = 1.46I, m_{\nu_{2M}} = m_{\nu_{3M}} = 10^3 m_{\nu_{1M}}, \\ &m_{\nu_{1M}} = 2.34 \times 10^{11}\ \mathrm{GeV}, \\ &m_{\nu_1} = 0.01\ \mathrm{eV}, m_\Phi = 2.67 \times 10^{13}\ \mathrm{GeV}, \\ &\langle\Phi\rangle = 23.6 m_P, \end{aligned} \qquad (118)$$

we get

$$\eta_B \simeq 8.92 \times 10^{-11}. \qquad (119)$$

This value of baryon asymmetry is in agreement with [22], $\eta_B = (8.75 \pm 0.23) \times 10^{-11}$.

Let us consider how η_B depends on the complex angles and CP phases one by one. Figure 7 shows η_B in the region $(5 \times 10^{-11}, 10^{-10})$ on the plane of the complex angle $\hat{\theta}$ for $m_{\nu_{2M}} = m_{\nu_{3M}} = 10^3 m_{\nu_{1M}}$, $m_{\nu_{1M}} = 10^{11}$ GeV (red), and $m_{\nu_{2M}} = m_{\nu_{3M}} = 10^5 m_{\nu_{1M}}$, $m_{\nu_{1M}} = 10^9$ GeV (blue) and all other parameters as given in (118). We see that in the red region $-2.05 < \mathrm{Im}[\hat{\theta}] < -1.68$ or $1.49 < \mathrm{Im}[\hat{\theta}] < 2.28$ and in the blue region $\mathrm{Im}[\hat{\theta}] \sim 3.3$ or $\mathrm{Im}[\hat{\theta}] \sim -3.4$ when varying $\mathrm{Re}[\hat{\theta}]$ even though if we extend the plot range for both axes. It means that it is free to choose the value of $\mathrm{Re}[\hat{\theta}]$ but $\mathrm{Im}[\hat{\theta}]$ is quite a strict constraint. η_B depends strongly on $\mathrm{Im}[\hat{\theta}]$, while it changes lightly when varying $\mathrm{Re}[\hat{\theta}]$. This conclusion is more clearly in Fig. 8, in which η_B is considered as a function of pure imaginary (red) and pure real (blue) $\hat{\theta}$.

Figure 9 shows η_B as a function of Dirac CP phase δ (left) and Majorana CP phase $\sigma = \rho$ (right) for $\hat{\theta} = 1.46I$

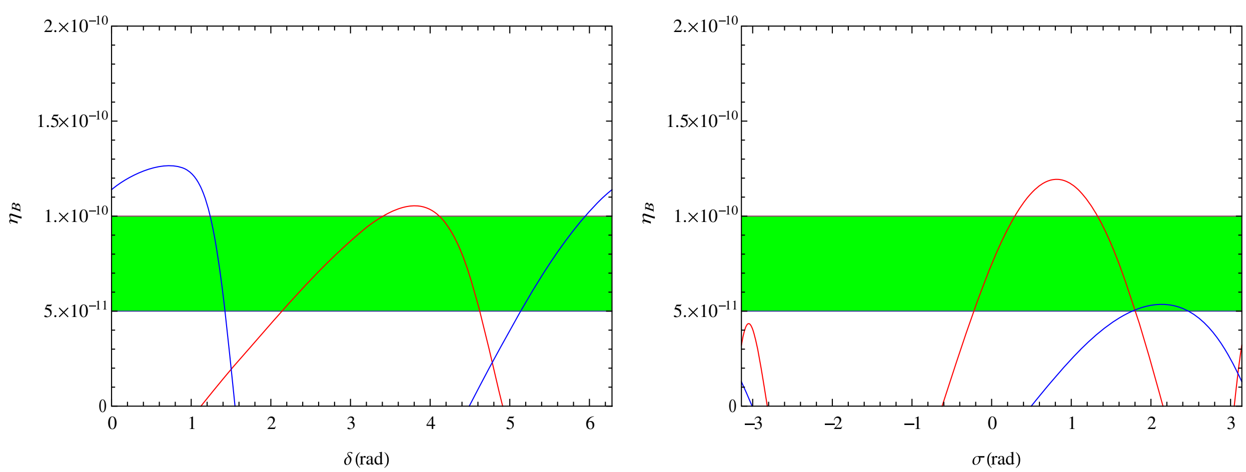

FIG. 9 (color online). η_B vs δ (left) and η_B vs $\sigma = \rho$ (right) for $\hat{\theta} = 1.46I$ (red) and $\hat{\theta} = -1.46I$ (blue).

(red) and $\hat{\theta} = -1.46I$ (blue), and the choice of other parameters given in (118). In brief, we see that η_B does not depend much on the *CP* phase but depend on the imaginary of the complex angle $\hat{\theta}$. This conclusion is the same as analysis in thermal scenario.

V. CONCLUSIONS

We have studied generation of inflation and leptogenesis in the 3-3-1-1 model by considering the symmetry breaking of the $U(1)_N$ gauge group at the GUT scale. The model contains two super heavy particles with mass proportional to Λ, the new gauge boson Z^N embedded to $U(1)_N$ and the scalar Higgs boson $H_3 \simeq S_4$. All other new massive particles get mass in order of ω. The singlet Higgs ϕ with $\langle\phi\rangle$ at the GUT scale can play the role of inflaton. The quantum corrections to the potential of inflaton is taken into account; thus there appears logarithm function of inflaton, making the presently considered model's inflation different from chaotic one. In this work, we have figured out the parameter spaces appeared in the inflaton potential matching the experiment on the spectrum index n_s, the tensor to scalar ratio r, the running index α as well as the amplitude of the curvature perturbation $\Delta^2_{\mathcal{R}}$. The inflaton mass is obtained in an order of 10^{13} GeV.

After the inflation, the heavy Majorana can be produced in a thermal bath or by decay of the inflaton. Depending on the Higgs couplings $\lambda_{10,11,12}$ in comparison with the Yukawa couplings h'^{ν}_{ij}, leptogenesis is considered in thermal or nonthermal scenario. We have shown how the 3-3-1-1 model generates lepton asymmetry then converts into baryon asymmetry in both cases. It is interesting that the model contains an extra channel contributing to the *CP* asymmetry. The heavy Majorana particles can decay into neutral neutrinos N_i and neutral complex Higgs H' with the coupling different by factor s_β from the original channel, $\nu_{kM} \to e_i^{\pm} H^{\mp}$. In thermal leptogenesis, the *CP* asymmetry generated by the new channel is considered flavor independent, while the ordinary channel is treated as flavor dependent due to the different lepton number of N_i and e_i. It leads the interference of the tree level with loop diagrams appeared gauge propagator to contribute to the *CP* asymmetry for the decay $\nu_{kM} \to e_i^{\pm} H^{\mp}$. This feature is new compared to other leptogenesis models.

The thermal and nonthermal leptogenesis have been calculated in detail. In order to get nonzero *CP* asymmetry we need to consider the complex Yukwa coupling matrix h^{ν} by expressing it in terms of the neutrino mass and mixing matrix and the orthogonal matrix R. We have presented how η_B depends on the *CP* phases δ, σ, ρ and complex angle $\hat{\theta} \equiv \hat{\theta}_1 = \hat{\theta}_2 = \hat{\theta}_3$. The baryon asymmetry is not much sensitive to the value of *CP* phases or pure real $\hat{\theta}$ but it alters a lot as a function of pure imaginary $\hat{\theta}$. This property is the same for both leptogenesis scenarios. Thank to the orthogonal matrix R and the complex angle $\hat{\theta}$, which makes the baryon symmetry completely in agreement with the experiment for both cases. One different thing of the two scenarios is that at any point of $\mathrm{Re}[\hat{\theta}]$ we always can find $\mathrm{Im}[\hat{\theta}]$ satisfying a fixed value of η_B in nonthermal case, but there is restriction of choosing pair of ($\mathrm{Im}[\hat{\theta}]$, $\mathrm{Re}[\hat{\theta}]$) in thermal scenario to match experiment on η_B. We know that the baryon asymmetry depends much on $\mathrm{Im}[\hat{\theta}]$ and it is easy to see that from the Fig. 5, there is an upper limit on the baryon asymmetry if we consider η_B as a function of $\mathrm{Im}[\hat{\theta}]$ in thermal scenario because of the effect of washout efficiency. However, there is no upper bound for $\eta_B(\mathrm{Im}[\hat{\theta}])$ in nonthermal case, see Fig. 8. By considering nonthermal leptogenesis, the reheating temperature T_R can be reduced much lower than the lightest heavy Majorana mass. In brief, the 3-3-1-1 model at the GUT scale successfully explains the baryon asymmetry of the Universe by studying both thermal and nonthermal leptogenesis mechanisms.

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