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Discriminating the minimal 3-3-1 models

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We show that due to the ρ parameter bound and the Landau pole limit, the reduced 3-3-1 model is unrealistic, while due to the ρ parameter and flavor-changing neutral current bounds, the simple 3-3-1 model is experimentally unfavored. All such conditions strictly constrain the gauge symmetry breaking scales of the minimal 3-3-1 model with three scalar triplets.

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I. INTRODUCTION

It is well known that the unsolved questions of the standard model, namely, the number of fermion generations, uncharacteristic heaviness of the top quark, the strong CP problem, electric charge quantization, neutrino masses, and dark matter, can be addressed by the 3-3-1 models [1–7]. Moreover, the $B-L$ dynamics (i.e., gauge symmetry of baryon minus lepton numbers) and resulting R parity, leptogenesis, and inflation can also be realized by these types of theories [8].

Recently, three versions of the minimal 3-3-1 model have emerged—the reduced 3-3-1 model [9], the simple 3-3-1 model [10], and the minimal 3-3-1 model with three scalar triplets [11]—which provide new theoretical and phenomenological aspects beyond the old versions. In this work, we will show experimentally favored degrees for such theories, which rely simply on their ρ parameter, flavor-changing neutral currents (FCNCs), and Landau pole. The Z and new Z' gauge boson mixing is also analyzed.

II. THE MINIMAL 3-3-1 MODELS

The gauge symmetry is given by $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (3-3-1), where the first factor is the color group and the last two factors are the extension of the electroweak symmetry. The electric charge operator takes the form $Q = T_3 - \sqrt{3}T_8 + X$, where $Y = -\sqrt{3}T_8 + X$ is the weak hypercharge. Here, $T_i (i = 1, 2, 3, \ldots, 8)$ and X are the $SU(3)_L$ and $U(1)_X$ charges, respectively (the color charges will be denoted by t_i). The fermions can be arranged as $\psi_{aL} = (\nu_{aL}, e_{aL}, e^c_{aR}) \sim (1,3,0)$, $Q_{\alpha L} = (d_{\alpha L}, -u_{\alpha L}, J_{\alpha L}) \sim (3, 3^*, -1/3)$, $Q_{3L} = (u_{3L}, d_{3L}, J_{3L}) \sim (3, 3, 2/3)$, $u_{aR} \sim (3, 1, 2/3)$, $d_{aR} \sim (3, 1, -1/3)$, $J_{\alpha R} \sim (3, 1, -4/3)$, and $J_{3R} \sim (3, 1, 5/3)$, where $a = 1, 2, 3$ and $\alpha = 1, 2$ are generation indices. Note that the values in parentheses present quantum numbers based upon the 3-3-1 symmetries, respectively.

The minimal 3-3-1 model with three scalar triplets works with the following scalar fields: $\eta = (\eta^0_1, \eta^-_2, \eta^+_3) \sim (1,3,0)$, $\rho = (\rho^+_1, \rho^0_2, \rho^{++}_3) \sim (1, 3, 1)$, and $\chi = (\chi^-_1, \chi^{--}_2, \chi^0_3) \sim (1, 3, -1)$. The reduced 3-3-1 model works with (ρ, χ) by excluding η, while the simple 3-3-1 model works with (η, χ) by excluding ρ. The vacuum expectation values of the scalars are given by $\langle\eta\rangle = (u/\sqrt{2}, 0, 0)$, $\langle\rho\rangle = (0, v/\sqrt{2}, 0)$, and $\langle\chi\rangle = (0, 0, w/\sqrt{2})$. The following calculations generally apply for all the models: for the reduced 3-3-1 model, $u = 0$, and for the simple 3-3-1 model, $v = 0$.

III. GAUGE BOSON MASSES AND MIXING

We now derive the mass spectrum of the gauge bosons, which arises from the Lagrangian $\sum_{\Phi=\eta,\rho,\chi}(D_\mu\langle\Phi\rangle)^\dagger (D^\mu\langle\Phi\rangle)$, where the covariant derivative takes the form $D_\mu = \partial_\mu + ig_s t_i G_{i\mu} + igT_i A_{i\mu} + ig_X X B_\mu$, with the gauge couplings (g_s, g, g_X) and gauge bosons $(G_{i\mu}, A_{i\mu}, B_\mu)$ associated with the respective 3-3-1 groups. We have physical, charged gauge bosons with respective masses,

$$W^\pm \equiv \frac{A_1 \mp iA_2}{\sqrt{2}}, \qquad X^\pm \equiv \frac{A_4 \pm iA_5}{\sqrt{2}},$$
$$Y^{\pm\pm} \equiv \frac{A_6 \pm iA_7}{\sqrt{2}}, \qquad m^2_W = \frac{g^2}{4}(u^2 + v^2),$$
$$m^2_X = \frac{g^2}{4}(u^2 + w^2), \qquad m^2_Y = \frac{g^2}{4}(v^2 + w^2). \tag{1}$$

To be consistent with the standard model, we impose $u, v \ll w$. The field W is identical to the standard model charged gauge boson, which implies $v^2_{\mathrm{w}} \equiv u^2 + v^2 = (246\ \mathrm{GeV})^2$, while X and Y are new gauge bosons with large masses in the w scale.

For the neutral gauge bosons, the photon, Z, and new Z' can be identified as

$$A = s_W A_3 + c_W\left(-\sqrt{3}t_W A_8 + \sqrt{1 - 3t^2_W}B\right),$$
$$Z = c_W A_3 - s_W\left(-\sqrt{3}t_W A_8 + \sqrt{1 - 3t^2_W}B\right), \tag{2}$$

$$Z' = \sqrt{1 - 3t^2_W}A_8 + \sqrt{3}t_W B, \tag{3}$$

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where $s_W = e/g = t/\sqrt{1+4t^2}$, with $t = g_X/g$, is the sine of the Weinberg angle [12]. The photon field A is physical ($m_A = 0$) and decoupled, whereas Z and Z' mix as given by the mass matrix,

$$\begin{pmatrix} m_Z^2 & m_{ZZ'}^2 \\ m_{ZZ'}^2 & m_{Z'}^2 \end{pmatrix}, \quad (4)$$

where

$$m_Z^2 = \frac{g^2}{4c_W^2}(u^2+v^2),$$
$$m_{ZZ'}^2 = \frac{g^2[(1-4s_W^2)u^2 - (1+2s_W^2)v^2]}{4\sqrt{3}c_W^2\sqrt{1-4s_W^2}},$$
$$m_{Z'}^2 = \frac{g^2[(1-4s_W^2)^2u^2 + (1+2s_W^2)^2v^2 + 4c_W^4w^2]}{12c_W^2(1-4s_W^2)}. \quad (5)$$

Hence, we obtain two physical neutral gauge bosons (besides the photon),

$$Z_1 = c_\varphi Z - s_\varphi Z', \qquad Z_2 = s_\varphi Z + c_\varphi Z', \quad (6)$$

with the Z-Z' mixing angle,

$$t_{2\varphi} = \frac{2m_{ZZ'}^2}{m_{Z'}^2 - m_Z^2} \simeq \frac{\sqrt{3(1-4s_W^2)}}{2c_W^4}\frac{[(1-4s_W^2)u^2 - (1+2s_W^2)v^2]}{w^2}, \quad (7)$$

and their masses,

$$m_{Z_1}^2 = \frac{1}{2}\left[m_Z^2 + m_{Z'}^2 - \sqrt{(m_Z^2 - m_{Z'}^2)^2 + 4m_{ZZ'}^4}\right] \simeq \frac{g^2}{4c_W^2}(u^2+v^2), \quad (8)$$

$$m_{Z_2}^2 = \frac{1}{2}\left[m_Z^2 + m_{Z'}^2 + \sqrt{(m_Z^2 - m_{Z'}^2)^2 + 4m_{ZZ'}^4}\right] \simeq \frac{g^2c_W^2}{3(1-4s_W^2)}w^2. \quad (9)$$

The approximations for the masses are given at the leading order. Because the mixing angle φ is small, we have $Z_1 \simeq Z$ and $Z_2 \simeq Z'$, which imply that the Z_1 is like the standard model Z boson, while Z_2 is a new neutral gauge boson with a large mass in the w scale.

IV. ρ PARAMETER

The experimental ρ parameter (or $\Delta\rho \equiv \rho - 1$ used below) that is contributed (or induced) only by the new physics comes from the following sources. The first one is given at tree level due to the Z-Z' mixing, which can be evaluated as

$$(\Delta\rho)_{\text{tree}} \equiv \frac{m_W^2}{c_W^2 m_{Z_1}^2} - 1 \simeq \frac{m_{ZZ'}^4}{m_Z^2 m_{Z'}^2} \simeq \frac{[(1-4s_W^2)u^2 - (1+2s_W^2)v^2]^2}{4c_W^4 v_{\text{w}}^2 w^2}. \quad (10)$$

The second one arises from the one-loop contributions of a heavy gauge boson doublet (X^-, Y^{--}). Note that the other new particles, such as the exotic quarks, Z', and new Higgs bosons, do not contribute [13]. Generalizing the results in [13] and using the X, Y masses in (1), we obtain

$$(\Delta\rho)_{\text{rad}} = \frac{3\sqrt{2}G_F}{16\pi^2}\left(m_Y^2 + m_X^2 - \frac{2m_Y^2m_X^2}{m_Y^2 - m_X^2}\ln\frac{m_Y^2}{m_X^2}\right) + \frac{\alpha}{4\pi s_W^2}\left(\frac{m_Y^2 + m_X^2}{m_Y^2 - m_X^2}\ln\frac{m_Y^2}{m_X^2} - 2 + 3t_W^2\ln\frac{m_Y^2}{m_X^2}\right)$$
$$= \frac{3g^2}{64\pi^2 v_{\text{w}}^2}\left(v_{\text{w}}^2 + 2w^2 - \frac{2(v^2+w^2)(u^2+w^2)}{v^2-u^2}\ln\frac{v^2+w^2}{u^2+w^2}\right) + \frac{g^2}{16\pi^2}\left(\frac{v_{\text{w}}^2 + 2w^2}{v^2-u^2}\ln\frac{v^2+w^2}{u^2+w^2} - 2 + 3t_W^2\ln\frac{v^2+w^2}{u^2+w^2}\right), \quad (11)$$

where $\sqrt{2}G_F = 1/v_{\text{w}}^2$ and $\alpha = g^2s_W^2/(4\pi)$. Summarizing the above results, we get the $\Delta\rho$ deviation due to the new physics contributions up to the one-loop level,

$$\Delta\rho = (\Delta\rho)_{\text{tree}} + (\Delta\rho)_{\text{rad}}. \quad (12)$$

Note that $(\Delta\rho)_{\text{rad}}$ can be negative or positive, depending on the sign and magnitude of the X and Y mass splitting, while $(\Delta\rho)_{\text{tree}}$ is always positive. Also, $(\Delta\rho)_{\text{rad}}$ and $(\Delta\rho)_{\text{tree}}$ are in the same order as $(u/w, v/w)^2$, and they can become comparable. On the other hand, it is well known that the quantum contributions (of the new physics) to ρ depend only on the X, Y mass splitting, which is used to break the vector part of weak $SU(2)$ (see [14]). Hence, the multiloop corrections to ρ are expected to be suppressed by the mass splitting $(m_Y^2 - m_X^2)/(m_Y^2 + m_X^2) \sim (v^2 - u^2)/w^2$ as well as the loop factor $1/(16\pi^2)$, which are only the subleading effects to the leading one-loop result. The following conclusions based on the one-loop calculations should remain unchanged.

V. NEW PHYSICS CONSTRAINTS

Because Z' nonuniversally couples to the ordinary quarks, it gives rise to tree-level FCNCs. These processes, which are completely identical to those in [10], can be evaluated and give the following bound: $w > 3.6$ TeV (see also [15] for other discussions and constraints on the 3-3-1 breaking scale). On the other hand, since $s_W^2 = g_X^2/(g^2 + 4g_X^2) < 1/4$, the model encounters a low Landau pole (Λ), at which $s_W^2(\Lambda) = 1/4$ or $g_X(\Lambda) = \infty$, which is about roundly $\Lambda = 4$–5 TeV, depending on the unfixed 3-3-1 breaking scale ($\mu_{331} < \Lambda$) [16]. Hereafter, $\Lambda = 5$ TeV will be taken into account. From the global fit, the ρ parameter is $\rho = 1.00040 \pm 0.00024$, which is 1.7σ above the standard model expectation, $\rho = 1$ [14].

Three remarks are in order.

(1) For the reduced 3-3-1 model ($u = 0$, $v = v_{\mathrm{w}}$), the deviation $\Delta\rho$ can be approximated as

$$\Delta\rho \simeq \left(\frac{1 + 2s_W^2}{2c_W^2}\right)^2 \frac{v_{\mathrm{w}}^2}{w^2}, \qquad (13)$$

which yields 9.243 TeV $< w <$ 18.487 TeV, provided that $0.00016 < \Delta\rho < 0.00064$ and $s_W^2 = 0.231$ [14]. The model is invalid due to the limit of the Landau pole, $w < 5$ TeV. In other words, due to the Landau pole limit, $w < 5$ TeV (assuming the model works), we have $\Delta\rho > 0.0022$, which is too large to be consistent with the experimental data [14].

(2) For the simple 3-3-1 model ($v = 0$, $u = v_{\mathrm{w}}$), the leading order for the $\Delta\rho$ deviation is

$$\Delta\rho \sim \left[\left(\frac{1 - 4s_W^2}{2c_W^2}\right)^2 + \frac{3\alpha}{4\pi s_W^2}\left(\frac{1}{4} - t_W^2\right)\right]\frac{v_{\mathrm{w}}^2}{w^2}, \quad (14)$$

which yields $w \sim 555$ GeV (by using the central value $\Delta\rho = 0.0004$, $s_W^2 = 0.231$, and $\alpha = 1/128$ [14]). The new physics is well defined below the Landau pole. However, as mentioned, the FCNCs constrain $w > 3.6$ TeV, which opposes the above regime. Thus, the model encounters an experimental discrepancy.

(3) For the minimal 3-3-1 model with three scalar triplets, because of $u^2 + v^2 = v_{\mathrm{w}}^2$, we can make a contour for $\Delta\rho$ (where $0.00016 < \Delta\rho < 0.00064$) as a function of only two variables (u, w). The Landau pole limit, $w < 5$ TeV, and the FCNC bound, $w > 3.6$ TeV, are also imposed. The result is shown in Fig. 1. For completeness, the mixing angle φ is shown in Fig. 2.

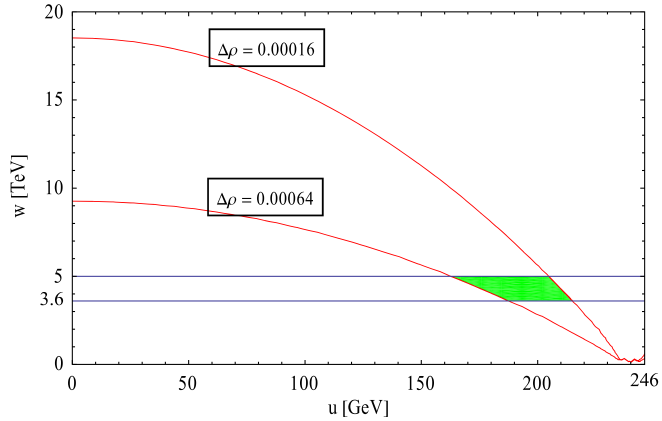

FIG. 1 (color online). The (u, w) region that is bounded by $0.00016 < \Delta\rho < 0.00064$ and 3.6 TeV $< w <$ 5 TeV. Note that u runs from 0 to 246 GeV.

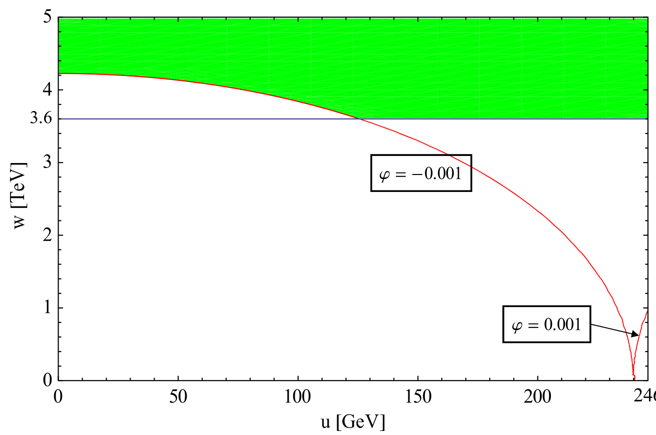

FIG. 2 (color online). The (u, w) region that is bounded by $-0.001 < \varphi < 0.001$ (the typical limits imposed by the electroweak measurements [14]) and 3.6 TeV $< w <$ 5 TeV. Note that u runs from 0 to 246 GeV.

VI. CONCLUSION

The reduced 3-3-1 model should be ruled out because it encounters either a large $\Delta\rho$ deviation or is mathematically inconsistent. The simple 3-3-1 model is experimentally unfavored due to the discrepancy between the FCNCs and the ρ parameter bounds. The minimal 3-3-1 model with three scalar triplets is consistent when 3.6 TeV $< w <$ 4–5 TeV and 162.5 GeV $< u <$ 215.6 GeV (or $0.55 < v/u < 1.14$). In all cases, we can always obtain the corresponding (u, w) values so that the Z-Z' mixing angle is small, consistent with the precision data.

The class of 3-3-1 models with $\beta = \pm\sqrt{3}$ (where β determines the embedding of the electric charge operator

$Q = T_3 + \beta T_8 + X$) and basic scalar triplets, which specifically consist of the above-mentioned ones and the 3-3-1 model with exotic charged leptons [17], could be the subject of these constraints. Moreover, although the minimal 3-3-1 model [1] is not considered in this work, the FCNCs, ρ parameter, and Landau pole may present similar bounds and the others [18] when including the additional contributions coming from the scalar sextet.

However, it is noted that the present constraints might be relaxed because the Landau pole can be lifted up by augmenting the matter content [19]. Also, for the 3-3-1 models with $|\beta| < \sqrt{3}$, where $s_W^2 = g_X^2/[g^2 + (1+\beta^2)g_X^2] < 1/(1+\beta^2)$, the Landau pole is lifted. For example, the 3-3-1 models without exotic charges, such as the one with right-handed neutrinos [2], may not encounter a Landau pole up to the Planck scale. Finally, note that the ρ parameter and Z-Z' mixing angle also depend on β, in addition to the breaking scales u, v, w.

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