PHYSICAL REVIEW A **90**, 062104 (2014)

Non-Markovianity of a two-level system transversally coupled to multiple bosonic reservoirs

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(Received 2 September 2014; published 1 December 2014)

Dynamics of a two-level open system transversally coupled to a single zero-temperature bosonic reservoir may be Markovian or non-Markovian, depending on whether the system-reservoir coupling is weak or strong. In this paper, we show that when the system is simultaneously coupled to N reservoirs its dynamics is always non-Markovian, provided that $N \geqslant N_{\rm cr}$, with $N_{\rm cr}$ a critical number depending on the reservoirs' parameters. Quantitatively, the non-Markovianity $\mathcal{N}$ is shown proportional to the number of contributed reservoirs. We explain our results in terms of the pseudomode theory, finding out that when $N \geqslant N_{\rm cr}$ the pseudomodes of all the reservoirs can always partially return information back to the system, no matter how strong the couplings between the system and each individual reservoir.

DOI: 10.1103/PhysRevA.90.062104 PACS number(s): 03.65.Yz, 03.67.−a

I. INTRODUCTION

Any realistic quantum system of interest should be treated as an open system because of its unavoidable coupling to various environments [1]. A thorough understanding of open system dynamics is not only a fundamental issue but also relevant to practical applications, such as in the development of quantum information technology [2]. In the usually used Markov approximation, the evolution of an open system is described by a family of completely positive trace-preserving reduced dynamical maps and a corresponding quantum master equation with a Lindblad generator [3]. Physically, the Markov approximation has priorly assumed a monotonic one-way decay of information from the system to the environment. However, in many situations quantum systems exhibit non-Markovian behavior [4–16] when there is a backflow of information from the environment to the system due to the memory effect. Correspondingly, some characteristics of the system (e.g., the coherence and the entanglement) would partially revive during the time evolution. This not only signifies a real physical phenomenon but also proves useful in practical schemes relying on non-Markovian evolutions, such as quantum-state engineering and quantum control [17].

So far several scenarios have been recognized under which the non-Markovian dynamics can happen, for example, strong system-environment coupling, structured reservoirs, low temperatures, and initial system-environment correlations [18–22]. In addition to those conventional mechanisms, some others have also been found able to induce non-Markovian dynamics of the system. As has been shown [23], revivals of quantum correlations of a composite system may occur when the environment is classical and does not backreact on the quantum system. Such a prediction has been realized by using an all-optical experiment [24]. Furthermore, for a bipartite open system where each subsystem locally interacts with a subsystem of a composite environment, the initial correlations between the subsystems of the environment can lead to non-Markovian behavior of the total open system, although the local dynamics of both subsystems of the system are Markovian [25]. Subsequently, an experimental demonstration of that phenomenon has also been achieved by using a photonic open system [26]. Although the unconventional mechanisms are few, there are still unknown strategies that would induce non-Markovian dynamics of an open quantum system.

A system undoubtedly exhibits Markovian behavior when it is weakly coupled to a single environment. However, what will happen if the system is coupled simultaneously to many environments? The consideration of simultaneous affections of multiple environments on a open system is relevant in many realistic situations where the system may be strongly coupled to a principal environment and meanwhile weakly to the minor ones [27,28]. For example, in a quantum dot the electron spin may be influenced strongly by the surrounding nuclei and weakly by the phonons [29]. The surrounding nitrogen impurities constitute the principal bath for a nitrogen-vacancy center, while the carbon-13 nuclear spins also have some influences on it [30]. A similar scenario occurs for a single-donor electron spin in silicon [31]. Intuitively, addition of an environment means addition of a decay channel and, hence, the system's information would monotonically flow along all those Markovian environments, retaining the Markovian dynamics of the system. Nevertheless, as we shall show here, this expectation is true only when the number of involved environments is not large enough. In order to make clear this question, we consider a two-level system (TLS) which is coupled to N reservoirs of field modes initially in the vacua. For $N = 1$, i.e., there is only one reservoir, the weak (strong) coupling regime leads to the Markovian (non-Markovian) dynamics. However, in the presence of $N > 1$ reservoirs, we find out that, independent of the coupling regime, the system always exhibits non-Markovian dynamics if N is equal to or exceeds a critical value $N_{\rm cr}$. That is, $N \geqslant N_{\rm cr}$ is the condition of triggering non-Markovian dynamics of the system, even in the weak-coupling regimes. Moreover, we shall show that the degree of the non-Markovian process, quantified by the so-called non-Markovianity $\mathcal{N}$, is proportional to the number N of the contributed reservoirs. Therefore it provides a possible

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way to trigger or enhance the non-Markovian dynamics of a TLS by increasing the number of reservoirs. To reveal how the system retrieves the decayed information from the reservoirs, particularly in the weak-coupling regimes, we shall examine the information flow between the system and reservoirs by means of the pseudomode theory [32,33]. We find that when $N \geqslant N_{\rm cr}$, the pseudomodes of the reservoirs can partially return information back to the system, regardless of magnitude of the couplings between the system and individual reservoirs.

We organize our paper as follows. In Sec. II, we first present our model of a TLS simultaneously interacting with N reservoirs and the notion of non-Markovianity. Then, focusing on the reservoirs with Lorentzian spectral densities, we demonstrate emergence of the system's non-Markovian dynamics under the condition $N \geqslant N_{\rm cr}$ as well as enhancement of non-Markovianity by increasing the number of reservoirs. In Sec. III, we make use of the pseudomode theory to explain our results. The final section, Sec. IV, is the conclusion.

II. THE MODEL AND THE NON-MARKOVIANITY

Let us for concreteness consider a TLS with ground state $|0\rangle$ and excited state $|1\rangle$ being coupled to N independent reservoirs of field modes assumed initially in the vacua. The total Hamiltonian is given by (with $\hbar = 1$)

$$\hat{H}=\frac{\omega_0}{2}\hat{\sigma}_z+\sum_{n=1}^{N}\sum_{k}[\omega_{n,k}\hat{a}_{n,k}^{\dagger}\hat{a}_{n,k}+g_{n,k}(\hat{a}_{n,k}^{\dagger}\hat{\sigma}_{-}+\hat{a}_{n,k}\hat{\sigma}_{+})], \tag{1}$$

where $\hat{a}_{n,k}$ ($\hat{a}_{n,k}^{\dagger}$) is the annihilation (creation) operator of field mode k with frequency $\omega_{n,k}$ in reservoir n, $\hat{\sigma}_{\pm}$ the raising and lowering operators of the TLS with transition frequency ω_0, and $g_{n,k}$ the coupling between the TLS and mode k in the nth reservoir.

Suppose the initial state of the TLS is of the form

$$|\phi(0)\rangle_S = c_0(0)\,|0\rangle_S + c_1(0)\,|1\rangle_S \tag{2}$$

and the state of N vacuum reservoirs reads $\prod_{n=1}^{N}|\bar{\mathbf{0}}\rangle_{n,r}$ with $|\bar{\mathbf{0}}\rangle_{n,r}=\prod_{k=1}|0_k\rangle_{n,r}$. Then the total state $|\Phi(0)\rangle = |\phi(0)\rangle_S \otimes \prod_{n=1}^{N}|\bar{\mathbf{0}}\rangle_{n,r}$ evolves after time $t>0$ into the state

$$|\Phi(t)\rangle = [c_0(0)|0\rangle_S + c_1(t)|1\rangle_S]\otimes\prod_{n=1}^{N}|\bar{\mathbf{0}}\rangle_{n,r} + |0\rangle_S\otimes\sum_{n=1}^{N}\sum_{k}c_{n,k}(t)|1_k\rangle_{n,r}, \tag{3}$$

in which the amplitude $c_0(0)$ is constant, while $c_1(t)$ and $c_{n,k}(t)$ are time dependent. From the Schrödinger equation, we obtain the time development of these amplitudes in the interaction picture, which are governed by $N+1$ differential equations:

$$\frac{d}{dt}c_1(t) = -i\sum_{n=1}^{N}\sum_{k}g_{n,k}e^{i(\omega_0-\omega_{n,k})t}c_{n,k}(t), \tag{4}$$

$$\frac{d}{dt}c_{n,k}(t) = -ig_{n,k}^{*}e^{-i(\omega_0-\omega_{n,k})t}c_1(t). \tag{5}$$

Integrating Eq. (5) with the initial condition $c_{n,k}(0)=0$ and inserting the solutions into Eq. (4), one obtains an integro-differential equation for the amplitude $c_1(t)$

$$\frac{d}{dt}c_1(t) = -\int_0^t\sum_{n=1}^{N}\sum_{k}|g_{n,k}|^2e^{i(\omega_0-\omega_{n,k})(t-t')}c_1(t')dt'. \tag{6}$$

The sum $\sum_k|g_{n,k}|^2e^{i(\omega_0-\omega_{n,k})(t-t')}$ in the above equation is recognized as a correlation function $f_n(t-t')$ of the nth reservoir, which in the limit of a large number of modes can be changed to an integration in terms of the spectral density $J_n(\omega)$ as

$$f_n(t-t') = \int d\omega J_n(\omega)\exp[i(\omega_0-\omega)(t-t')]. \tag{7}$$

Therefore the amplitude $c_1(t)$ can be reexpressed as

$$\frac{d}{dt}c_1(t) = -\int_0^t dt' c_1(t')F(t-t'), \tag{8}$$

with the kernel $F(t-t')\equiv\sum_{n=1}^{N}f_n(t-t')$. The correlation function $f_n(t-t')$ in Eq. (7) has actually been written as a Fourier transform of the nth reservoir's spectral density and by virtue of the linearity of which we note that the kernel $F(t-t')$ can be expressed as a Fourier transform of $\mathbf{J}(\omega)=\sum_{n=1}^{N}J_n(\omega)$, namely,

$$F(t-t') = \int d\omega\mathbf{J}(\omega)\exp[i(\omega_0-\omega)(t-t')]. \tag{9}$$

The implication of expression (9) is clear: the N reservoirs that interact simultaneously with one and the same system are tantamount to a "single" reservoir with the "combined spectral density" $\mathbf{J}(\omega)$ being the sum of spectral densities $J_n(\omega)$ of all the N individual reservoirs. For convenience, we express the dynamics of the TLS by the reduced density matrix in the system's basis $\{|1\rangle\,,|0\rangle\}$ as

$$\rho(t) = \begin{pmatrix}\rho_{11}(0)|c_1(t)|^2 & \rho_{10}(0)c_1(t)\\ \rho_{01}(0)c_1^{*}(t) & \rho_{00}(0)+\rho_{11}(0)(1-|c_1(t)|^2)\end{pmatrix}, \tag{10}$$

where $\rho_{11}(0)=|c_1(0)|^2$, $\rho_{00}=|c_0(0)|^2$, and $\rho_{10}(0)=\rho_{01}^{*}(0)=c_1(0)c_0^{*}(0)$.

The non-Markovianity characterizing the degree of a non-Markovian process can be quantified by different measures, such as the Breuer-Laine-Piilo measure based on the distinguishability between different initial states of the system [34], the Lorenzo-Plastina-Paternostro measure based on the volume of accessible states of the system [35], and the Rivas-Huelga-Plenio measure based on the entanglement that the system shares with an ancilla [36]. It is known that in general these measures are not always equivalent, and cases have been found where one of them vanishes while another one does not [37]. However, as for our considered situation, the dynamical map described by the form (10) is recognized as an amplitude damping channel, for which a reliable condition to indicate the onset of non-Markovianity has been given as $d|c_1(t)|/dt>0$, $\forall\ t>0 \Leftrightarrow$ the system's dynamics is non-Markovian [37–39]. Actually, the dynamics of form (10) is indivisible—a fundamental property of non-Markovianity—if and only if (iff) $d|c_1(t)|/dt>0$ at any time [37]. Moreover, some relevant non-Markovianity measures, such as those given in Refs. [34–36], vanish at the same time iff the above condition does not hold [39]; therefore they give the same results for the occurrence of non-Markovianity.

The dynamics of trace distance between two different initial states $\rho_1(0)$ and $\rho_2(0)$ of an open system is one of the most employed quantifiers. A Markovian evolution can never increase the trace distance; hence the violation of the contractiveness of the trace distance would signify the non-Markovian dynamics of the system. Based on this concept, the non-Markovianity can be quantified by a measure $\mathcal{N}$ defined as [34]

$$\mathcal{N} = \max_{\rho_1(0),\rho_2(0)} \int_{\sigma>0} \sigma[t,\rho_1(0),\rho_2(0)]dt, \tag{11}$$

in which $\sigma[t,\rho_1(0),\rho_2(0)] = dD[\rho_1(t),\rho_2(t)]/dt$ is the rate of change of the trace distance given by

$$D[\rho_1(t),\rho_2(t)] = \tfrac{1}{2}\mathrm{Tr}|\rho_1(t)-\rho_2(t)|, \tag{12}$$

where $|A| = \sqrt{A^\dagger A}$. In order to evaluate the non-Markovianity $\mathcal{N}$, we have to find a specific pair of optimal initial states to maximize the time derivative of the trace distance. In Ref. [40] it is proved that the pair of optimal states is associated with two antipodal pure states on the surface of the Bloch sphere. We thus adopt a pair of initial states $\rho_{1,2}(0) = |\psi_{1,2}(0)\rangle\langle\psi_{1,2}(0)|$ with $|\psi_{1,2}(0)\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$ as the optimal one. In this way, the time derivative of the trace distance can be derived in a simple form as $\sigma[t,\rho_1(0),\rho_2(0)] = d|c_1(t)|/dt$. Hence, the variation of trace distance is consistent with that of $|c_1(t)|$, which is in turn related to the sum of the correlation functions of all the involved reservoirs [cf. Eq. (8)].

Now consider the case when the spectral densities of all the reservoirs are Lorentzian, which for an nth reservoir takes the form $J_n(\omega) = \gamma_n\lambda_n^2/\{2\pi[(\omega-\omega_0)^2+\lambda_n^2]\}$, with γ_n the system-reservoir coupling strength and λ_n^{-1} the reservoir's correlation time. Then the two-point correlation function of an nth reservoir can be expressed as $f_n(\tau) = \frac{1}{2}\gamma_n\lambda_n\exp(-\lambda_n|\tau|)$ and the function $c_1(t)$ that determines the system's dynamics can be derived by solving Eq. (8) using the Laplace transform technique. Here, to derive an analytical expression for the system's dynamics and visually present the effects through the parameter N (the reservoirs' number), we assume identical reservoirs for simplicity. The calculation for the less trivial case of nonidentical reservoirs (that actually matters in practice) is more cumbersome and, as we shall show below by numerical simulations, the result remains unchanged qualitatively. With identical system-reservoir couplings, i.e., $\gamma_n/\lambda_n = \gamma/\lambda\ \forall n$, we obtain the function $c_1(t)$ as

$$c_1(t) = e^{-\lambda t}\left[\cosh\left(\frac{Gt}{2}\right) + \frac{\lambda}{G}\sinh\left(\frac{Gt}{2}\right)\right], \tag{13}$$

with $G = \sqrt{\lambda^2 - 2N\gamma\lambda}$. Before dealing with an arbitrary N, we first recall the case of $N = 1$ (i.e., the TLS interacts with one reservoir), that $\gamma < \lambda/2$ ($\gamma > \lambda/2$) represents the weak (strong) system-reservoir coupling regime resulting in the system's Markovian (non-Markovian) dynamics. In the presence of more than one reservoir, the system's dynamics would vary with N. As can be verified from Eq. (13), if $1 \leqslant N < [\lambda/2\gamma + 1]$ ($[x]$ the integer party of x) and $\gamma < \lambda/2$, then $c_1(t)$ monotonically decays to zero, indicating Markovian behavior of the system. That is, if initially a TLS is in contact with N Markovian reservoirs, each of which has $\gamma < \lambda/2$, then the system dynamics remains Markovian when $1 \leqslant N < [\lambda/2\gamma + 1]$. Yet, if more reservoirs are added so that $N \geqslant [\lambda/2\gamma + 1]$, then $c_1(t)$ oscillates in time, signifying non-Markovian behavior of the system, no matter how large the ratio $\lambda/2\gamma$. Therefore, $N_{\rm cr} = [\lambda/2\gamma + 1]$ can be regarded as a critical number of reservoirs in the sense that the condition $N \geqslant N_{\rm cr}$ guarantees the system's non-Markovian dynamics in any coupling regime. For instance, in a trivial case of strong-coupling regime with $\gamma > \lambda/2$ (or, the same, $\lambda/2\gamma < 1$), $N_{\rm cr}$ is found to be 1, i.e., just a single such reservoir suffices to trigger the non-Markovian dynamics of the system. Also, the greater the number of reservoirs (in either strong- or weak-coupling regimes), the stronger the non-Markovianity, as we shall show later. The interesting fact arises when $\gamma < \lambda/2$ (i.e., when all the N reservoirs are all in the weak-coupling regimes), as the system's dynamics can still become non-Markovian if $N \geqslant [\lambda/2\gamma + 1] = N_{\rm cr}$. Moreover, the weaker the system coupled to each reservoir (i.e., the smaller the value of $2\gamma/\lambda$), the greater the number N of reservoirs is required to guarantee the onset of non-Markovian dynamics. In principle, the system's non-Markovian dynamics can always be obtained by just adding a sufficiently large number of reservoirs as long as $\lambda/2\gamma$ is finite.

We plot the time evolution of trace distance $D[\rho_1(t),\rho_2(t)]$ of a TLS interacting with N identical reservoirs in Figs. 1(a) and 1(b), while the reservoirs are different in Fig. 1(c). We first choose $\gamma = 3\lambda/8$ for the coupling of the TLS, with each reservoir implying a Markovian dynamics if $N = 1$. (See the dashed curve in Fig. 1(a); $D[\rho_1(t),\rho_2(t)]$ asymptotically decays, as it should be.) However, if the TLS is allowed to be simultaneously coupled to more reservoirs, $D[\rho_1(t),\rho_2(t)]$ becomes an oscillatory function of time, signifying an emergence of the system's non-Markovian dynamics. (In Fig. 1(a) this happens already for N starting from 2.) In Fig. 1(b), where a weaker coupling $\gamma = 5\lambda/24$ is chosen, the corresponding critical number of reservoirs is found to be $N_{\rm cr} = 3$ and, as seen from the figure, the system dynamics remains Markovian for $N = 2$ but becomes non-Markovian for $N \geqslant 3$. Although in the above discussions the reservoirs are chosen to be identical, the results are indeed general, as we demonstrate in Fig. 1(c) for different system-reservoir couplings. Suppose the TLS interacts simultaneously with $N = 2$ distinct reservoirs, one has $\gamma_1 = \lambda/8$ and the other has $\gamma_2 = 3\lambda/8$, then the system's dynamics is Markovian [blue curve in Fig. 1(c)]. However, if one more pair ($N = 4$) or two more pairs ($N = 6$) of such reservoirs are added, then the transition of the dynamics from Markovian to non-Markovian occurs [see the red and green curves in Fig. 1(c)]. In Fig. 1(d) we display the non-Markovianity $\mathcal{N}$ as a function of the number N of the involved reservoirs. Evidently from Fig. 1(d), $\mathcal{N}$ becomes positive starting from $N = N_{\rm cr} = 2$ for $\gamma = 3\lambda/8$ (black squares), but from $N = N_{\rm cr} = 3$ for $\gamma = 5\lambda/24$ (black circles). We thus encounter a sudden emergence of non-Markovian dynamics at $N = N_{\rm cr}$. Furthermore, $\mathcal{N}$ is increasing with N.

Next, we switch to the situation when $\gamma_n/\lambda_n = \gamma/\lambda > 1/2\ \forall n$, i.e., when the TLS dynamics is readily non-Markovian for any $N \geqslant 1$. We are concerned with the

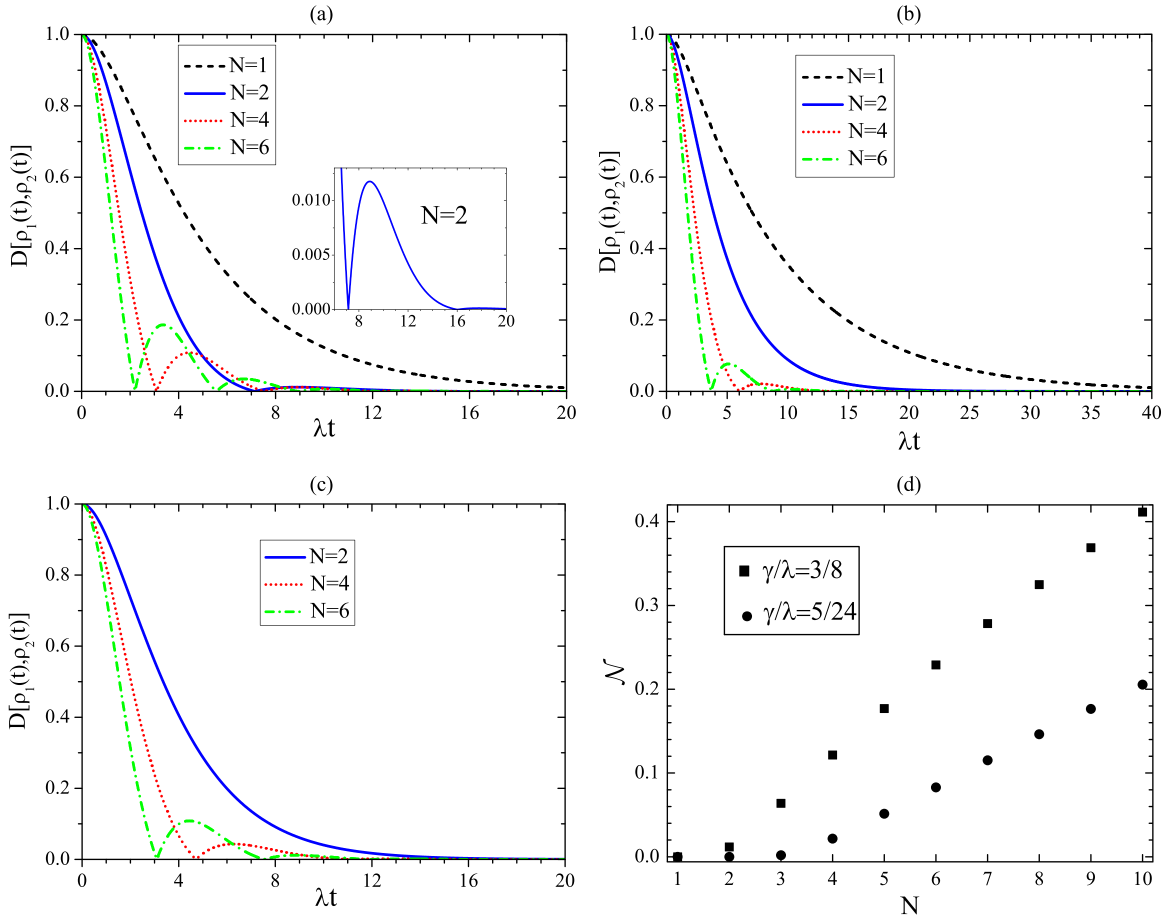

FIG. 1. (Color online) The trace distance $D[\rho_1(t),\rho_2(t)]$, with $\rho_{1,2}(0)=\frac{1}{2}(|0\rangle \pm |1\rangle)(\langle 0| \pm \langle 1|)$, as a function of λt and the non-Markovianity $\mathcal{N}$ as a function of N for the case of a TLS coupled simultaneously to N reservoirs. In (a) and (b) the reservoirs are identical with $\gamma = 3\lambda/8$, $N_{cr}=2$ and $\gamma = 5\lambda/24$, $N_{cr}=3$, respectively. In (c) the reservoirs are nonidentical: the blue curve is for $N=2$ reservoirs with $\gamma_1=\lambda/8$ and $\gamma_2 = 3\lambda/8$, the red curve is for $N=4$ reservoirs with $\gamma_{1,2}=\lambda/8$ and $\gamma_{3,4}=3\lambda/8$, and the green curve is for $N=6$ reservoirs with $\gamma_{1,2,3}=\lambda/8$ and $\gamma_{4,5,6}=3\lambda/8$. In (d) the reservoirs are identical with $\gamma = 3\lambda/8$ (black squares) and $\gamma=5\lambda/24$ (black circles).

change of non-Markovianity when the number of reservoirs increases. For that purpose, in Fig. 2 we plot $\mathcal{N}$ versus γ/λ for various values of N. Besides the increase of $\mathcal{N}$ with γ/λ that should occur, what is worthy to note here is enhancement of the non-Markovianity due to the addition of reservoirs and, more importantly, the enhancement holds for whatever values of γ and λ, as visualized from Fig. 2.

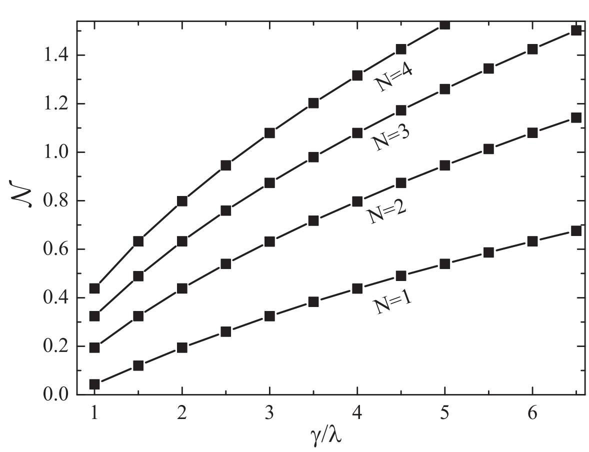

FIG. 2. The non-Markovianity $\mathcal{N}$ as a function of γ/λ for various numbers N of identical reservoirs.

III. EXPLANATION IN TERMS OF THE PSEUDOMODE THEORY

So far, we have shown that a TLS always exhibits non-Markovian dynamics when $N \geqslant N_{cr}$. In particular, non-Markovian dynamics can even be triggered when the couplings of the system with all the reservoirs are weak, which would contradict the common thinking that in the weak-coupling regime no information can be returned from the reservoirs back to the system. In order to shed some light on this issue, we make use of the so-called pseudomode theory developed in Refs. [32] and [33] to clarify how the decayed information can flow back to the system in the weak-coupling regimes. According to the pseudomode theory [32,33], pseudomodes of a reservoir are auxiliary variables that are introduced in terms of the position of the poles of the reservoir's spectral distribution. The coupling of the system with a reservoir can be thought of as coherent interaction between the system and the pseudomodes, which are in turn surrounded by external Markovian reservoirs called the pseudomodes' reservoirs. By treating the system of interest plus the pseudomodes as an extended system, one can derive a corresponding Markovian master equation. For the present model with a TLS coupled to N reservoirs with Lorentzian spectral densities (each reservoir contains one pseudomode), the pseudomode theory leads to the following master equation for the density matrix $\rho^{\rm SP}(t)$ of the extended system:

$$\dot{\rho}^{\rm SP}(t) = -i[\hat{H}_0,\rho^{\rm SP}(t)] - \sum_{n=1}^{N}\frac{\Gamma_n}{2}[\hat{b}_n^\dagger\hat{b}_n\rho^{\rm SP}(t) -2\hat{b}_n\rho^{\rm SP}(t)\hat{b}_n^\dagger + \rho^{\rm SP}(t)\hat{b}_n^\dagger\hat{b}_n], \quad (14)$$

where $\hat{H}_0 = \frac{\omega_0}{2}\hat{\sigma}_z + \sum_{n=1}^{N}\omega_0\hat{b}_n^\dagger\hat{b}_n + \sum_{n=1}^{N}\Omega_n(\hat{b}_n^\dagger\hat{\sigma}_- + \hat{b}_n\hat{\sigma}_+)$, with $\hat{b}_n^\dagger$ ($\hat{b}_n$) being the creation (annihilation) operator of the nth pseudomode whose constant of coupling to the TLS is $\Omega_n = \sqrt{\lambda_n\gamma_n/2}$, and $\Gamma_n = 2\lambda_n$ denotes the decay rate of the nth pseudomode. By tracing out Eq. (14) over all the pseudomodes we obtain a master equation for the density matrix $\rho(t)$ of the TLS:

$$\dot{\rho}(t) = \Gamma_S(t)\big[\hat{\sigma}_-\rho(t)\hat{\sigma}_+ - \tfrac{1}{2}\{\hat{\sigma}_+\hat{\sigma}_-,\rho(t)\}\big], \quad (15)$$

with the time-dependent decay rate $\Gamma_S(t)$ given by

$$\Gamma_S(t) = \frac{1}{|c(t)|^2}\left(\sum_{n=1}^{N}2\Omega_n{\rm Im}\{c(t)b_n^*(t)\}\right), \quad (16)$$

where $c(t)$ and $b_n(t)$, amplitudes of the excited state of the TLS and the nth pseudomode, satisfy the following differential equations:

$$i\dot{c}(t) = \omega_0 c(t) + \sum_{n=1}^{N}\Omega_n b_n(t), \quad (17)$$

$$i\dot{b}_n(t) = \left(\omega_0 - \frac{i}{2}\Gamma_n\right)b_n(t) + \Omega_n c(t). \quad (18)$$

In terms of the pseudomode theory, when a TLS interacts with a single reservoir, its information first flows to the pseudomode and then from the pseudomode to the pseudomode's reservoir. The flow from the pseudomode to its reservoir is one-way, but the flow between the system and the pseudomode is two-way. In other words, the pseudomode could play a role of memory supplier, which can be revealed, for the case of $N = 1$, by the equation [33]

$$\frac{d|b_1(t)|^2}{dt} + \Gamma_1|b_1(t)|^2 = \Gamma_S(t)|c(t)|^2. \quad (19)$$

This equation connects the compensated rate of change of the pseudomode population (CRCPP) and the system's decay rate. If the pseudomode population is reduced ($d|b_1(t)|^2/dt < 0$) and meanwhile that reduction cannot be accounted for by the decay to its reservoir due to the term $\Gamma_1|b_1(t)|^2$, then the left-hand side of Eq. (19) is negative and so is the system's decay rate $\Gamma_S(t)$. Physically, this means that the energy of the populated pseudomode is transferred not only to its reservoir but also to the system and, as a consequence, the system's dynamics becomes non-Markovian. Therefore the evolution of CRCPP can be used as a witness to specify the directions of the information flow between system and reservoirs. In particular, its negativity indicates the backflow of information to the system. For an arbitrary $N > 1$ we have derived the following equation associated with the total CRCPP (see the Appendix):

$$\sum_{n=1}^{N}\left(\frac{d|b_n(t)|^2}{dt} + \Gamma_n|b_n(t)|^2\right) = \Gamma_S(t)|c(t)|^2. \quad (20)$$

This equation implies that the system's decay rate $\Gamma_S(t)$ is determined by the total compensated rates of change of all the pseudomodes' populations. Notwithstanding, we should make clear the information flow in an individual pseudomode, particularly when the system is weakly coupled to all the reservoirs. For this purpose, by solving the differential equations (17) and (18) we derive for an arbitrary nth pseudomode of N identical reservoirs (i.e., $\gamma_n/\lambda_n = \gamma/\lambda\ \forall n$) an analytical expression for its CRCPP, $d|b_n(t)|^2/dt + \Gamma_n|b_n(t)|^2$, as

$$\frac{d|b_n(t)|^2}{dt} + \Gamma_n|b_n(t)|^2 = Q(t)\left[\lambda\sinh\left(\frac{Gt}{2}\right) + G\cosh\left(\frac{Gt}{2}\right)\right], \quad (21)$$

with $Q(t) = 2\lambda\gamma e^{-\lambda t}\sinh(Gt/2)/G^2$ and $G = \sqrt{\lambda(\lambda - 2N\gamma)}$ as given previously. Obviously, if $1 \leqslant N < N_{\rm cr} = [\lambda/2\gamma + 1]$ (here $\gamma < \lambda/2$, i.e., weak-coupling regimes are assumed), then the CRCPP is always positive for $t > 0$, implying a one-way flow of the information from the system to the pseudomode and eventually to the pseudomode's reservoir. Nevertheless, if $N \geqslant N_{\rm cr}$, then $G = i\sqrt{\lambda(2N\gamma - \lambda)}$ is purely imaginary and the CRCPP can attain a negative value inside several periods of time during the evolution, implying the information of the pseudomode flows back to the system and this backaction is independent of the coupling strength of the system with all the reservoirs. Remarkably, the condition $N \geqslant N_{\rm cr}$ is consistent with that for the onset of system's non-Markovian dynamics, as shown in the previous section. Therefore we conclude that if $N \geqslant N_{\rm cr}$ the information in the pseudomodes of N identical reservoirs can flow back to the system, by which means the system can partially retrieve the previously lost information that corresponds to the non-Markovian dynamics. Moreover, the total returned information of all the pseudomodes determines the degree of the non-Markovian process, explaining why the non-Markovianity $\mathcal{N}$ is proportional to the number N of involved reservoirs, given a definite coupling strength [cf. Fig. 1(d)]. Although in the above discussion we considered identical reservoirs, the results are valid to the general situations of nonidentical reservoirs too.

We plot in Fig. 3 the time dependence of the nth mode CRCPP, $d|b_n(t)|^2/dt + \Gamma_n|b_n(t)|^2$, for both cases of identical and nonidentical reservoirs. Figure 3(a) represents the case when the TLS is weakly coupled with N identical reservoirs with $\gamma = 3\lambda/8$. When there is only one reservoir ($N = 1$), the values of the CRCPP are always positive [cf. solid line in Fig. 3(a)]. As we have shown above, for $\gamma = 3\lambda/8$, $d|b_n(t)|^2/dt + \Gamma_n|b_n(t)|^2$ can sometimes be negative starting

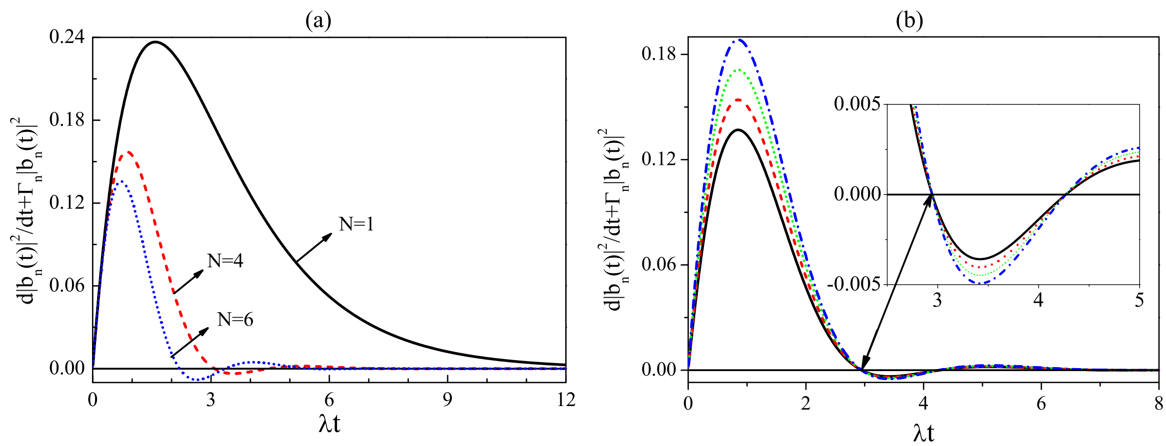

FIG. 3. (Color online) The compensated rate of change of an nth pseudomode population, $d|b_n(t)|^2/dt + \Gamma_n|b_n(t)|^2$, as a function of γ/λ. (a) All the N reservoirs are identical with a coupling strength $\gamma = 3\lambda/8$. (b) $N = 4$ reservoirs with different coupling strengths to the system: $\gamma_1 = 8\lambda/24$ (black solid curve), $\gamma_2 = 9\lambda/24$ (red dashed curve), $\gamma_3 = 10\lambda/24$ (green dotted curve), and $\gamma_4 = 11\lambda/24$ (blue dash-dotted curve).

from $N = N_{cr} = 2$, as is visualized in Fig. 3(a), for example, for $N = 4$ (red curve) and $N = 6$ (blue curve). Furthermore, the time moment at which the CRCPP first becomes negative is earlier for $N = 6$ than for $N = 4$. This fact implies that a sudden emergence of non-Markovian dynamics takes place sooner when the number of contributed reservoirs is larger, which is also consistent with the behavior of the trace distance displayed in Fig. 1(a) for $N = 4$ and $N = 6$. In Fig. 3(b), we consider four reservoirs with different coupling constants: $\gamma_1 = 8\lambda/24$, $\gamma_2 = 9\lambda/24$, $\gamma_3 = 10\lambda/24$, and $\gamma_4 = 11\lambda/24$ for the first, second, third, and fourth reservoir, respectively. Although all four reservoirs are only weakly coupled to the system, the CRCPP $d|b_n(t)|^2/dt + \Gamma_n|b_n(t)|^2$ $\forall n$ can become negative during the time evolution. It is interesting to note that the four pseudomodes begin to return information at the same time [here at $t \approx 2.95/\lambda$, as seen from Fig. 3(b) and, more evidently, from its inset], despite their distinct coupling strengths to the system.

IV. CONCLUSION

In conclusion, we have studied a model consisting of one TLS transversely coupled simultaneously with N zero-temperature bosonic reservoirs, which could be realized by the configuration sketched in Fig. 4: a TLS, such as a two-level atom, is located at the center of N lossy cavities. The fundamental mode ω_0 supported by a cavity displays a Lorentzian broadening due to the nonperfect reflectivity of the cavity mirrors. The effective spectral density of the intracavity field can thus be treated as Lorentzian. In experiment, instead of the dynamics of trace distance, a simple and reliable witness of non-Markovianity is the atomic average energy whose growth at some stage of time evolution signifies the non-Markovian dynamics, which greatly simplifies the experimental implementation. In the presence of N reservoirs, we explore the emergence and enhancement of non-Markovian dynamics of the TLS. We find out that, provided N is not smaller than a critical value N_{cr} (i.e., $N \geqslant N_{cr}$), the system dynamics is always non-Markovian for whatever system-reservoir coupling strengths. In particular, non-Markovian dynamics even emerges when the system is weakly coupled with all the reservoirs, exhibiting a fundamental difference to the situation with $N < N_{cr}$, under which the system's dynamics remains Markovian all the time. Although the analytical formula of N_{cr} is derived for identical reservoirs, our results are valid also to the general cases of nonidentical ones. Moreover, we show quantitatively that the non-Markovianity $\mathcal{N}$ is proportional to the number N of the contributed reservoirs in both weak- and strong-coupling regimes. This means that if, for a given N, the system's dynamics is already non-Markovian, then the non-Markovian behavior can be enhanced by further increasing the reservoirs' number. To interpret the obtained results we examine the information flow between the system and the reservoirs by taking advantage of the pseudomode theory. We find that the pseudomodes of all the reservoirs can always return information back to the system during certain periods of time in the course of evolution if $N \geqslant N_{cr}$, regardless of the system-reservoir coupling strengths. It is the total returned information of all the pseudomodes that determines the non-Markovianity $\mathcal{N}$, in agreement with the proportional relation of $\mathcal{N}$ versus N.

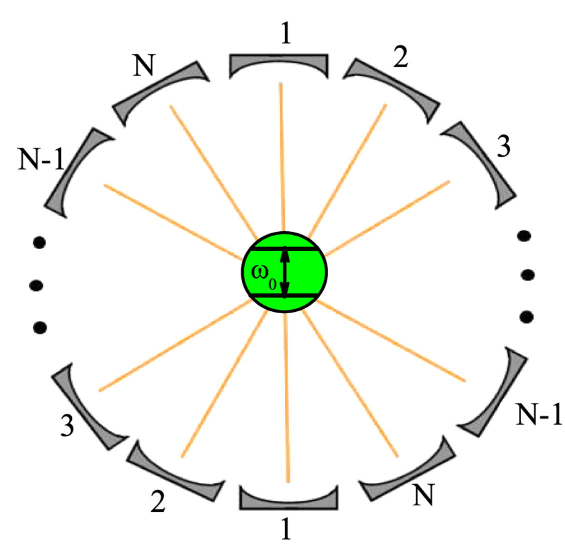

FIG. 4. (Color online) A possible configuration to simultaneously couple a TLS with N lossy cavities (served as reservoirs).

Among other mechanisms by which non-Markovian dynamics of an open system can be induced or enhanced, as mentioned in the Introduction, our results provide an alternate one, namely, by letting more reservoirs be coupled to the system of interest. Finally, we would like to recall that in this work we only consider a very specific model of open quantum systems—a TLS with transversal coupling to zero-temperature bosonic reservoirs. Generalization of our results to other more complex models, such as the longitudinal coupling, the hybrid of transversal and longitudinal couplings, the general thermal reservoirs, and so on, needs further elaborate efforts.

ACKNOWLEDGMENTS

In this work, Z.X.M. and Y.J.X. are supported by the National Natural Science Foundation (China) under Grants No. 11204156, No. 61178012, and No. 11247240, and the Promotive Research Fund for Excellent Young and Middle-Aged Scientists of Shandong Province (China) under Project No. BS2013DX034, while N.B.A. is funded by the National Foundation for Science and Technology Development (NAFOSTED, Vietnam) under Project No. 103.01-2014.02.

APPENDIX: DERIVATION OF EQ. (20)

By virtue of Eq. (18),

$$\begin{aligned}\frac{d}{dt}|b_n(t)|^2 &= b_n^*(t)\frac{d}{dt}b_n(t) + b_n(t)\frac{d}{dt}b_n^*(t)\\ &= -i\left(\omega_0 - \frac{i}{2}\Gamma_n\right)|b_n(t)|^2 - i\Omega_n c(t)b_n^*(t)\\ &\quad + i\left(\omega_0 + \frac{i}{2}\Gamma_n\right)|b_n(t)|^2 + i\Omega_n c^*(t)b_n(t)\\ &= -\Gamma_n|b_n(t)|^2 + 2\Omega_n \mathrm{Im}\{c(t)b_n^*(t)\}. \end{aligned}\tag{A1}$$

Therefore we have $d|b_n(t)|^2/dt+\Gamma_n|b_n(t)|^2=2\Omega_n\mathrm{Im}\{c(t)b_n^*(t)\}$. On the other hand, from the time-dependent decay rate $\Gamma_S(t)$ given by Eq. (16),

$$\Gamma_S(t)|c(t)|^2 = \sum_{n=1}^{N} 2\Omega_n \mathrm{Im}\{c(t)b_n^*(t)\}. \tag{A2}$$

Then, by comparison, we obtain

$$\sum_{n=1}^{N}\left(\frac{d|b_n(t)|^2}{dt} + \Gamma_n|b_n(t)|^2\right) = \Gamma_S(t)|c(t)|^2, \tag{A3}$$

which is Eq. (20).

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