Generation of two-mode photon-added displaced squeezed states

2014 Adv. Nat. Sci: Nanosci. Nanotechnol. 5 035012

(http://iopscience.iop.org/2043-6262/5/3/035012)

IOP Publishing | Vietnam Academy of Science and Technology
Advances in Natural Sciences: Nanoscience and Nanotechnology
Adv. Nat. Sci.: Nanosci. Nanotechnol. **5** (2014) 035012 (6pp)
doi:10.1088/2043-6262/5/3/035012

Generation of two-mode photon-added displaced squeezed states

Thi Xuan Hoai Nguyen[1] **and Ba An Nguyen**[2]

[1] Danang University's College of Education, 459 Ton Duc Thang, Da Nang, Vietnam
[2] Center for Theoretical Physics, Institute of Physics, Vietnam Academy of Science and Technology (VAST), 18 Hoang Quoc Viet, Cau Giay, Ha Noi, Vietnam

E-mail: xuanhoai.ph@gmail.com and nban@iop.vast.ac.vn

Received 16 July 2014
Accepted for publication 8 August 2014
Published 5 September 2014

Abstract
Two-mode photon-added displaced squeezed states were previously introduced and their various interesting nonclassical as well as nonlocal properties were studied in great detail. In this paper we present experimental schemes for generating those states using optical devices such as nondegenerate parametric downconverters, displacers, beam-splitters and photo-detectors. The purpose is to see how close to the desired state the generated state can be and how high the success probability corresponding to a given fidelity is.

Keywords: photon addition, beam-splitter, nondegenerate parametric downconverter, fidelity, success probability
Classification numbers: 3.01

1. Introduction

Quantum optics distinguishes nonclassical from classical states and quantum information science distinguishes entangled from unentangled (or separable) states. Although developed almost independently and at different times (while the study of nonclassical states appeared just after the introduction of coherent states in the 1960s [1], the notion of entangled states had been encountered already in 1935 with respect to the completeness of quantum mechanics [2]), nonclassicality and entanglement are closely related. Most evident is the fact that no entanglement can be created at all by a beam-splitter from a classical source. In other words, for the output of a beam-splitter to be entangled the input should be nonclassical. Recently, a unified quantification of nonclassicality and entanglement has been put forward in [3] according to which the amount of nonclassicality of a (multimode) input field is exactly equal to the amount of (multipartite) entanglement of the output field. While nonclassical properties gave birth to new developments of the quantum coherence theory [4], entanglement has been recognized as a necessarily vital resource for quantum information processing and quantum computing, which are totally new ways of handling communication and computation [5].

There are many classes of nonclassical states: cat states [6], squeezed states [7], antibunched states [8], fan states [9], pair coherent states [10], trio coherent states [11], just to name a few. An interesting one called displaced squeezed state was introduced in [12]. Its extension to the two-mode case is of the form [13]

$$|\,\alpha, \beta, s\rangle_{ab} = D_a(\alpha)D_b(\beta)S_{ab}(s)\,|\,00\rangle_{ab}, \quad (1)$$

where $D_a(\alpha) = \exp(\alpha a^\dagger - \alpha^* a)$, $D_b(\beta) = \exp(\beta b^\dagger - \beta^* b)$ are the single-mode displacement operators (or displacers for short), $S_{ab}(s) = \exp\left(s^* ab - s a^\dagger b^\dagger\right)$ the two-mode squeezer, $|\,00\rangle_{ab}$ the state containing no photons in both modes a and b, $a^\dagger(a)$ the photon creation (annihilation) operator for mode a, $b^\dagger(b)$ the same for mode b and α, β, s are complex in general. The authors of [13] also considered new states obtained by adding photons to either of the modes, focusing on the nonclassical behaviors of mean numbers, Mandel Q parameter, cross-correlation function and Wigner function. Such states have recently been generalized in [14] to the situation when photons are added to both modes, whose sum-squeezing, difference-squeezing, higher-order antibunching and entanglement were analyzed in detail. These generalized states called two-mode photon-added displaced squeezed states have the form

$$|\,\alpha,\,\beta,\,s,\,m,\,n\rangle_{ab}=N_{mn}(\alpha,\,\beta,\,s)a^{\dagger m}b^{\dagger n}\,|\,\alpha,\,\beta,\,s\rangle_{ab},\tag{2}$$

where

$$N_{mn}(\alpha,\,\beta,\,s)=\frac{1}{\sqrt{C_{mn}(\alpha,\,\beta,\,s)}},\tag{3}$$

with

$$C_{mn}(\alpha,\,\beta,\,s)={}_{ab}\langle\alpha,\,\beta,\,s\,|\,b^{n}a^{m}a^{\dagger m}b^{\dagger n}\,|\,\alpha,\,\beta,\,s\rangle_{ab}\tag{4}$$

is the normalization coefficient. By virtue of the operatoric identities (α, β and s are assumed real for simplicity)

$$S_{ab}^{\dagger}(s)D_{a}^{\dagger}(\alpha)a^{\dagger}D_{a}(\alpha)S_{ab}(s)=a^{\dagger}\cosh s-b\sinh s+\alpha,\tag{5}$$

$$S_{ab}^{\dagger}(s)D_{a}^{\dagger}(\alpha)aD_{a}(\alpha)S_{ab}(s)=a\cosh s-b^{\dagger}\sinh s+\alpha,\tag{6}$$

$$S_{ab}^{\dagger}(s)D_{b}^{\dagger}(\beta)b^{\dagger}D_{b}(\beta)S_{ab}(s)=b^{\dagger}\cosh s-a\sinh s+\beta,\tag{7}$$

$$S_{ab}^{\dagger}(s)D_{b}^{\dagger}(\beta)bD_{b}(\beta)S_{ab}(s)=b\cosh s-a^{\dagger}\sinh s+\beta,\tag{8}$$

we can derive the expression of $C_{mn}(\alpha,\,\beta,\,s)$ in the form which is useful for later numerical calculations as follows

$$\begin{aligned}C_{mn}(\alpha,\,\beta,\,s)&=(m!n!)^{2}(\cosh s)^{m+n}\alpha^{m}\beta^{n}\\&\times\sum_{l=0}^{m}\sum_{p=0}^{m}\sum_{q=0}^{p}\sum_{l'=0}^{n}\sum_{p'=0}^{n}\sum_{q'=0}^{p'}\delta_{l,m-p+q'}\delta_{l',n-p'+q}\\&\times\frac{(\cosh s)^{l-p+l'-p'}(-\sinh s)^{q+q'}\alpha^{p-l-q}\beta^{p'-l'-q'}}{\begin{array}{l}[(m-l)!(m-p)!(p-q)!\\ q!(n-l')!(n-p')!(p'-q')!q'!]\end{array}}.\end{aligned}\tag{9}$$

The study of photon addition has recently emerged as a topic of lively interest because this proves to be a plausible way to engineer on-demand quantum states (see, e.g. [15], and a nice review [16] with the references therein). In particular, adding photons always makes a classical state nonclassical. Also, many multimode photon-added states including the two-mode states (2) at the same time possess a certain amount of inter-mode entanglement that could be exploited to perform various useful global quantum tasks only by means of local operation combined with classical communication.

In this paper we are concerned with the techniques of how to add a desired number of photons to each of the two modes of the displaced squeezed state given by equation (1), i.e., how to generate the two-mode photon-added displaced squeezed states given by equation (2). Two different schemes for that purpose will be presented in the next section to see how close the generated state is to the target one and how the corresponding success probability could be, depending on the parameters involved. We shall briefly summarize and outline possible related problems in the final section.

2. The generation schemes

Although photon-added states were introduced many years ago [17], they had to wait until 2004 when a single-photon addition was actually realized for the first time in the laboratory [18]. Before going into any detail, it is worth at this point clarifying the real meaning of the terminologies 'photon-added' or 'adding photons' which may be misleading. A non-specialist would think that a photon-added state $|\,\tilde{\psi}\rangle_a=a^{\dagger}\,|\,\psi\rangle_a\big/\sqrt{\langle\hat{n}\rangle+1}$, with $\langle\hat{n}\rangle\equiv{}_a\langle\psi\,|\,\hat{n}\,|\,\psi\rangle_a$ $\left(\hat{n}=a^{\dagger}a\right)$ being the averaged photon number in the original state $|\,\psi\rangle_a$, should be the state in which the averaged photon number becomes $\tilde{n}={}_a\langle\tilde{\psi}\,|\,\hat{n}\,|\,\tilde{\psi}\rangle_a=\langle\hat{n}\rangle+1$, i.e., exactly one photon should be added. However, this is not true. In fact, $\tilde{n}$ can be calculated to be $\tilde{n}=\left\langle(\hat{n}+1)^{2}\right\rangle\big/\left(\langle\hat{n}\rangle+1\right)$ so that $\tilde{n}-\left(\langle\hat{n}\rangle+1\right)\propto V_n$, where $V_n\equiv\left\langle\hat{n}^{2}\right\rangle-\langle n\rangle^{2}$ is the photon number variance, which vanishes only for a Fock state with a definite photon number. This implies $\tilde{n}\neq\langle\hat{n}\rangle+1$ in general. In view of the above elucidation, we now correctly understand that 'adding m photons' to a state $|\,\psi\rangle_a$ simply means m times of application of $a^{\dagger}$ on that state. In this spirit and making use of the properties of currently available optical devices we shall in this section consider two schemes for generating the two-mode photon-added displaced squeezed states of the form (2). For implementing photon addition, the first scheme uses beam-splitters, while the second one employs nondegenerate parametric downconverters. Both schemes require photo-detectors which are assumed ideal photon-number-resolving for simplicity in theoretical calculations.

2.1. Photon addition by beam-splitters

Beam-splitter (BS), a device that transforms two input modes to two new output modes, is described by the unitary operator

$$B_{ab}(t)=\exp\left[\theta\left(a^{\dagger}b-b^{\dagger}a\right)\right],\tag{10}$$

with $t=\cos\theta$ the transmittivity and $r=\sin\theta$ the reflectivity. Figure 1 sketches the arrangement of the devices necessary for the generation process. DC denotes a nondegenerate parametric downconverter described by the two-mode squeezer $S_{ab}(s)$. In the downconversion process, a mother-photon from the beam that pumps a $\chi^{(2)}$ nonlinear medium gives birth to two daughter-photons belonging to two different modes labeled a and b. Mode a (b) is then displaced by $D_a(\alpha)$ $\left(D_b(\beta)\right)$ which can be implemented by means of a low-reflectivity (low-r) and an intense coherent beam with amplitude α' (β') such that $r\alpha'=\alpha\left(r\beta'=\beta\right)$. As a result, the state $|\,\alpha,\,\beta,\,s\rangle_{ab}$ of the form (1) appears after such operations. To simulate the action of $a^{\dagger m}b^{\dagger n}$, let mode a of state $|\,\alpha,\,\beta,\,s\rangle_{ab}$ be an input to a beam-splitter (BS1) having transmittivity t and, at the same time, mode b be an input to another beam-splitter (BS2) having the same transmittivity. As for the other input modes a' and b' of BS1 and BS2, they are Fock states $|\,m\rangle_{a'}$ and $|\,n\rangle_{b'}$, respectively. Behind the beam-splitters there are two photo-detectors, PD1 and PD2, to detect photons of outgoing modes a' and b', respectively. We are interested in the case when neither detectors click,

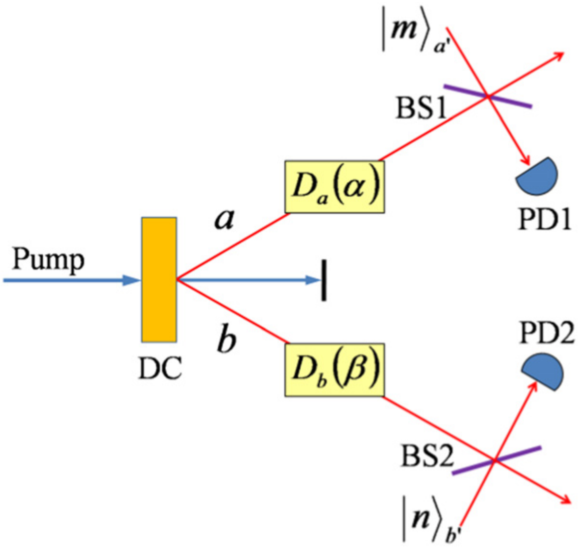

Figure 1. Schematic setup for generation of two-mode photon-added displaced squeezed states using beam-splitters for photon addition. A nondegenerate parametric downconverter DC followed by two displacers $D_a(\alpha)$ and $D_b(\beta)$ are used to create states $|\,\alpha,\,\beta,\,s\rangle_{ab}$ in equation (1), while a Fock state $|\,m\rangle_{a'}$ $\left(|\,n\rangle_{b'}\right)$, a beam-splitter BS1 (BS2) combined with a photo-detector PD1 (PD2) are used to add photons to mode a (b). Success is heralded when no photons are registered by both the photo-detectors.

projecting the two modes a and b onto

$$\left|\psi_{BS}\right\rangle_{ab} = \frac{\left|\psi'_{BS}\right\rangle_{ab}}{\sqrt{P_{BS}}}, \tag{11}$$

with

$$\left|\psi'_{BS}\right\rangle_{ab} = {}_{a'}\langle 0\,|\,B_{aa'}(t)\,|\,m\rangle_{a'} \times {}_{b'}\langle 0\,|\,B_{bb'}(t)\,|\,n\rangle_{b'}\,|\,\alpha,\,\beta,\,s\rangle_{ab} \tag{12}$$

and

$$P_{BS} = {}_{ab}\left\langle \psi'_{BS} | \psi'_{BS} \right\rangle_{ab} \tag{13}$$

being the success probability (i.e., the probability that there are no clicks at all). Using the decomposition of the beam-splitter [19]

$$B_{aa'}(t) = \exp\left(-\frac{r}{t}a'^{\dagger}a\right)t^{a'^{\dagger}a' - a^{\dagger}a}\exp\left(\frac{r}{t}a^{\dagger}a'\right), \tag{14}$$

we have

$${}_{a'}\langle 0\,|\,B_{aa'}(t)\,|\,m\rangle_{a'} = \frac{r^m}{t^m\sqrt{m!}}t^{a^{\dagger}a}a^{\dagger m} \tag{15}$$

and

$${}_{b'}\langle 0\,|\,B_{bb'}(t)\,|\,n\rangle_{b'} = \frac{r^n}{t^n\sqrt{n!}}t^{b^{\dagger}b}b^{\dagger n}. \tag{16}$$

Substituting equations (15) and (16) into equation (12) yields

$$\left|\psi'_{BS}\right\rangle_{ab} = \frac{r^{m+n}}{t^{m+n}\sqrt{m!n!}}t^{a^{\dagger}a}a^{\dagger m}t^{b^{\dagger}b}b^{\dagger n}\,|\,\alpha,\,\beta,\,s\rangle_{ab}. \tag{17}$$

Putting $t^{a^{\dagger}a}$ and $t^{b^{\dagger}b}$ in equation (17) in the form $\exp\left(a^{\dagger}a\ln t\right)$ and $\exp\left(b^{\dagger}b\ln t\right)$ then using the equalities [20]

$$\exp\left(a^{\dagger}a\ln t\right) = \frac{1}{t}\sum_{j=0}^{\infty}\frac{\left(1-t^{-1}\right)^j}{j!}a^j a^{\dagger j}, \tag{18}$$

$$\exp\left(b^{\dagger}b\ln t\right) = \frac{1}{t}\sum_{j'=0}^{\infty}\frac{\left(1-t^{-1}\right)^{j'}}{j'!}b^{j'} b^{\dagger j'}, \tag{19}$$

we can write P_{BS}, equation (13), in the explicit form ready for numerical calculations as

$$P_{BS} = \frac{\left(1-t^2\right)^{m+n}}{m!n!t^{2(m+n+2)}} \times \sum_{j=0}^{\infty}\sum_{j'=0}^{\infty}\frac{\left(1-t^{-2}\right)^{j+j'}}{j!j'!}C_{m+j,n+j'}(\alpha,\,\beta,\,s). \tag{20}$$

As for the fidelity $F_{BS} = \left|{}_{ab}\left\langle \psi_{BS} | \alpha,\,\beta,\,s,\,m,\,n\right\rangle_{ab}\right|^2$ of the generated state (11) with respect to the target state (2), its explicit form reads

$$F_{BS} = \frac{\left|\sum_{j=0}^{\infty}\sum_{j'=0}^{\infty}\frac{(1-t^{-1})^{j+j'}}{j!j'!}C_{m+j,n+j'}(\alpha,\,\beta,\,s)\right|^2}{C_{m,n}(\alpha,\,\beta,\,s)\sum_{j=0}^{\infty}\sum_{j'=0}^{\infty}\frac{(1-t^{-2})^{j+j'}}{j!j'!}C_{m+j,n+j'}(\alpha,\,\beta,\,s)} \tag{21}$$

Precisely speaking, from equation (17), the net effect of the beam-splitters BS1 and BS2 in figure 1, conditioned on the outcome that no photons at all are detected by both detectors, is the action of $t^{a^{\dagger}a}a^{\dagger m}t^{b^{\dagger}b}b^{\dagger n}$ on $|\,\alpha,\,\beta,\,s\rangle_{ab}$. So, a naïve inference would tell us that the desired addition of photons (i.e., the action of $a^{\dagger m}b^{\dagger n}$) would be achieved if $t = 1$. However, such a mathematical 'result' cannot be accepted since it is unphysical: in fact, a beam-splitter with $t = 1$ is tantamount to nothing and thus the state $|\,\alpha,\,\beta,\,s\rangle_{ab}$ remains itself without any photons added. What can be expected is that the generated state would more resemble the intended one if t is getting closer to 1. To consolidate this we plot in figures 2 and 3 the fidelity (21) as a function of t for several sets of the other parameters. As expected, the figures indicate that, though the fidelity can never be one, it always increases with t and asymptotically tends to 1 in the limit $t \to 1$. The bad thing however is reduction of the corresponding success probability with increasing t, as seen from their curves (see the bottom parts of figures 2 and 3), which also agrees with physical intuition. Namely, the greater t becomes, the easier the photons of mode a' $\left(b'\right)$ get through the beam-splitter BS1 (BS2) towards the detector PD1 (PD2) and, as a consequence, detection of no photons is less possible.

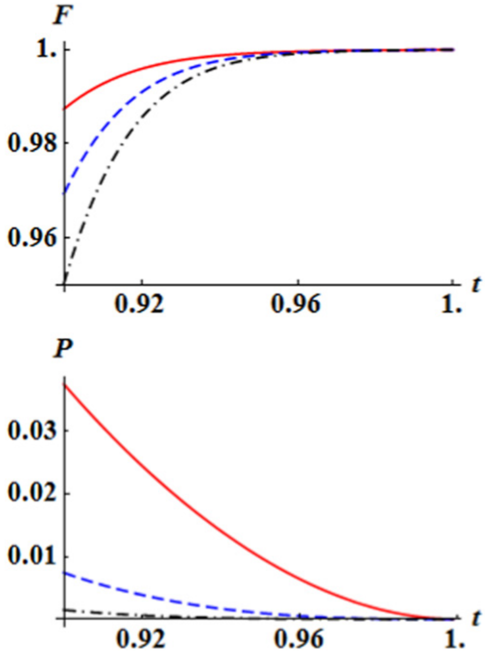

Figure 2. The fidelity $F \equiv F_{BS}$, equation (21), and the corresponding success probability $P \equiv P_{BS}$, equation (20), as functions of the beam-splitters' transmittivity t for $\alpha = \beta = s = 0.1$ with $\{m,\ n\} = \{1,\ 1\}$ (red solid), $\{1,\ 2\}$ (blue dashed), and $\{2,\ 2\}$ (black dash-dotted).

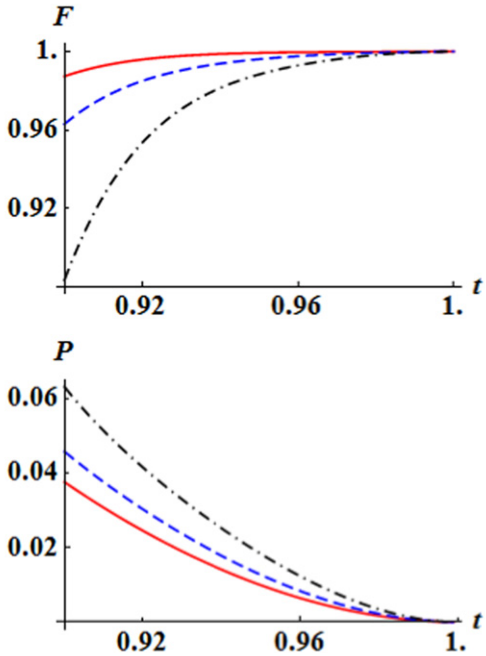

Figure 3. The fidelity $F \equiv F_{BS}$, equation (21), and the corresponding success probability $P \equiv P_{BS}$, equation (20), as functions of the beam-splitters' transmittivity t for $m = n = 1$ with $\alpha = \beta = s = 0.1$ (red solid), 0.3 (blue dashed), and 0.5 (black dash-dotted).

Furthermore, as follows from figure 2, for given $\alpha,\ \ \beta,\ \ s$ and t, both the fidelity and the corresponding generation probability decrease with increasing m or/and n. Such a behavior of probability is due to the fact that the chance for a detector to click is proportional to the number of incoming photons.

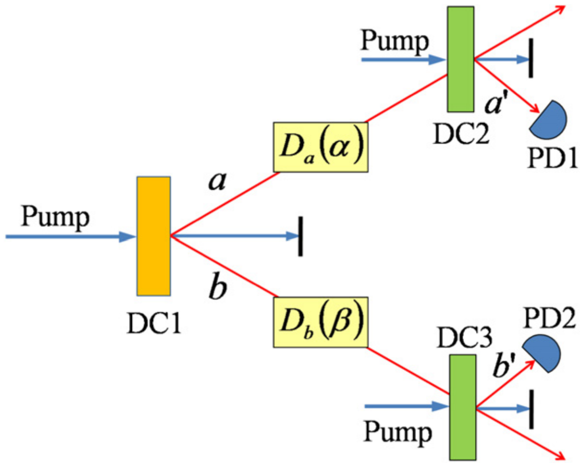

Figure 4. Schematic setup for generation of two-mode photon-added displaced squeezed states using downconverters for photon addition. A nondegenerate parametric downconverter DC1 combined with two displacers $D_a(\alpha)$ and $D_b(\beta)$ are used to create states $|\,\alpha,\,\beta,\,s\rangle_{ab}$ in equation (1), while a downconverter DC2 (DC3) combined with a photo-detector PD1 (PD2) are used to add photons to mode a (b). Success is heralded when m photons are registered by PD1 and n photons are registered by PD2, simultaneously.

Practically, this implies that adding more photons is more challenging and, even when it succeeded, the price to pay is the reduction of fidelity. Finally, as seen from figure 3, for given $m,\ \ n$ and t, the fidelity decreases but the corresponding success probability increases with increasing $\alpha,\ \ \beta,\ \ s$. Physically, these properties can be interpreted as follows. Here the state to which we add photons is $|\,\alpha,\,\beta,\,s\rangle_{ab}$ defined by equation (1). Its total averaged number of photons in both modes can be calculated to be $\left\langle \hat{n}_{\rm total} \right\rangle = {}_{ab}\langle \alpha,\,\beta,\,s\,|\,\left(a^\dagger a + b^\dagger b\right)|\,\alpha,\,\beta,\,s\rangle_{ab} = 2\sinh^2 s + \alpha^2 + \beta^2$. Then, an increase in any of $\alpha,\ \ \beta$ or/and s certainly raises $\left\langle \hat{n}_{\rm total} \right\rangle$, thus triggering a decrease in the fidelity $\propto t^{\langle \hat{n}_{\rm total} \rangle}$ since $t < 1$. As for the success probability behavior, when t is high enough, a larger number of photons in mode a (b) may cause fewer photons from both modes a and a' (b and b') to come into the photo-detector PD1 (PD2), thus increasing the possibility of no clicks. As a concrete situation, consider two states $|\,01\rangle_{aa'}$ and $|\,11\rangle_{aa'}$ as inputs to BS1. The corresponding outputs will be $t\,|\,01\rangle_{aa'} + {\rm i}r\,|\,10\rangle_{aa'}$ and ${\rm i}tr\sqrt{2}\left(|\,02\rangle_{aa'} + |\,20\rangle_{aa'}\right) + \left(t^2 - r^2\right)|\,11\rangle_{aa'}$. For the first input state the probability of no clicks is $P\left(|\,01\rangle_{aa'}\right) = r^2$, while for the second input state the probability of no clicks is $P\left(|\,11\rangle_{aa'}\right) = 2t^2r^2$. Clearly, $P\left(|\,11\rangle_{aa'}\right) > P\left(|\,01\rangle_{aa'}\right)$ if $t > 1/\sqrt{2} \approx 0.707$.

2.2. Photon addition by downconverters

The Fock states $|\,m\rangle_{a'},\,|\,n\rangle_{b'}$ in the previous scheme are difficult to prepare and detections of no photons are also difficult to observe due to the always present vacuum noise. So, in this subsection we present an alternative way to add photons. Namely, nondegenerate parametric downconverters will be used instead of beam-splitters. The associated schematic setup is sketched in figure 4. Let the squeezing degree of both the downconverters DC2 and DC3 be z which is also assumed real for simplicity. The generation is regarded to be successful when at the same time PD1 registers m photons and PD2 n photons. If this event happens, it projects the states of modes a and b onto

$$\left|\psi_{DC}\right\rangle_{ab} = \frac{\left|\psi'_{DC}\right\rangle_{ab}}{\sqrt{P_{DC}}}, \tag{22}$$

with

$$\left|\psi'_{DC}\right\rangle_{ab} = {}_{a'}\langle m \mid S_{aa'}(z) \mid 0\rangle_{a'} \times {}_{b'}\langle n \mid S_{bb'}(z) \mid 0\rangle_{b'} \mid \alpha, \beta, s\rangle_{ab} \tag{23}$$

and

$$P_{DC} = {}_{ab}\left\langle \psi'_{DC} | \psi'_{DC} \right\rangle_{ab} \tag{24}$$

being the probability of successful generation. Using the decomposition of the two-mode squeezers [19]

$$\begin{aligned} S_{aa'}(z) = & \exp\left[-\tanh(z) a^{\dagger} a'^{\dagger}\right] \\ & \times \exp\left[-\ln(\cosh z)\left(a^{\dagger} a + a' a'^{\dagger}\right)\right] \\ & \times \exp\left[\tanh(z) a a'\right], \end{aligned} \tag{25}$$

$$\begin{aligned} S_{bb'}(z) = & \exp\left[-\tanh(z) b^{\dagger} b'^{\dagger}\right] \\ & \times \exp\left[-\ln(\cosh z)\left(b^{\dagger} b + b' b'^{\dagger}\right)\right] \\ & \times \exp\left[\tanh(z) b b'\right], \end{aligned} \tag{26}$$

we can calculate the matrix elements in equation (23) and obtain

$${}_{a'}\langle m \mid S_{aa'}(z) \mid 0\rangle_{a'} = \frac{(-\tanh z)^m}{\cosh z \sqrt{m!}} a^{\dagger m} (\cosh z)^{-a^{\dagger} a} \tag{27}$$

and

$${}_{b'}\langle n \mid S_{bb'}(z) \mid 0\rangle_{b'} = \frac{(-\tanh z)^n}{\cosh z \sqrt{n!}} b^{\dagger n} (\cosh z)^{-b^{\dagger} b}. \tag{28}$$

Substituting equations (27) and (28) into equation (23) yields

$$\begin{aligned} \left|\psi'_{DC}\right\rangle_{ab} = & \frac{(-\tanh z)^{m+n}}{(\cosh z)^2 \sqrt{m! n!}} a^{\dagger m} (\cosh z)^{-a^{\dagger} a} \\ & \times b^{\dagger n} (\cosh z)^{-b^{\dagger} b} \mid \alpha, \beta, s\rangle_{ab}. \end{aligned} \tag{29}$$

Using again the equalities (18) and (19) for $(\cosh z)^{-a^{\dagger} a}$ and $(\cosh z)^{-b^{\dagger} b}$ we can write P_{DC}, equation (24), explicitly as

$$\begin{aligned} P_{DC} = & \frac{(\sinh z)^{2(m+n)}}{m! n!} \\ & \times \sum_{j=0}^{\infty} \sum_{j'=0}^{\infty} \frac{\left(-\sinh^2 z\right)^{j+j'}}{j! j'!} C_{m+j, n+j'}(\alpha, \beta, s). \end{aligned} \tag{30}$$

and the fidelity $F_{DC} = \left| {}_{ab}\left\langle \psi_{DC} | \alpha, \beta, s, m, n \right\rangle_{ab} \right|^2$ of the generated state (22) with respect to the target state (2) as

$$F_{DC} = \frac{\left| \sum_{j=0}^{\infty} \sum_{j'=0}^{\infty} \frac{(1-\cosh z)^{j+j'}}{j! j'!} C_{m+j, n+j'}(\alpha, \beta, s) \right|^2}{C_{m,n}(\alpha, \beta, s) \sum_{j=0}^{\infty} \sum_{j'=0}^{\infty} \frac{\left(-\sinh^2 z\right)^{j+j'}}{j! j'!} C_{m+j, n+j'}(\alpha, \beta, s)} \tag{31}$$

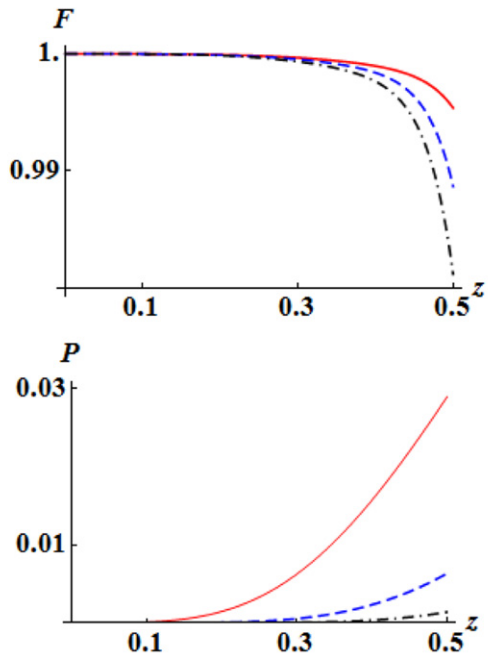

Figure 5. The fidelity $F \equiv F_{DC}$, equation (31), and the corresponding success probability $P \equiv P_{DC}$, equation (30), as functions of squeezing degree z of the two two-mode squeezers for $\alpha = \beta = s = 0.1$ with $\{m, n\} = \{1, 1\}$ (red solid), $\{1, 2\}$ (blue dashed), and $\{2, 2\}$ (black dash-dotted).

As seen from equation (29), the net effect of the downconverter DC2 (DC3) in figure 4, conditioned on the outcome that m (n) photons are detected by the photo-detector PD1 (PD2), is the implementation of the action of $a^{\dagger m} (\cosh z)^{-a^{\dagger} a}$ $(b^{\dagger m} (\cosh z)^{-b^{\dagger} b})$ on mode a (b) of the state $|\alpha, \beta, s\rangle_{ab}$. So, the smaller the value of z is the better $a^{\dagger m} (\cosh z)^{-a^{\dagger} a}$ $(b^{\dagger n} (\cosh z)^{-b^{\dagger} b})$ simulates $a^{\dagger m} (b^{\dagger n})$. This explains why in figures 5 and 6 the fidelity always decreases with increasing z. For $z \to 0$ the fidelity is approaching but never reaches 1 because when $z = 0$ nothing will occur. An increase in the fidelity when z is decreasing is, however, accompanied with a decrease in the corresponding success probability, as shown by the bottom parts of figures 5 and 6. This is due to the properties of two-mode squeezers which are reflected by equations (27) and (28). Furthermore, figure 5 shows that, for given α, β, s and z, both the fidelity and generation probability decrease with increasing m or/and n. This agrees with the case of using beam-splitters in the previous subsection that adding more photons is both worse in quality and more difficult in realization. Finally, similar to the beam-splitter-based scheme, the fidelity decreases but the

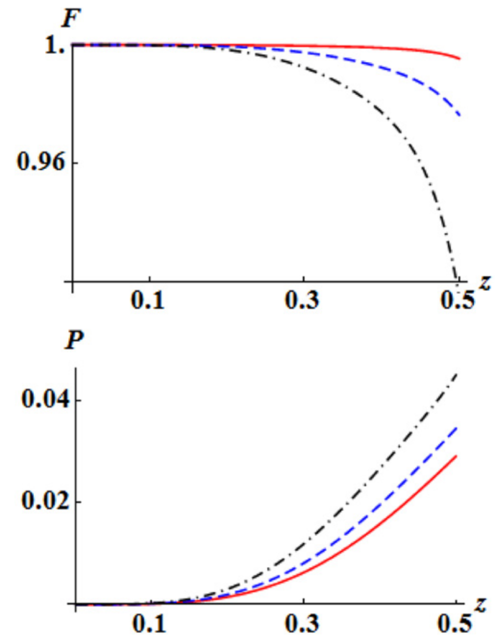

Figure 6. The fidelity $F \equiv F_{DC}$, equation (31), and the corresponding success probability $P \equiv P_{DC}$, equation (30), as functions of squeezing degree z of the two two-mode squeezers for $m = n = 1$ with $\alpha = \beta = s = 0.1$ (red solid), 0.3 (blue dashed), and 0.5 (black dash-dotted).

corresponding success probability increases with increasing α, β, s for given m, n and z, as observed from figure 6.

3. Conclusion

In summary, we have presented possible experimental setups for generation of the two-mode photon-added displaced squeezed state whose properties were investigated in detail previously. The focus is on the techniques of how to add arbitrary numbers of photons to each of the two modes of the displaced squeezed states. We considered two schemes for photon addition: one uses high-transmittivity (high-t) beam-splitters and the other uses small-squeezing degree (small-z) nondegenerate parametric downconverters. For each scheme the fidelity of the generated state with respect to the target one as well as the corresponding success probability were derived analytically and displayed graphically in dependence on all the parameters involved. We show that, though perfect generation (i.e., unit fidelity with unit success probability) is impossible, the fidelity is approaching 1 for t tending to 1 or z tending to 0, at the cost of vanishing success probability. As a consequence, a tradeoff should be made towards achieving sufficiently high fidelity accompanied with reasonably not too low success probability. All the devices used are currently available except the ideal number-resolving photo-detectors. Use of real nonideal photo-detectors with certain efficiency results in mixed states with much lower fidelity. Therefore, additional methods to purify the generated states as well as to improve their fidelity are welcome before one can apply them in quantum information processing. Besides, people are now also interested in photon subtraction [21] which is also a very useful tool to engineer quantum states. Hence, studying two-mode photon-subtracted displaced squeezed states seems to be a good topic for this work to follow. This would also provide a chance to closely compare differences and similarities between adding photons to and subtracting photons from the two-mode displaced squeezed states.

Acknowledgements

T X H N is grateful to Danang University's College of Education, Hue University's College of Education and Hue Center for Theoretical and Computational Physics for encouraging her in pursuing research work.

References

[1] Klauder J R and Sudarshan E C G 1968 *Fundamentals of Quantum Optics* (New York: W A Benjamin)
[2] Einstein A, Rosen N and Podolsky B 1935 *Phys. Rev.* **47** 777
[3] Vogel W and Sperling J 2014 *Phys. Rev.* A **89** 052302
[4] Mandel L and Wolf E 1995 *Optical Coherence and Quantum Optics* (Cambridge: Cambridge University Press)
[5] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)
[6] Schrödinger E 1935 *Naturwissenschaften* **23** 807
[7] Walls D 1983 *Nature* **306** 141
[8] Paul H 1982 *Rev. Mod. Phys.* **54** 1061
[9] An N B 2001 *Phys. Lett.* A **284** 72
[10] Agarwal G S 1986 *Phys. Rev. Lett.* **57** 827
Agarwal G S (ed) 1988 *J. Opt. Soc. Am.* B **5** 1940
[11] An N B and Duc T M 2002 *J. Opt. B: Quantum Semiclass. Opt.* **4** 80
[12] Caves C M 1981 *Phys. Rev.* D **23** 1639
[13] Wang X B, Kwek L C, Liu Y and Oh C H 2001 *J. Phys. B: At. Mol. Opt. Phys.* **34** 1059
[14] Truong M D, Nguyen T X H and Nguyen B A 2014 *Int. J. Theor. Phys.* **53** 899
[15] Zhang S L, Dong Y L, Zou X B, Shi B S and Guo G C 2013 *Phys. Rev.* A **88** 032324
[16] Kim M S 2008 *J. Phys. B: At. Mol. Opt. Phys.* **41** 133001
[17] Agarwal G S and Tara K 1991 *Phys. Rev.* A **43** 492
[18] Zavatta A, Viciani S and Bellini M 2004 *Science* **306** 660
[19] Barnett S M and Radmore P M 1997 *Methods in Theoretical Quantum Optics* (Oxford: Oxford University Press)
[20] Robert A F, Michael M N and Vernon D S 1984 *Phys. Rev.* D **29** 1107
[21] Hong L 1999 *Phys. Lett.* A **264** 265
Chen X Y 2008 *Phys. Lett.* A **372** 2976
Chang P, Shao B and Long G L 2008 *Phys. Lett.* A **372** 7124