Deterministic controlled bidirectional remote state preparation

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Abstract
In this paper we propose a new protocol which allows two distant parties to simultaneously and deterministically exchange their states under control of a third remote party in such a way that it cannot succeed without permission of the controller. The original non-local quantum resource *a priori* shared among the three parties is a quintet of qubits in a so-called linear cluster state.

1. Introduction

One of the most specific and also most important concepts in quantum information sciences is quantum entanglement [1]. With the aid of it, a number of classically impossible tasks can be done quantumly (i.e. in a way based on the postulates of quantum mechanics), such as quantum cryptography [2], quantum dense coding [3], quantum information splitting [4] and so on. In 1993, Bennett *et al* [5] first suggested a quantum method called quantum teleportation to securely and faithfully teleport an unknown quantum state of a single qubit from a sender to a distant receiver by using an Einstein–Podolsky–Rosen pair [6] as the shared quantum channel. Soon after that event, related works appeared widely and achieved great developments in both theoretical [7] and experimental [8] domains. In 2001, Huelga *et al* [9] proposed a modified scheme of quantum teleportation, called quantum remote control, by using bidirectional quantum teleportation method to implement an arbitrary unitary operation upon a distant quantum system. Since then, the method of bidirectional quantum teleportation has been used in many related quantum remote control protocols [10–12]. In a quantum teleportation protocol, one party, Alice, can transmit any unknown quantum state to another party, Bob. In the case where Alice knows her state, the task can be completed by a simpler method called remote state preparation [13, 14]. The principal concern of remote state preparation is to study whether the required

entanglement and classical communication cost can be traded off with respect to quantum teleportation. As we know, in original versions of quantum teleportation and remote state preparation the entanglement is shared only among two parties (Alice and Bob). As an obvious extension, many modified versions have been established using various different non-local resources which are shared among more than two parties to perform global multiparty tasks. For example, Karlsson and Bourennane [15] first proposed a scheme, called controlled quantum teleportation, making use of the so-called three-qubit Greenberger–Horne–Zeilinger state [16] shared among three parties (Alice, Bob and a controller Charlie) as the quantum channel. Afterwards, entangled states other than the Greenberger–Horne–Zeilinger one have also been employed to perform controlled quantum teleportation [17–19]. The merit of controlled quantum teleportation is that Alice and Bob are unable to complete the task without permission of the controller Charlie. Very recently, bidirectional quantum teleportation has been extended to include quantum control [20–22]. Such protocols have been referred to as controlled bidirectional quantum teleportation. In these protocols, by using multi-qubit entangled states, Alice and Bob are able to simultaneously teleport an unknown quantum state to each other under the control of Charlie.

In this work, we devise a protocol called controlled bidirectional remote state preparation which has not been dealt with so far. For the non-local quantum resource shared beforehand among the participants, we use the five-qubit linear cluster state, i.e. the same entangled state as was used in [20, 21]. In order to achieve unit success probability, we follow the adaptive measurement strategy similar to that

in [14]. In section 2, we set up the task and briefly mention the type of cluster states we shall use to fulfill the task. Section 3 presents in detail our protocol which consists of four sequential steps. Finally, we summarize our work in section 4.

2. The task setup and the cluster state

Suppose that Alice has a quantum state

$$|\psi_A\rangle = a\,|0\rangle + b\mathrm{e}^{\mathrm{i}\phi}\,|1\rangle \qquad (1)$$

with $a, b = \sqrt{1-a^2}$ and ϕ being real numbers known to her, while Bob has another quantum state

$$|\psi_B\rangle = x\,|0\rangle + y\mathrm{e}^{\mathrm{i}\varphi}\,|1\rangle \qquad (2)$$

with $x, y = \sqrt{1-x^2}$ and ϕ being real numbers known to him. Alice wishes to remotely prepare $|\psi_A\rangle$ at Bob's lab, Bob wishes to remotely prepare $|\psi_B\rangle$ at Alice's lab, and these are required to be done securely under quantum control of a single third party, Charlie, who need not know any information about $|\psi_A\rangle$ and $|\psi_B\rangle$ but plays a decisive role upon the task completion.

Trivially, of course, this task could be done awkwardly by two independent probabilistic controlled remote state preparation protocols, each consumes one Greenberger–Horne–Zeilinger trio as the quantum channel and one controller, involving in total a non-local resource of six qubits and two separate controllers. Our aim here is to design a single protocol that allows Alice and Bob to exchange their own states simultaneously and deterministically using only a non-local resource of five qubits and only one common controller.

The five qubits we shall use as the original non-local resource in our protocol belong to a special class of multi-particle entangled states, the cluster states, which were introduced in 2001 by Briegel and Raussendorf [23], with the primary purpose of carrying out a new paradigm of quantum computation based on measurements but not on conventional circuits of unitary operations. Cluster states quickly attracted much attention and various schemes for generating them via different physical systems have been proposed [24–27]. A linear cluster state $|C_N\rangle_{12...N}$ of N qubits can be represented compactly in the form

$$|C_N\rangle_{12...N} = \frac{1}{\sqrt{2^N}} \bigotimes_{n=1}^{N} \left(|0\rangle_n + |1\rangle_n\, \sigma_z^{(n+1)}\right), \qquad (3)$$

where $\sigma_z^{(n)}$ is the Z-Pauli matrix acting on qubit n ($\sigma_z^{(n)}|j\rangle_n = (-1)^j|j\rangle_n$, $\forall j \in \{0,1\}$, with the convention $\sigma_z^{(N+1)} \equiv 1$). For $N = 5$,

$$|C_5\rangle_{12345} = \tfrac{1}{2}\,(|00000\rangle + |00111\rangle + |11010\rangle$$
$$+ |11101\rangle)_{12345}, \qquad (4)$$

which is the state we shall use as the shared non-local resource in our protocol to be described in detail in section 3.
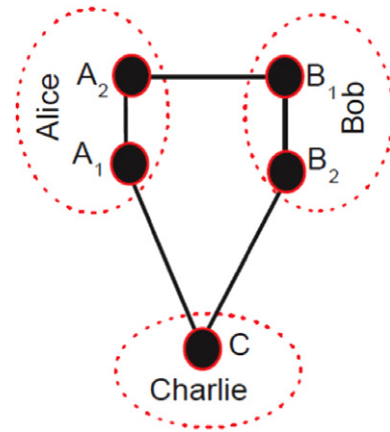

Figure 1. The distribution of qubits of the original cluster state, equation (5), among the three participants Alice, Bob and Charlie, for our controlled bidirectional remote state preparation protocol. Qubits are represented by dots and entangled qubits are connected by solid lines.

3. The protocol

For our convenience we rename the qubits in the state $|C_5\rangle_{12345}$ given in equation (4) as

$$|Q\rangle_{A_1A_2B_1B_2C} = \tfrac{1}{2}\,(|00000\rangle + |01011\rangle + |10101\rangle$$
$$+ |11110\rangle)_{A_1A_2B_1B_2C} \qquad (5)$$

of which, intuitively, both Alice and Bob should hold two qubits and the remaining qubit is held by Charlie. However, the distribution of qubits among the three parties is subtle: a wrong distribution will lead to a failure. After examining all the possibilities we have come up with the following. While qubit C should always be distributed to Charlie, qubits A_1, A_2, B_1 and B_2 can be distributed as follows: either (i) A_1, A_2 to Alice and B_1, B_2 to Bob or (ii) B_1, B_2 to Alice and A_1, A_2 to Bob. Other distributions, say, A_1, B_1 to Alice and A_2, B_2 to Bob, etc do not work. Here we shall work with the qubits' distribution (i) which is sketched in figure 1. Our protocol is composed of four sequential steps.

In the first step, Alice and Bob independently perform the following actions. Alice (Bob) first takes an ancillary qubit $|0\rangle_{A_1'}$ ($|0\rangle_{B_2'}$), then performs a $\mathrm{CNOT}_{A_1A_1'}$ ($\mathrm{CNOT}_{B_2B_2'}$) on qubits A_1 and A_1'. (B_2 and B_2'), where CNOT_{XY} is the controlled-NOT gate acting on two qubits X and Y as $\mathrm{CNOT}_{XY}|i\rangle_X|j\rangle_Y = |i\rangle_X|i \oplus j\rangle_Y$ with $\oplus$ an addition mod 2. After those actions, qubits A_1', B_2' become entangled with those in state (5), i.e. $|Q\rangle\,|j\rangle_{A_1A_2B_1B_2C} \to |Q'\rangle_{A_1A_1'A_2B_1B_2B_2'C}$:

$$|Q'\rangle_{A_1A_1'A_2B_1B_2B_2'C} = \tfrac{1}{2}\,(|0000000\rangle + |0010111\rangle + |1101001\rangle$$
$$+ |1111110\rangle)_{A_1A_1'A_2B_1B_2B_2'C}. \qquad (6)$$

Note that, though $|Q'\rangle_{A_1A_1'A_2B_1B_2B_2'C}$ is a seven-qubit entangled state, the actual non-local resource costs just five qubits because the entanglement of qubits A_1' and B_2' with those in state (5) is made locally.

In the second step, Alice first measures qubit A_1 in the basis $\{|v_k\rangle_{A_1}, k = \{0, 1\}\}$ determined by $\{a, b\}$ as

$$\begin{pmatrix} |v_0\rangle_{A_1} \\ |v_1\rangle_{A_1} \end{pmatrix} = \begin{pmatrix} a & b \\ b & -a \end{pmatrix} \begin{pmatrix} |0\rangle_{A_1} \\ |1\rangle_{A_1} \end{pmatrix}. \qquad (7)$$

Then, if the outcome is k (i.e. $|v_k\rangle_{A_1}$ is found), she measures qubit A_1' in the basis $\{|v_{k'}'\rangle_{A_1'}, k' = \{0, 1\}\}$ determined by $\{k, \varphi\}$ as

$$\begin{pmatrix} |v_0'\rangle_{A_1'} \\ |v_1'\rangle_{A_1'} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & e^{-i\phi} \\ e^{2ik\phi} & -e^{i(2k-1)\phi} \end{pmatrix} \begin{pmatrix} |0\rangle_{A_1'} \\ |1\rangle_{A_1'} \end{pmatrix}. \qquad (8)$$

As for Bob, he first measures qubit B_2 in the basis $\{|u_l\rangle_{B_2}, l = \{0, 1\}\}$ determined by $\{x, y\}$ as

$$\begin{pmatrix} |u_0\rangle_{B_2} \\ |u_1\rangle_{B_2} \end{pmatrix} = \begin{pmatrix} x & y \\ y & -x \end{pmatrix} \begin{pmatrix} |0\rangle_{B_2} \\ |1\rangle_{B_2} \end{pmatrix}. \qquad (9)$$

Then, if the outcome is l (i.e. $|u_l\rangle_{B_2}$ is found), he measures qubit B_2' in the basis $\{|u_{l'}'\rangle_{B_2'}, l' = \{0, 1\}\}$ determined by $\{l, \phi\}$ as

$$\begin{pmatrix} |u_0'\rangle_{B_2'} \\ |u_1'\rangle_{B_2'} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & e^{-i\varphi} \\ e^{2il\varphi} & -e^{i(2l-1)\varphi} \end{pmatrix} \begin{pmatrix} |0\rangle_{B_2'} \\ |1\rangle_{B_2'} \end{pmatrix}. \qquad (10)$$

In terms of the basic states $|v_k\rangle_{A_1}$, $|v_{k'}'\rangle_{A_1'}$, $|u_l\rangle_{B_2}$ and $|u_{l'}'\rangle_{B_2'}$, we can reexpress state (6) as follows:

$$\begin{aligned} &|Q'\rangle_{A_1 A_1' A_2 B_1 B_2 B_2' C} \\ &= \frac{1}{4} \sum_{k,k',l,l'=0}^{1} |v_k\rangle_{A_1} |v_{k'}'\rangle_{A_1'} |u_l\rangle_{B_2} |u_{l'}'\rangle_{B_2} |D_{klk'l'}\rangle_{CA_2 B_1}, \end{aligned} \qquad (11)$$

where

$$\begin{aligned} |D_{0000}\rangle_{CA_2 B_1} &= \big(ax\,|000\rangle + aye^{i\varphi}\,|110\rangle + bxe^{i\phi}\,|101\rangle \\ &\quad + bye^{i(\phi+\varphi)}\,|011\rangle\big)_{CA_2 B_1}, \end{aligned} \qquad (12)$$

$$\begin{aligned} |D_{0001}\rangle_{CA_2 B_1} &= \big(ax\,|000\rangle - aye^{i\varphi}\,|110\rangle + bxe^{i\phi}\,|101\rangle \\ &\quad - bye^{i(\phi+\varphi)}\,|011\rangle\big)_{CA_2 B_1}, \end{aligned} \qquad (13)$$

$$\begin{aligned} |D_{0010}\rangle_{CA_2 B_1} &= \big(ax\,|000\rangle + aye^{i\varphi}\,|110\rangle - bxe^{i\phi}\,|101\rangle \\ &\quad - bye^{i(\phi+\varphi)}\,|011\rangle\big)_{CA_2 B_1}, \end{aligned} \qquad (14)$$

$$\begin{aligned} |D_{0011}\rangle_{CA_2 B_1} &= \big(ax\,|000\rangle - aye^{i\varphi}\,|110\rangle - bxe^{i\phi}\,|101\rangle \\ &\quad + bye^{i(\phi+\varphi)}\,|011\rangle\big)_{CA_2 B_1}, \end{aligned} \qquad (15)$$

$$\begin{aligned} |D_{0100}\rangle_{CA_2 B_1} &= \big(aye^{i\varphi}\,|000\rangle - ax\,|110\rangle + bye^{i(\phi+\varphi)}\,|101\rangle \\ &\quad - bxe^{i\phi}\,|011\rangle\big)_{CA_2 B_1}, \end{aligned} \qquad (16)$$

$$\begin{aligned} |D_{0101}\rangle_{CA_2 B_1} &= \big(-aye^{i\varphi}\,|000\rangle - ax\,|110\rangle + bye^{i(\phi+\varphi)}\,|101\rangle \\ &\quad + bxe^{i\phi}\,|011\rangle\big)_{CA_2 B_1}, \end{aligned} \qquad (17)$$

$$\begin{aligned} |D_{0110}\rangle_{CA_2 B_1} &= \big(aye^{i\varphi}\,|000\rangle - ax\,|110\rangle - bye^{i(\phi+\varphi)}\,|101\rangle \\ &\quad + bxe^{i\phi}\,|011\rangle\big)_{CA_2 B_1}, \end{aligned} \qquad (18)$$

$$\begin{aligned} |D_{0111}\rangle_{CA_2 B_1} &= \big(-aye^{i\varphi}\,|000\rangle - ax\,|110\rangle - bye^{i(\phi+\varphi)}\,|101\rangle \\ &\quad - bxe^{i\phi}\,|011\rangle\big)_{CA_2 B_1}, \end{aligned} \qquad (19)$$

$$\begin{aligned} |D_{1000}\rangle_{CA_2 B_1} &= \big(bxe^{i\phi}\,|000\rangle + bye^{i(\phi+\varphi)}\,|110\rangle - ax\,|101\rangle \\ &\quad - aye^{i\varphi}\,|011\rangle\big)_{CA_2 B_1}, \end{aligned} \qquad (20)$$

$$\begin{aligned} |D_{1001}\rangle_{CA_2 B_1} &= \big(bxe^{i\phi}\,|000\rangle - bye^{i(\phi+\varphi)}\,|110\rangle - ax\,|101\rangle \\ &\quad + aye^{i\varphi}\,|011\rangle\big)_{CA_2 B_1}, \end{aligned} \qquad (21)$$

$$\begin{aligned} |D_{1010}\rangle_{CA_2 B_1} &= \big(-bxe^{i\phi}\,|000\rangle - bye^{i(\phi+\varphi)}\,|110\rangle - ax\,|101\rangle \\ &\quad - aye^{i\varphi}\,|011\rangle\big)_{CA_2 B_1}, \end{aligned} \qquad (22)$$

$$\begin{aligned} |D_{1011}\rangle_{CA_2 B_1} &= \big(-bxe^{i\phi}\,|000\rangle + bye^{i(\phi+\varphi)}\,|110\rangle - ax\,|101\rangle \\ &\quad + aye^{i\varphi}\,|011\rangle\big)_{CA_2 B_1}, \end{aligned} \qquad (23)$$

$$\begin{aligned} |D_{1100}\rangle_{CA_2 B_1} &= \big(bye^{i(\phi+\varphi)}\,|000\rangle - bxe^{i\phi}\,|110\rangle - aye^{i\varphi}\,|101\rangle \\ &\quad + ax\,|011\rangle\big)_{CA_2 B_1}, \end{aligned} \qquad (24)$$

$$\begin{aligned} |D_{1101}\rangle_{CA_2 B_1} &= \big(-bye^{i(\phi+\varphi)}\,|000\rangle - bxe^{i\phi}\,|110\rangle + aye^{i\varphi}\,|101\rangle \\ &\quad + ax\,|011\rangle\big)_{CA_2 B_1}, \end{aligned} \qquad (25)$$

$$\begin{aligned} |D_{1110}\rangle_{CA_2 B_1} &= \big(-bye^{i(\phi+\varphi)}\,|000\rangle + bxe^{i\phi}\,|110\rangle - aye^{i\varphi}\,|101\rangle \\ &\quad + ax\,|011\rangle\big)_{CA_2 B_1} \end{aligned} \qquad (26)$$

and

$$\begin{aligned} |D_{1111}\rangle_{CA_2 B_1} &= \big(bye^{i(\phi+\varphi)}\,|000\rangle + bxe^{i\phi}\,|110\rangle + aye^{i\varphi}\,|101\rangle \\ &\quad + ax\,|011\rangle\big)_{CA_2 B_1}. \end{aligned} \qquad (27)$$

After their measurements, Alice and Bob should let each other know the outcomes kk' and ll'. What is interesting is that it is not necessary for them to send secret massages. Instead, they just need to broadcast their outcomes via any public media since these outcomes in fact mean nothing to any outside parties.

It is worth noting that in the second step Alice and Bob used the adaptive measurement strategy. Namely, the choice of bases for measuring qubits A_1' and B_2' depends essentially on the outcomes of the prior measurements on qubits A_1 and B_2, respectively. Clearly from equation (11), if the outcomes of Alice's and Bob's measurements are $klk'l'$, then the three unmeasured qubits C, A_2 and B_1 are projected onto the state $|D_{klk'l'}\rangle_{CA_2 B_1}$ (see equations (12)–(27)) with an equal probability of 1/16. Note also that at this stage Alice and Bob are still unable to complete the task without Charlie's participation since their qubits A_2 and B_1 are still entangled with qubit C. The role of the controller Charlie will be seen in the next step.

In the third step, Charlie measures qubit C in the basis

$$\begin{pmatrix} |-\rangle_{\mathrm{C}} \\ |+\rangle_{\mathrm{C}} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} |0\rangle_{\mathrm{C}} \\ |1\rangle_{\mathrm{C}} \end{pmatrix}, \qquad (28)$$

then publicly announces the outcome $m = 1$ if qubit C is found in the state $|+\rangle_{\mathrm{C}}$ and $m = 0$ if it is in the state $|-\rangle_{\mathrm{C}}$. Depending on the outcomes $klk'l'm$ of all the measurements described above, the state of qubits A_2 and B_1, with an equal probability of 1/32, collapses into $|\Psi_{klk'l'm}\rangle_{A_2B_1}$, which have the following explicit forms:

$$|\Psi_{00000}\rangle_{A_2B_1} = |\Psi_{00111}\rangle_{A_2B_1}$$
$$= \left(x\,|0\rangle + y\mathrm{e}^{\mathrm{i}\varphi}\,|1\rangle\right)_{A_2} \otimes \left(a\,|0\rangle + b\mathrm{e}^{\mathrm{i}\phi}\,|1\rangle\right)_{B_1}, \qquad (29)$$

$$|\Psi_{00001}\rangle_{A_2B_1} = |\Psi_{00110}\rangle_{A_2B_1}$$
$$= \left(x\,|0\rangle - y\mathrm{e}^{\mathrm{i}\varphi}\,|1\rangle\right)_{A_2} \otimes \left(a\,|0\rangle - b\mathrm{e}^{\mathrm{i}\phi}\,|1\rangle\right)_{B_1}, \qquad (30)$$

$$|\Psi_{00010}\rangle_{A_2B_1} = |\Psi_{00101}\rangle_{A_2B_1}$$
$$= \left(x\,|0\rangle - y\mathrm{e}^{\mathrm{i}\varphi}\,|1\rangle\right)_{A_2} \otimes \left(a\,|0\rangle + b\mathrm{e}^{\mathrm{i}\phi}\,|1\rangle\right)_{B_1}, \qquad (31)$$

$$|\Psi_{00100}\rangle_{A_2B_1} = |\Psi_{00011}\rangle_{A_2B_1}$$
$$= \left(x\,|0\rangle + y\mathrm{e}^{\mathrm{i}\varphi}\,|1\rangle\right)_{A_2} \otimes \left(a\,|0\rangle - b\mathrm{e}^{\mathrm{i}\phi}\,|1\rangle\right)_{B_1}, \qquad (32)$$

$$|\Psi_{01000}\rangle_{A_2B_1} = |\Psi_{01011}\rangle_{A_2B_1}$$
$$= \left(y\mathrm{e}^{\mathrm{i}\varphi}\,|0\rangle - x\,|1\rangle\right)_{A_2} \otimes \left(a\,|0\rangle + b\mathrm{e}^{\mathrm{i}\phi}\,|1\rangle\right)_{B_1}, \qquad (33)$$

$$|\Psi_{01100}\rangle_{A_2B_1} = |\Psi_{01111}\rangle_{A_2B_1}$$
$$= \left(y\mathrm{e}^{\mathrm{i}\varphi}\,|0\rangle - x\,|1\rangle\right)_{A_2} \otimes \left(a\,|0\rangle - b\mathrm{e}^{\mathrm{i}\phi}\,|1\rangle\right)_{B_1}, \qquad (34)$$

$$|\Psi_{01001}\rangle_{A_2B_1} = |\Psi_{01010}\rangle_{A_2B_1}$$
$$= \left(y\mathrm{e}^{\mathrm{i}\varphi}\,|0\rangle + x\,|1\rangle\right)_{A_2} \otimes \left(a\,|0\rangle - b\mathrm{e}^{\mathrm{i}\phi}\,|1\rangle\right)_{B_1}, \qquad (35)$$

$$|\Psi_{01101}\rangle_{A_2B_1} = |\Psi_{01110}\rangle_{A_2B_1}$$
$$= \left(y\mathrm{e}^{\mathrm{i}\varphi}\,|0\rangle + x\,|1\rangle\right)_{A_2} \otimes \left(a\,|0\rangle + b\mathrm{e}^{\mathrm{i}\phi}\,|1\rangle\right)_{B_1}, \qquad (36)$$

$$|\Psi_{10000}\rangle_{A_2B_1} = |\Psi_{10111}\rangle_{A_2B_1}$$
$$= \left(x\,|0\rangle + y\mathrm{e}^{\mathrm{i}\varphi}\,|1\rangle\right)_{A_2} \otimes \left(b\mathrm{e}^{\mathrm{i}\phi}\,|0\rangle - a\,|1\rangle\right)_{B_1}, \qquad (37)$$

$$|\Psi_{10001}\rangle_{A_2B_1} = |\Psi_{10110}\rangle_{A_2B_1}$$
$$= \left(x\,|0\rangle - y\mathrm{e}^{\mathrm{i}\varphi}\,|1\rangle\right)_{A_2} \otimes \left(b\mathrm{e}^{\mathrm{i}\phi}\,|0\rangle + a\,|1\rangle\right)_{B_1}, \qquad (38)$$

$$|\Psi_{10100}\rangle_{A_2B_1} = |\Psi_{10011}\rangle_{A_2B_1}$$
$$= \left(x\,|0\rangle + y\mathrm{e}^{\mathrm{i}\varphi}\,|1\rangle\right)_{A_2} \otimes \left(b\mathrm{e}^{\mathrm{i}\phi}\,|0\rangle + a\,|1\rangle\right)_{B_1}, \qquad (39)$$

$$|\Psi_{10010}\rangle_{A_2B_1} = |\Psi_{10101}\rangle_{A_2B_1}$$
$$= \left(x\,|0\rangle - y\mathrm{e}^{\mathrm{i}\varphi}\,|1\rangle\right)_{A_2} \otimes \left(b\mathrm{e}^{\mathrm{i}\phi}\,|0\rangle - a\,|1\rangle\right)_{B_1}, \qquad (40)$$

$$|\Psi_{11000}\rangle_{A_2B_1} = |\Psi_{11111}\rangle_{A_2B_1}$$
$$= \left(y\mathrm{e}^{\mathrm{i}\varphi}\,|0\rangle - x\,|1\rangle\right)_{A_2} \otimes \left(b\mathrm{e}^{\mathrm{i}\phi}\,|0\rangle - a\,|1\rangle\right)_{B_1}, \qquad (41)$$

$$|\Psi_{11001}\rangle_{A_2B_1} = |\Psi_{11110}\rangle_{A_2B_1}$$
$$= \left(y\mathrm{e}^{\mathrm{i}\varphi}\,|0\rangle + x\,|1\rangle\right)_{A_2} \otimes \left(b\mathrm{e}^{\mathrm{i}\phi}\,|0\rangle + a\,|1\rangle\right)_{B_1}, \qquad (42)$$

$$|\Psi_{11011}\rangle_{A_2B_1} = |\Psi_{11100}\rangle_{A_2B_1}$$
$$= \left(y\mathrm{e}^{\mathrm{i}\varphi}\,|0\rangle - x\,|1\rangle\right)_{A_2} \otimes \left(b\mathrm{e}^{\mathrm{i}\phi}\,|0\rangle + a\,|1\rangle\right)_{B_1},$$

and
$$\qquad (43)$$

$$|\Psi_{11010}\rangle_{A_2B_1} = |\Psi_{11101}\rangle_{A_2B_1}$$
$$= \left(y\mathrm{e}^{\mathrm{i}\varphi}\,|0\rangle + x\,|1\rangle\right)_{A_2} \otimes \left(b\mathrm{e}^{\mathrm{i}\phi}\,|0\rangle - a\,|1\rangle\right)_{B_1}. \qquad (44)$$

As is evident from equations (29)–(44), for any possible collection $klk'l'm$ of outcomes, $|\Psi_{klk'l'm}\rangle_{A_2B_1}$ turns out to be a product state, but it is not yet readily in the desired form $|\psi_A\rangle_{A_2} \otimes |\psi_B\rangle_{B_1}$. So, a final step, the fourth step, is needed for Alice and Bob to locally reconstruct the target state.

In the fourth step, Alice (Bob) should apply a proper unitary operator $R^{\mathrm{A}}_{ll'm}$ ($R^{\mathrm{B}}_{kk'm}$), if it exists, on qubit $A_2(B_1)$ to transform its state to $|\psi_A\rangle_{A_2}(|\psi_B\rangle_{B_1})$. That is, to be successful, Alice (Bob) needs to know not only the outcome of Bob's (Alice's) measurement in the second step, but also the outcome of Charlie's measurement in the third step, certifying the controller's role in our protocol. Should Charlie, by some important reasons, decline to carry out the measurement or to disclose the measurement outcome, the task remains unfulfilled. Carefully analyzing the data in equations (29)–(44), we have, for any possible outcomes $klk'l'm$, come up with the general formulae for $R^{\mathrm{A}}_{ll'm}$ and $R^{\mathrm{B}}_{kk'm}$ as

$$R^{\mathrm{A}}_{ll'm} = \sigma_x^l \sigma_z^{l \oplus l' \oplus m} \qquad (45)$$

and

$$R^{\mathrm{B}}_{kk'm} = \sigma_x^k \sigma_z^{k \oplus k' \oplus m}, \qquad (46)$$

where σ_x is the X-Pauli matrix $(\sigma_x|j\rangle = |j \oplus 1\rangle;\ j = \{0, 1\})$. Since Alice and Bob are always able to reconstruct the desired state by the operators $R^{\mathrm{A}}_{ll'm}$ and $R^{\mathrm{B}}_{kk'm}$ defined above, our controlled bidirectional remote state preparation protocol is deterministic, i.e. the success probability is 1.

4. Conclusion

To summarize, we have put forward an idea of how two distant parties (Alice and Bob) can simultaneously exchange their quantum states securely, deterministically and under the same control (by Charlie) using only local operations and classical

communication. Since each party knows his/her own state, we call our protocol deterministic controlled bidirectional remote state preparation. The local operations involved in the main steps (i.e. from the second step) are simple single-qubit von Neumann measurements. The original quantum channel (5) whose qubits should be *a priori* distributed through space among the participants consists only of five qubits which are here assumed to be provided off-line in the linear cluster state. The actually working quantum channel (6), though it consumes two more auxiliary qubits and two controlled-NOT gates, can be made locally in the preliminary step (i.e. the first step). The reason for extending the original to the working quantum channel is to make room for adaptive measurements as described in the second step, thanks to which unit success probability is achieved. The classical message each of Alice and Bob has to broadcast costs 2 bits, whereas that of Charlie costs just 1 bit, resulting in the total classical communication of 5 bits. Taken altogether, the presented protocol is feasible within the reach of current technologies.

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