Physics Letters A 378 (2014) 2063–2069

Contents lists available at ScienceDirect

Physics Letters A

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Controllable entanglement transfer via two parallel spin chains

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ARTICLE INFO

Article history:
Received 9 May 2014
Accepted 20 May 2014
Available online 4 June 2014
Communicated by V.M. Agranovich

Keywords:
Entanglement transfer
Spin chains
Weak measurements

ABSTRACT

Transferring quantum states between nearby quantum processors is important for building up a powerful quantum computer. In this paper, we propose a controllable scheme to transfer bipartite entangled states using two open-ended spin-$\frac{1}{2}$ chains in parallel as a dual-rail quantum channel. We perform two sets of operations, one on one end of the chains at the beginning of the system evolution and the other on the other end of the chains at the time the transferred entanglement needs to be picked up. Among the operations employed in the scheme there are weak measurements with controllable strengths. By suitably choosing the strengths of these weak measurements, the entanglement transferability is pronouncedly improved, compared to that due to the spin chains' natural dynamics. In principle, the entanglement amount at the receiving site can be made arbitrarily close to that at the sending site, i.e., perfect entanglement transfer could be achieved asymptotically.

1. Introduction

The transfer of quantum states is undoubtedly very important in future's quantum information processing technology [1]. For long-distance quantum communication, photons are the most suited candidates to play the role of quantum information carriers flying from one to another in a faraway spatial location. For example, in quantum key distribution, the photons encoding a secret key via their polarization freedoms can easily travel along long optical fibers or through free space and can then be readily measured at an arriving location. However, in distributed quantum computation [2–4], not only transferring quantum states between quantum computers is important but also interfacing a quantum computer (say, arrays of spins or trapped ions) with optics is necessary. An idea to avoid such interfacing problems is to use the same physical systems for both the quantum computers and the quantum channels. For short distances, it is more suitable to adopt the collective phenomena, such as the natural dynamical evolution, of a permanently coupled chain of quantum systems to connect different nearby quantum processors or registers to build up a powerful quantum computer. In fact, by using a 1D spin chain as the data bus, Bose proposed a quantum state transfer protocol in which an unknown state can be efficiently transferred from one to another spin with certain fidelity via the spin chain's natural evolution [5]. Nevertheless, for a spin chain governed by a uniformly coupled Heisenberg Hamiltonian [5], perfect quantum state transfer is only possible for systems with two or three spins [6]. Subsequently, a number of approaches, such as engineered couplings [7–13], Gaussian wave-packet encoding [14–16], employment of specific pulses [17], weak coupling of the sending and receiving qubits to a quantum many-body system [12,18,19] and so on, have been proposed to achieve perfect or near perfect quantum state transfer. In addition to these strategies, Burgarth and Bose also suggested a dual-rail channel by adding an auxiliary spin chain to improve transfer capability of single-spin states [20–22]. With enough measurements carried out, their protocol will achieve conclusively perfect quantum state transfer with a success probability close to 1. The adding of an additional spin chain is actually not problematic and is even much easier in many experiments [23–25] that realize a whole bunch of parallel uncoupled chains rather than just a single one.

As is well known, entanglement is a key resource to realize various intriguing tasks in quantum information processing and quantum computing [1]. The capability of on-demand transfer of entanglement through spin chains is, of course, significant [26,27]. In particular, it is practically interesting to obtain entanglement between two independent spins at a receiving site through the process of transferring the entanglement as a whole prepared between two spins at a sending site. To achieve this task, the dual-rail channel based on using two parallel spin chains (cf. Fig. 1) is

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http://dx.doi.org/10.1016/j.physleta.2014.05.046
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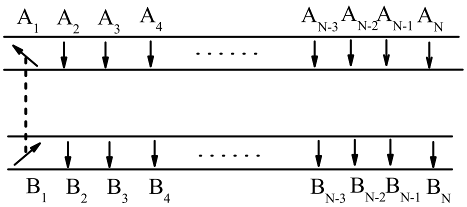

Fig. 1. The schematic setup for entanglement transfer through two parallel spin-$\frac{1}{2}$ chains each contains N spins. An entangled state to be transferred is encoded in spins A_1 and B_1 on each of which the sender performs a weak measurement before the system starts to evolve. Later, at a desired moment of time, the receiver performs a suitable set of operations on spins A_N and B_N to get them in the intended entangled state.

a naturally occurring setting. However, with respect to the issue of entanglement transfer, the answer to the question "Can the entanglement of an arbitrarily prepared bipartite entangled state be perfectly transferred through a pair of parallel spin chains?" is still not known completely. Our study in this work shows that in fact not all the bipartite entangled states can be transferred via two parallel spin chains. More concretely, we find out that there are states of two spins that are initially entangled at a sending site but later become always unentangled at a receiving site (i.e., the two spins at the receiving site remain separable during the entire time evolution). Also, there are states whose entanglement can be transferred to a destination, but during the system's natural evolution their entanglement appears with some delay [28], then suddenly vanishes, and after some time reappears again, etc. Here we propose a controllable scheme that allows us to improve the entanglement transfer in terms of the dual-rail protocol, especially to renew the transferability of those states whose entanglement cannot be transferred by natural evolution. Namely, we find that for bipartite entangled states of the form $\alpha|00\rangle+\beta|11\rangle$ ($|0\rangle\equiv|\downarrow\rangle$: spin-down state, $|1\rangle\equiv|\uparrow\rangle$: spin-up state and $|\alpha|^2+|\beta|^2=1$), a large weight of the $|11\rangle$ component hinders its entanglement transfer. Therefore, in our scheme, we first lower the weight of the $|11\rangle$ component by means of weak measurements [29–39] with strength p on each of the two spins at the sending site. The weak measurement differs from the projective measurement in that the former does not completely collapse the system's measured state. Actually, such kind of measurements has been experimentally realized in several physical contexts [40–45]. Next, we let the system evolve as it should. And, finally, at a desired receiving site, we perform on each spin another weak measurement with strength q. By suitably choosing q we shall be able to transfer entanglement of any bipartite entangled states. In principle, the entanglement degree at the receiving site can be made in our scheme exactly equal to that at the sending site, i.e., perfect entanglement transfer could be achieved.

We structure our paper as follows. After this Introduction, in Section 2 we deal with a solvable model consisting of two parallel open-ended spin-$\frac{1}{2}$ chains each of which is characterized by nearest neighbor interactions and under a common uniform magnetic field. By means of dual-rail encoding, the process of entanglement transfer along the chains is investigated. It is shown that, due to natural evolution, not any entangled states can transfer their entanglement and the entanglement transfer, if it happens, cannot be perfect. Then, in Section 3, we propose a controllable scheme to circumvent such limitations imposed by the system's natural dynamics. By performing appropriate prior and posterior unsharp measurements, the entanglement transferability is considerably enhanced and, in principle, can be made asymptotically perfect. Finally, we conclude in Section 4.

2. Dual-rail transfer of entanglement

Consider, for generality, two 1D spin-$\frac{1}{2}$ graphs A and B, each of which contains N spins. The spins in graph A (B) are labeled $A_1, A_2, \ldots$ and A_N ($B_1, B_2, \ldots$ and B_N). There are no interactions between the graphs so the total Hamiltonian of the system can be written as [20]

$$H=H^{(A)}\otimes I^{(B)}+I^{(A)}\otimes H^{(B)}, \tag{1}$$

where $H^{(S)}$ ($S=A,B$) is the Hamiltonian of spin graph S and $I^{(S)}$ the identity operator. The authors of Refs. [46–48] studied spin rings, so for precise analytical formulation they had to introduce the cyclic boundary conditions which are a good approximation only for rings with a large radius. Here we are interested in linear open-ended spin chains (see Fig. 1), which represent the most natural geometry for an information transfer channel. Assuming the nearest neighbor Heisenberg interactions of equal strength and the common uniform magnetic field h, the Hamiltonians $H^{(A)}$ and $H^{(B)}$ in Eq. (1) are identical in form, i.e., for both $S=A$ and B,

$$H^{(S)}=-\frac{J}{2}\sum_{j=1}^{N-1}\left(\sigma_x^j\sigma_x^{j+1}+\sigma_y^j\sigma_y^{j+1}+\sigma_z^j\sigma_z^{j+1}\right)-h\sum_{j=1}^{N}\sigma_z^j, \tag{2}$$

where $\sigma_{x(y,z)}^j$ are the $x(y,z)$ Pauli matrices for the jth spin and $J>0$ the coupling strength between nearest neighbors.

Let the two-spin entangled state to be transferred has the form

$$\left|\psi(0)\right\rangle_{A_1B_1}=\cos\theta|0\rangle_{A_1}|0\rangle_{B_1}+e^{i\phi}\sin\theta|1\rangle_{A_1}|1\rangle_{B_1}, \tag{3}$$

with $0<\theta<\pi/2$ and $0<\phi<\pi$. Unlike the transfer of single-spin states, the transfer of two-spin entangled states would be more subtle since the entanglement dynamics due to decoherence is very rich and sensitive to the form of the entangled state to be transferred (see, e.g., [28]). Hence, we should consider the whole range of possible values of ϕ and θ to explore the dependence of entanglement transferability on those parameters. The form (3) of the input state means that a dual-rail encoding is adopted: information is encoded in states of the first spin pair A_1B_1 of the two chains. As for the other spins, they are all prepared in the unexcited (i.e., spin-down) state $|0\rangle\equiv|\downarrow\rangle$. As $H^{(S)}$ commutes with $\sum_{j=1}^N\sigma_z^j$, there exists at most one excitation (i.e., one spin-up state) in each chain. For convenience, we denote by $|\mathbf{0}\rangle^{(S)}=|0\ldots0\ldots0\rangle_{S_1\ldots S_j\ldots S_N}$ the state with all the spins being unexcited and by $|\mathbf{j}\rangle^{(S)}=|0\ldots1\ldots0\rangle_{S_1\ldots S_j\ldots S_N}$ ($\mathbf{j}=\mathbf{1},\mathbf{2},\ldots,\mathbf{s},\ldots,\mathbf{r},\ldots,\mathbf{N}$) the state with only spin j being excited. The eigenstates $|\widetilde{m}\rangle^{(S)}$ and eigenenergies $E_m=E_m^{(A)}=E_m^{(B)}$ of the Hamiltonian (2) relevant to our problem can be derived as [5] $|\widetilde{m}\rangle^{(S)}=\{[\sqrt{2}+\delta_{m,1}(1-\sqrt{2})]/\sqrt{N}\}\sum_{j=1}^N\cos[\pi(m-1)(2j-1)/2N]|\mathbf{j}\rangle^{(S)}$ and $E_m=2h+2J\{1-\cos[\pi(m-1)/N]\}$, with $m=1,2,\ldots,N$. Since the two parallel spin chains do not have any direct interactions, the excitation transfer in each chain can be dealt with independently. In this case, the transition amplitude of an excitation from a sth to an rth site in each chain takes the same form as

$$\begin{aligned}c_{sr}^{(S)}(t)&={}^{(S)}\langle\mathbf{r}|e^{-iH^{(S)}t}|\mathbf{s}\rangle^{(S)}\\&=\sum_{m=1}^{N}{}^{(S)}\langle\mathbf{r}|\widetilde{m}\rangle^{(S)(S)}\langle\widetilde{m}|\mathbf{s}\rangle^{(S)}e^{-iE_mt}.\end{aligned} \tag{4}$$

As the values of $c_{sr}^{(A)}(t)$ and $c_{sr}^{(B)}(t)$ of the two identical chains are the same for all possible s and r, we ignore their superscripts (A) and (B) throughout the paper. For concreteness, we set $s=1$ and $r=N$ in the following (i.e., the sender and the receiver are located near the opposite ends of the chains).

In terms of the aforementioned notations $|\mathbf{0}\rangle^{(S)}$ and $|\mathbf{j}\rangle^{(S)}$, the initial state of the whole system reads

$$\begin{aligned}\big|\Psi(0)\big\rangle &\equiv \big|\Psi(0)\big\rangle^{(A)(B)}\\ &= \cos\theta|\mathbf{0}\rangle^{(A)}|\mathbf{0}\rangle^{(B)} + e^{i\phi}\sin\theta|\mathbf{1}\rangle^{(A)}|\mathbf{1}\rangle^{(B)},\end{aligned} \tag{5}$$

which at time $t > 0$ evolves to

$$\begin{aligned}\big|\Psi(t)\big\rangle &\equiv \big|\Psi(t)\big\rangle^{(A)(B)}\\ &= \cos\theta|\mathbf{0}\rangle^{(A)}|\mathbf{0}\rangle^{(B)}\\ &\quad + e^{i\phi}\sin\theta\sum_{j=1}^{N}\sum_{k=1}^{N}c_{1j}(t)c_{1k}(t)|\mathbf{j}\rangle^{(A)}|\mathbf{k}\rangle^{(B)}.\end{aligned} \tag{6}$$

The reduced density operator of the Nth spin pair can be obtained from Eq. (6) as

$$\begin{aligned}\rho_{A_NB_N}(t) &= \rho_{00,00}(t)|00\rangle_{A_NB_N}\langle 00| + \rho_{01,01}(t)|01\rangle_{A_NB_N}\langle 01|\\ &\quad + \rho_{10,10}(t)|10\rangle_{A_NB_N}\langle 10| + \rho_{11,11}(t)|11\rangle_{A_NB_N}\langle 11|\\ &\quad + \rho_{00,11}(t)|00\rangle_{A_NB_N}\langle 11| + \rho_{11,00}(t)|11\rangle_{A_NB_N}\langle 00|,\end{aligned} \tag{7}$$

where

$$\rho_{00,00}(t) = \cos^2\theta + \sin^2\theta\big(1 - \big|c_{1N}(t)\big|^2\big)^2, \tag{8}$$

$$\rho_{01,01}(t) = \rho_{10,10}(t) = \sin^2\theta\big|c_{1N}(t)\big|^2\big(1 - \big|c_{1N}(t)\big|^2\big), \tag{9}$$

$$\rho_{11,11}(t) = \sin^2\theta\big|c_{1N}(t)\big|^4 \tag{10}$$

and

$$\rho_{11,00}(t) = \rho^*_{00,11}(t) = \sin\theta\cos\theta\, e^{i\phi}c^2_{1N}(t). \tag{11}$$

In deriving the above expressions we have made use of the equality $\sum_{j=1}^{N}|c_{1j}(t)|^2 = 1$.

We first analyze the closeness of the transferred state $\rho_{A_NB_N}(t)$ to the initial state $|\psi(0)\rangle_{A_1B_1}\langle\psi(0)|$ by the fidelity $F(t)$ which is defined as $F(t) = {}_{A_1B_1}\langle\psi(0)|\rho_{A_NB_N}(t)|\psi(0)\rangle_{A_1B_1}$. By virtue of Eqs. (3) and (7), we have obtained

$$F(t) = \cos^2\theta + \big|c_{1N}(t)\big|^4\sin^2\theta, \tag{12}$$

when the magnetic field was chosen appropriately as in Ref. [5] to ensure a maximal fidelity. Obviously, the fidelity $F(t)$ depends not only on the excitation transition probability $|c_{1N}(t)|^2$ but also on the initial state in terms of θ. To assess the mean quality of the state transfer process we then average $F(t)$ over all the possible values of θ and ϕ to obtain the averaged fidelity $F_{\text{av}}(t)$ in the form

$$F_{\text{av}}(t) = \frac{1}{2}\big(1 + \big|c_{1N}(t)\big|^4\big), \tag{13}$$

which remains of course a function of the length N of the spin chains. In Fig. 2 we plot the maximum averaged fidelity $F^{\text{Max}}_{\text{av}}$ versus N within a certain time interval. Since during the system's natural evolution the maximal value of $|c_{1N}(t)|^2$ (and so of $F^{\text{Max}}_{\text{av}}$) can be reached at different times for different N, we have chosen a quite long time interval of $t \in [0, 1000/J]$. As seen from the figure, the quantum state transfer is near perfect for $N = 4$ in which case $F^{\text{Max}}_{\text{av}} \approx 0.9997$, pretty good for $N = 5, 7, 8$ in which case $F^{\text{Max}}_{\text{av}} > 0.9$ and not so good for other values of N.

However, since fidelity tells nothing about inseparability of the transferred state, to evaluate the performance of entanglement transfer through the two spin chains we need another figure of merit, the one that measures the amount of entanglement at any

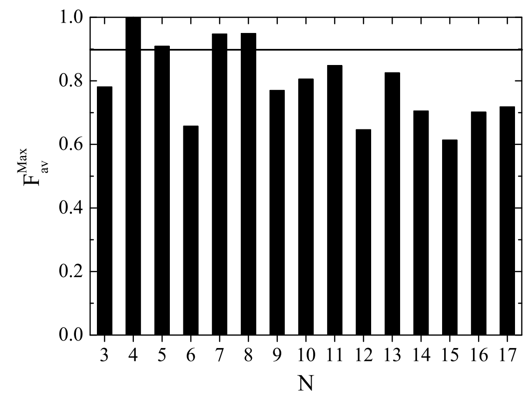

Fig. 2. The maximum averaged fidelity $F^{\text{Max}}_{\text{av}}$ achieved within a time interval $[0, 1000/J]$ as a function of the chain length N. The horizontal line at 0.9 is just to guide the eye.

site. In what follows, as such a figure of merit, we shall use concurrence [49], which for the spins at the sending site is $C_{A_1B_1}(0) = \sin(2\theta)$ and for the spins at the receiving site is

$$C_{A_NB_N}(t) = 2\sin\theta\big|c_{1N}(t)\big|^2\max\big\{0, \widetilde{C}_{A_NB_N}(t)\big\} \tag{14}$$

with

$$\widetilde{C}_{A_NB_N}(t) = \cos\theta - \big(1 - \big|c_{1N}(t)\big|^2\big)\sin\theta. \tag{15}$$

Clearly, the possession of entanglement at site N is determined by the sign of $\widetilde{C}_{A_NB_N}(t)$: the spin pair A_NB_N is entangled if $\widetilde{C}_{A_NB_N}(t) > 0$, otherwise it is unentangled. We now show that character of the dual-rail transfer of entanglement is qualitatively distinguished for three different classes of the input states of the form (3). Analytically, $\widetilde{C}_{A_NB_N}(t) > 0$ iff

$$\cot\theta > 1 - \big|c_{1N}(t)\big|^2. \tag{16}$$

Since $|c_{1N}(t)|^2$ is a transition probability, it must satisfy the inequalities $0 < |c_{1N}(t)|^2 < 1$. The RHS of Eq. (16) is therefore also bounded between zero and one. As for the LHS of Eq. (16), it decreases with increasing θ and is evidently not smaller than one for $0 < \theta \le \pi/4$. This implies that $\widetilde{C}_{A_NB_N}(t)$ is always positive for $\theta \in (0, \pi/4]$. However, the behavior of $\widetilde{C}_{A_NB_N}(t)$ is delicate for $\theta > \pi/4$. The transition probability $|c_{1N}(t)|^2$ itself oscillates with time as the system is evolving. Denoting by $|c^{\max}_{1N}(t)|^2$ the maximum value of $|c_{1N}(t)|^2$ during the time evolution, it can be verified that $\widetilde{C}_{A_NB_N}(t)$ is always non-positive for $\theta \ge \theta_{\max}$ with

$$\theta_{\max} = \arctan\left(\frac{1}{1 - |c^{\max}_{1N}(t)|^2}\right). \tag{17}$$

Finally, for θ lying in between $\pi/4$ and $\theta_{\max}$, $\widetilde{C}_{A_NB_N}(t)$ may be positive within some time intervals sandwiched between other time intervals within which $\widetilde{C}_{A_NB_N}(t)$ turns out to be non-positive.

Physically, the above analysis indicates that the dual-rail transfer of entanglement is state-dependent. For those input states (of the form (3)) that have θ as small as $0 < \theta \le \pi/4$ some amount of entanglement is always transferred from the first to the last end of the spin chains. Nevertheless, for those states that have θ as large as $\theta_{\max} \le \theta < \pi/2$ the entanglement transfer does not occur at all. This property can be interpreted as strong vulnerability of the large-weight $|11\rangle$ component of state (3) due to decoherence brought about by the two spin chains which act as efficient amplitude damping quantum channels. For intermediate values of θ, such as $\pi/4 < \theta < \theta_{\max}$, the entanglement transfer is somewhat delayed at first, then suddenly becomes active for some period of time, then suddenly ceases the action for another period of time

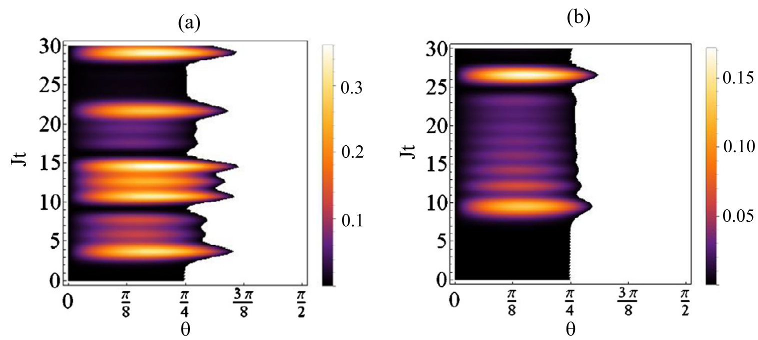

Fig. 3. Density plot of the concurrence of the spin pair $A_N B_N$ as a function of θ and Jt when (a) $N = 6$ for which $\theta_{\max} \approx \frac{2.94\pi}{8}$ and (b) $N = 17$ for which $\theta_{\max} \approx \frac{2.49\pi}{8}$. In both (a) and (b) there is no entanglement at all in the region with $\theta > \theta_{\max}$. Note that the values of $\theta_{\max}$ in this figure were derived only within the time interval $[0, 30/J]$.

and so on. In other words, within this intermediate domain of θ, the spin pair $A_N B_N$ undergoes alternating entanglement sudden birth and entanglement sudden death during the course of time. All the above-mentioned features of the dual-rail transfer of entanglement through two parallel spin chains can be visualized in Fig. 3 where we plot the concurrence $C_{A_N B_N}(t)$ versus Jt and θ for a couple of fixed values of N.

3. Controllable scheme

As learned from the previous section, not all the two-spin entangled states can transfer their entanglement from the first to the last spin pair by virtue of the system's natural evolution. Moreover, because of intrinsic amplitude damping effect along the spin chains, even in case entanglement is transferable the degree of entanglement at the receiving site is always lower than that at the sending site, i.e., the entanglement transfer is always imperfect.

To enhance the transfer process of entanglement [27,46] or quantum correlations [48] a so-called phase-shift control via Aharonov–Casher effect [50] or Dzyaloshinskii–Moriya interaction [51] was employed for the spin rings, which, however, does not apply to the open-ended chains considered here. In this section, we propose another strategy to control the evolution so that the entanglement transfer is improved considerably. Our scheme in principle allows transferring 100% amount of entanglement from one end to the other end of the spin chains. Of interest is the fact that using our scheme all the entangled states of the form (3) (including those with $\theta > \theta_{\max}$) could perfectly transfer their entanglement down the spin chains. Concretely, we shall perform two sets of operations, one at the sending site before the system's evolution starts and the other at the receiving site after some desired period of time during the evolution. The first set of operations comprises measurements that aim at reducing the vulnerability of the input state to the decoherence caused by the subsequent evolution. As for the second set of operations, we could design it so as to recover the input state and thus the transferred amount of entanglement would be perfect.

Let us now go into detail. At time $t = 0$ we perform on each of the spins A_1 and B_1 a weak measurement $\mathcal{W}_S(p)$ ($S = A_1$ or B_1) with strength p ($0 \le p < 1$). Such a weak measurement can be implemented by watching an outside detector which indirectly measures the spin state. If the detector produces a click, the measured state is irreversibly destroyed, but if there are no clicks (null outcome), the measured state is only partially collapsed and could be recovered to the original state with some probability. In essence, null-outcome weak measurements are fuzzy ones and it is their fuzziness what we exploit to improve the entanglement transfer in a controllable fashion. Mathematically, a weak measurement with strength p corresponds to the map

$$\mathcal{W}_S(p)|n\rangle_S \to (1-p)^{n/2}|n\rangle_S, \tag{18}$$

where $|n\rangle_S$ ($n = 0, 1$) is a basic state of spin S. Consider again the state (3) of the first spin pair and we wish to transfer the entanglement contained in that state to the Nth spin pair as perfectly as possible. First, before the system starts to evolve, we perform on each of the spins A_1 and B_1 a weak measurement $\mathcal{W}_S(p)$. The measurements are regarded as successful when we obtain the null outcomes in which case the measured state (3) becomes

$$\begin{aligned} \big|\psi(0,p)\big\rangle &= \mathcal{W}_{A_1}(p)\mathcal{W}_{B_1}(p)\big|\psi(0)\big\rangle_{A_1B_1} \\ &= \frac{1}{\sqrt{P_1(0,p)}}\big[\cos\theta|00\rangle + (1-p)e^{i\phi}\sin\theta|11\rangle\big]_{A_1B_1}, \end{aligned} \tag{19}$$

with

$$P_1(0,p) = \cos^2\theta + (1-p)^2\sin^2\theta \tag{20}$$

the success probability. Note that the measurement strength p is controllable through adjusting the time of detecting the spin state. Therefore, the state of the whole system right after the weak measurements with null outcomes can be written in the form

$$\begin{aligned} \big|\Psi(0,p)\big\rangle = \frac{1}{\sqrt{P_1(0,p)}}\big[&\cos\theta|\mathbf{0}\rangle^{(A)}|\mathbf{0}\rangle^{(B)} \\ &+ (1-p)e^{i\phi}\sin\theta|\mathbf{1}\rangle^{(A)}|\mathbf{1}\rangle^{(B)}\big], \end{aligned} \tag{21}$$

which then starts their course of evolution. Governed by the Hamiltonian (1), the state (21) evolves at time $t > 0$ to

$$\begin{aligned} \big|\Psi(t,p)\big\rangle = \frac{1}{\sqrt{P_1(0,p)}}\Bigg[&\cos\theta|\mathbf{0}\rangle^{(A)}|\mathbf{0}\rangle^{(B)} \\ &+ (1-p)e^{i\phi}\sin\theta\sum_{\mathbf{j}=1}^{\mathbf{N}}\sum_{\mathbf{k}=1}^{\mathbf{N}} c_{1j}(t)c_{1k}(t)|\mathbf{j}\rangle^{(A)}|\mathbf{k}\rangle^{(B)}\Bigg], \end{aligned} \tag{22}$$

with $c_{1n}(t) = |c_{1n}(t)|e^{i\varphi_n(t)}$ the transition amplitudes which are model-dependent. Suppose that after some time t we need to use the Nth spin pair the entanglement amount of which we want to be as much as possible to that of the first spin pair. For that purpose we perform at the desired time t the second set of operations as follows. We first bit-flip both spins A_N and B_N, then carry out on each of them a null-outcome weak measurement with strength q, and finally bit-flip them again. Such operations transform state (22) to

$$\begin{aligned}
|\Psi(t,p,q)\rangle &= \bigotimes_{S=A_N,B_N} \mathrm{NOT}_S \mathcal{W}_S(q)\, \mathrm{NOT}_S |\Psi(t,p)\rangle \\
&= \frac{1}{\sqrt{P_2(t,p,q)}} \Bigg[(1-q)\cos\theta |\mathbf{0}\rangle^{(A)} |\mathbf{0}\rangle^{(B)} \\
&\quad + (1-p)e^{i\phi}\sin\theta \\
&\quad \times \left(c_{1N}(t)|\mathbf{N}\rangle^{(A)} + \sqrt{1-q}\sum_{\mathbf{j}=1}^{\mathbf{N-1}} c_{1j}(t)|\mathbf{j}\rangle^{(A)} \right) \\
&\quad \times \left(c_{1N}(t)|\mathbf{N}\rangle^{(B)} + \sqrt{1-q}\sum_{\mathbf{k}=1}^{\mathbf{N-1}} c_{1k}(t)|\mathbf{k}\rangle^{(B)} \right) \Bigg],
\end{aligned} \tag{23}$$

where $\mathrm{NOT}_S|n\rangle_S = |1-n\rangle_S$ and

$$\begin{aligned}
P_2(t,p,q) &= (1-q)^2\cos^2\theta \\
&\quad + (1-p)^2\big[1 - q\big(1 - |c_{1N}(t)|^2\big)\big]^2 \sin^2\theta
\end{aligned} \tag{24}$$

the total success probability (i.e., the probability of obtaining the null-outcomes by both the weak measurements implemented at $t=0$ and $t>0$). The pragmatic benefit of our scheme is the ability in choosing the strengths of each of the weak measurements. Given the strength p of the first weak measurements and the time t the system has evolved, we are able to determine the strength q of the second weak measurements in order to improve the entanglement transferability. If we choose q to satisfy the condition

$$q = q_r = 1 - (1-p)|c_{1N}(t)|^2, \tag{25}$$

then the state (23) will have the form

$$\begin{aligned}
|\Psi_r(t,p)\rangle &\equiv |\Psi(t,p,q_r)\rangle \\
&= \frac{1}{\sqrt{P_2^r(t,p)}} \Bigg\{ (1-p)|c_{1N}(t)|^2 |\Phi\rangle^{(A)(B)} \\
&\quad + \Bigg[(1-p)^{3/2} c_{1N}(t) \sum_{\mathbf{k}=1}^{\mathbf{N-1}} c_{1k}(t)\big(|\mathbf{N}\rangle^{(A)}|\mathbf{k}\rangle^{(B)} \\
&\quad + |\mathbf{k}\rangle^{(A)}|\mathbf{N}\rangle^{(B)}\big) + (1-p)^2|c_{1N}(t)| \\
&\quad \times \sum_{\mathbf{j},\mathbf{k}=1}^{\mathbf{N-1}} c_{1j}(t)c_{1k}(t)|\mathbf{j}\rangle^{(A)}|\mathbf{k}\rangle^{(B)} \Bigg] \\
&\quad \times e^{i\phi}\sin\theta |c_{1N}(t)| \Bigg\},
\end{aligned} \tag{26}$$

where

$$|\Phi\rangle^{(A)(B)} = \cos\theta|\mathbf{0}\rangle^{(A)}|\mathbf{0}\rangle^{(B)} + e^{i[\phi+2\varphi_N(t)]}\sin\theta|\mathbf{N}\rangle^{(A)}|\mathbf{N}\rangle^{(B)} \tag{27}$$

and

$$\begin{aligned}
P_2^r(t,p) &\equiv P_2(t,p,q_r) \\
&= (1-p)^2|c_{1N}(t)|^4\big[1-(1-p)\big(1-|c_{1N}(t)|^2\big)^2\sin^2\theta\big].
\end{aligned} \tag{28}$$

It is important to observe from the RHS of Eq. (26) that the first term scales as $(1-p)$ while the second and the third ones as $(1-p)^{3/2}$, and the fourth one as $(1-p)^2$. Therefore, in the limit of $p\to 1$ only the first term contributes, i.e., $|\Psi_r(t,p)\rangle \to |\Phi\rangle^{(A)(B)}$. Observe also that, compared with the input state (5), state $|\Phi\rangle^{(A)(B)}$ has an additional relative phase $2\varphi_N(t)$ which is brought in during the transfer process. We can, however, phase-shift each spin of the Nth pair by the unitary operator $\mathcal{P}_S(-\varphi_N(t)) = \{\{1,0\},\{0,e^{-i\varphi_N(t)}\}\}$ to remove such a discrepancy in the relative phase. Thus we can in principle make the state of the Nth spin pair arbitrarily close to the input state of the first pair (i.e., perfect transfer of the two-spin state (3) from the first to the last spin pair can be achieved), by approaching p close enough to 1. It is worthy to note at this point that the entanglement amount contained in state $|\Phi\rangle^{(A)(B)}$ does not depend on the relative phase, so the above-mentioned phase-shift operations can be skipped if our concern is only with the process of entanglement transfer.

Based on the formulae obtained above when the two set of operations have been applied, we now analyze how the transfer of entanglement is improved by our scheme as p increases. Concerning the transfer of entanglement of the state (3), we have known in the previous section that by natural evolution the transfer character depends on the state's parameters. That is, for $0<\theta\le\pi/4$ some amount of entanglement can always be transferred, though the transfer is not perfect (i.e., the entanglement amount at the receiving site is always smaller than that at the sending site), for $\pi/4<\theta<\theta_{\max}$ the entanglement of Nth spin pair undergoes a sequence of alternating sudden birth and sudden death, and for $\theta_{\max}\le\theta<\pi/2$ the entanglement transfer does not occur at all. Here, we shall show that by our scheme all those three situations can be improved. The concurrence $C_{A_NB_N}(t)$, Eq. (14), of the spin pair A_NB_N at time $t>0$ due to natural evolution now becomes

$$C^r_{A_NB_N}(t,p) = 2\sin\theta\max\big\{0, \widetilde{C}^r_{A_NB_N}(t,p)\big\}, \tag{29}$$

with

$$\widetilde{C}^r_{A_NB_N}(t,p) = \frac{\cos\theta - (1-p)(1-|c_{1N}(t)|^2)\sin\theta}{1-(1-p)(1-|c_{1N}(t)|^2)^2\sin^2\theta}. \tag{30}$$

Transparently, in the ideal case of $p\to 1$, the concurrence $C^r_{A_NB_N}(t,p\to 1)$ is approaching $\sin(2\theta) = C_{A_1B_1}(0)$, that happens independently of the value of θ and of the chain's length N. This means that perfect entanglement transfer could in principle be achieved at any desired time t, for whatever the parameters of the input state (3) and the number of spins in the parallel chains. This also implies that the approaching of $C^r_{A_NB_N}(t,p\to 1)$ to $C_{A_1B_1}(0)$ remains in theory valid in the thermodynamic limit $N\to\infty$. Another salient pragmatic advantage of our scheme is that, for values of $\theta>\pi/4$, for which due to natural evolution the entanglement of A_NB_N may suffer alternating sudden births and sudden deaths or may not exist at all, we are still able to manipulate the system's dynamics so that entanglement is always transferable (even for $\theta\ge\theta_{\max}$). At that aim we have to perform the first weak measurements with a strength p larger than a minimum value $p_{\min}$, which is determined from Eqs. (29) and (30) as

$$p_{\min} = 1 - \frac{\cot\theta}{1-|c^{\min}_{1N}(t)|^2}, \tag{31}$$

where $|c^{\min}_{1N}(t)|^2$ is the minimum excitation transition probability from the first to the Nth spin within the evolution time. Generally, $|c^{\min}_{1N}(t)|^2\approx 0$ regardless of N, hence $p_{\min}\approx 1-\cot\theta$, i.e., $p_{\min}$ increases with increasing θ, as could be expected. For instance, if $\theta = 5\pi/16\approx 0.98$, then $p_{\min}\approx 0.33$ and if $\theta = 3\pi/8\approx 1.18$, then $p_{\min}\approx 0.59$.

Fig. 4(a) displays the time evolution of the improved concurrence of the spin pair A_NB_N in dependence on p for $N=6$ and $\theta=\pi/4$. Evidently, at any given moment t during the time evolution the concurrence $C^r_{A_NB_N}(t,p)$ increases, i.e., the entanglement transfer is improved, with increasing p. Fig. 4(b) is similar to Fig. 4(a), but for $\theta = 5\pi/16\approx 0.98<\theta_{\max}\approx 1.16$. Without controlling (i.e., just by natural evolution) the entanglement of the spin

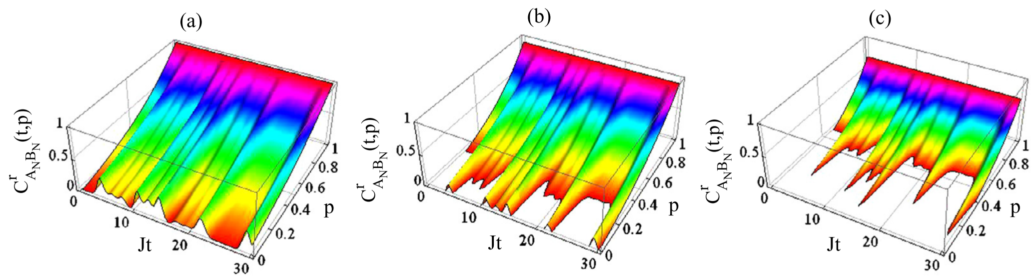

Fig. 4. Time evolution of the improved concurrence $C^r_{A_N B_N}(t,p)$, Eq. (29), with $N = 6$ in dependence of p under the condition (25) for (a) $\theta = \pi/4$, (b) $\theta = 5\pi/16$ and (c) $\theta = 3\pi/8$.

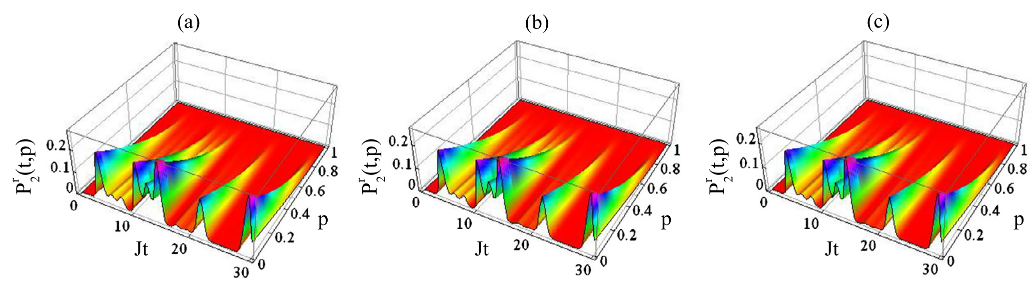

Fig. 5. Time evolution of the success probability $P^r_2(t,p)$, Eq. (28), with $N = 6$ in dependence of p under the condition (25) for (a) $\theta = \pi/4$, (b) $\theta = 5\pi/16$ and (c) $\theta = 3\pi/8$.

pair $A_N B_N$ suffers from a delayed appearance followed by blank periods in the course of evolution. However, with proper controlling, these weaknesses can be circumvented for $p > p_{\min} \approx 0.33$, as seen from Fig. 4(b). Most interesting result is shown in Fig. 4(c) for the situation when $\theta = 3\pi/8 \approx 1.18 > \theta_{\max} \approx 1.16$. In this situation entanglement transfer is absolutely impossible by natural evolution, but turns out to be absolutely possible in our scheme if p is chosen so that $p > p_{\min} \approx 0.59$, as visualized in Fig. 4(c). Although the advantage of our scheme in enhancing the entanglement transferability is pronounced, it is not for free. The price to pay is that our scheme is probabilistic. Generally, a higher entanglement transferability, which corresponds to a larger value of p, is accompanied by a lower success probability, as shown in Fig. 5 for $P^r_2(t,p)$, Eq. (28), versus Jt and p. For a definite p the success probabilities also vary with evolution time and there exist time windows within which the probabilities are very small. Therefore, in order to achieve a reasonably high success probability it is important to choose the time to implement the second set of operations on the Nth spin pair. We also note that the success probabilities are not sensitive to the input states at the sending site in terms of θ, as seen from Fig. 5, where the shapes of $P^r_2(t,p)$ are quite similar for different values of θ.

4. Conclusion

In this work, we have studied the transfer of entanglement contained in entangled states of the form $|\psi\rangle = \cos\theta|0\rangle|0\rangle + e^{i\phi}\sin\theta|1\rangle|1\rangle$, Eq. (3), through two identical open-ended spin-$\frac{1}{2}$ chains with nearest neighbor interactions within each chain. The state $|\psi\rangle$ is encoded by a sender in the two leftmost spins of the chains and will be retrieved by a receiver at the two rightmost spins of the chains. The desired target is that the entanglement amount contained in the receiver's state would be as close as possible to that contained in the sender's state. We examined the entanglement transferability due to the system's natural dynamics and found out that for θ in between $\pi/4$ and some $\theta_{\max}$ entanglement transfer is not always possible, for $\theta \geq \theta_{\max}$ no entanglement at all can be transferred, and only for $0 < \theta \leq \pi/4$ the transfer occurs but not perfectly. To enhance the entanglement transferability we keep making use of the system's dynamics but manage it by performing null-outcome weak measurements with strength p on the sender's state before the evolution starts and other null-outcome weak measurements with strength q on the receiver's state at a later suitable time. By suitably choosing q, conditioned on p and the time the state is retrieved by the sender, our scheme allows to make entanglement always transferable for any value of θ, provided that p is chosen to be greater than some $p_{\min}$ whose value depends on θ. In general, the amount of entanglement of the receiver's state is pronouncedly improved by our scheme in comparison to that without any controls. In particular, perfect entanglement transfer could be, in principle, achieved asymptotically for any ϕ, θ and N.

Acknowledgements

In this work Z.X.M. and Y.J.X. are supported by National Natural Science Foundation of China under Grants Nos. 11204156, 61178012 and 11304179, and Scientific Research Foundation for Outstanding Young Scientists of Shandong Province under project No. BS2013DX034, while N.B.A. is supported by the Vietnam National Foundation for Science and Technology Development (NAFOSTED) via project No. 103.01-2014.02 and J.K. by the IT R&D program of MOTIE/KEIT No. 0043464 (2012).

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