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Universal scheme for finite-probability perfect transfer of arbitrary multispin states through spin chains

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HIGHLIGHTS

- Scheme that can achieve perfect quantum state transfer is devised.
- The scheme is state-independent and applicable to any spin-interaction models.
- The scheme allows perfect transfer of arbitrary multispin states.
- Applications to two typical models are considered in detail.

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ABSTRACT

In combination with the theories of open system and quantum recovering measurement, we propose a quantum state transfer scheme using spin chains by performing two sequential operations: a projective measurement on the spins of 'environment' followed by suitably designed quantum recovering measurements on the spins of interest. The scheme allows perfect transfer of arbitrary multispin states through multiple parallel spin chains with finite probability. Our scheme is universal in the sense that it is state-independent and applicable to any model possessing spin–spin interactions. We also present possible methods to implement the

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required measurements taking into account the current experimental technologies. As applications, we consider two typical models for which the probabilities of perfect state transfer are found to be reasonably high at optimally chosen moments during the time evolution.

1. Introduction

Efficient and on-demand transfer of quantum states from one location to another is an essential prerequisite in processing quantum information. For long-distance quantum communication and quantum cryptography photons are the best carriers that can freely fly back and forth between different remote locations. However, photons turn out not suited for distributed quantum computation when nearby quantum processors need to be connected to build up a powerful quantum computer because one faces challenging problems associated with mapping states of the processor qubits to states of photons at the sending location and vice versa at the receiving location [1–3]. In an on-chip architecture quantum computer, short-distance quantum state transfer from one processor to another can preferably be realized by the natural dynamical evolution of permanently coupled spins in spin chains that connect the two processors, circumventing the problems of interfacing with the flying photons as encountered in long-distance quantum communication.

The first quantum state transfer mechanism using a spin chain was proposed by Bose [4] in which an unknown quantum state encoded in a sending spin can be transferred to a desired receiving spin by means of natural evolution of the spins in the chain. However, due to the amplitude damping effect caused by the spin chain, perfect quantum state transfer cannot be achieved for arbitrary length of the spin chain [5]. In the last few years, a number of approaches, such as engineered couplings [6–12], Gaussian wave-packet encoding [13–15], employment of global pulses for specific spin chain Hamiltonian [16], weak coupling of the sending and receiving qubits to a quantum many-body system [11,17,18] and employment of measurement [19–23] have been proposed to achieve perfect or near-perfect quantum state transfer. Recently, Yao et al. [24,25] have proposed a high-fidelity quantum state transfer through certain classes of random, unpolarized (infinite temperature) spin chains. Subsequently, the practicality of such spin chain wiring has been analyzed in diamond quantum technologies [26] and a scheme for high-dimensional quantum state transfer has also been considered [27]. Furthermore, Burgarth and Bose have suggested a dual-rail channel by adding an auxiliary spin chain to improve transfer capability of the state of a single spin [28–30]. If enough measurements can be carried out, their protocol will achieve conclusively perfect quantum state transfer whose success probability might reach 1. Although many strategies have been made to improve quantum state transfer, they apply only to special situations. For example, the engineering of couplings [6–12] is valid only in those physical implementations where interaction strengths can be tuned to appropriate values, as opposed to being "given" [30]. The schemes using global pulses [16] are restricted to a spin chain with a nearest neighbor Ising coupling. Concerning the diversity of practical settings, it is meaningful to develop a general approach that is not only independent of the concrete spin–spin interactions but also provides finite-probability perfect state transfer.

It has been recognized that the quantum state transfer along a spin chain has some similarity to the open system dynamics. For example, the effect of Heisenberg anti-ferromagnetic chain can be thought as a depolarizing channel [31], while a ferromagnetic spin chain acts essentially as an amplitude damping quantum channel converting an input state $\rho(0)$ to $\rho(t) = K_0(t)\rho(0)K_0^\dagger(t)+K_1(t)\rho(0)K_1^\dagger(t)$, where $K_0(t) = |0\rangle\langle 0| + |c_{sr}(t)|\,|1\rangle\langle 1|$ and $K_1(t) = \sqrt{1-|c_{sr}(t)|^2}\,|0\rangle\langle 1|$, with $|0\rangle$ ($|1\rangle$) denoting the spin-down (spin-up) state, are the Kraus operators with $c_{sr}(t)$ the transition amplitude of an spin-up state from a sending site s to a receiving site r of the spin chain [4]. Hence, we can draw lessons from the open system theory to fight against the "environment noise" on the state transfer process, yet the separation between system and environment in the spin chains' case is rather delicate. Namely, at each moment of time, one spin, the spin of concern, serves as system and all the other spins as environment. For example, at $t = 0$ the sth spin, in which quantum information is encoded, is the system but at a later desired $t > 0$ the rth spin, whose state we wish to be identical to that of the sth spin, will be treated as the system. In the context of open system dynamics, a number of controlling schemes have been developed to fight against the environmental noises (cf. [32] and references therein). Aiming at the suppression of decoherence of a single qubit in the amplitude damping channel, a scheme using prior weak measurement [33] followed by posterior quantum reversal measurement has been proposed [34] and experimentally demonstrated [35]. The idea of using weak and quantum reversal measurements has also been generalized to improve quantum state transfer through spin chains [36,37]. However, the fidelity of the recovered state is found to be inversely proportional to the success probability. In particular, complete restoration of the initial state occurs with zero probability, i.e., perfect state transfer with finite probability is impossible. As shown in [38], if the non-unitary evolution of the interested system can be randomly decomposed into many unitary quantum processes (i.e., each of the Kraus operators is proportional to a unitary operator), then the recovery of an unknown quantum state would be possible by performing a measurement on the environment followed by a quantum restoration operation on the system conditioned on the environment measurement outcome. Compared to the schemes based on weak and quantum reversal measurements [34,35], an advantage of the latter is that it can achieve a complete restoration of the initial state with a nonzero probability. Unfortunately, the evolution of the spin chain in terms of the Kraus operators K_0 and K_1 does not satisfy this 'unitary random decomposition' condition. Nevertheless, still motivated by this idea [32,38], we develop an efficient strategy to restore the transferred state along the spin chains.

In this paper, we present a general scheme that aims at controlling the dynamics of the spin chains so as to achieve perfect quantum state transfer with finite success probability. After encoding necessary quantum information into the state of the spins at the sending site and preparing the spins at all the other sites in their spin-down states, the natural evolution is let to start. Later, at an intended time $t > 0$, at which we decide to retrieve the original information from the spins at a receiving site, we measure the environment (i.e., perform a projective measurement on all the spins except those at the receiving site). If, with some finite probability, we find the environment in a state with all its spins being in spin-down states, we proceed with a proper action, otherwise we fail. It is worthy noting that, in the case of success, the spins at the receiving site would appear in a state as though a weak measurement [33] were made on them, namely, the input state was transferred to the receiving site with only partial collapse. Since, unlike projective measurements, a weak measurement can be undone, we are able to completely restore the original state with a finite success probability. In fact, undoing weak measurements is viable and experimental realizations have been reported in various physical contexts [35,39–41]. Concerning feasibility of perfect state transfer the present method of 'strongly' measuring the environment at time $t > 0$ gains a pronounced advantage over that of 'weakly' measuring the system at $t = 0$ [36,37] because in the latter method perfect state transfer is achieved only asymptotically with vanishingly small probability.

This paper contains 4 sections. After this introduction section, Section 2 describes in detail the scheme for finite-probability perfect transfer of an arbitrary M-spin state through M independent spin-chains. For clarity, the cases of $M = 1$ and any $M \geq 2$ are considered separately. In Section 3 the scheme developed in Section 2 is applied to two models with different types of spin–spin interactions in a spin chain. It is shown that in each model there can be found optimal times during the evolution at which success probabilities of perfect state transfer are remarkably high. A comparison between the two models is also demonstrated in terms of perfect state transfer success probabilities. Finally, conclusion is drawn in Section 4.

2. The scheme

As was already known, the intrinsic dynamics of coupled spins in spin chains allows a state of spins at a sending site to be transferred to other spins at a receiving site, but the resulting averaged fidelity is generally much less than 1. It has also been shown recently in [37] that the transferability of bipartite entanglement via dual-rail quantum channel served by two parallel spin chains is statedependent [37]. Here, we propose a method of how to use multiple spin chains to transfer arbitrary multispin states both perfectly and with well finite success probability. The key idea behind our scheme is the observation that, when at $t > 0$ during the time evolution the environment is measured and a certain outcome is obtained, then the system at that time is projected onto a state that still contains partial information of the system state at $t = 0$ and, thus, proper operations could be designed to manipulate the system so that the original quantum information will be recovered fully. Our scheme to be presented here is universal in the sense that it applies to any multispin states and to any types of spin–spin interactions.

2.1. The case of single-spin states

First, let us for clarity consider the scheme for finite-probability perfect transfer of a single-spin state through a 1D spin-$\frac{1}{2}$ graph containing N coupled spins. The state of interest, which is encoded in an sth spin at time $t = 0$ and needed to be transferred to an rth spin ($r \neq s$; $s, r \in \{1, 2, \ldots, N\}$), has the most general form

$$|\psi_1(0)\rangle_s = \sum_{j=0}^{1} \alpha_j \, |j\rangle_s \,, \tag{1}$$

with $\sum_{j=0}^{1} |\alpha_j|^2 = 1$ to satisfy the normalization, while all the other spins are in their spin-down states. The total state of all the N spins at $t = 0$ can be written as

$$|\Psi_1(0)\rangle = |\psi_1(0)\rangle_s \, |0\ldots00\ldots0\rangle_{1\ldots s-1s+1\ldots N} = \alpha_0 \, |\mathbf{0}\rangle + \alpha_1 \, |\mathbf{s}\rangle \,, \tag{2}$$

where the notations $|\mathbf{0}\rangle \equiv |0\ldots0\rangle_{1\ldots N}$ and $|\mathbf{s}\rangle \equiv |0\ldots1\ldots0\rangle_{1\ldots s\ldots N}$ are introduced for short. To keep the consideration generic at this stage we do not specify any concrete model for the interaction between spins in the chain, but, as in most situations, we do assume that the spin-chain's Hamiltonian H conserves the total number of spin-up states. Such an assumption implies that the state (2) will evolve at time $t > 0$ to

$$|\Psi_1(t)\rangle = \alpha_0 \, |\mathbf{0}\rangle + \alpha_1 \sum_{k=1}^{N} c_{sk}(t) \, |\mathbf{k}\rangle \,, \tag{3}$$

with $c_{sk}(t) = \langle \mathbf{k}| \exp(-iHt) \, |\mathbf{s}\rangle$ being the transition amplitude of a spin-up state from the sth site to the kth site whose actual value is model-dependent.

Our scheme runs as follows. Suppose that at a desired time $t > 0$ during the natural evolution we need to use the spin at a receiving site r whose state we wish to be identical to that at the sending site s at $t = 0$. Aiming at that target, we measure the environment spins which at that time t comprises all the spins other than the rth one. This measurement may either consist of $(N-1)$ measurements on $(N-1)$ individual spins of the environment to find out the exact state ($|0\rangle$ or $|1\rangle$) of a spin or be just a single coarse-grained 'collective' measurement on all the spins of the environment to find out the 'total' state without knowledge of the precise state of each spin. In both kinds of the above-mentioned measurements we are interested in two outcomes: $n_{up} = 0$ corresponding to finding no spins being in state $|1\rangle$ or to finding the environmental 'total' state being in $|0\ldots00\ldots0\rangle_{1\ldots r-1,r+1\ldots N}$ and $n_{up} = 1$ otherwise. If the outcome is $n_{up} = 1$, we fail because the spin at site r is projected onto $|0\rangle_r$, erasing all the information of the original state at site s. Yet, if the outcome is $n_{up} = 0$, which happens with a probability

$$\widetilde{P}_1(t) = |\alpha_0|^2 + |\alpha_1 c_{sr}(t)|^2, \tag{4}$$

the spin r is projected onto state

$$\left|\widetilde{\psi}_1(t)\right\rangle_r = \frac{1}{\sqrt{\widetilde{P}_1(t)}} \left(\alpha_0 \, |0_r\rangle + \alpha_1 |c_{sr}(t)| e^{i\varphi_{sr}(t)} \, |1_r\rangle\right). \tag{5}$$

Note that in Eq. (5) we have decomposed $c_{sr}(t)$ as $|c_{sr}(t)|e^{i\varphi_{sr}(t)}$ with $\varphi_{sr}(t)$ being a relative phase that has arisen during the evolution due to a governing Hamiltonian. The merit of measuring the environment is that the rth spin state $\left|\widetilde{\psi}_1(t)\right\rangle_r$ associated with the outcome $n_{up} = 0$ contains some information about the input state at site s in terms α_0 and α_1. It means that the input state at the sensing site after being transferred to the receiving site was only partially collapsed and is therefore possible to be restored with a finite probability by a suitable operation. The partial collapse of a quantum system is also referred to as a weak measurement [33–35] on the system. After obtaining the outcome $n_{up} = 0$, what makes the state $\left|\widetilde{\psi}_1(t)\right\rangle_r$, Eq. (5), different from the input state $|\psi_1(0)\rangle_s$, Eq. (1), is the presence of $|c_{sr}(t)|$ and $\varphi_{sr}(t)$ in it. As for $\varphi_{sr}(t)$, we can remove it by applying on the rth spin a unitary phase-shift operator of the form

$$U = \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\varphi_{sr}(t)} \end{pmatrix}, \tag{6}$$

thus transforming $\left|\widetilde{\psi}_1(t)\right\rangle_r$ to

$$\left|\overline{\psi}_1(t)\right\rangle_r = \frac{1}{\sqrt{\widetilde{P}_1(t)}} \left(\alpha_0 \left|0\right\rangle_r + \alpha_1 |c_{sr}(t)| \left|1\right\rangle_r\right). \tag{7}$$

To get rid of $|c_{sr}(t)|$ we perform on the spin a recovering measurement described by two operators R_a and R_b $\left(R_a^+ R_a + R_b^+ R_b = I\right)$ of the forms

$$R_a = \sqrt{1 - |c_{sr}(t)|^2} \left|0\right\rangle \left\langle 0\right|, \tag{8}$$

$$R_b = |c_{sr}(t)| \left|0\right\rangle \left\langle 0\right| + \left|1\right\rangle \left\langle 1\right|. \tag{9}$$

The outcome "a" occurs with probability $P_a(t) = (1 - |c_{sr}(t)|^2)|\alpha_0|^2/\widetilde{P}_1(t)$, in which situation the spin state is irreversibly collapsed into $|0\rangle_r$ and all the information is lost. Otherwise, if the outcome is "b", which occurs with probability $P_b(t) = |c_{sr}(t)|^2/\widetilde{P}_1(t)$, the state of spin r at time $t > 0$ becomes identical to that of spin s at $t = 0$, i.e., perfect quantum state transfer is achieved. The total success probability (for obtaining the outcomes $n_{up} = 0$ in measuring the environment and "b" in measuring spin r) is

$$P_1(t) = \widetilde{P}_1(t) P_b(t) = |c_{sr}(t)|^2, \tag{10}$$

which is obviously greater than zero and may be quite high in models with large enough modulus of transition amplitude $|c_{sr}(t)|$.

In the following, taking the current experimental technology into account, we present two possible methods to realize the recovering measurement. We note that R_b can be decomposed as

$$R_b \equiv NOT \cdot W_0 \cdot NOT \tag{11}$$

where

$$NOT = \left|0\right\rangle \left\langle 1\right| + \left|1\right\rangle \left\langle 0\right| \tag{12}$$

and

$$W_0 = \left|0\right\rangle \left\langle 0\right| + |c_{sr}(t)| \left|1\right\rangle \left\langle 1\right|. \tag{13}$$

This decomposition suggests an indirect way to realize the above quantum recovering measurement as follows. First, the spin is bit-flipped by a NOT. Then, it is measured by two measurement operators W_0 and $W_1 = \sqrt{1 - |c_{sr}(t)|^2} \left|1\right\rangle \left\langle 1\right|$. If the outcome corresponds to application of W_1, the state recovering process fails. Otherwise, if the outcome corresponds to application of W_0 (null-outcome), the qubit is bit-flipped again by another NOT. Such probabilistic measurements, i.e., null-outcome weak measurement and its reversal have been successfully implemented in various physical contexts, such as for the superconducting phase qubit [39], all-optical apparatuses [40] and atomic ions [41]. The quantum recovering measurement mentioned above can also be done rather directly by letting the spin go through an empty filter. The filter should have the following 'selective' properties: it is transparent for spins in spin-up states, but with the probability $1 - |c_{sr}(t)|^2$ absorbs spins in spin-down states. If so, when we check the filter and find a spin there, we fail. Luckily enough, however, if the filter remains empty we are sure that perfect transfer of the state of interest has been fulfilled from site s to site r. The use of such 'selective' filtering was known as the Procrustean method and was used for probabilistically producing a Bell state from a non-maximally entangled qubit pair. It is straightforward to verify that in both above indirect and direct implementations of the quantum recovering measurement the total success probability of perfect quantum state transfer is the same.

2.2. The case of multispin states

Next, we proceed to deal with multispin states. To transfer M-spin states from one processor/register to another, the processors/registers are connected by M 1D spin-$\frac{1}{2}$ graphs named $A^{(1)}, A^{(2)}, \ldots$ and $A^{(M)}$. Each graph has N spins. The spins in graph $A^{(m)}$ ($m = 1, 2, \ldots$ and M) are labeled $A_1^{(m)}, A_2^{(m)}, \ldots$ and $A_N^{(m)}$. The requirement of availability of M graphs at the same time causes no problems at all. In fact, it is even easier to produce a whole bunch of independent spin graphs rather than just a single one [42–44]. The graphs do not interact with each other so their total Hamiltonian reads

$$H = \oplus_{m=1}^{M} H_{A^{(m)}}, \tag{14}$$

where $H_{A^{(m)}}$ is the Hamiltonian of graph $A^{(m)}$. The working dynamics is of course governed by the Hamiltonians. But, in this section, we would like to develop a qualitative theory of quantum state transfer which is valid for graphs with any kinds of spin–spin interactions. In the next section we will specify the model by explicitly studying concrete types of the Hamiltonian.

The M-spin state to be transferred is encoded in M spins $A_s^{(1)}, A_s^{(2)}, \ldots$ and $A_s^{(M)}$ and has the most general form

$$|\psi_M(0)\rangle_{A_s^{(1)}\ldots A_s^{(M)}} = \sum_{k_1,k_2,\ldots,k_M=0}^{1} \alpha_{k_1k_2\ldots k_M} |k_1\rangle_{A_s^{(1)}} |k_2\rangle_{A_s^{(2)}} \ldots |k_M\rangle_{A_s^{(M)}}, \tag{15}$$

with the normalization condition $\sum_{k_1,k_2,\ldots,k_M=0}^{1} |\alpha_{k_1k_2\ldots k_M}|^2 = 1$. In terms of the abbreviated notations $|\mathbf{0}\rangle_{A^{(m)}} \equiv |0\ldots0\rangle_{A_1^{(m)}\ldots A_N^{(m)}}$ and $|\mathbf{j}\rangle_{A^{(m)}} \equiv |0\ldots1\ldots0\rangle_{A_1^{(m)}\ldots A_j^{(m)}\ldots A_N^{(m)}}$, the total initial state of the M graphs can be written as

$$\begin{aligned}
|\Psi_M(0)\rangle &\equiv |\Psi_M(0)\rangle_{A^{(1)}\ldots A^{(M)}} \\
&= \alpha_{00\ldots00} |\mathbf{0}\rangle_{A^{(1)}} |\mathbf{0}\rangle_{A^{(2)}} \ldots |\mathbf{0}\rangle_{A^{(M-1)}} |\mathbf{0}\rangle_{A^{(M)}} \\
&\quad + \alpha_{10\ldots00} |\mathbf{s}\rangle_{A^{(1)}} |\mathbf{0}\rangle_{A^{(2)}} \ldots |\mathbf{0}\rangle_{A^{(M-1)}} |\mathbf{0}\rangle_{A^{(M)}} \\
&\quad + \alpha_{01\ldots00} |\mathbf{0}\rangle_{A^{(1)}} |\mathbf{s}\rangle_{A^{(2)}} \ldots |\mathbf{0}\rangle_{A^{(M-1)}} |\mathbf{0}\rangle_{A^{(M)}} \\
&\quad + \cdots \\
&\quad + \alpha_{00\ldots01} |\mathbf{0}\rangle_{A^{(1)}} |\mathbf{0}\rangle_{A^{(2)}} \ldots |\mathbf{0}\rangle_{A^{(M-1)}} |\mathbf{s}\rangle_{A^{(M)}} \\
&\quad + \alpha_{11\ldots00} |\mathbf{s}\rangle_{A^{(1)}} |\mathbf{s}\rangle_{A^{(2)}} \ldots |\mathbf{0}\rangle_{A^{(M-1)}} |\mathbf{0}\rangle_{A^{(M)}} \\
&\quad + \cdots \\
&\quad + \alpha_{10\ldots10} |\mathbf{s}\rangle_{A^{(1)}} |\mathbf{0}\rangle_{A^{(2)}} \ldots |\mathbf{s}\rangle_{A^{(M-1)}} |\mathbf{0}\rangle_{A^{(M)}} \\
&\quad + \cdots \\
&\quad + \alpha_{00\ldots11} |\mathbf{0}\rangle_{A^{(1)}} |\mathbf{0}\rangle_{A^{(2)}} \ldots |\mathbf{s}\rangle_{A^{(M-1)}} |\mathbf{s}\rangle_{A^{(M)}} \\
&\quad + \cdots \\
&\quad + \alpha_{11\ldots11} |\mathbf{s}\rangle_{A^{(1)}} |\mathbf{s}\rangle_{A^{(2)}} \ldots |\mathbf{s}\rangle_{A^{(M-1)}} |\mathbf{s}\rangle_{A^{(M)}}.
\end{aligned} \tag{16}$$

As usual, each of the Hamiltonians $H_{A^{(m)}}$, though being not specified, is assumed to conserve the total number of spin-up states. Also, we assume for simplicity identical dynamics in all the graphs, i.e., $H_{A^{(1)}} \equiv H_{A^{(2)}} \equiv \cdots \equiv H_{A^{(M)}}$. Therefore, at time $t > 0$ state (16) will evolve to

$$\begin{aligned}
|\Psi_M(t)\rangle &\equiv |\Psi_M(t)\rangle_{A^{(1)}A^{(2)}\ldots A^{(M)}} \\
&= \alpha_{00\ldots00} |\mathbf{0}\rangle_{A^{(1)}} |\mathbf{0}\rangle_{A^{(2)}} \ldots |\mathbf{0}\rangle_{A^{(M-1)}} |\mathbf{0}\rangle_{A^{(M)}}
\end{aligned}$$

$$
\begin{aligned}
&+\sum_{j=1}^{N} c_{sj}(t)\left[\alpha_{10\ldots00}\,|\mathbf{j}\rangle_{A^{(1)}}\,|\mathbf{0}\rangle_{A^{(2)}}\ldots|\mathbf{0}\rangle_{A^{(M-1)}}\,|\mathbf{0}\rangle_{A^{(M)}}\right.\\
&+\alpha_{01\ldots00}\,|\mathbf{0}\rangle_{A^{(1)}}\,|\mathbf{j}\rangle_{A^{(2)}}\ldots|\mathbf{0}\rangle_{A^{(M-1)}}\,|\mathbf{0}\rangle_{A^{(M)}}\\
&+\cdots\\
&+\left.\alpha_{00\ldots01}\,|\mathbf{0}\rangle_{A^{(1)}}\,|\mathbf{0}\rangle_{A^{(2)}}\ldots|\mathbf{0}\rangle_{A^{(M-1)}}\,|\mathbf{j}\rangle_{A^{(M)}}\right]\\
&+\sum_{j,j'=1}^{N} c_{sj}(t)c_{sj'}(t)\left[\alpha_{11\ldots00}\,|\mathbf{j}\rangle_{A^{(1)}}\,\left|\mathbf{j}'\right\rangle_{A^{(2)}}\ldots|\mathbf{0}\rangle_{A^{(M-1)}}\,|\mathbf{0}\rangle_{A^{(M)}}\right.\\
&+\cdots\\
&+\alpha_{10\ldots10}\,|\mathbf{j}\rangle_{A^{(1)}}\,|\mathbf{0}\rangle_{A^{(2)}}\ldots\left|\mathbf{j}'\right\rangle_{A^{(M-1)}}\,|\mathbf{0}\rangle_{A^{(M)}}\\
&+\cdots.\\
&+\left.\alpha_{00\ldots11}\,|\mathbf{0}\rangle_{A^{(1)}}\,|\mathbf{0}\rangle_{A^{(2)}}\ldots|\mathbf{j}\rangle_{A^{(M-1)}}\,\left|\mathbf{j}'\right\rangle_{A^{(M)}}\right]\\
&+\cdots\\
&+\sum_{j,j',\ldots,j'',j'''=1}^{N} c_{sj}(t)c_{sj'}(t)\ldots c_{sj''}(t)c_{sj'''}(t)\alpha_{11\ldots11}\,|\mathbf{j}\rangle_{A^{(1)}}\\
&\times\left|\mathbf{j}'\right\rangle_{A^{(2)}}\ldots\left|\mathbf{j}''\right\rangle_{A^{(M-1)}}\,\left|\mathbf{j}'''\right\rangle_{A^{(M)}},
\end{aligned}
\tag{17}
$$

in which $c_{sj}(t) \equiv c_{sj}^{A^{(1)}}(t) \equiv \cdots \equiv c_{sj}^{A^{(M)}}(t)$. To achieve perfect transfer of the M-spin state (15) to that of M spins $A_r^{(1)}, A_r^{(2)}, \ldots$ and $A_r^{(M)}$, we measure the environment which at time t is a set of all the spins in the M graphs except $A_r^{(1)}, A_r^{(2)}, \ldots$ and $A_r^{(M)}$. Also in the case of multispin states, measurements on the environment may be of two kinds. For the first kind, $M(N-1)$ measurements on $M(N-1)$ individual spins of the environment are performed to find out which exact state each spin is in. For the second kind, all the spins of the environment are measured jointly to find out their 'total' state without the need to know which one in which state. Here, again we are only interested in two outcomes: $n_{up} = 0$ corresponding to finding no spins at all being in state $|1\rangle$ for the first kind of measurement or to finding the environment being in the 'total' state $|0\ldots00\ldots0\ldots0\ldots00\ldots0\rangle_{A_1^{(1)}\ldots A_{r-1}^{(1)}A_{r+1}^{(1)}\ldots A_N^{(1)}\ldots A_1^{(M)}\ldots A_{r-1}^{(M)}A_{r+1}^{(M)}\ldots A_N^{(M)}}$ for the second kind of measurement and $n_{up} = 1$ otherwise. The measurement is regarded successful only if the outcome is $n_{up} = 0$, which happens with a probability

$$
\begin{aligned}
\widetilde{P}_M(t) = {} & |\alpha_{00\ldots00}|^2 + (|\alpha_{10\ldots00}|^2 + |\alpha_{01\ldots00}|^2 + \cdots |\alpha_{00\ldots01}|^2)|c_{sr}(t)|^2\\
&+ (|\alpha_{11\ldots00}|^2 + \cdots + |\alpha_{10\ldots10}|^2 + \cdots + |\alpha_{00\ldots11}|^2)|c_{sr}(t)|^4\\
&+ \cdots + |\alpha_{11\ldots11}|^2|c_{sr}(t)|^{2M}.
\end{aligned}
\tag{18}
$$

As for the unmeasured spins $A_r^{(1)}, A_r^{(2)}, \ldots$ and $A_r^{(M)}$, they are correspondingly projected onto state

$$
\begin{aligned}
\left|\widetilde{\psi}_M(t)\right\rangle_{A_r^{(1)}\ldots A_r^{(M)}} = {} & \frac{1}{\sqrt{\widetilde{P}_M(t)}}\left\{\alpha_{00\ldots00}\,|00\ldots00\rangle + |c_{sr}(t)|e^{i\varphi_{sr}(t)}\left[\alpha_{10\ldots00}\,|10\ldots00\rangle\right.\right.\\
&+ \left.\alpha_{01\ldots00}\,|01\ldots00\rangle + \cdots + \alpha_{00\ldots01}\,|00\ldots01\rangle\right]\\
&+ |c_{sr}(t)|^2 e^{2i\varphi_{sr}(t)}\left[\alpha_{11\ldots00}\,|11\ldots00\rangle + \cdots + \alpha_{00\ldots11}\,|00\ldots11\rangle\right]\\
&+ \cdots\\
&+ \left.|c_{sr}(t)|^N e^{Ni\varphi_{sr}(t)}\alpha_{11\ldots11}\,|11\ldots11\rangle\right\}_{A_r^{(1)}A_r^{(2)}\ldots A_r^{(M-1)}A_r^{(M)}}.
\end{aligned}
\tag{19}
$$

Similarly to the single-graph case, the phase $\varphi_{sr}(t)$ that has arisen during the evolution can be removed by individually phase-shifting the M spins by a unitary operator (6). As a consequence of such phase-shifts, state $\left|\widetilde{\psi}_M(t)\right\rangle_{A_r^{(1)}A_r^{(2)}\ldots A_r^{(M)}}$ becomes

$$
\left|\overline{\psi}_M(t)\right\rangle_{A_r^{(1)}\ldots A_r^{(M)}} = \frac{1}{\sqrt{\widetilde{P}_M(t)}}\left\{\alpha_{00\ldots00}\,|00\ldots00\rangle + |c_{sr}(t)|\left[\alpha_{10\ldots00}\,|10\ldots00\rangle\right.\right.
$$

$$+\ \alpha_{01\ldots00}\,|01\ldots00\rangle+\cdots+\alpha_{00\ldots01}\,|00\ldots01\rangle]$$
$$+\,|c_{sr}(t)|^2\,[\alpha_{11\ldots00}\,|11\ldots00\rangle+\cdots+\alpha_{00\ldots11}\,|00\ldots11\rangle]$$
$$+\cdots$$
$$+\ |c_{sr}(t)|^N\alpha_{11\ldots11}\,|11\ldots11\rangle\big\}_{A_r^{(1)}A_r^{(2)}\ldots A_r^{(M-1)}A_r^{(M)}}\,. \tag{20}$$

Finally, we perform on each of spins $A_r^{(1)}, A_r^{(2)}, \ldots$ and $A_r^{(M)}$ a quantum recovering measurement described by the operators ((8)) and (9). If all the outcomes are "b", which occur with probability $P_{b\ldots b}(t) = |c_{sr}(t)|^{2M}/\widetilde{P}_M(t)$, the state of spins $A_r^{(1)}, A_r^{(2)}, \ldots$ and $A_r^{(M)}$ at $t > 0$ turns out to be exactly that of spins $A_s^{(1)}, A_s^{(2)}, \ldots$ and $A_s^{(M)}$ at $t = 0$. The total success probability is clearly equal to

$$P_M(t) = \widetilde{P}_M(t)P_{b\ldots b}(t) = |c_{sr}(t)|^{2M}. \tag{21}$$

Since spin–spin interaction is necessarily present in any model for transferring spin states, the quantity $|c_{sr}(t)|$ is guaranteed always greater than zero. Our scheme, therefore, guarantees quantum state transfer to be perfect with a well finite probability, as opposed to previous schemes [36] in which perfect transfer of quantum states appears possible only asymptotically with vanishingly small probability.

3. Application

So far we have presented our general scheme to realize perfect state transfer though spin chains without invoking concrete types of spin–spin interactions. Although the perfect state transfer process in our scheme is state-independent, its success probability depends on the transition amplitude $c_{sr}(t)$ which is model-dependent. As the quantity $|c_{sr}(t)|$ is greater than zero in any model with whatever spin–spin interactions, the success probability of our scheme is guaranteed to greater than zero too. Yet, we are still concerned with the characters of success probability depending on various parameters, such as the evolution time, the length of spin chains and the concrete interactions between spins. Therefore, in this section we apply our scheme to two models with typical types of spin–spin interaction in a spin chain. The first model we consider is that with ferromagnetic Heisenberg interactions between nearest neighbor spins of equal strengths. For each spin-chain $A^{(m)}$ $(m = 1, 2, \ldots, M)$, which is a linear open ended chain consisting of N spins, the underlying interaction Hamiltonian is given by

$$H_{XYZ} = -\frac{J}{2}\sum_{j=1}^{N-1}(\sigma_x^j\sigma_x^{j+1} + \sigma_y^j\sigma_y^{j+1} + \sigma_z^j\sigma_z^{j+1}) - h\sum_{j=1}^{N}\sigma_z^j + E_0, \tag{22}$$

where E_0 denotes the spin-chain ground state energy, $\sigma^j_{x(y,z)}$ are the $x(y, z)$ Pauli matrices for the jth spin, $J > 0$ the coupling strength between nearest neighbors, and h a common uniform magnetic field used to prevent thermal excitations. Setting $E_0 = 0$ for simplicity, the Hamiltonian (22) can be exactly diagonalized yielding the eigenstates and eigenenergies $\{|\widetilde{\mu}\rangle\,, E_\mu;\, \mu = 1, 2, \ldots, N\}$ of the form [4]

$$|\widetilde{\mu}\rangle = a_\mu\sum_{j=1}^{N}\cos\left[\frac{\pi}{2N}(\mu-1)(2j-1)\right]|\mathbf{j}\rangle\,, \tag{23}$$

with $a_\mu = \left[\sqrt{2} + \delta_{\mu,1}(1-\sqrt{2})\right]/\sqrt{N}$ and

$$E_\mu = 2h + 2J\left\{1 - \cos\left[\frac{\pi}{N}(\mu-1)\right]\right\}. \tag{24}$$

Then the transition amplitude for each spin-chain takes the same form as

$$c_{sr}^{XYZ}(t) \equiv \left\langle\mathbf{r}|e^{-itH_{XYZ}}|\mathbf{s}\right\rangle = \sum_{\mu=1}^{N}\langle\mathbf{r}|\widetilde{\mu}\rangle\,\langle\widetilde{\mu}|\mathbf{s}\rangle\,e^{-iE_\mu t}. \tag{25}$$

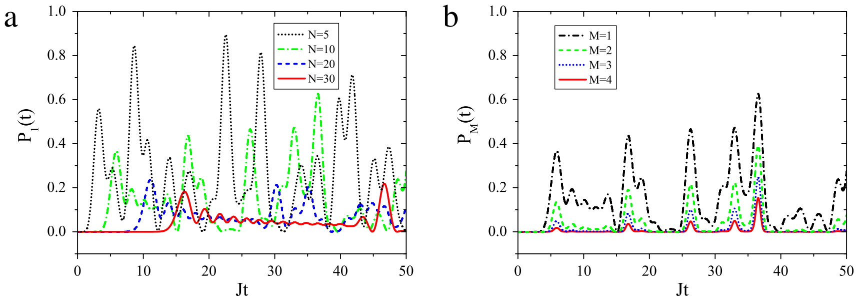

Fig. 1. (Color online) The success probability $P_M(t)$ for perfectly transferring an arbitrary M-spin state via M spin-chains as a function of scaled time Jt within the model described by the interaction Hamiltonian (22) for (a): $M = 1$ and different spin-chain lengths N and (b): different M and the same spin-chain lengths $N = 10$.

For concreteness, in what follows we are interested in the case when the sender is located near one end of the chains and the receiver near the other end, i.e., $s = 1$ and $r = N$ apply. The interested transition amplitude is thus

$$c_{1N}^{XYZ}(t) = \sum_{\mu=1}^{N} a_\mu^2 \cos\left[\frac{\pi}{2N}(\mu-1)\right]\cos\left[\frac{\pi}{2N}(\mu-1)(2N-1)\right]e^{-iE_\mu t}. \tag{26}$$

Another relevant model we would like to consider is described by the so-called XY Hamiltonian of the form

$$H_{XY} = \frac{J}{2}\sum_{j=1}^{N-1}(\sigma_x^j\sigma_x^{j+1} + \sigma_y^j\sigma_y^{j+1}) + E_0. \tag{27}$$

Setting again $E_0 = 0$, the corresponding eigenstates and eigenenergies $\{|\widetilde{\nu}\rangle, E_\nu; \nu = 1, 2, \ldots, N\}$ of the Hamiltonian (27) can be derived as [45]

$$|\widetilde{\nu}\rangle = \sqrt{\frac{2}{N+1}}\sum_{j=1}^{N}\sin\left(\frac{\pi\nu j}{N+1}\right)|\mathbf{j}\rangle \tag{28}$$

and

$$E_\nu = -2J\cos\left(\frac{\pi\nu}{N+1}\right). \tag{29}$$

Using Eqs. (28) and (29) the transition amplitude for $s = 1$ and $r = N$ reads

$$c_{1N}^{XY}(t) \equiv \langle\mathbf{N}|\, e^{-itH_{XY}}\,|\mathbf{1}\rangle = \frac{2}{N+1}\sum_{\nu=1}^{N}\sin\left(\frac{\pi\nu}{N+1}\right)\sin\left(\frac{\pi\nu N}{N+1}\right)e^{-iE_\nu t}. \tag{30}$$

Time-dependences of the success probabilities for perfect transfer of single-spin and multispin states are plotted in Fig. 1 using the model Hamiltonian (22) and in Fig. 2 using the model Hamiltonian (27), respectively. In both models the probabilities oscillate in time showing peaks of different heights at different moments. We observe that the position of peaks changes dramatically with the chains' length N (see Figs. 1(a) and 2(a)), but unchanged with the number M of spins in the state to be transferred (see Figs. 1(b) and 2(b)), and, in addition, the whole probability profile is, on average, lowered with increasing M or/and N, a fact that could be expected physically. Generally, the receiver should carry out his/her proper operations at an optimal time t_{opt} corresponding to the highest peak in order to achieve the best performance. Though the optimal time in each model is specifically sensitive only to N, it is difficult to derive t_{opt} in the form of an analytic function of N. Instead, for a concrete model, it is rather sufficient to determine it numerically. For instance, as seen from Figs. 1 and 2, within

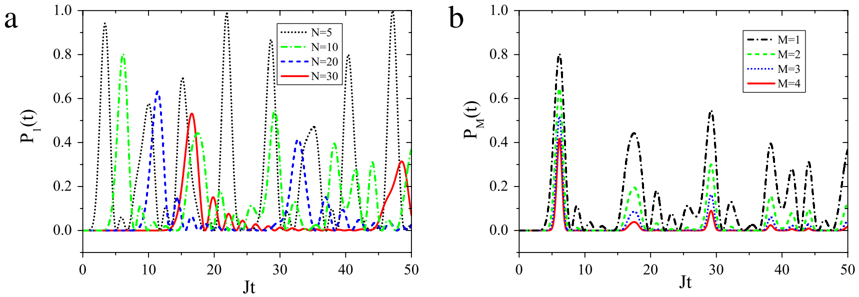

Fig. 2. (Color online) The success probability $P_M(t)$ for perfectly transferring an arbitrary M-spin state via M spin-chains as a function of scaled time Jt within the model described by the interaction Hamiltonian (27) for (a): $M = 1$ and different spin-chain lengths N and (b): different M and the same spin-chain lengths $N = 10$.

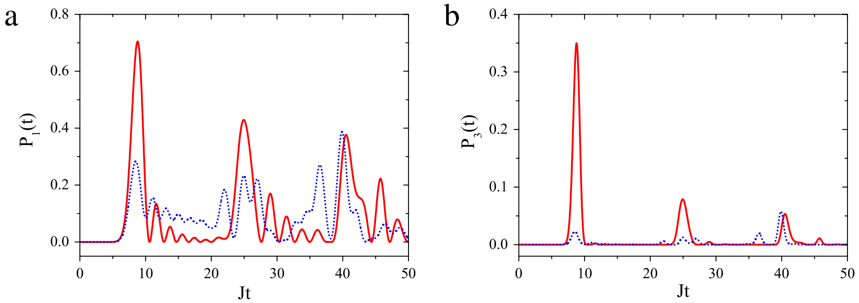

Fig. 3. (Color online) The success probability $P_M(t)$ for perfectly transferring an arbitrary M-spin state via M spin-chains as a function of scaled time Jt for the spin-chain lengths $N = 15$ using the interaction Hamiltonian (22) (dotted curves) and the interaction Hamiltonian (27) (solid curves) with (a): $M = 1$ and (b): $M = 3$.

the time period up to $Jt = 50$, $t_{opt} \simeq 22.55/J$ for the model (22) whereas $t_{opt} \simeq 47.15/J$ for the model (27) when $N = 5$, but when $N = 10$, $t_{opt} \simeq 36.55/J$ for the model (22) whereas $t_{opt} \simeq 6.12/J$ for the model (27). Also remarkable is the fact that success probabilities at optimal times may be quite high.

To compare the two models, we show $P_1(t)$ in Fig. 3(a) and $P_3(t)$ in Fig. 3(b) as functions of Jt for the same $N = 15$ in both the models. With regard only to the success probability of perfect state transfer, the model (27) seems better than the model (22). However, regarding an overall evaluation, it might not be so, if taking also into account other factors such as fabrication feasibility and stability of the spin-chains.

4. Conclusion

In conclusion, we have studied the problem of transferring quantum states through spin-chains which serve as quantum channels to connect different processors within a powerful quantum computer. This problem was dealt with previously by many authors, either by making use of the spin-chains' natural evolution or by combining it with null-outcome weak measurements performed on the interested spins at the beginning and the end of the evolution. However, these strategies only provide low fidelity of the transferred state with respect to the desired one or near-perfect state transfer with tiny success probability. Here, motivated by theory of open systems, we have proposed another strategy that allows ones to perfectly transfer an arbitrary multispin state with apparently finite success probability. The key idea behind the strategy is that after the sender encoded the input state in the spins at his/her site the whole system is let to evolve naturally until an optimal time at which the receiver measures the environment followed by quantum recovering measurements on individual spins at his/her site. The nice feature is that conditioned on an outcome of the environment's measurement, which happens with a finite probability, the receiver is able to design appropriate measurements to perform on his/her spins to convert them to be in the state identical to that of the sender, i.e., perfect state transfer is achieved with finite probability. We have developed the general theory showing that the perfection is state-independent and holds in any model with non-zero transition amplitudes, but the success probability is model-dependent. By considering two models with typical spin–spin interactions we have seen in both models that, besides reproducing the common tendency of decrease of the probability of perfect state transfer with increasing the chain length or/and the number of spins in the to-be-transferred state, the success probability can be reasonably high at an optimal time. This is the merit of the present scheme compared to all the previous ones. We have also discussed methods to implement the necessary recovering measurements. Our scheme is thus feasible within today's experiment technologies and would be useful in future practical applications.

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