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Improved quantum state transfer via quantum partially collapsing measurements

Zhong-Xiao Man [a,*], Nguyen Ba An [b], Yun-Jie Xia [a]

[a] *Shandong Provincial Key Laboratory of Laser Polarization and Information Technology, Department of Physics, Qufu Normal University, Qufu 273165, China*

[b] *Center for Theoretical Physics, Institute of Physics, Vietnam Academy of Science and Technology (VAST), 18 Hoang Quoc Viet, Cau Giay, Hanoi, Viet Nam*

HIGHLIGHTS

- A scheme using weak/reversal measurements is devised to improve quantum state transfer.
- It can suppress dissipation allowing optimal quantum state transfer in open system.
- Explicit condition for achieving near-perfect quantum state transfer is established.
- Applications to spin chain and cavity array are considered in detail.

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ABSTRACT

In this work, we present a general scheme to improve quantum state transfer (QST) by taking advantage of quantum partially collapsing measurements. The scheme consists of a weak measurement performed at the initial time on the qubit encoding the state of concern and a subsequent quantum reversal measurement at a desired time on the destined qubit. We determine the strength q_r of the post quantum reversal measurement as a function of the strength p of the prior weak measurement and the evolution time t so that near-perfect QST can be achieved by choosing p close enough to 1, with a finite success probability, regardless of the evolution time and the distance over which the QST takes place. The merit of our scheme is twofold: it not only improves QST, but also suppresses the energy dissipation, if any.

* Corresponding author. Tel.: +86 05375051319.
E-mail addresses: manzhongxiao@163.com, zxman@mail.qfnu.edu.cn (Z.-X. Man), nban@iop.vast.ac.vn (N. Ba An), yjxia@mail.qfnu.edu.cn (Y.-J. Xia).

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1. Introduction

Transferring quantum states from one to another location is an important phase in quantum information processing and distributed quantum computing [1]. The transfer can rely either on individual carriers, such as photons, or on collective phenomena, such as the natural dynamical evolution of a permanently coupled chain of quantum systems. In long distance quantum communication, photons are the most suited candidates to carry quantum states as they easily travel far away along optical fibers or through free space and can be readily measured at an arriving location. However, the interconnections of separate quantum processors or registers in a scalable, solid-state quantum computer [2–4] require mapping quantum states between two locations over relatively short distances. In this case, the employment of "quantum wire" made out of many interacting components is more suitable for quantum states' transferring. The first short-distance quantum state transfer (QST) protocol was proposed by Bose in which an unknown state can be efficiently transferred through a spin chain (data bus) via natural evolution [5]. In this protocol, the unknown state is encoded in the sth spin and will be transferred to the rth spin with certain fidelity after waiting for a specific amount of time depending on the length of the chain [5]. Subsequently, perfect QST in spin chains was experimentally realized using liquid nuclear magnetic resonance [6]. For a chain of spins subject to a uniformly coupled Heisenberg Hamiltonian [5], perfect QST is only possible for two or three qubits [7]. In order to achieve a perfect QST, several schemes have been proposed. It was found that, by appropriately engineering the couplings in a spin chain, a perfect or near-perfect QST can be accomplished for arbitrarily long chains [7–14]. However, the engineering of couplings is only be applicable in those physical implementations where interaction strengths can be tuned to appropriate values, as opposed to being "given" [15]. In Refs. [16,17], the QST with Gaussian wave-packet encoding was proposed for a ring of N spins and for open ended spin chains, which have also been suggested for communication through spin-chains under various static external fields [13,18]. The realization of wave-packet encoding should, however, involve several qubits for encoding or continuous time control. Another approach is to couple the sending and receiving qubits weakly to a quantum many-body system [13,19,20], which, however, will result in a slower transfer. By using a spin chain Hamiltonian with a nearest neighbor Ising coupling, in conjunction with "global" pulses (i.e., pulses that act on each spin of the chain in exactly the same manner) at regular intervals, a perfect transport of a state from one end to the other can be obtained [21]. Obviously, the schemes using global pulses are restricted to Ising chains. Burgarth and Bose also suggested a dual-rail channel by adding an auxiliary spin chain to improve transfer capability [15,22]. If enough measurements can be carried out, their protocol will achieve conclusively perfect transfer with certain success probability accessible to 1. Recently, Yao et al. [23,24] have proposed a high-fidelity QST through certain classes of random, unpolarized (infinite temperature) spin chains. Subsequently, the practicality of such spin chain wiring has been analyzed in diamond quantum technologies [25] and a high-dimensional quantum state transfer scheme has also been considered [26]. In addition to spin chains, the schemes to implement QST in other physical contexts have also been proposed [27].

In fact, the spin chain acts as an amplitude damping quantum channel converting the input state $\rho(0)$ to $\rho(t) = M_0\rho(0)M_0^\dagger + M_1\rho(0)M_1^\dagger$, where $M_0 = |0\rangle\langle 0| + |c_r(t)|\,|1\rangle\langle 1|$ and $M_1 = \sqrt{1-|c_r(t)|^2}\,|0\rangle\langle 1|$ are the Kraus operators with $c_r(t)$ the transition amplitude of an excitation (the $|1\rangle$ state) from the sending site to the receiving site of a spin chain [1,5]. Motivated by this fact, we recognize that schemes that can suppress the decoherence effects due to the zero-temperature energy relaxation are beneficial to improve the fidelity of QST through a spin chain. Therefore, in this work, we propose a general scheme by applying two sequential quantum measurements on the sending and the receiving spins, respectively, to boost the QST fidelity without modulating the spins (the data bus) between them. According to the quantum mechanics postulates, quantum (strong) measurement cannot be undone, because it totally collapses the measured system. However, if the measurement is weak (i.e., it only partially collapses the measured system) [1,28], it turns out possible to recover the measured state probabilistically [29,30] through another partially collapsing measurement called quantum reversal measurement. Such schemes based on quantum partially collapsing measurements have been demonstrated experimentally in various contexts, such as the superconducting phase qubit [31], the single-photon qubit [32], and the single trapped and laser-cooled $^{40}Ca^+$ [33]. The idea of combined usage of weak measurement and quantum reversal measurement has also been developed to fight against decoherence of a quantum system due to zero-temperature energy relaxation [34] and experimentally demonstrated in optical system [35]. Such strategy has also been adopted to protect entanglement from degradation in various decoherence scenarios [36–39].

A weak measurement $\mathcal{W}_X(p)$ with strength p ($0 \leq p < 1$) maps the state $|n\rangle_X$ ($n = 0, 1$) of qubit X as

$$\mathcal{W}_X(p)|n\rangle_X \to \sqrt{(1-p)^n}|n\rangle_X, \tag{1}$$

whereas a quantum reversal measurement $\mathcal{R}_X(q)$ with strength q ($0 \leq q < 1$) corresponds to the mapping

$$\mathcal{R}_X(q)|n\rangle_X \to \sqrt{(1-q)^{1-n}}|n\rangle_X. \tag{2}$$

It is straightforward to verify that $NOT \cdot \mathcal{W}_X(q) \cdot NOT|n\rangle_X \equiv \sqrt{(1-q)^{1-n}}|n\rangle_X$, with NOT the NOT gate acting on the state $|n\rangle_X$ as $NOT|n\rangle_X = |1-n\rangle_X$. This identity suggests a practical way to implement a $\mathcal{R}_X(q)$: first bit-flip the qubit X, then perform on it a weak measurement $\mathcal{W}_X(q)$ and, finally, bit-flip it again. A prior weak measurement on a qubit, if successful (i.e., with "null outcome"), reduces amplitude of its excited state making the qubit more robust against the decoherence process. After the qubit underwent the decoherence, a post quantum reversal measurement with suitably chosen strength can, in theory, make the qubit state arbitrarily close to the initial one. To restore the initial state of a quantum system, two suitable sequential measurements are to be implemented on one and the same system. As for QST, the state should be transferred between systems, which are located at different locations. Therefore, the above-described strategy cannot be applied in a straight way. In our scheme, we first make a weak measurement on the sending system with strength p and later, at any desired time $t > 0$, recover the transferred state by making a quantum reversal measurement with strength q on the destined system. We derive a relationship between p, q and t so that a perfect QST can be achieved. Our scheme is applicable not only to the usual QST in a spin chain (e.g., the Bose's model [5]), but also to the QST among qubits which experience energy dissipations.

2. The general scheme

We first present our general scheme to improve the fidelity of QST via a combination of partially collapsing measurements, namely, a weak measurement, followed by a quantum reversal measurement. The system we consider consists of N qubits (spin-$\frac{1}{2}$ ones or two-level atoms, ...) with a Hamiltonian H. Suppose that the state of concern,

$$|\psi(0,0)\rangle = \cos(\theta/2)\,|0\rangle + e^{i\phi}\sin(\theta/2)\,|1\rangle\,, \tag{3}$$

is encoded in a sth qubit and will be transferred to an rth qubit ($r \neq s;\; s, r \in \{1, 2, \ldots, N\}$) through the data bus served by the other in-between qubits. We assume that except the sth qubit all the other qubits are in their ground states. For convenience, we denote by $|\mathbf{0}\rangle = |0\cdots0\rangle_{1\cdots N}$ the state with all the qubits being unexcited and by $|\mathbf{j}\rangle = |0\cdots1\cdots0\rangle_{1\cdots j\cdots N}$ ($\mathbf{j} = \mathbf{1}, \mathbf{2}, \ldots, \mathbf{s}, \ldots, \mathbf{r}, \ldots, \mathbf{N}$) the state with only qubit j being excited. To run our scheme, we perform a weak measurement on the sth qubit with strength p before the whole system starts to evolve. By such a measurement, the state $|\psi(0,0)\rangle$ becomes $|\psi(0,p)\rangle = \mathcal{W}_s(p)\,|\psi(0,0)\rangle$,

$$|\psi(0,p)\rangle = \frac{1}{\sqrt{P_1(0,p)}}\left[\cos(\theta/2)\,|0\rangle + e^{i\phi}\sin(\theta/2)\sqrt{1-p}\,|1\rangle\right], \tag{4}$$

with $P_1(0,p) = \cos^2(\theta/2) + (1-p)\sin^2(\theta/2)$ being the success probability without completely collapsing the measured state via the weak measurement. Therefore, the state of the whole system of N qubits after the weak measurement can be written in the form

$$|\Psi(0,p)\rangle = \frac{1}{\sqrt{P_1(0,p)}}\left[\cos(\theta/2)\,|\mathbf{0}\rangle + e^{i\phi}\sin(\theta/2)\sqrt{1-p}\,|\mathbf{s}\rangle\right]. \tag{5}$$

In the closed system, the excitation number (the total z component in the language of spin) of the system is usually conserved, so does the open system if we take the environment and the system as a whole. On the other hand, the ground state $|\mathbf{0}\rangle$ keeps invariant in the time evolution, therefore at time $t > 0$ the system state $|\Psi(0,p)\rangle$ evolves to

$$|\Psi(t,p)\rangle = \frac{1}{\sqrt{P_1(0,p)}}\Bigg[\cos(\theta/2)\,|\mathbf{0}\rangle + e^{i\phi}\sin(\theta/2)\sqrt{1-p} \times \Bigg(c_r(t)\,|\mathbf{r}\rangle + \sum_{\mathbf{k}(\neq\mathbf{r})=\mathbf{1}}^{\mathbf{N}} c_k(t)\,|\mathbf{k}\rangle\Bigg)\Bigg], \tag{6}$$

where $c_j(t) = |c_j(t)|e^{i\varphi_j(t)}$ are time-dependent coefficients to be determined by the concrete model. At an arbitrary time we want to retrieve the state of concern (3) at the rth qubit, we perform a quantum reversal measurement with strength q on the rth qubit transforming the state (6) to $|\Psi(t,p,q)\rangle = \mathcal{R}_r(q)\,|\Psi(t,p)\rangle$,

$$|\Psi(t,p,q)\rangle = \frac{1}{\sqrt{P_2(t,p,q)}}\Bigg[\cos(\theta/2)\sqrt{1-q}\,|\mathbf{0}\rangle + e^{i\phi}\sin(\theta/2)\sqrt{1-p} \times \Bigg(c_r(t)\,|\mathbf{r}\rangle + \sqrt{1-q}\sum_{\mathbf{k}(\neq\mathbf{r})=\mathbf{1}}^{\mathbf{N}} c_k(t)\,|\mathbf{k}\rangle\Bigg)\Bigg], \tag{7}$$

in which $P_2(t,p,q) = (1-q)\cos^2(\theta/2) + (1-p)\sin^2(\theta/2)[|c_r(t)|^2 + (1-q)\sum_{\mathbf{k}(\neq\mathbf{r})=\mathbf{1}}^{\mathbf{N}} |c_k(t)|^2]$ is the success probability of the two measurements without completely collapsing the system's state. To perfectly recover the state of concern at the rth qubit, we will judiciously determine the strength q of the post quantum reversal measurement, depending on the prior weak measurement's strength p and the evolution time t. If we choose $q = q_r = 1-(1-p)|c_r(t)|^2$, then the state (7) is changed to

$$|\Psi(t,p,q_r)\rangle = \frac{1}{\sqrt{P_2(t,p)}}\Bigg[\sqrt{1-p}|c_r(t)|\;|\psi_r\rangle + (1-p)e^{i\phi}\sin(\theta/2) \times |c_r(t)|\sum_{\mathbf{k}(\neq\mathbf{r})=\mathbf{1}}^{\mathbf{N}} c_k(t)\,|\mathbf{k}\rangle\Bigg], \tag{8}$$

where $|\psi_r\rangle = \cos(\theta/2)\,|\mathbf{0}\rangle + e^{i[\phi+\varphi_r(t)]}\sin(\theta/2)\,|\mathbf{r}\rangle$ and $P_2(t,p,q)$ is changed to $P_2(t,p) = (1-p)|c_r(t)|^2 + (1-p)^2|c_r(t)|^2\sin^2(\theta/2)\sum_{\mathbf{k}(\neq\mathbf{r})=\mathbf{1}}^{\mathbf{N}} |c_k(t)|^2$. Note that $|\psi_r\rangle$ can be written as $|\psi_r\rangle = |0\cdots0\rangle_{1\cdots r-1}\,[\cos(\theta/2)\,|0\rangle + e^{i[\phi+\varphi_r(t)]}\sin(\theta/2)\;|1\rangle]_r\,|0\cdots0\rangle_{r+1\cdots N}$. Thus, in order to have the state of the rth qubit coincide with that of the sth qubit, a phase shift realized by the unitary operator $\mathcal{P}(-\varphi_r(t)) = \{\{1,0\},\{0,e^{-i\varphi_r(t)}\}\}$ should be generated for the rth qubit. In other words, our action to be done on the rth qubit at the desired time t is $\mathcal{P}(-\varphi_r(t))\;\mathcal{R}_r(q_r)$, transforming the state $|\Psi(t,p)\rangle$ in Eq. (6) to

$$\big|\widetilde{\Psi}(t,p,q_r)\big\rangle = \frac{1}{\sqrt{P_2(t,p)}}\Bigg[\sqrt{1-p}|c_r(t)|\;|0\cdots0\rangle_{1,\ldots,r-1}\,|\psi(0,0)\rangle_r\,|0\cdots0\rangle_{r+1\cdots N} + (1-p)e^{i\phi}\sin(\theta/2)|c_r(t)|\sum_{\mathbf{k}(\neq\mathbf{r})=\mathbf{1}}^{\mathbf{N}} c_k(t)\,|\mathbf{k}\rangle\Bigg]. \tag{9}$$

An important observation followed from the RHS of Eq. (9) is that, the first term scales as $\sqrt{1-p}$, while the second one as $(1-p)$. Therefore, the state of qubit r, at any intended time and for any qubits' number N, can be made arbitrarily close to the state of qubit s (i.e., the state $|\psi(0,0)\rangle$ in Eq. (3)) in the limit of $p \to 1$. This implies that perfect QST from the sth qubit to the rth qubit can, in principle, be achieved by choosing p close enough to 1.

In the following, we analyze the closeness of the transferred state to the state of concern by the fidelity defined as $F(p,t) = \langle\psi(0,0)|\,\rho_r(p,t)\,|\psi(0,0)\rangle$ with $\rho_r(p,t)$ the actual reduced density matrix of the rth qubit. If no measurements are performed at all (i.e., $p = q = 0$) but still with the phase shift operation, the fidelity is

$$\begin{aligned} F(0,t) &\equiv F(p=q=0,t) \\ &= \cos^2(\theta/2)[1-\sin^2(\theta/2)|c_r(t)|^2] + \sin^4(\theta/2)|c_r(t)|^2 + \frac{1}{2}\sin^2(\theta)|c_r(t)|. \end{aligned} \tag{10}$$

Yet, with the quantum partially collapsing measurements as in our scheme the fidelity takes the form

$$F(p,t) = \frac{1+\frac{1}{4}(1-p)\sin^2(\theta)(1-|c_r(t)|^2)}{1+(1-p)\sin^2(\theta/2)(1-|c_r(t)|^2)}. \tag{11}$$

Although our scheme does not require knowledge of the state to be transferred (i.e., it is valid to any unknown state), the fidelity turns out state-dependent. Of interest is then the averaged fidelity $F_{\text{av}}(p,t)$ which is obtained by averaging $F(p,t)$ over all the pure states on the Bloch sphere. As a result, we obtain

$$F_{\text{av}}(p,t) = \frac{1}{2A^2}[A^2 + 2A - 2\ln(1+A)], \tag{12}$$

with $A = (1-p)(1-|c_r(t)|^2)$. For comparison, we also derive the averaged fidelity $F_{\text{av}}(0,t)$ in the case when the system evolves naturally,

$$F_{\text{av}}(0,t) = \frac{1}{2} + \frac{1}{3}|c_r(t)| + \frac{1}{6}|c_r(t)|^2. \tag{13}$$

As for the success probability of our scheme, obviously, it is also state-dependent. The averaged success probability is

$$P_{\text{av}}(p,t) = \frac{1}{2}(1-p-A)(2+A). \tag{14}$$

For $\varepsilon = 1-p \ll 1$,

$$F_{\text{av}}(p,t) = 1 - \frac{\varepsilon}{3}|c_r(t)|^2 + \mathcal{O}(\varepsilon^2) \tag{15}$$

and

$$P_{\text{av}}(p,t) = (1-p)|c_r(t)|^2 + \mathcal{O}(\varepsilon^2). \tag{16}$$

As seen from Eq. (15), the averaged fidelity $F_{\text{av}}(p,t)$ is approaching 1 for p tending to 1, but at the price that the averaged success probability $P_{\text{av}}(p,t)$, Eq. (16), is becoming vanishingly small. Generally, for increasing p, $F_{\text{av}}(p,t)$ is increasing and $P_{\text{av}}(p,t)$ is decreasing. The merit of our scheme is that a rather high averaged fidelity can still be reached with a reasonable averaged success probability for p not very close to 1. The details depend on the concrete model, as we will illustrate in the next section.

3. Applications

In this section we apply our general scheme described above to two physical models: one is a finite linear spin chain under a constant magnetic field and the other is a cavity array with dissipation.

3.1. Spin chain

Consider a linear chain of N spins with the nearest neighbor Heisenberg interactions of equal strength and under a common uniform magnetic field. The Hamiltonian is given by [5]

$$H = -\frac{J}{2}\sum_{j=1}^{N-1}(\sigma_x^j\sigma_x^{j+1} + \sigma_y^j\sigma_y^{j+1} + \sigma_z^j\sigma_z^{j+1}) - B\sum_{j=1}^{N}\sigma_z^j, \tag{17}$$

where $\sigma_{x(y,z)}^j$ are the Pauli matrices for the jth spin, $J > 0$ is the coupling strength between nearest neighbors and B is the magnetic field. The eigenstates of this Hamiltonian can be obtained as [5] $|\widetilde{m}\rangle = a_m \sum_{j=1}^{N} \cos[\pi(m-1)(2j-1)/2N]\,|\mathbf{j}\rangle$, with $m = 1, 2, \ldots, N$ and $a_m = [\sqrt{2} + \delta_{m,1}(1-\sqrt{2})]/\sqrt{N}$, while the corresponding eigenenergies are given by $E_m = 2B + 2J\{1-\cos[\pi(m-1)/N]\}$. In this case, $c_r(t) = \langle \mathbf{r}|\, e^{-iHt}\,|\mathbf{s}\rangle = \sum_{m=1}^{N} \langle \mathbf{r}|\widetilde{m}\rangle\,\langle \widetilde{m}|\mathbf{s}\rangle\, e^{-iE_m t}$.

According to intrinsic dynamics governed by the Hamiltonian (17), the averaged fidelity $F_{av}(0, t)$ (with the proper phase shift performed on the rth spin) exhibits a series of maxima at different moments and its overall oscillation is sensitive to the number N of spins in the chain. Specifically, within a finite time interval of length $T_{max} = 4000/J$, it was shown [5] that only for a limited number of N (namely, $N = 2, 4, 7, 8, 10, 11, 13$ and 14) can the fidelity exceed 0.9. As a general tendency, the larger the N the smaller the maximal value at which the fidelity can arrive within a finite (but long) time interval and the longer the time it takes for achieving the maximum fidelity. Contrastingly, by our scheme (i.e., a prior weak measurement with strength p on the sth spin followed by a reversal measurement with strength $q_r = 1 - (1 - p)|c_r(t)|^2$ and a proper phase shift on the rth spin) the fidelity can in principle be boosted to one (i.e., perfect QST is achieved) at an arbitrary time, independent of the chain length. The price to pay for near-perfect QST, as mentioned in Section 2, is the tiny success probability. Nevertheless, compared to the situation of natural evolution [5], our scheme can provide a considerably improved fidelity with a reasonable success probability. This is demonstrated in Fig. 1, where we show the time-dependence of the averaged fidelity and the corresponding averaged success probability on the prior weak measurement strength p for spin chains with $N = 6$ and $N = 60$ when $s = 1$ and $r = N$ (i.e., the encoded spin and the destined spin are at opposite ends of the chain). As evident from Fig. 1(a) and (b), the fidelities $F_{av}(p, t)$ (solid lines) obtained in our scheme are manifestly larger than $F_{av}(0, t)$ (dotted line) due to natural dynamics and, the closer p is to 1 the better the fidelity is improved. For a fixed value of p, the improvement in fidelity changes over time, but the tendency of oscillation of $F_{av}(p, t)$ mimics that of $F_{av}(0, t)$. Moreover, the oscillation amplitudes of $F_{av}(p, t)$ are largely shrunk in comparison to those of $F_{av}(0, t)$, implying that with our scheme the information of the state of concern is retained at the destined spin most of the time without being transported to the other spins. By comparing the fidelities $F_{av}(0, t)$ (dotted lines) in Fig. 1(a) and (b) for relatively short ($N = 6$) and long ($N = 60$) spin chains, we can see that in the former case the information of the concerned state at the first spin is transferred to the Nth spin quickly, while in the latter case the Nth spin remains in its ground state ($F_{av}(0, t) = 0.5$) for a long time before evolving to a state which is closest to the concerned state (here, for $N = 60$, $F_{av}(0, t) = 0.701$) at $t \approx 3473.06/J$. Such a sensitivity to the chain length is not pronounced in our scheme as the fidelities $F_{av}(p, t)$ (solid lines) in Fig. 1(a) and (b) show. The corresponding averaged success probabilities plotted in Fig. 1(c) and (d) reveal that, though oscillating in time, $P_{av}(p = p_1, t) < P_{av}(p = p_2, t)$ if $p_1 > p_2$ all the time. For a definite value of p, the fidelity and success probability peak together. Therefore, at the time the information of the concerned state is best transferred to the destined spin (fidelity reaches the highest peak), our scheme can be implemented with the largest success probability, and vice versa. As is particularly visualized from Fig. 1(b) and (d), at $t \approx 3473.06/J$ the fidelity $F_{av}(p, t)$ and the success probabilities $P_{av}(p, t)$ arrive at their maximum values simultaneously for all the chosen values of p.

3.2. Cavity array with dissipation

In the previous subsection the spins are not coupled to any dissipative environments. Our general scheme also works for the case with dissipation. In this subsection we will show that, in addition to improving QST, our scheme also suppresses energy dissipation, if any. Due to the possibility of individual addressing, an array of coupled cavities is probably a promising candidate for simulating spin chains [40–42], which also have naturally nearest neighbor without assumption by contrast to the dipolar spin chains [24]. In Ref. [40], a binary transmission scheme was presented by using the array of coupled cavities with each cavity containing a three-level atom, where either atoms or photons can be used as a channel to transfer information. The coupled cavities have been experimentally realized in various contexts [43], including photonic crystals [44], superconducting resonators [45], and cavity-fiber-cavity systems [46]. To implement our QST scheme in the presence of dissipation let us consider an array of N coupled cavities described by the Hamiltonian [47] ($\hbar = 1$)

$$H_c = \sum_{j=1}^{N} \omega_c \hat{a}_j^\dagger \hat{a}_j + \sum_{j=1}^{N-1} J(\hat{a}_j^\dagger \hat{a}_{j+1} + \hat{a}_j \hat{a}_{j+1}^\dagger), \tag{18}$$

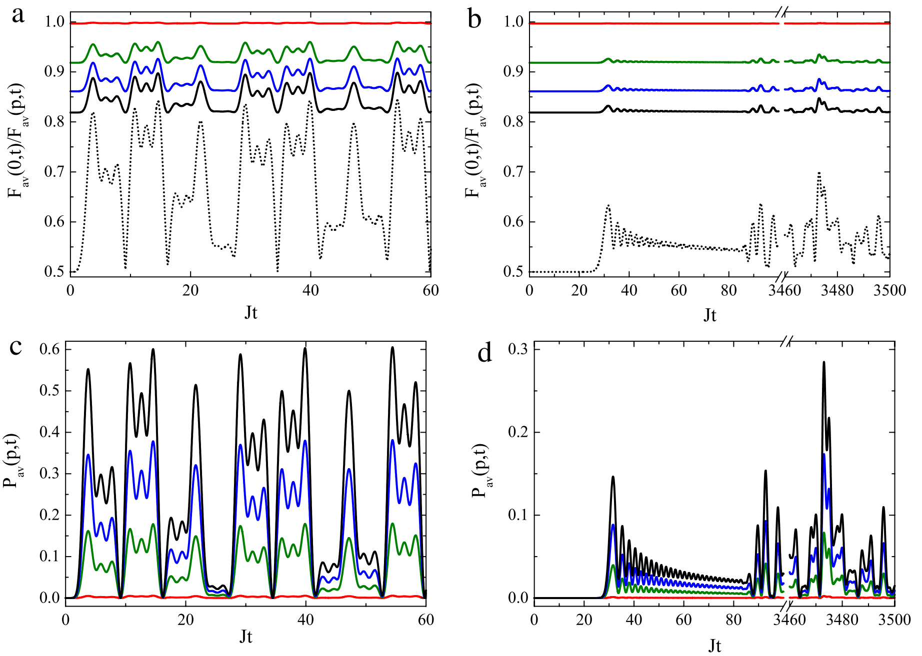

Fig. 1. (Color online) (a)–(b) The dependence on scaled time Jt of the averaged fidelities $F_{av}(0, t)$ (dotted curves), Eq. (13), under natural evolution and $F_{av}(p, t)$ (solid curves), Eq. (12), under measurement-controlled evolution for the prior weak measurement strengths $p = 0.1, 0.4, 0.7$ and 0.99 (from bottom to top). (c)–(d) The averaged success probabilities $P_{av}(p, t)$, Eq. (14), for $p = 0.1, 0.4, 0.7$ and 0.99 (from top to bottom). In (a) and (c) the number of spins is $N = 6$, while in (b) and (d) $N = 60$. In particular, we show in (b) the dynamics in the time interval from $Jt = 3460$ to $Jt = 3500$ during which the fidelity $F_{av}(0, t)$ reaches a highest maximum of 0.701 at $Jt \approx 3473.06$.

where $\hat{a}_j^\dagger$ ($\hat{a}_j$) is the creation (annihilation) operator of the photon with frequency ω_c localized in cavity j and J the photon hopping rate between neighboring cavities. The dissipation is accounted for by assuming that each cavity photon interacts with an independent multimode reservoir described by the interaction Hamiltonian [48]

$$H_{int} = \sum_{j=1}^{N} \sum_{k_j} (g_{k_j} \hat{a}_j^\dagger \hat{b}_{k_j} + g_k^* \hat{a}_j \hat{b}_{k_j}^\dagger), \tag{19}$$

with $\hat{b}_{k_j}^\dagger$ ($\hat{b}_{k_j}$) the creation (annihilation) operator of mode k_j of the reservoir j with frequency ω_{k_j} and g_{k_j} the interaction strength between the jth cavity photon and reservoir mode k_j.

Assuming that the state of concern (3) is encoded in the photon state of cavity s, while all the other cavities as well as all the reservoirs are prepared initially in the vacuum states, then the total system contains up to one excitation (either a photon in a cavity or an excited mode in a reservoir), which is conserved during the evolution. Governed by the total Hamiltonian $H_c + H_{int}$, the initial state of the total system, $|\Psi(0)\rangle = \cos(\theta/2)\,|0\cdots 0\rangle \otimes |\mathbf{0}\cdots\mathbf{0}\rangle + e^{i\phi}\sin(\theta/2)\,|0\cdots 1_s \cdots 0\rangle \otimes |\mathbf{0}\cdots\mathbf{0}\rangle$, will evolve into

$$|\Psi(t)\rangle = \cos(\theta/2)\,|0\cdots 0\rangle \otimes |\mathbf{0}\cdots\mathbf{0}\rangle + e^{i\phi}\sin(\theta/2)\left\{\sum_{j=1}^{N}\left[c_j(t)\,\big|0\cdots 1_j\cdots 0\big\rangle \otimes |\mathbf{0}\cdots\mathbf{0}\rangle \right.\right.$$
$$\left.\left. + \sum_{k_j} c_{k_j}(t)\,|0\cdots 0\rangle \otimes \big|\mathbf{0}\cdots\mathbf{1}_{k_j}\cdots\mathbf{0}\big\rangle\right]\right\}, \tag{20}$$

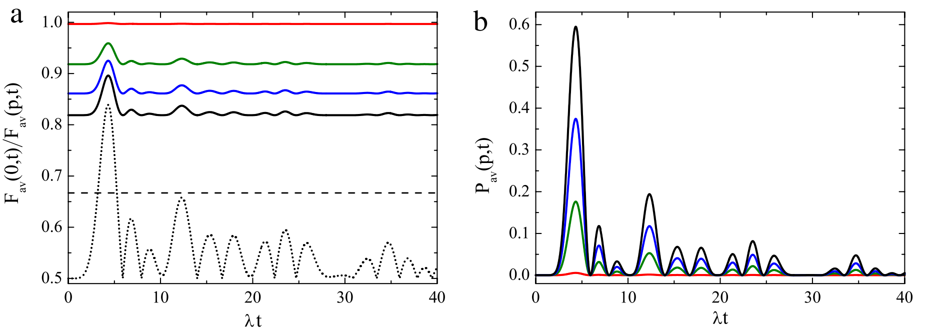

Fig. 2. (Color online) (a) The dependence on scaled time λt of the averaged fidelities $F_{av}(0, t)$ (dotted curves), Eq. (13), under natural evolution and $F_{av}(p, t)$ (solid curves), Eq. (12), under measurement-controlled evolution for the prior weak measurement strengths $p = 0.1, 0.4, 0.7, 0.99$ (from bottom to top). (b) The averaged success probabilities $P_{av}(p, t)$, Eq. (14), for $p = 0.1, 0.4, 0.7, 0.99$ (from top to bottom). The number of cavities is $N = 7$. The other parameters used are $J/\lambda = 1$ and $R/\lambda = 0.3$. The strait dashed line in (a) at 2/3 marks the highest possible fidelity achievable classically.

where $|0\cdots 0\rangle$ ($|\mathbf{0}\cdots\mathbf{0}\rangle$) denotes the state with no photons in cavities (no excited modes in reservoirs) and $\left|0\cdots 1_j\cdots 0\right\rangle$ $\left(\left|\mathbf{0}\cdots\mathbf{1}_{k_j}\cdots\mathbf{0}\right\rangle\right)$ the state with only one photon in cavity j (only one excited mode k_j in reservoir j). As the evolved state (20) shows, the cavity photon may hop from the sth cavity to the jth cavity or to the jth reservoir with mode k_j excited, with a probability amplitude $c_j(t)$ or $c_{k_j}(t)$, respectively (expressions of the interested $c_j(t)$ are given in the Appendix). From this point of view, the N reservoirs could be thought of as another set of N nodes of the data bus although they play detrimental roles for the QST problem. Therefore, the formulas derived previously (i.e., the averaged fidelity $F_{av}(p, t)$ (12), the averaged success probability $P_{av}(p, t)$ (14)), remain applicable in the presence of the dissipative reservoirs. In other words, our scheme applies as well to the situation when dissipation due to environment is taken into account. We display in Fig. 2 the dynamics of the averaged fidelity $F_{av}(0, t)$ under natural evolution with only phase shift on the rth qubit, the averaged fidelity $F_{av}(p, t)$ due to our scheme (i.e., a prior weak measurement with strength p on the sth qubit followed by a quantum reversal measurement with strength $q_r = 1-(1-p)|c_r(t)|^2$ and a phase shift on the rth qubit) and the corresponding averaged success probabilities $P_{av}(p, t)$ for an array of $N = 7$ cavities for QST from the first to the last cavity. As Fig. 2(a) indicates, $F_{av}(0, t)$ (dotted curves) will eventually decay to 0.5 (that is, the 7th qubit ends up at its ground state because of the dissipation caused by the reservoirs). This is in clear contrast to the closed spin chain discussed above where the fidelity keeps on oscillating in time and reaches its maximal value even after a very long time of waiting. Moreover, for the considered cavity array with dissipation, except the first peak, all the subsequent peaks of $F_{av}(0, t)$ are lower than 2/3, which is the highest possible fidelity for classical transmission of a quantum state [49]. The advantage of our scheme is pronounced by the fact that not only all the peaks of $F_{av}(p, t)$ (solid curves in Fig. 2(a)) are always higher than those of $F_{av}(0, t)$, but also their heights can be made all the time higher than 2/3 by choosing a proper (not so close to 1) value of p. For example, even for p as small as 0.1, the averaged fidelity is already greater than 0.8 at any time. The averaged success probabilities $P_{av}(p, t)$ are plotted in Fig. 2(b), from which it follows that $P_{av}(p, t)$ decreases with p for any evolution moments, while for a definite p the oscillations of $P_{av}(p, t)$ in time are in step with that of $F_{av}(0, t)$ (dotted curves in Fig. 2(a)). In the long time limit, $P_{av}(p, t)$ decays to zero implying that the fidelity cannot be improved any more once the destined cavity has decayed to its vacuum state. Therefore, in the presence of dissipation, a considerable improvement of the fidelity with a reasonable success probability can be achieved by choosing a modest p.

4. Conclusion

In conclusion, we have proposed a general scheme to improve the QST by applying two quantum partially collapsing measurements: one is weak measurement, which is performed on the sending qubit at the initial time, and the other one is quantum reversal measurement, which is performed on the receiving qubit at a desired time during the system evolution. Our scheme applies to both closed and open systems, thus, besides improving QST, energy dissipation, if any, is also suppressed. Effectiveness of our scheme depends on the strengths of the two quantum partially collapsing measurements. At any desired time t and for a given strength p of the prior weak measurement, we are able to determine the strength q_r of the post quantum reversal measurement so that QST is best. It is also shown that the performance is better for larger p. In theory, the QST could be near-perfect (i.e., with the fidelity approaching 1) at any time if p is chosen close enough to 1, at the price of vanishingly reduced success probability. The merits of our scheme are demonstrated through two concrete applications, one to a closed chain of spins and the other to an open array of cavities. In both cases, the advantages of our scheme are pronouncedly seen, as compared to the cases under natural evolution.

Finally, we would like to discuss briefly on the practical implementation of weak measurement and quantum reversal measurement. The aim of a weak measurement is to drive the qubit toward its ground state in a coherent way. If the measurement completely collapses the qubit to its ground state, we discard the result. The quantum reversal measurement is in a sense opposite to the weak measurement since it drives the qubit from its ground state toward its excited state. A practical way to implement the quantum reversal measurement includes a bit-flip of the system followed by a weak measurement on it, and finally bit-flip it again. In practice, implementations of the two measurements depend on concrete physical system. A reversal of the weak measurement was demonstrated experimentally in a superconducting phase qubit [31] as well as in a single-photon qubit [32]. Very recently, an experimental recovery of a qubit from partial collapse was demonstrated using a single trapped and laser-cooled $^{40}Ca^{+}$ [33]. By using all-optical apparatuses the decoherence suppression of a single qubit [35] as well as entanglement protection of two qubits [37] via weak measurement and quantum reversal measurement was also achieved experimentally.

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Appendix. Derivation of $c_j(t)$ in Eq. (20)

According to the total Hamiltonian $H_c + H_{int}$, given by Eqs. (18) and (19), the equations of motion for the time-dependent coefficients $c_j(t)$ and $c_{k_j}(t)$, with $j = 1, 2, \ldots, N$, in the evolved state (20) are

$$i\frac{\partial c_j(t)}{\partial t} = \sum_{k_j} g_{k_j} e^{-i(\omega_{k_j}-\omega_0)t} c_{k_j}(t) + J[c_{j-1}(t) + c_{j+1}(t)], \tag{21}$$

$$i\frac{\partial c_{k_j}(t)}{\partial t} = g^*_{k_j} e^{i(\omega_{k_j}-\omega_0)t} c_j(t), \tag{22}$$

with the conventions $c_0(t) = c_{N+1}(t) = 0$. Integrating Eqs. (22) with the initial conditions $c_{k_j}(0) = 0$ and inserting their solutions into Eqs. (21) yield a closed set of integro-differential equations for the amplitudes $c_j(t)$:

$$\frac{\partial c_j(t)}{\partial t} = -\int_0^t dt' \sum_{k_j} |g_{k_j}|^2 e^{-i(\omega_{k_j}-\omega_0)(t-t')} c_j(t') - iJ[c_{j-1}(t) + c_{j+1}(t)]. \tag{23}$$

In the limit of large number of reservoir modes, the sum $\sum_{k_j} |g_{k_j}|^2 e^{-i(\omega_{k_j}-\omega_0)(t-t')}$ can be well approximated by the integration $\int d\omega S(\omega) e^{-i(\omega-\omega_0)(t-t')}$, where $S(\omega)$ is the effective spectral density assumed for definiteness to be Lorentzian, $S(\omega) = (\lambda R^2/\pi)/[(\omega - \omega_c)^2 + \lambda^2]$, with λ being half-width at half-height of the field spectrum profile and R the cavity photon–reservoir coupling. Hence, Eqs. (23) read

$$\frac{\partial c_j(t)}{\partial t} = -\int_0^t dt' \int d\omega S(\omega) e^{-i(\omega-\omega_0)(t-t')} c_j(t') - iJ[c_{j-1}(t) + c_{j+1}(t)], \tag{24}$$

from which we obtain

$$c_j(t) = \sum_m \lim_{\tau \to \tau_m} (\tau - \tau_m) \widetilde{c}_j(\tau) e^{\tau_m t}, \tag{25}$$

where $\widetilde{c}_j(\tau)$ are solutions of the Laplace-transformed Eqs. (24) and τ_m a pole of $\widetilde{c}_j(\tau)$.

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