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On-demand control of coherence transfer between interacting qubits surrounded by a dissipative environment

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On-demand measurement-based schemes to control coherence transfer between interacting qubits inside a dissipative environment are considered. The measurements employed are of the types of quantum weak measurement and quantum reversal measurement of which one is carried out at the beginning of the system evolution $t = 0$ and the other at a later intended time $t > 0$. The relevant questions are which type of measurement should be performed at $t = 0$ and which type at $t > 0$, as well as which strength of the second measurement should be chosen in dependence on the strength of the first measurement to optimally achieve a demand? We answer such questions in this work as to meet two concrete demands, namely, to make optimal (i) the transfer of coherence from the sth qubit to the rth qubit and (ii) the preservation of coherence at the sth qubit. Particularly, independent of the coherence degree of the sth qubit at $t = 0$, the coherence degree of either the rth qubit or the sth qubit at any $t > 0$ can be made arbitrarily close to the maximum value equal to $1/2$, but at the cost of vanishingly reduced success probability. As an application, we discuss the coherence transfer between two qubits of a bipartite system. We also analyze the influence of the controlling schemes for optimal coherence transfer on the degree of entanglement created at $t > 0$ between the two qubits.

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I. INTRODUCTION

According to the quantum mechanics postulates, quantum (strong) measurement cannot be undone: once the wave function of an original state collapses, it is gone forever. However, if the measurement is weak [1,2], it turns out to be possible to resuscitate the measured state, yet with a less-than-one probability [3,4]. Thus, a weak measurement contrasts with an ordinary (strong) projective one in that it is probabilistically reversible because of its unsharpness (i.e., it does not totally collapse the measured system). Hence, after a weak measurement, the measured state could be recovered by a proper action called quantum reversal measurement. The scheme for such a reversal measurement has been demonstrated experimentally for a superconducting phase qubit [5,6] as well as for a single-photon qubit [7]. The idea of combined usage of a prior weak measurement followed by a posterior reversal measurement has been developed to cope with decoherence of a quantum system due to zero-temperature energy relaxation [8] and experimentally demonstrated in an optical system [9]. A prior weak measurement on a qubit before it is exposed to a dissipative environment moves the qubit towards its ground state to strengthen the robustness of the system against the decoherence process. A posterior reversal measurement after the system undergoes the decoherence can, in principle, make the system arbitrarily close to the initial state, but at a cost of decreased probability. As for a bipartite system, the quantum entanglement contained therein serves as a key resource in quantum information processing and quantum computing. Therefore, protection of entanglement from degradation becomes increasingly important. As shown in Ref. [10], the change of the initial entangled state of two qubits due to prior weak measurements on each of them can be exactly recovered with a less-than-one probability by means of proper posterior reversal measurements. However, if the two qubits experience two independent amplitude-damping channels, the reversal measurements alone can only partially improve the damped entanglement under most conditions. In Ref. [11], a scheme using weak measurements before and reversal measurements after the amplitude-damping decoherence is proposed and experimentally realized to protect entanglement. Namely, the scheme in Ref. [11] shows that the decayed entanglement can be greatly enhanced and the sudden vanishing of entanglement can be effectively circumvented. In addition, in Ref. [11], the concept of maximal improvement of entanglement is introduced. Furthermore, an optimal condition for the strength of prior weak measurements, the strength of posterior reversal measurements, and the decoherence time have been established under which the system's entanglement can be maximally improved [12,13].

In this work, we study the possibility of taking advantage of weak and reversal measurements to control the coherence dynamics of N interacting qubits embedded in a common dissipative environment. We design efficient schemes to realize different demands, namely, (i) to maximally transfer the coherence from one to the other qubit and (ii) to maximally preserve the initial coherence of a definite qubit. For each demand, we find the optimal conditions to be satisfied by the strengths of the weak and reversal measurements. As

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a concrete application, we study the control of coherence dynamics in a bipartite system consisting of two qubits (qubit A and qubit B), which corresponds to the case of $N = 2$ of our general schemes. In such a system, one may wish, under certain circumstances, to enhance the transfer of coherence from qubit A to qubit B, while, under other circumstances, to preserve the initial coherence of qubit A.

We structure our paper as follows. Section II consists of two parts. In Sec. II A we present the general scheme to maximally transfer the coherence from one qubit to another one and in Sec. II B we present the general scheme to maximally preserve the coherence of a definite qubit. In Sec. III, we derive the exact solution for a bipartite system surrounded by a dissipative environment using a nonperturbative approach and apply our schemes to this model. The influence of each of the controlling schemes on the bipartite entanglement dynamics is also analyzed. In Sec. IV, the possible experimental realization of the schemes is briefly discussed. Finally, Sec. V is the conclusion.

II. THE CONTROLLING SCHEMES

The model we consider consists of N interacting qubits surrounded by a dissipative environment. The Hamiltonian $\hat{H}$ of the total system is composed of three parts,

$$\hat{H} = \hat{H}_0 + \hat{H}_I + \hat{H}_J, \tag{1}$$

where $\hat{H}_0$, $\hat{H}_I$, and $\hat{H}_J$ represent, respectively, the free Hamiltonian, the qubit-environment interaction and the qubit-qubit interaction. At the initial time $t = 0$, we assume that only one qubit, which we call, without loss of generality, the sth qubit ($s \in \{1,2,\ldots,N\}$), possesses some coherence, i.e., its initial state is $|\psi(0)\rangle_s = c_0(0)|0\rangle_s + c_1(0)|1\rangle_s$, with $|c_0(0)|^2 + |c_1(0)|^2 = 1$, while all the other $N-1$ qubits and the environment E are in their ground states. For convenience, we denote by $|\mathbf{0}\rangle_{S(E)}$ the ground state of the N-qubit system (the multimode environment) and by $|\mathbf{j}\rangle_S$ ($\mathbf{j} = \mathbf{1},\mathbf{2},\ldots,\mathbf{s},\ldots,\mathbf{r},\ldots,\mathbf{N}$) we denote the state in which only qubit j is excited to the state $|1\rangle_j$. The total initial state of the qubit system S and the environment E can then be expressed as $|\Psi(0)\rangle_{SE} = [c_0(0)|\mathbf{0}\rangle_S + c_1(0)|\mathbf{s}\rangle_S] \otimes |\mathbf{0}\rangle_E$. For the above closed qubit-environment system, the total excitation number is conserved. Therefore, the single excitation at the sth qubit can either hop to another qubit j or be transformed to a mode of the environment. In the following, we present the general schemes to realize two tasks, namely, to optimally transfer the coherence from the sth qubit to the rth qubit ($r \neq s$) and to optimally retain the coherence of the sth qubit.

A. Optimal control for coherence transfer

Besides the obvious detrimental effect of the environment, the intermediate qubits are also the obstacles to inhibit the coherence transfer between two sites r and s. In this sense, the initial coherence degree of the sth qubit cannot be recovered at the rth site without external manipulation. However, as will be seen, our scheme allows one not only to retrieve the coherence degree of the sth site at the rth site, but also to make the coherence at the rth site exceed that at the sth site with a finite success probability. Therefore, the merit of our scheme is twofold: one is to fight against the dissipative effects of the environment and the other is to further increase the coherence degree at the rth site over the initial value.

To start our scheme, we perform a weak measurement on the sth qubit with strength p before launching the interaction of the whole system. Mathematically, a weak measurement $\mathcal{W}_X(p)$ with strength p ($0 \leqslant p < 1$) maps the state $|n\rangle_X$ of qubit X as

$$\mathcal{W}_X(p)|n\rangle_X \to \sqrt{(1-p)^n}|n\rangle_X, \quad (n = 0,1). \tag{2}$$

Thus, the weak measurement $\mathcal{W}_s(p)$ transforms the state of qubit s to $|\psi(p,0)\rangle_s = [c_0(0)|0\rangle_s + c_1(0)\sqrt{1-p}|1\rangle_s]/\sqrt{P_1(p)}$ with $P_1(p) = 1 - p|c_1(0)|^2$ the probability of success, defined as the probability of not completely collapsing the state by the measurement. As shown above, the weak measurement drives a qubit towards its ground state, making it more robust in the later decoherence process and thus less coherence of the qubit will be lost to the dissipative environment. Therefore, the first weak measurement plays an important role in the whole scheme to enhance the coherence transfer or preservation. With given strength of the first weak measurement and "strength" (in terms of the evolution time) of dissipation, an optimal strength of the second measurement can be constructed to achieve a definite demand. The initial state of the total system after the weak measurement can be described as

$$|\Psi(p,0)\rangle_{SE} = \frac{1}{\sqrt{P_1(p)}}[c_0(0)|\mathbf{0}\rangle_S + c_1(0)\sqrt{1-p}|\mathbf{s}\rangle_S] \otimes |\mathbf{0}\rangle_E. \tag{3}$$

Governed by the Hamiltonian (1), the state (3) evolves at $t > 0$ into

$$\begin{aligned}|\Psi(p,t)\rangle_{SE} = \frac{1}{\sqrt{P_1(p)}}\Bigg\{\Bigg[&c_0(0)|\mathbf{0}\rangle_S + c_1(0)\sqrt{1-p}\,F_r(t)|\mathbf{r}\rangle_S\\ &+ c_1(0)\sqrt{1-p}\sum_{i(\neq r)=1}^{N} F_i(t)|\mathbf{i}\rangle_S\Bigg] \otimes |\mathbf{0}\rangle_E\\ &+ c_1(0)\sqrt{1-p}\,b(t)|\mathbf{0}\rangle_S \otimes |\bar{\mathbf{1}}\rangle_E\Bigg\},\end{aligned} \tag{4}$$

where $|\bar{\mathbf{1}}\rangle_E$ denotes the collective excited state of the environment, while $F_j(t)$ and $b(t) = \sqrt{1 - \sum_{j=1}^{N}|F_j(t)|^2}$ denote the transition amplitudes of the excitation to the jth qubit and to the environment.

As time goes on, the coherence of the rth qubit becomes nonzero and we are able to manipulate it at any time by applying the second measurement on it. It is cautioned that the type of second measurement is situation dependent: it is either a quantum reversal measurement if $|c_1(0)\sqrt{1-p}\ F_r(t)|^2/P_1(p) < 1/2$ or a weak measurement if $|c_1(0)\sqrt{1-p}\ F_r(t)|^2/P_1(p) \geqslant 1/2$. A quantum reversal measurement $\mathcal{R}_X(p_r)$ with strength p_r ($0 \leqslant p_r < 1$) on the state $|n\rangle_X$ of qubit X corresponds to the map

$$\mathcal{R}_X(p_r)|n\rangle_X \to \sqrt{(1-p_r)^{n\oplus 1}}|n\rangle_X, \tag{5}$$

with $\oplus$ being an addition mod 2. If $|c_1(0)\sqrt{1-p}\ F_r(t)|^2/P_1(p) < 1/2$, the quantum reversal measurement $\mathcal{R}_r(p_r)$ will be performed on the rth qubit, transforming the state (4) to

$$|\Psi_{\rm I}(p,t,p_r)\rangle_{SE} = \frac{1}{\sqrt{P_{\rm I}(p,t,p_r)}}\Bigg\{\Bigg[c_0(0)\sqrt{1-p_r}|\mathbf{0}\rangle_S + c_1(0)\sqrt{1-p}\,F_r(t)|\mathbf{r}\rangle_S + \sum_{i(\neq r)=1}^{N} c_1(0)\sqrt{(1-p)(1-p_r)}\,F_i(t)|\mathbf{i}\rangle_S\Bigg]\otimes|\mathbf{0}\rangle_E + c_1(0)\sqrt{(1-p)(1-p_r)}\,b(t)|\mathbf{0}\rangle_S\otimes|\bar{\mathbf{1}}\rangle_E\Bigg\}, \quad (6)$$

with $P_{\rm I}(p,t,p_r) = (1-p_r)[1-p|c_1(0)|^2] + (1-p)p_r|c_1(0)|^2|F_r(t)|^2$ the success probability of the combined process of both $\mathcal{W}_s(p)$ at $t=0$ and $\mathcal{R}_r(p_r)$ at $t>0$. Otherwise [i.e., if $|c_1(0)\sqrt{1-p}\,F_r(t)|^2/P_1(p) \geqslant 1/2$], the weak measurement $\mathcal{W}_r(p_r)$ will be performed on the rth qubit, which transforms the state (4) to

$$|\Psi_{\rm II}(p,t,p_r)\rangle_{SE} = \frac{1}{\sqrt{P_{\rm II}(p,t,p_r)}}\Bigg\{\Bigg[c_0(0)|\mathbf{0}\rangle_S + c_1(0)\sqrt{(1-p)(1-p_r)}\,F_r(t)|\mathbf{r}\rangle_S + \sum_{i(\neq r)=1}^{N} c_1(0)\sqrt{1-p}\,F_i(t)|\mathbf{i}\rangle_S\Bigg]\otimes|\mathbf{0}\rangle_E + c_1(0)\sqrt{1-p}\,b(t)|\mathbf{0}\rangle_S\otimes|\bar{\mathbf{1}}\rangle_E\Bigg\}, \quad (7)$$

with $P_{\rm II}(p,t,p_r) = 1 - p|c_1(0)|^2 - |c_1(0)|^2(1-p)p_r|F_r(t)|^2$ the success probability of the combined process of both $\mathcal{W}_s(p)$ at $t=0$ and $\mathcal{W}_r(p_r)$ at $t>0$.

The coherence degree of the rth qubit being in the states (6) or (7) can be expressed as

$$C_r^{(\rm I,II)}(p,t,p_r) = \frac{|c_0(0)c_1(0)|\sqrt{(1-p)(1-p_r)}|F_r(t)|}{P_{\rm I,II}(p,t,p_r)}. \quad (8)$$

From the manipulation procedure described above, there are two parameters, p and p_r, that we can manage at will. The optimal strength $p_r^{(\rm I,II)}$ of the second measurement that maximizes the coherence degree of the rth qubit at time t can be derived from the conditions

$$\frac{\partial C_r^{(\rm I,II)}(p,t,p_r)}{\partial p_r}\bigg|_{p_r^{(\rm I,II)}} = 0 \quad \text{and} \quad \frac{\partial^2 C_r^{(\rm I,II)}(p,t,p_r)}{\partial p_r^2}\bigg|_{p_r^{(\rm I,II)}} < 0, \quad (9)$$

from which we obtain

$$p_r^{(\rm I)} = \frac{1-2|c_1(0)|^2|F_r(t)|^2(1-p) - |c_1(0)|^2 p}{1-|c_1(0)|^2|F_r(t)|^2(1-p) - |c_1(0)|^2 p}, \quad (10)$$

and

$$p_r^{(\rm II)} = \frac{2(1-p)|c_1(0)|^2|F_r(t)|^2 + p|c_1(0)|^2 - 1}{(1-p)|c_1(0)|^2|F_r(t)|^2}. \quad (11)$$

The optimal coherence degree of the rth qubit at time t, in accordance with the two optimal conditions (10) and (11), has the same form,

$$C_r^{\rm opt}(p,t) = \frac{|c_0(0)|}{2\sqrt{1-|c_1(0)|^2[(1-p)|F_r(t)|^2+p]}}, \quad (12)$$

but the two corresponding success probabilities differ:

$$P_{\rm I}^{\rm opt}(p,t) = 2|c_1(0)|^2(1-p)|F_r(t)|^2 \quad (13)$$

and

$$P_{\rm II}^{\rm opt}(p,t) = 2[1-p|c_1(0)|^2 - (1-p)|c_1(0)|^2|F_r(t)|^2]. \quad (14)$$

It is interesting to observe from Eq. (12) that regardless of the initial coherence degree of the sth qubit, we always have $\lim_{p\to 1} C_r^{\rm opt}(p,t) = 1/2$, which is the largest possible coherence degree of a superposition state of a two-level system, but the success probability of such ideal performance is tending to zero, as is evident from (13). Nevertheless, this implies that our scheme can provide near maximal transfer of coherence from qubit s to qubit r with a finite success probability.

B. Optimal control for coherence preservation

Different from the previous section, in this section we would like to control the dynamics so that the initial coherence degree of the sth qubit is maintained as much as possible at any desired time during the evolution. Our optimal control scheme again requires two steps of operation: a prior weak measurement on the sth qubit at $t=0$ and a posterior measurement also on the sth qubit (not on the rth qubit as in the previous section) at $t>0$. To achieve the goal, the type of second measurement should be chosen appropriately. As already known from the previous section, after the operation of $\mathcal{W}_s(p)$ at $t=0$, the total system state takes the form (3), which evolves into the form (4) at time $t>0$. Now, at the desired time t, what type of second measurement is to be performed depends on the sign of $|c_1(0)\sqrt{1-p}F_s(t)|^2/P_1(p) - 1/2$ [not $|c_1(0)\sqrt{1-p}F_r(t)|^2/P_1(p) - 1/2$ as in the previous section]. Namely, if $|c_1(0)\sqrt{1-p}F_s(t)|^2/P_1(p) - 1/2 < 0$, then $\mathcal{R}_s(p_r)$ is the right choice, otherwise $\mathcal{W}_s(p_r)$ should be chosen. Indeed, the coherence preservation of the sth qubit can be thought of as the coherence transfer discussed in Sec. II A with $r=s$. Therefore, the formula of the optimal coherence degree of qubit s is the same as that in Eq. (12), but with a replacement of $F_r(t)$ with $F_s(t)$:

$$C_s^{\rm opt}(p,t) = \frac{|c_0(0)|}{2\sqrt{1-|c_1(0)|^2[(1-p)|F_s(t)|^2+p]}}. \quad (15)$$

The corresponding success probabilities, denoted as $P_{\rm III}^{\rm opt}(p,t)$ and $P_{\rm IV}^{\rm opt}(p,t)$ with respect to the two types of second measurements, can be constructed in the same way from (13) and (14) as

$$P_{\mathrm{III}}^{\mathrm{opt}}(p,t) = 2|c_1(0)|^2(1-p)|F_s(t)|^2 \tag{16}$$

and

$$P_{\mathrm{IV}}^{\mathrm{opt}}(p,t) = 2[1 - p|c_1(0)|^2 - (1-p)|c_1(0)|^2|F_s(t)|^2]. \tag{17}$$

Quite surprisingly, as seen from Eq. (15), we could make the coherence degree of the sth qubit at $t > 0$ to be almost maximum (i.e., almost equal to 1/2) regardless of its value at $t = 0$ by using p very close to one. Of course, this would succeed only with a vanishingly small probability.

III. APPLICATION TO THE BIPARTITE SYSTEM

So far we have presented the general controlling schemes without specifying the concrete physical models of the qubits' system and the environment. In this section, we apply the schemes to a system of two qubits to manipulate the coherence transfer therein. The system may be modeled as two two-level atoms A and B coupled to each other by a dipole-dipole-like interaction: qubit A can formally be regarded as the donor of the coherence and qubit B can be regarded as the acceptor. In practice, we can consider Rydberg atoms that possess clean two levels and interact via dipolar couplings. The environment surrounding the two qubits is modeled as a bath of harmonic oscillators with a specified spectral density. The three parts of the Hamiltonian in (1) are given as follows ($\hbar = 1$). The first part,

$$\hat{H}_0 = \omega_A \hat{\sigma}_+^{(A)}\hat{\sigma}_-^{(A)} + \omega_B \hat{\sigma}_+^{(B)}\hat{\sigma}_-^{(B)} + \sum_k \omega_k \hat{a}_k^+ \hat{a}_k, \tag{18}$$

with $\hat{\sigma}_\pm^{(X)}(\omega_X)$ being the inversion operators (transition frequencies) of the qubit $X = A,B$ and $\hat{a}_k$ ($\hat{a}_k^\dagger$) being the annihilation (creation) operator for the bath's kth mode with frequency ω_k, is the free Hamiltonian. The second part,

$$\hat{H}_{\mathrm{I}} = \left(\alpha_A \hat{\sigma}_+^{(A)} + \alpha_B \hat{\sigma}_+^{(B)}\right) \sum_k g_k \hat{a}_k + \mathrm{H.c.}, \tag{19}$$

describes the qubit-environment interaction with constant couplings g_k. As the interaction of a qubit to the environment depends on the value of the bath field at the qubit's position, a dimensionless real constant α_X is introduced to individualize the qubit: the actual coupling strength between the Xth qubit and the kth mode is thus characterized by $\alpha_X|g_k|$. For simplicity, we assume that α_X are real numbers. Finally, the third part,

$$\hat{H}_J = J\left(\hat{\sigma}_+^{(A)}\hat{\sigma}_-^{(B)} + \hat{\sigma}_-^{(A)}\hat{\sigma}_+^{(B)}\right), \tag{20}$$

with a constant (real) coupling J, is the interaction Hamiltonian between the qubits.

At the initial time $t = 0$, we assume that only the donor qubit A possesses some degree of coherence, i.e., its initial state is $|\psi(0)\rangle_A = c_0(0)|0\rangle_A + c_1(0)|1\rangle_A$, with $|c_0(0)|^2 + |c_1(0)|^2 = 1$, while the acceptor qubit B and the environment E are in their ground states $|0\rangle_B$ and $|\mathbf{0}\rangle_E = \otimes_k |0_k\rangle_E$, respectively. Governed by the Hamiltonians (18)–(20), the initial state $|\Psi(0)\rangle_{ABE} = c_0(0)|00\mathbf{0}\rangle_{ABE} + c_1(0)|10\mathbf{0}\rangle_{ABE}$ of the total system evolves at $t > 0$ into

$$\begin{aligned} |\Psi(t)\rangle_{ABE} = {} & [c_0(0)|00\rangle_{AB} + c_1(0)F_A(t)|10\rangle_{AB} \\ & + c_1(0)F_B(t)|01\rangle_{AB}]|\mathbf{0}\rangle_E \\ & + \sum_k c_1(0)b_k(t)|001_k\rangle_{ABE}, \end{aligned} \tag{21}$$

where $|1_k\rangle$ denotes the bath state with one excitation in mode k. The exact solution of the system evolution can be derived via the formal procedure [14]. In the interaction picture, we obtain the motion equations for the involved coefficients as

$$\frac{\partial F_A(t)}{\partial t} = -i\alpha_A \sum_k g_k e^{-i(\omega_k-\omega_0)t} b_k(t) - iJF_B(t), \tag{22}$$

$$\frac{\partial F_B(t)}{\partial t} = -i\alpha_B \sum_k g_k e^{-i(\omega_k-\omega_0)t} b_k(t) - iJF_A(t), \tag{23}$$

$$\frac{\partial b_k(t)}{\partial t} = -ig_k^* e^{i(\omega_k-\omega_0)t}[\alpha_A F_A(t) + \alpha_B F_B(t)], \tag{24}$$

where we assumed $\omega_A = \omega_B = \omega_0$ for simplicity. By integrating Eq. (24) with the initial condition $b_k(0) = 0$ and inserting its solution into Eqs. (22) and (23), we obtain two integro-differential equations for $F_A(t)$ and $F_B(t)$ as

$$\begin{aligned} \frac{\partial F_A(t)}{\partial t} = {} & -\int_0^t dt' \sum_k |g_k|^2 e^{-i(\omega_k-\omega_0)(t-t')} \\ & \times \left[\alpha_A^2 F_A(t') + \alpha_A\alpha_B F_B(t')\right] - iJF_B(t), \end{aligned} \tag{25}$$

$$\begin{aligned} \frac{\partial F_B(t)}{\partial t} = {} & -\int_0^t dt' \sum_k |g_k|^2 e^{-i(\omega_k-\omega_0)(t-t')} \\ & \times \left[\alpha_A\alpha_B F_A(t') + \alpha_B^2 F_B(t')\right] - iJF_A(t). \end{aligned} \tag{26}$$

In Eqs. (25) and (26), the sum $\sum_k |g_k|^2 e^{-i(\omega_k-\omega_0)(t-t')}$ is recognized as the bath correlation function $f(t-t') = {}_E\langle\bar{\mathbf{0}}|A(t)A^\dagger(t')|\bar{\mathbf{0}}\rangle_E$, with $A(t) = \sum_k g_k b_k e^{-i(\omega_k-\omega_0)t}$. In the large-number-of-mode limit, that sum can be replaced by an integral $\int d\omega S(\omega) e^{-i(\omega-\omega_0)(t-t')}$, where $S(\omega)$ is referred to as the bath effective spectral density. Then, Eqs. (25) and (26) become

$$\begin{aligned} \frac{\partial F_A(t)}{\partial t} = {} & -\int_0^t dt' f(t-t')\left[\alpha_A^2 F_A(t') + \alpha_A\alpha_B F_B(t')\right] \\ & - iJF_B(t), \end{aligned} \tag{27}$$

$$\begin{aligned} \frac{\partial F_B(t)}{\partial t} = {} & -\int_0^t dt' f(t-t')\left[\alpha_A\alpha_B F_A(t') + \alpha_B^2 F_B(t')\right] \\ & - iJF_A(t). \end{aligned} \tag{28}$$

To specify the structure of the bath, we consider a Lorentzian spectral density of the form [15–17]

$$S(\omega) = \frac{W^2}{\pi} \frac{\lambda}{(\omega_0 - \delta - \omega)^2 + \lambda^2}, \tag{29}$$

where λ is half width at half height of the bath field spectrum profile, W measures the qubit-environment coupling, and δ is the detuning between the bath center frequency ω_c and the qubit transition frequency ω_0. Taking the Laplace transform of both sides of Eqs. (27) and (28), with $S(\omega)$ given in (29), we obtain

$$\widetilde{F}_A(l) = \frac{\left[l(l+\lambda-i\delta)+R^2\beta_B^2\right]}{(l^2+J^2)(l+\lambda-i\delta)+R^2 l-2iR^2\beta_A\beta_B J}, \quad (30)$$

$$\widetilde{F}_B(l) = \frac{-[R^2\beta_A\beta_B+iJ(l+\lambda-i\delta)]}{(l^2+J^2)(l+\lambda-i\delta)+R^2 l-2iR^2\beta_A\beta_B J}, \quad (31)$$

where $\widetilde{F}_{A(B)}(l)$ is the Laplace transform of $F_{A(B)}(t)$, $\alpha_T = \sqrt{\alpha_A^2+\alpha_B^2}$, $\beta_{A(B)} = \alpha_{A(B)}/\alpha_T$, and $R = W\alpha_T$. Note that $\beta_A^2 + \beta_B^2 = 1$, by definition. Inverse Laplace transforming Eqs. (30) and (31) yields the explicit time-dependent solutions:

$$F_{A(B)}(t) = \sum_m \lim_{l\to l_m}(l-l_m)\widetilde{F}_{A(B)}(l)e^{l_m t}, \quad (32)$$

with l_m being an mth pole of $\widetilde{F}_{A(B)}(l)$. In particular, for $\beta_A = \beta_B = 1/\sqrt{2}$, we can derive exact analytical expressions for $F_{A(B)}(t)$ as

$$F_A(t) = \tfrac{1}{2}e^{iJt}[Q(t)+1], \quad (33)$$

$$F_B(t) = \tfrac{1}{2}e^{iJt}[Q(t)-1], \quad (34)$$

where

$$Q(t) = e^{-(\lambda-i\delta+3iJ)t/2}\left[\cosh(\Omega t/2) + \frac{\lambda-i\delta-iJ}{\Omega}\sinh(\Omega t/2)\right], \quad (35)$$

with $\Omega = \sqrt{(\lambda-i\delta-iJ)^2-4R^2}$.

Without any controls, the degree of coherence of qubit A (B) can be characterized by $C_A(t) = |c_0(0)c_1(0)F_A(t)|$ $[C_B(t) = |c_0(0)c_1(0)F_B(t)|]$. The entanglement amount between qubits A and B can be assessed by Wootters's concurrence [18] as $E_{AB}(t) = 2|c_1^2(0)F_A(t)F_B(t)|$. From Eq. (35), we see that $\lim_{t\to\infty} Q(t) = 0$. Hence, for the case of $\beta_A = \beta_B = 1/\sqrt{2}$, we have $C_A(t\to\infty) = C_B(t\to\infty) = \frac{1}{2}|c_0(0)c_1(0)|$, implying a balanced distribution of the initial coherence of qubit A among qubits A and B in the long-time limit, while the steady entanglement takes the form $E_{AB}(t\to\infty) = \frac{1}{2}|c_1(0)|^2$. In other words, one-half of the initial coherence of the donor qubit will be transferred to the acceptor qubit eventually, while another half is trapped in the donor qubit. As for $\beta_A \neq \beta_B$, it can be verified numerically that both $|F_A(t)|$ and $|F_B(t)|$ will decay to zero, implying that the coherence of both qubits A and B as well as their entanglement will eventually vanish.

We first exhibit the optimal control of the coherence transfer from the donor qubit A to the acceptor qubit B with our scheme described in Sec. II A. The optimal coherence degree $C_B^{\text{opt}}(p,t)$ of qubit B and the corresponding success probabilities are formally given in Eqs. (12)–(14) by setting $s = A$ and $r = B$. In Fig. 1, we plot the time dependence of the coherence degree of qubit B and the success probabilities resulting from our control scheme. Figures 1(a) and 1(b) demonstrate that at any desired time during the evolution, the coherence degree of qubit B, i.e., $C_B^{\text{opt}}(p,t)$, is pronouncedly increased by our scheme as compared to the situation without any controls ($p = p_r = 0$), in both the weak and strong system-environment coupling regimes. Also pronounced is the effect that $C_B^{\text{opt}}(p,t)$ is getting larger for increasing p. Because of the presence of interaction between qubits A and B dictated by $\hat{H}_J$ in Eq. (20), the dynamics experiences damped oscillation even in the weak system-environment coupling regime [cf. Fig. 1(a)]. The oscillation amplitude is reduced when p is bigger since the "stronger weak" measurement preserves a stronger coherence at qubit B and inhibits the coherence exchange between the two qubits. As for the strong-coupling regime, additional oscillation caused by environmental memory effect comes into play, which interferes with that due to qubit-qubit interaction, resulting in irregular damped oscillations, as seen from Fig. 1(b). Interestingly, the oscillations' interfering in the strong-coupling regime makes the saturation time much shorter than that in the weak-coupling regime. Figures 1(c) and 1(d) show the corresponding success probabilities. Generally, the probability decreases with increasing p and tends to zero in the limit of $p\to 1$. It is worthwhile to note that the probability exhibits large-amplitude transient oscillations before reaching the steady value. In fact, the coherence is exchanged between the two qubits during the evolution and, therefore, if at the time we perform the second control action the coherence is mostly located at qubit B, then the probability of successfully enhancing the coherence degree of qubit B is certainly large, and vice versa. This can be confirmed from two factors. The first one is the consistency of the oscillations of coherence and the success probability [cf. the one-to-one correspondence between the peaks and valleys in Figs. 1(a) and 1(c) as well as in Figs. 1(b) and 1(d)]. The second one is that the oscillation amplitude of the success probability shrinks for a larger p, as in this case the coherence exchange is inhibited. In the inset of Fig. 1(c), we show the alternation of the manner of the second measurement: for a small prior weak measurement strength $p = 0.1$, the situation of $|c_1(0)\sqrt{1-p}F_B(t)|^2/P_1(p) \geqslant 1/2$ occurs just for an initial short period of time, during which the second measurement should be a weak measurement instead of a reversal measurement.

Intuitively, if the coherence degree of qubit B increases, then the coherence degree of qubit A would decrease. To see whether it is true under our controlling actions, we derive the expression for the coherence degree of qubit A when the optimal conditions of $p_r^{(\mathrm{I})}$ in Eq. (10) and $p_r^{(\mathrm{II})}$ in Eq. (11) are satisfied, which has the form

$$C_A(p,t) = \frac{|c_0(0)c_1(0)||F_A(t)|\sqrt{1-p}}{2-2|c_1(0)|^2[|F_B(t)|^2(1-p)+p]}. \quad (36)$$

From this expression, it follows $\lim_{p\to 1} C_A(p,t) = 0$, which is consistent with $\lim_{p\to 1} C_B^{\text{opt}}(p,t) = 1/2$. We plot $C_A(p,t)$ versus λt and p in Figs. 2(a) and 2(b). Generally, the coherence degree of qubit A is largely diminished compared to the situation with no controls.

Although controlling entanglement is not our purpose in this work, one is wondering about how the optimal control for $A\to B$ coherence transfer affects the entanglement between the two qubits as compared to the case without any controls. In terms of concurrence, the entanglement amount between qubits A and B with respect to the two types of posterior

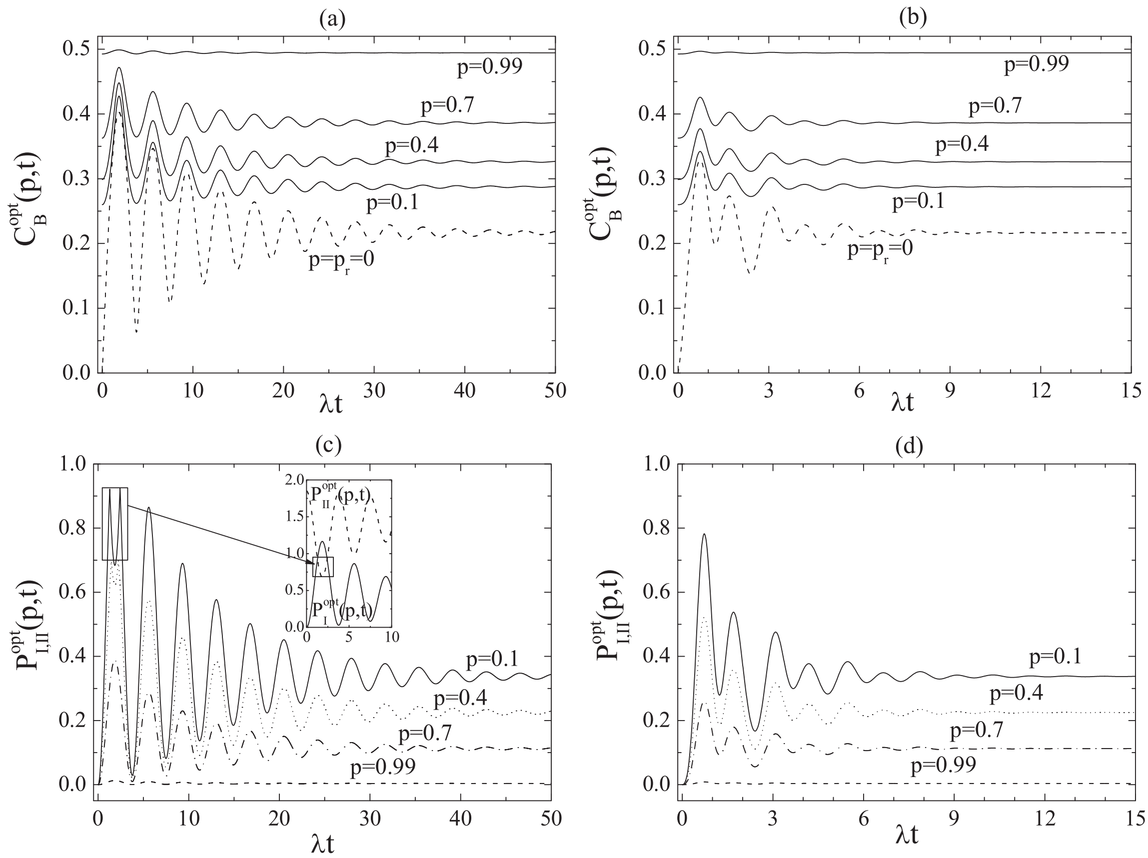

FIG. 1. The optimal coherence degree of the acceptor qubit B, $C_B^{\rm opt}(p,t)$ in Eq. (12), and the corresponding success probability vs scaled time λt for various strengthes p of the null-outcome prior weak measurement on the donor qubit A in the (a),(c) weak qubit-environment coupling regime with $R/\lambda = 0.4$ and (b),(d) strong-coupling regime with $R/\lambda = 4$. The other parameters are chosen as $c_0(0) = 1/2$, $c_1(0) = \sqrt{3}/2$, $J/\lambda = 0.8$, $\beta_A = \beta_B = 1/\sqrt{2}$, and $\delta = 0$.

measurements is

$$E_{AB}^{\rm (I,II)}(p,t,p_r) = \frac{2|c_1(0)|^2(1-p)\sqrt{1-p_r}|F_A(t)F_B(t)|}{P_{\rm I,II}(p,t,p_r)}, \tag{37}$$

which depends explicitly on t, p, and p_r. Replacing p_r by $p_r^{\rm (I)}$ or $p_r^{\rm (II)}$ in Eq. (37) yields a single formula,

$$E_{AB}^{\rm opt}(p,t) = \frac{|F_A(t)||c_1(0)|\sqrt{1-p}}{\sqrt{1-[|F_B(t)|^2(1-p)+p]|c_1(0)|^2}}, \tag{38}$$

that depends only on t and p. Note that the superscript "opt" in $E_{AB}^{\rm opt}(p,t)$ just indicates the optimal control for $A \to B$ coherence transfer, but not the optimal control for entanglement itself. Figures 2(c) and 2(d) show that, compared to the case without controlling (i.e., $p = p_r = 0$), the A-B entanglement amount may be larger for a small value of p or smaller for a large value of p. Nevertheless, as can be expected, in the limit of $p \to 1$, the entanglement $E_{AB}^{\rm opt}(p,t)$ should tend to zero, consistent with the behaviors of coherence degrees of qubits A and B in this limit.

Next, we would like to demonstrate the control scheme described in Sec. II B, by which the coherence degree of qubit A is maintained as much as possible at any desired time during the evolution. Figures 3(a) and 3(b) reveal that, by our method of optimal controlling, the coherence degree of qubit A becomes larger than that without any controls (i.e., $p = p_r = 0$) at any desired time t, for any $p > 0$ and any ratio R/λ. More concretely, at a given time t and for a given value of R/λ, the coherence degree of qubit A increases with p. Concerning the coherence degree of qubit B [i.e., $C_B(p,t)$] and the A-B entanglement [i.e., $E_{AB}^{\rm (III,IV)}(p,t,p_r)$] under the conditions of the optimal strengths of posterior measurements, we have

$$C_B(p,t) = \frac{|c_0(0)c_1(0)||F_B(t)|\sqrt{1-p}}{2-2|c_1(0)|^2[|F_A(t)|^2(1-p)+p]} \tag{39}$$

and

$$E_{AB}^{\rm opt}(p,t) = \frac{|F_B(t)||c_1(0)|\sqrt{1-p}}{\sqrt{1-[|F_A(t)|^2(1-p)+p]|c_1(0)|^2}}. \tag{40}$$

As followed from Eq. (39), $\lim_{p\to 1} C_B(p,t) = 0$, implying that in the limit $p \to 1$, the coherence degree of qubit A is approaching the maximum value 1/2, which will be trapped in qubit A without being transferred to qubit B. As for the amount of A-B entanglement, measured by the concurrence $E_{AB}^{\rm opt}(p,t)$ in Eq. (40), it may be greater or smaller than that without controls depending on p, similar to the situation discussed in the previous section.

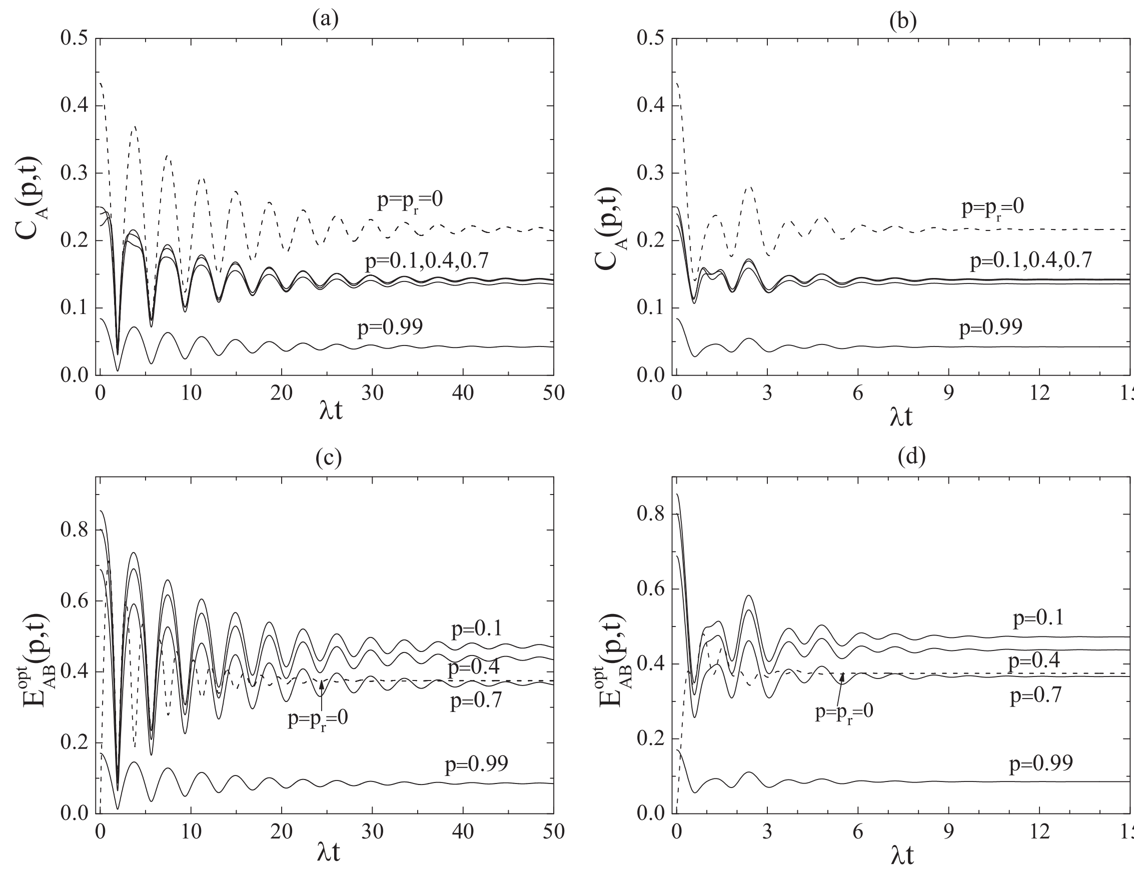

FIG. 2. The same as in Fig. 1, but for the coherence degree of the donor qubit A, $C_A(p,t)$ in Eq. (36) and the A-B entanglement amount $E_{AB}^{\rm opt}(p,t)$ in Eq. (38), under the condition for optimal control of coherence transfer.

IV. EXPERIMENTAL REALIZATION

In this section, we would like to briefly review the possible experimental realization of our schemes. Indeed, the two main components here, i.e., the weak measurement and the quantum reversal measurement, have been realized in various contexts. A reversal of the weak measurement was demonstrated experimentally in a superconducting phase qubit [5,6] as well as in a single-photon qubit [7]. Very recently, an experimental recovery of a qubit from partial collapse was demonstrated using a single trapped and laser-cooled $^{40}Ca^+$ [19]. The demonstration features a qubit implementation that permits both partial collapse (weak measurement) and coherent manipulation with high fidelity [19]. In this experiment [19], the initial states were prepared by first optically pumping to $|0\rangle$, then applying a $\pi/2$ or π pulse, when necessary, to the radio-frequency coil. They tested a set of four different initial states and employed quantum process tomography

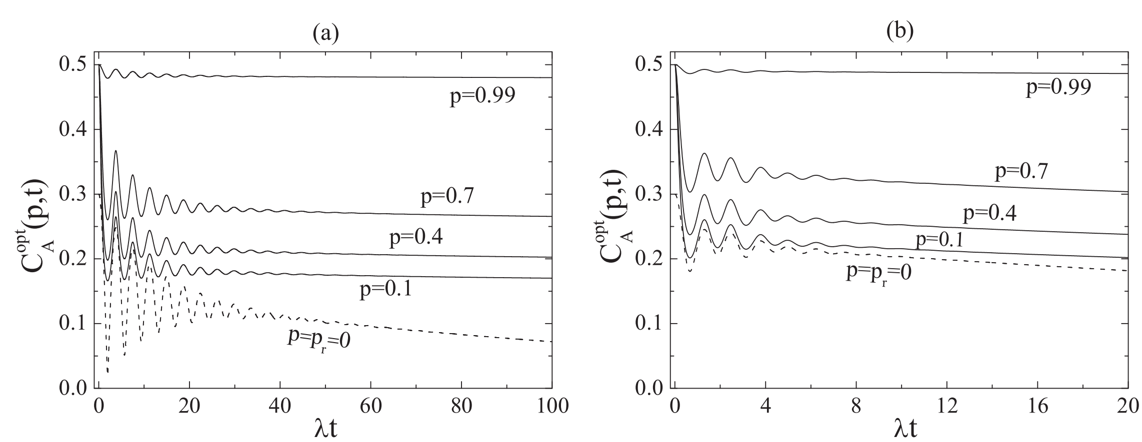

FIG. 3. The optimal coherence degree of the donor qubit, $C_A^{\rm opt}(p,t)$ in Eq. (15) vs scaled time λt for various p in the (a) weak and (b) strong system-environment coupling regimes with $R/\lambda = 0.4$ and $R/\lambda = 4$, respectively. The other parameters are chosen as $c_0(0) = 1/\sqrt{10}$, $c_1(0) = 3/\sqrt{10}$, $J/\lambda = 0.8$, $\beta_A = 1/2$, $\beta_B = \sqrt{3}/2$, and $\delta = 0$.

[1,20] protocol to fully characterize state preparation, the weak measurement, and the reversal process [19]. By using all-optical apparatuses, the decoherence suppression of a single qubit via weak measurement and quantum reversal measurement was also achieved experimentally in Ref. [9]. The protection of entanglement of two qubits from independent decoherences via the combined weak measurements and reversal measurements was realized also by exploiting all-optical apparatuses [11].

As for the considered bipartite system, the two qubits interact with the same environment; therefore, the contexts of cavity QED experiments with trapped ions and circuit QED experiments would be more suitable. Field coupling and coherent quantum state storage between two Josephson phase qubits were realized via a microwave cavity on a chip [21,22]. Therefore, within current technologies, our schemes could be realized, say, for solid-state or superconducting phase qubits [5]. For the phase qubit, the weak measurement can be implemented by allowing the state $|1\rangle$ to tunnel out of the quantum well with the probability p, while the state $|0\rangle$ cannot tunnel out [6]. As for the quantum reversal measurement with the mathematical expression in Eq. (5), it is straightforward to verify that $\sqrt{(1-p_r)^{n\oplus 1}}|n\rangle_X \equiv \text{NOT}\cdot\mathcal{W}_X(p_r)\cdot\text{NOT}|n\rangle_X$, with NOT the NOT gate and $\mathcal{W}_X(p_r)$ the weak measurement on X with strength p_r. This identity suggests a practical way to implement a $\mathcal{R}_X(p_r)$: first bit-flip the X system, then perform a $\mathcal{W}_X(p_r)$ on it, and finally bit-flip it again. For the phase qubit, this process consists of a π pulse, second weak measurement, and one more π pulse.

V. CONCLUSION

In conclusion, we proposed two general schemes to control the coherence dynamics of N interacting qubits embedded in a dissipative environment by means of suitably choosing a combination of quantum weak measurement and quantum reversal measurement. We are interested in the situation when at $t=0$, only the sth qubit possesses some degree of coherence. While the first scheme aims at optimal transfer of coherence from the sth qubit to the rth qubit, the aim of the second scheme is to preserve the coherence of the sth qubit. In both of the schemes, two sequential measurements are to be performed. The first measurement is a weak measurement with strength p to be performed on the sth qubit at the time ($t=0$) the system starts to evolve. Later, at a desired moment $t>0$ during evolution, the second measurement with strength p_r is to be carried out on the rth (sth) qubit in the first (second) scheme. However, the second measurement should be chosen either as a quantum reversal measurement or as a weak measurement depending on the system's state at the desired time t. For a given strength p of the prior weak measurement, we established the explicit conditions for the second measurement strengths p_r to achieve an optimal on-demand control mentioned above. In theory, when p_r satisfies the established condition, the coherence degree of the rth (sth) qubit at the desired time t in the first (second) scheme can be made arbitrarily close to $1/2$ by choosing the strength p of the first weak measurement close enough to 1. However, as a rule, in the limit of $p\rightarrow 1$, the success probability is tending to zero. As an application, we apply our general schemes to the simplest case of $N=2$, i.e., the case with two qubits embedded in a dissipative environment modeled as a structured bath with Lorentzian spectral density. We also examined the influence of the controlling schemes on the entanglement dynamics of the pair of qubits. We found out that depending on the value of the prior weak measurement strength p, the entanglement degree can be increased or decreased in comparison to the situation without any controls. Of course, when $p\rightarrow 1$, the entanglement is shown to vanish, consistent with the individual qubit's coherence behavior in this limit. Within current technology, our control schemes would be applicable in several experimental settings including cavity QED, circuit QED, superconducting rings, etc. Furthermore, we suggest that it might be possible to apply them also in artificial molecular systems when quantum coherence turns out to play an important role [23–28]. For example, a photo cell was proposed [28] based on the nanoscale architecture of photosynthetic reaction centers that explicitly harnesses the quantum coherence. Whether the setup of our schemes could be adopted to preserve the coherence (or facilitate its transfer) of the interested system in the dissipative environment deserves further investigation.

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