Phenomenology of the 3-3-1-1 model

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In this work we discuss a new $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X \otimes \mathrm{U}(1)_N$ (3-3-1-1) gauge model that overhauls the theoretical and phenomenological aspects of the known 3-3-1 models. Additionally, we derive the outcome of the 3-3-1-1 model from precise electroweak bounds to dark matter observables. We firstly advocate that if the $B-L$ number is conserved as the electric charge, the extension of the standard model gauge symmetry to the 3-3-1-1 one provides a minimal, self-contained framework that unifies all the weak, electromagnetic, and $B-L$ interactions, apart from the strong interaction. The W parity (similar to the R parity) arises as a remnant subgroup of the broken 3-3-1-1 symmetry. The mass spectra of the scalar and gauge sectors are diagonalized when the scale of the 3-3-1-1 breaking is compatible to that of the ordinary 3-3-1 breaking. All the interactions of the gauge bosons with the fermions and scalars are obtained. The standard model Higgs (H) and gauge (Z) bosons are realized at the weak scales with consistent masses despite their respective mixings with the heavier particles. The 3-3-1-1 model provides two forms of dark matter that are stabilized by W-parity conservation: one fermion which may be either a Majorana or Dirac fermion, and one complex scalar. We conclude that in the fermion dark matter setup the Z_2 gauge-boson resonance sets the dark matter observables, whereas in the scalar one the Higgs portal dictates them. The standard model Glashow-Iliopoulos-Maiani mechanism works in the model because of W-parity conservation. Hence, the dangerous flavor-changing neutral currents due to the ordinary and exotic quark mixing are suppressed, while those coming from the nonuniversal couplings of the Z_2 and Z_N gauge bosons are easily evaded. Indeed, the $K^0-\bar{K}^0$ and $B_s^0-\bar{B}_s^0$ mixings limit $m_{Z_{2,N}} > 2.037$ TeV and $m_{Z_{2,N}} > 2.291$ TeV, respectively, while the LEPII searches provide a rather close bound, $m_{Z_{2,N}} > 2.737$ TeV. The violation of Cabibbo-Kobayashi-Maskawa unitarity due to the loop effects of the Z_2 and Z_N gauge bosons is negligible.

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I. INTRODUCTION

The standard model [1] has been extremely successful. However, it describes only about 5% of the mass-energy density of our Universe. What remains is roughly 25% dark matter and 70% dark energy, which lies beyond the standard model. In addition, the standard model cannot explain the nonzero small masses and mixing of the neutrinos, the matter-antimatter asymmetry of the Universe, and the inflationary expansion of the early Universe. On the theoretical side, the standard model cannot show how the Higgs mass is stabilized against radiative corrections, what makes electric charge exist in discrete amounts, and why there are only three generations of fermions observed in nature.

Among the standard model's extensions that attempt to address these issues, the recently proposed $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X \otimes \mathrm{U}(1)_N$ (3-3-1-1) gauge model has the following interesting features [2]. (i) The theory arises as a necessary consequence of the 3-3-1 models [3–5] that respects the conservation of lepton and baryon numbers. (ii) The $B-L$ number is naturally gauged because it is a combination of the $\mathrm{SU}(3)_L$ and $\mathrm{U}(1)_N$ charges. And, the resulting theory yields a unification of the electroweak and $B-L$ interactions, apart from the strong interaction. (iii) The right-handed neutrinos emerge as fundamental fermion constituents, and consequently the small masses of the active neutrinos are generated by the type I seesaw mechanism. (iv) W parity—which has a similar form to the R parity in supersymmetry—naturally arises as a conserved remnant subgroup of the broken 3-3-1-1 gauge symmetry. (v) Dark matter automatically exists in the model and is stabilized due to W parity. It is the lightest particle among

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the new particles which characteristically have incorrect lepton numbers, transforming as odd fields under W parity (so-called W particles). The dark matter candidate may be a neutral fermion (N) or a neutral complex scalar (H').

The 3-3-1-1 model includes all the good features of the 3-3-1 models. The number of fermion families is just three, a consequence of anomaly cancellation and the QCD asymptotic freedom condition [6]. The third quark generation transforms differently under $\mathrm{SU}(3)_L$ than the first two. This explains why the top quark is uncharacteristically heavy [7]. The strong CP problem is solved by just its particle content with an appropriate Peccei-Quinn symmetry [8]. The electric charge quantization is due to a special structure of the gauge symmetry and the fermion content [9]. Additionally, the model also provides the mentioned dark matter candidates similarly to Refs. [10,11]. The 3-3-1-1 model can solve the potential issues of the 3-3-1 models because the unwanted interactions and vacuums that lead to the dangerous tree-level flavor-changing neutral currents (FCNCs) [12] as well as the CPT violation [13] are all suppressed due to W-parity conservation [2].

In a previous paper [2] the 3-3-1-1 model and its direct consequence—dark matter—was proposed. In the current work, we will deliver a detailed study of this new model. Particularly, we consider the new physics consequences besides the dark matter that are implied by the new extended sectors beyond those of the 3-3-1 model. These sectors include the new neutral gauge boson (C) associated with $\mathrm{U}(1)_N$, and the new scalar (ϕ) required for the total $\mathrm{U}(1)_N$ breaking with necessary mass generations. The total $\mathrm{U}(1)_N$ breaking that consequently breaks the $B-L$ symmetry—where $B-L$ is a residual charge related to the N charge and a $\mathrm{SU}(3)_L$ generator—can happen close to the 3-3-1 breaking scale of order TeV. This leads to a finite mixing and an interesting interplay between the new neutral gauge bosons—such as the Z' of the 3-3-1 model—and the C of $\mathrm{U}(1)_N$. Notice that our previous work only considered a special case when the $B-L$ breaking scale was very high [similar to the grand unified theory (GUT) scale] [14], so that the new physics beyond the ordinary 3-3-1 symmetry was decoupled, neglecting its imprint at low energy [2]. Indeed, the stability of the proton is already ensured by the 3-3-1-1 gauge symmetry; there is no reason why this scale is not present at the 3-3-1 scale. Similarly to the new neutral gauge bosons, there is an interesting mixing among the new neutral scalars that is used to break the 3-3-1 and $B-L$ symmetries.

It is interesting to note that the new scalars and new gauge bosons as well as the new fermions can give significant contributions to the production and decay of the standard model Higgs boson. They might also modify the well-measured standard model couplings, such as those of the photon and W and Z bosons with the fermions. There exist hadronic FCNCs due to the contribution of the new neutral gauge bosons. These gauge bosons can also take part in electron-positron collisions [such as those at LEPII and the International Linear Collider (ILC)] as well as in dark matter observables. The presence of the new neutral gauge bosons also induces the apparent violation of Cabibbo-Kobayashi-Maskawa (CKM) unitarity. In some cases, the new scalar responsible for the $\mathrm{U}(1)_N$ breaking may act as an inflaton. The decays of some new particles can solve the matter-antimatter asymmetry via leptogenesis mechanisms.

The scope of this work is as follows. The 3-3-1-1 model is calculated in detail, namely, the scalar potential and the gauge-boson sector are in a general case diagonalized. All the interactions of the gauge bosons with the fermions as well as with the scalars are derived. The new physics processes arising from the FCNCs, the LEPII Collider, the violation of CKM unitarity, and dark matter observables are analyzed. Particularly, we perform a phenomenological study of the dark matter, taking into account the current data as well as the new contributions of the physics at $\Lambda \sim \omega$ that were seen in Ref. [2]. The constraints on the new gauge-boson and dark matter masses are also obtained.

The rest of this work is organized as follows. In Sec. II, we give a review of the model. Sections III and IV are devoted, respectively, to the scalar and gauge sectors. In Sec. V we obtain all the gauge interactions of the fermions and scalars. Section VI aims at studying the new physics processes and constraints. Finally, we summarize our results and make concluding remarks in Sec. VII.

II. A REVIEW OF THE 3-3-1-1 MODEL

The 3-3-1-1 model [2] is based on the gauge symmetry

$$\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X \otimes \mathrm{U}(1)_N, \qquad (1)$$

where the first three groups are the ordinary gauge symmetry of the 3-3-1 models [3–5], while the last one is a necessary gauge extension of the 3-3-1 models that respects the conservation of lepton (L) and baryon (B) numbers. Indeed, the 3-3-1 and $B-L$ symmetries do not commute and are not algebraically closed. To be concrete, for a lepton triplet (see below) we have $B-L = \mathrm{diag}(-1,-1,0)$, which does not commute with the $\mathrm{SU}(3)_L$ generators as $T_i = \frac{1}{2}\lambda_i$ for $i = 4,5,6,7$. It is easily checked that

$$[B-L, T_4 \pm iT_5] = \mp(T_4 \pm iT_5) \neq 0,$$
$$[B-L, T_6 \pm iT_7] = \mp(T_6 \pm iT_7) \neq 0.$$

The nonclosed algebras can be deduced from the fact that in order for $B-L$ to be some generator of $\mathrm{SU}(3)_L$, we must have a linear combination $B-L = x_i T_i$ ($i = 1,2,3,\ldots,8$) and thus $\mathrm{Tr}(B-L) = 0$, which is invalid for the lepton triplet, $\mathrm{Tr}(B-L) = -2 \neq 0$ (and even for other particle multiplets). In other words, $B-L$ and T_i by themselves do not make a symmetry on which we can base our theory. Therefore, to have a closed algebra, we must introduce at least a new Abelian charge N so that $B-L$ is a residual symmetry of the closed group $\mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N$, i.e., $B-L = x_i T_i + yN$, where the embedding coefficients $x_i, y \neq 0$ are given below. [The existence of N can also be understood by a current-algebra approach for T_i and $B-L$—similarly to the case of the hypercharge Y—when we combine $\mathrm{SU}(2)_L$ with $\mathrm{U}(1)_Q$ to create the $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ electroweak symmetry.] Note that N cannot be identified as X (which defines the electric charge operator) because they generally differ for the particle multiplets (see below); thus, they are independent charges. In fact, the normal Lagrangian of the 3-3-1 models (including the gauge interactions, minimal Yukawa Lagrangian, and minimal scalar potential) always preserves a $\mathrm{U}(1)_N$ Abelian symmetry that, along with $\mathrm{SU}(3)_L$, realizes $B-L$ as a conserved (noncommuting) residual charge; this has been investigated in the literature and given in terms of $B = \mathcal{B}$ and $L = bT_8 + \mathcal{L}$, where b is dependent on the 3-3-1 model class and $N = \mathcal{B} - \mathcal{L}$ [2,15]. Note also that a violation in N due to some unwanted interaction, by contrast, would lead to a corresponding violation in $B-L$, and vice versa. Because T_i are gauged charges, $B-L$ and N must be gauged charges [by contrast, $T_i \sim (B-L) - yN$ are global, which is incorrect]. The gauging of $B-L$ is a consequence of the fact that $B-L$ and $\mathrm{SU}(3)_L$ do not commute (which is unlike the standard model case). The theory is only consistent if it includes $\mathrm{U}(1)_N$ as a gauge symmetry, which also necessarily makes the resulting theory free from all the nontrivial leptonic and baryonic anomalies [2]. Otherwise, the 3-3-1 models must contain (abnormal) interactions that explicitly violate $B-L$ (or N). Equivalently, the 3-3-1 models only survive if $B-L$ (which is actually recognized as an approximate symmetry) is not a symmetry of such theories, which was explicitly shown in Ref. [16]. Thus, assuming that the $B-L$ charge is conserved (a condition that is respected by experiment, the standard model, and even the typical 3-3-1 models [1,3–5]), the Abelian factor $\mathrm{U}(1)_N$ must be included so that the algebras are closed, which is necessary in order to have a self-consistent theory. Apart from the strong interaction with the $\mathrm{SU}(3)_C$ group, the $\mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X \otimes \mathrm{U}(1)_N$ framework thus presents a unification of the electroweak and $B-L$ interactions, in the same manner that the standard model electroweak theory does for the weak and electromagnetic interactions.

The two Abelian factors of the 3-3-1-1 symmetry associated with the $\mathrm{SU}(3)_L$ group correspondingly determine the electric charge Q and the $B-L$ operators as residual symmetries, given by

$$Q = T_3 - \frac{1}{\sqrt{3}}T_8 + X, \qquad B-L = -\frac{2}{\sqrt{3}}T_8 + N, \tag{2}$$

where $T_i (i = 1,2,3,\ldots,8)$, and X and N are the charges of $\mathrm{SU}(3)_L$, $\mathrm{U}(1)_X$ and $\mathrm{U}(1)_N$, respectively [the $\mathrm{SU}(3)_C$ charges will be denoted by t_i]. Note that the above definitions of Q and $B-L$ embed the 3-3-1 model with neutral fermions [5] in the theory considered. However, the coefficients of T_8 might be different depending on which class of the 3-3-1 models it is embedded in [15].

The conserved charge Q is responsible for the electromagnetic interaction, whereas $B-L$ must be broken so that the $\mathrm{U}(1)_N$ gauge boson gets a large enough mass to escape from the detectors. Indeed, $B-L$ is broken down to a parity (i.e., a Z_2 symmetry),

$$P = (-1)^{3(B-L)+2s} = (-1)^{-2\sqrt{3}T_8+3N+2s}, \tag{3}$$

which consequently makes "incorrect $B-L$ particles" stable, providing dark matter candidates [2]. We see that this R parity is a residual symmetry of the broken $\mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N$ gauge symmetry, which is unlike the R symmetry in supersymmetry [17]. That being said, the parity P automatically exists, and due to its nature it will play an important role in the model in addition to stabilizing the dark matter candidates, as is shown throughout the paper.

The fermion content of the 3-3-1-1 model that is anomaly free is given as [2]

$$\psi_{aL} = \begin{pmatrix} \nu_{aL} \\ e_{aL} \\ (N_{aR})^c \end{pmatrix} \sim (1, 3, -1/3, -2/3), \tag{4}$$

$$\nu_{aR} \sim (1,1,0,-1), \qquad e_{aR} \sim (1,1,-1,-1), \tag{5}$$

$$Q_{\alpha L} = \begin{pmatrix} d_{\alpha L} \\ -u_{\alpha L} \\ D_{\alpha L} \end{pmatrix} \sim (3, 3^*, 0, 0),$$
$$Q_{3L} = \begin{pmatrix} u_{3L} \\ d_{3L} \\ U_L \end{pmatrix} \sim (3, 3, 1/3, 2/3), \tag{6}$$

$$u_{aR} \sim (3,1,2/3,1/3), \qquad d_{aR} \sim (3,1,-1/3,1/3), \tag{7}$$

$$U_R \sim (3,1,2/3,4/3), \qquad D_{\alpha R} \sim (3,1,-1/3,-2/3), \tag{8}$$

where the quantum numbers located in parentheses are defined using the gauge symmetries ($\mathrm{SU}(3)_C$, $\mathrm{SU}(3)_L$, $\mathrm{U}(1)_X$, $\mathrm{U}(1)_N$), respectively. The family indices are $a = 1,2,3$ and $\alpha = 1,2$.

The exotic fermions N_R, U, and D have been included to complete the fundamental representations of the $\mathrm{SU}(3)_L$ group, respectively. By the embedding, their electric charges take the usual values, $Q(N_R) = 0$, $Q(U) = 2/3$, and $Q(D) = -1/3$. However, their $B-L$ charges take the values $[B-L](N_R) = 0$, $[B-L](U) = 4/3$, and $[B-L](D) = -2/3$, which are abnormal in comparison to those of the

TABLE I. The W parity (P) separates the model particles into two classes: (i) W particles that possess $P=-1$, and (ii) ordinary particles that have $P=+1$. The first class includes a large portion of the new particles, while the second class is dominated by the standard model particles.

Particle	ν	e	u	d	G	γ	W	Z	Z'	C	$\eta_{1,2}$	$\rho_{1,2}$	χ_3	ϕ	N	U	D	X	Y	η_3	ρ_3	$\chi_{1,2}$
L	1	1	0	0	0	0	0	0	0	0	0	0	0	−2	0	−1	1	1	1	−1	−1	1
P	+	+	+	+	+	+	+	+	+	+	+	+	+	+	−	−	−	−	−	−	−	−

standard model particles. These exotic fermions (and the associated bosons) have ordinary baryon numbers; however, they possess anomalous lepton numbers and are P odd (see Table I for details) [2]. Such particles are generally called wrong-lepton particles (or W particles for short) and the parity P is thus called W parity. All other particles of the model, including the standard model ones (which either have both ordinary baryon and lepton numbers, or only differ from the ordinary lepton number by an even lepton number, such as the ϕ scalar given below) are even under W parity, and they can be considered as ordinary particles.

Let us recall that the neutral fermions N_{aR} might have left-handed counterparts, N_{aL}, which transform as singlets under any gauge symmetry group including $\mathrm{U}(1)_N$. In this way, the N_{aL} are truly sterile, which is unlike the ν_{aR} usually considered in the literature. Interestingly, the sterile fermions N_{aL} are W particles like the N_{aR}. If the N_{aL} are not included, the N_{aR} are Majorana fermions. Otherwise, the presence of the N_{aL} yields $N_{aL,R}$ as generic fermions (which may be Dirac ones). Further, we will exploit this matter by deriving the dark matter observables for the cases of the Dirac or Majorana fermions.

To break the gauge symmetry and generate the masses for the particles in a correct way, the 3-3-1-1 model needs the following scalar multiplets [2]:

$$\eta=\begin{pmatrix}\eta_1^0\\ \eta_2^-\\ \eta_3^0\end{pmatrix}\sim(1,3,-1/3,1/3),$$
$$\rho=\begin{pmatrix}\rho_1^+\\ \rho_2^0\\ \rho_3^+\end{pmatrix}\sim(1,3,2/3,1/3),$$
$$\chi=\begin{pmatrix}\chi_1^0\\ \chi_2^-\\ \chi_3^0\end{pmatrix}\sim(1,3,-1/3,-2/3),$$
$$\phi\sim(1,1,0,2), \tag{9}$$

with the vacuum expectation values (VEVs) that conserve Q and P being given by, respectively,

$$\langle\eta\rangle=\frac{1}{\sqrt{2}}(u,0,0)^T,\qquad \langle\rho\rangle=\frac{1}{\sqrt{2}}(0,v,0)^T,$$
$$\langle\chi\rangle=\frac{1}{\sqrt{2}}(0,0,\omega)^T,\qquad \langle\phi\rangle=\frac{1}{\sqrt{2}}\Lambda. \tag{10}$$

The VEVs of η, ρ, and χ only break $\mathrm{SU}(3)_C\otimes\mathrm{SU}(3)_L\otimes\mathrm{U}(1)_X\otimes\mathrm{U}(1)_N$ to $\mathrm{SU}(3)_C\otimes\mathrm{U}(1)_Q\otimes U(1)_{B-L}$, which leaves the $B-L$ invariant. The ϕ breaks $\mathrm{U}(1)_N$ as well as the $B-L$ that defines the W parity, $U(1)_{B-L}\to P$, with the form as given in [2]. It also provides the mass for the $\mathrm{U}(1)_N$ gauge boson as well as the Majorana masses for ν_{aR}. Note that ρ_3, η_3, and $\chi_{1,2}$ are the W particles, while the others including ϕ are not (i.e., they are ordinary particles). The electrically neutral fields η_3 and χ_1 cannot develop a VEV due to W-parity conservation. To be consistent with the standard model, we suppose $u,v\ll\omega,\Lambda$.

Up to the gauge fixing and ghost terms, the Lagrangian of the 3-3-1-1 model is given by

$$\begin{aligned}\mathcal{L}=&\sum_{\text{fermion multiplets}}\bar{\Psi}\,i\gamma^\mu D_\mu\Psi+\sum_{\text{scalar multiplets}}(D^\mu\Phi)^\dagger(D_\mu\Phi)\\&-\frac{1}{4}G_{i\mu\nu}G_i^{\mu\nu}-\frac{1}{4}A_{i\mu\nu}A_i^{\mu\nu}-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}-\frac{1}{4}C_{\mu\nu}C^{\mu\nu}\\&-V(\rho,\eta,\chi,\phi)+\mathcal{L}_{\text{Yukawa}},\end{aligned} \tag{11}$$

with the covariant derivative

$$D_\mu=\partial_\mu+ig_st_iG_{i\mu}+igT_iA_{i\mu}+ig_XXB_\mu+ig_NNC_\mu, \tag{12}$$

and the field strength tensors

$$\begin{aligned}G_{i\mu\nu}&=\partial_\mu G_{i\nu}-\partial_\nu G_{i\mu}-g_sf_{ijk}G_{j\mu}G_{k\nu},\\A_{i\mu\nu}&=\partial_\mu A_{i\nu}-\partial_\nu A_{i\mu}-gf_{ijk}A_{j\mu}A_{k\nu},\\B_{\mu\nu}&=\partial_\mu B_\nu-\partial_\nu B_\mu,\qquad C_{\mu\nu}=\partial_\mu C_\nu-\partial_\nu C_\mu.\end{aligned} \tag{13}$$

Ψ denotes the fermion multiplets, such as ψ_{aL}, Q_{3L}, u_{aR}, and so on, whereas Φ stands for scalar multiplets ϕ, η, ρ, and χ. The coupling constants (g_s,g,g_X,g_N) and the gauge bosons $(G_{i\mu},A_{i\mu},B_\mu,C_\mu)$ are defined as coupled to the generators (t_i,T_i,X,N), respectively. It is noted that in a mass basis the $W^\pm$ bosons are associated with $T_{1,2}$, the photon γ is associated with Q, and the Z, Z' are associated with generators that are orthogonal to Q. All these fields, including C and the gluons G, are even under W parity. However, the new non-Hermitian gauge bosons—$X^{0,0*}$ as coupled to $T_{4,5}$ and $Y^\pm$ as coupled to $T_{6,7}$—are the W particles.

The scalar potential and Yukawa Lagrangian mentioned above are obtained as follows [2]:

$$\mathcal{L}_{\text{Yukawa}} = h^e_{ab}\bar{\psi}_{aL}\rho e_{bR} + h^\nu_{ab}\bar{\psi}_{aL}\eta\nu_{bR} + h'^\nu_{ab}\bar{\nu}^c_{aR}\nu_{bR}\phi + h^U\bar{Q}_{3L}\chi U_R + h^D_{\alpha\beta}\bar{Q}_{\alpha L}\chi^* D_{\beta R} + h^u_a\bar{Q}_{3L}\eta u_{aR} + h^d_a\bar{Q}_{3L}\rho d_{aR} + h^d_{\alpha a}\bar{Q}_{\alpha L}\eta^* d_{aR} + h^u_{\alpha a}\bar{Q}_{\alpha L}\rho^* u_{aR} + \text{H.c.}, \tag{14}$$

$$\begin{aligned} V(\rho,\eta,\chi,\phi) = {} & \mu_1^2\rho^\dagger\rho + \mu_2^2\chi^\dagger\chi + \mu_3^2\eta^\dagger\eta + \lambda_1(\rho^\dagger\rho)^2 + \lambda_2(\chi^\dagger\chi)^2 + \lambda_3(\eta^\dagger\eta)^2 \\ & + \lambda_4(\rho^\dagger\rho)(\chi^\dagger\chi) + \lambda_5(\rho^\dagger\rho)(\eta^\dagger\eta) + \lambda_6(\chi^\dagger\chi)(\eta^\dagger\eta) \\ & + \lambda_7(\rho^\dagger\chi)(\chi^\dagger\rho) + \lambda_8(\rho^\dagger\eta)(\eta^\dagger\rho) + \lambda_9(\chi^\dagger\eta)(\eta^\dagger\chi) + (f\epsilon^{mnp}\eta_m\rho_n\chi_p + \text{H.c.}) \\ & + \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 + \lambda_{10}(\phi^\dagger\phi)(\rho^\dagger\rho) + \lambda_{11}(\phi^\dagger\phi)(\chi^\dagger\chi) + \lambda_{12}(\phi^\dagger\phi)(\eta^\dagger\eta). \end{aligned} \tag{15}$$

Because of the 3-3-1-1 gauge symmetry, the Yukawa Lagrangian and scalar potential take the standard forms that contain no lepton-number-violating interactions.

If such violating interactions as well as nonzero VEVs of η_3 and χ_1 were present (as in the 3-3-1 model), they would be the sources for the hadronic FCNCs at tree level [12]. The FCNC problem is partially solved by the 3-3-1-1 symmetry and W-parity conservation. Also, the presence of the η_3 and χ_1 VEVs would imply a mass hierarchy between the real and imaginary components of the X^0 gauge boson due to their different mixings with the neutral gauge bosons. This leads to CPT violation, which is experimentally unacceptable [13]. The CPT violation encountered with the 3-3-1 model is thus solved by the 3-3-1-1 symmetry and W-parity conservation as well.

Table I lists all the model particles with their parity values explicitly provided. The lepton numbers have also been included for convenience. However, the baryon numbers are not listed since they can be obtained as usual (all the quarks u, d, U, and D have $B = 1/3$, whereas the other particles have $B = 0$). As shown in Ref. [2], the X^0 gauge boson cannot be dark matter. However, the neutral fermion (a combination of N_a fields) or the neutral complex scalar (a combination of η_3^0 and χ_1^0 fields) can be dark matter depending on which one of them is the lightest wrong-lepton particle, in agreement with Ref. [11].

The fermion masses that are obtained from the Yukawa Lagrangian after the gauge symmetry breaking have been presented in Ref. [2] in detail. Below, we will calculate the masses and physical states of the scalar and gauge boson sectors when the Λ scale of the $U(1)_N$ breaking is comparable to the ω scale of the 3-3-1 breaking, which was neglected in Ref. [2]. Also, all the gauge interactions of fermions and scalars as well as the constraints on the new physics are derived. We stress again that in the regime $\Lambda \gg \omega$ the $B-L$ and 3-3-1 symmetries decouple, whereas—when these scales become comparable—the new physics associated with the $B-L$ and that of the 3-3-1 model are correlated, possibly at the TeV scale, all of which may be proven at the LHC or ILC.

III. SCALAR SECTOR

Since W parity is conserved, only the neutral scalar fields that are even under this parity symmetry can develop the VEVs given in Eq. (10). We expand the fields around these VEVs as

$$\eta = \langle\eta\rangle + \eta' = \begin{pmatrix} \frac{u}{\sqrt{2}} \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{S_1+iA_1}{\sqrt{2}} \\ \eta_2^- \\ \frac{S_3'+iA_3'}{\sqrt{2}} \end{pmatrix},$$

$$\rho = \langle\rho\rangle + \rho' = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \\ 0 \end{pmatrix} + \begin{pmatrix} \rho_1^+ \\ \frac{S_2+iA_2}{\sqrt{2}} \\ \rho_3^+ \end{pmatrix}, \tag{16}$$

$$\chi = \langle\chi\rangle + \chi' = \begin{pmatrix} 0 \\ 0 \\ \frac{\omega}{\sqrt{2}} \end{pmatrix} + \begin{pmatrix} \frac{S_1'+iA_1'}{\sqrt{2}} \\ \chi_2^- \\ \frac{S_3+iA_3}{\sqrt{2}} \end{pmatrix},$$

$$\phi = \langle\phi\rangle + \phi' = \frac{\Lambda}{\sqrt{2}} + \frac{S_4+iA_4}{\sqrt{2}}, \tag{17}$$

where in each expansion the first and last terms are denoted as the VEVs and physical fields, respectively. Note that $S_{1,2,3,4}$ and $A_{1,2,3,4}$ are W even, while those with primed signs, $S'_{1,3}$ and $A'_{1,3}$, are W odd. There is no mixing between the W-even and W-odd fields due to W-parity conservation. On the other hand, the f parameter in the scalar potential can be complex (the remaining parameters, such as the μ^2's and λ's, are all real). However, its phase can be removed by redefining the fields η, ρ, χ appropriately. Consequently, the scalar potential conserves the CP symmetry. Assuming that the CP symmetry is also conserved by the vacuum, the VEVs and f can simultaneously be considered as the real parameters. There is no mixing between the scalars (CP even) and pseudoscalars (CP odd) due to CP conservation.

To find the mass spectra of the scalar fields, let us expand all the terms of the potential up to the second-order contributions of the fields:

$$\mu_1^2(\rho^\dagger\rho) = \mu_1^2(\langle\rho\rangle^\dagger\langle\rho\rangle + \langle\rho\rangle^\dagger\rho' + \rho'^\dagger\langle\rho\rangle + \rho'^\dagger\rho') = \mu_1^2\left(\frac{v^2}{2} + vS_2 + \rho_1^+\rho_1^- + \rho_3^+\rho_3^- + \frac{S_2^2 + A_2^2}{2}\right),$$
$$\mu_2^2(\chi^\dagger\chi) = \mu_2^2\left(\frac{\omega^2}{2} + \omega S_3 + \chi_2^-\chi_2^+ + \frac{S_1'^2 + A_1'^2 + S_3^2 + A_3^2}{2}\right),$$
$$\mu_3^2(\eta^\dagger\eta) = \mu_3^2\left(\frac{u^2}{2} + uS_1 + \eta_2^-\eta_2^+ + \frac{S_1^2 + A_1^2 + S_3'^2 + A_3'^2}{2}\right),$$
$$\mu^2(\phi^\dagger\phi) = \mu^2\left(\frac{\Lambda^2}{2} + \Lambda S_4 + \frac{S_4^2 + A_4^2}{2}\right),$$

$$\lambda(\phi^\dagger\phi)^2 = \lambda\left[\frac{\Lambda^4}{4} + \Lambda^2 S_4^2 + \Lambda^3 S_4 + \frac{\Lambda^2}{2}(S_4^2 + A_4^2) + \text{interaction}\right],$$
$$\lambda_1(\rho^\dagger\rho)^2 = \lambda_1\left[\frac{v^4}{4} + v^2 S_2^2 + v^3 S_2 + v^2\left(\rho_1^+\rho_1^- + \rho_3^+\rho_3^- + \frac{S_2^2 + A_2^2}{2}\right) + \text{interaction}\right],$$
$$\lambda_2(\chi^\dagger\chi)^2 = \lambda_2\left[\frac{\omega^4}{4} + \omega^2 S_3^2 + \omega^3 S_3 + \omega^2\left(\chi_2^-\chi_2^+ + \frac{S_1'^2 + A_1'^2 + S_3^2 + A_3^2}{2}\right) + \text{interaction}\right],$$
$$\lambda_3(\eta^\dagger\eta)^2 = \lambda_3\left[\frac{u^4}{4} + u^2 S_1^2 + u^3 S_1 + u^2\left(\eta_2^-\eta_2^+ + \frac{S_1^2 + A_1^2 + S_3'^2 + A_3'^2}{2}\right) + \text{interaction}\right],$$

$$\lambda_4(\rho^\dagger\rho)(\chi^\dagger\chi) = \lambda_4\left[\frac{v^2\omega^2}{4} + \frac{\omega v^2}{2}S_3 + \frac{v\omega^2}{2}S_2 + v\omega S_2 S_3 + \frac{v^2}{2}\left(\chi_2^-\chi_2^+ + \frac{S_1'^2 + A_1'^2 + S_3^2 + A_3^2}{2}\right)\right.$$
$$\left. + \frac{\omega^2}{2}\left(\rho_1^+\rho_1^- + \rho_3^+\rho_3^- + \frac{S_2^2 + A_2^2}{2}\right) + \text{interaction}\right],$$
$$\lambda_5(\rho^\dagger\rho)(\eta^\dagger\eta) = \lambda_5\left[\frac{v^2u^2}{4} + \frac{uv^2}{2}S_1 + \frac{vu^2}{2}S_2 + vuS_1S_2 + \frac{v^2}{2}\left(\eta_2^-\eta_2^+ + \frac{S_1^2 + A_1^2 + S_3'^2 + A_3'^2}{2}\right)\right.$$
$$\left. + \frac{u^2}{2}\left(\rho_1^+\rho_1^- + \rho_3^+\rho_3^- + \frac{S_2^2 + A_2^2}{2}\right) + \text{interaction}\right],$$
$$\lambda_6(\chi^\dagger\chi)(\eta^\dagger\eta) = \lambda_6\left[\frac{\omega^2u^2}{4} + \frac{u\omega^2}{2}S_1 + \frac{\omega u^2}{2}S_3 + u\omega S_1S_3 + \frac{\omega^2}{2}\left(\eta_2^-\eta_2^+ + \frac{S_1^2 + A_1^2 + S_3'^2 + A_3'^2}{2}\right)\right.$$
$$\left. + \frac{u^2}{2}\left(\chi_2^+\chi_2^- + \frac{S_1'^2 + A_1'^2 + S_3^2 + A_3^2}{2}\right) + \text{interaction}\right],$$

$$\lambda_7(\rho^\dagger\chi)(\chi^\dagger\rho) = \frac{\lambda_7}{2}(v\chi_2^- + \omega\rho_3^-)(\omega\rho_3^+ + v\chi_2^+) + \text{interaction},$$
$$\lambda_8(\rho^\dagger\eta)(\eta^\dagger\rho) = \frac{\lambda_8}{2}(v\eta_2^- + u\rho_1^-)(u\rho_1^+ + v\eta_2^+) + \text{interaction},$$
$$\lambda_9(\chi^\dagger\eta)(\eta^\dagger\chi) = \lambda_9\left[\frac{\omega}{2}(S_3' + iA_3') + \frac{u}{2}(S_1' - iA_1')\right]\left[\frac{u}{2}(S_1' + iA_1') + \frac{\omega}{2}(S_3' - iA_3')\right] + \text{interaction},$$

$$\lambda_{10}(\phi^\dagger\phi)(\rho^\dagger\rho)=\lambda_{10}\left[\frac{\Lambda^2v^2}{4}+\frac{v\Lambda^2}{2}S_2+\frac{\Lambda v^2}{2}S_4+v\Lambda S_2S_4+\frac{v^2}{2}\left(\frac{S_4^2+A_4^2}{2}\right)+\frac{\Lambda^2}{2}\left(\rho_1^+\rho_1^-+\rho_3^+\rho_3^-+\frac{S_2^2+A_2^2}{2}\right)+\text{interaction}\right],$$

$$\lambda_{11}(\phi^\dagger\phi)(\chi^\dagger\chi)=\lambda_{11}\left[\frac{\Lambda^2\omega^2}{4}+\frac{\omega\Lambda^2}{2}S_3+\frac{\Lambda\omega^2}{2}S_4+\omega\Lambda S_3S_4+\frac{\omega^2}{2}\left(\frac{S_4^2+A_4^2}{2}\right)+\frac{\Lambda^2}{2}\left(\chi_2^+\chi_2^-+\frac{S_1'^2+A_1'^2+S_3^2+A_3^2}{2}\right)+\text{interaction}\right],$$

$$\lambda_{12}(\phi^\dagger\phi)(\eta^\dagger\eta)=\lambda_{12}\left[\frac{\Lambda^2u^2}{4}+\frac{u\Lambda^2}{2}S_1+\frac{\Lambda u^2}{2}S_4+u\Lambda S_1S_4+\frac{u^2}{2}\left(\frac{S_4^2+A_4^2}{2}\right)+\frac{\Lambda^2}{2}\left(\eta_2^+\eta_2^-+\frac{S_1^2+A_1^2+S_3'^2+A_3'^2}{2}\right)+\text{interaction}\right],$$

$$f\epsilon^{mnp}\eta_m\rho_n\chi_p+\text{H.c.}=f\left[\frac{uv\omega}{\sqrt{2}}+\frac{uv}{\sqrt{2}}S_3+\frac{u\omega}{\sqrt{2}}S_2+\frac{v\omega}{\sqrt{2}}S_1+\frac{u}{\sqrt{2}}(S_2S_3-A_2A_3-\rho_3^+\chi_2^--\rho_3^-\chi_2^+)+\frac{v}{\sqrt{2}}(S_1S_3-A_1A_3-S_1'S_3'+A_1'A_3')+\frac{\omega}{\sqrt{2}}(S_1S_2-A_1A_2-\eta_2^-\rho_1^+-\eta_2^+\rho_1^-)\right]+\text{interaction}.$$

The scalar potential—which is the sum of all the above terms—can be rearranged as

$$V(\rho,\eta,\chi,\phi)=V_{\text{min}}+V_{\text{linear}}+V_{\text{mass}}+V_{\text{interaction}},\tag{18}$$

where the interactions stored in $V_{\text{interaction}}$ do not need to be explicitly obtained. V_{min} contains the terms that are independent of the scalar fields,

$$V_{\text{min}}=\mu_1^2\frac{v^2}{2}+\mu_2^2\frac{\omega^2}{2}+\mu_3^2\frac{u^2}{2}+\mu^2\frac{\Lambda^2}{2}+\lambda_1^2\frac{v^4}{4}+\lambda_2^2\frac{\omega^4}{4}+\lambda_3^2\frac{u^4}{4}+\lambda^2\frac{\Lambda^4}{4}+\lambda_4^2\frac{v^2\omega^2}{4}+\lambda_5^2\frac{v^2u^2}{4}+\lambda_6^2\frac{u^2\omega^2}{4}+\lambda_{10}^2\frac{v^2\Lambda^2}{4}+\lambda_{11}^2\frac{\Lambda^2\omega^2}{4}+\lambda_{12}^2\frac{u^2\Lambda^2}{4}+f\frac{uv\omega}{\sqrt{2}},$$

which only contributes to the vacuum energy; it does not affect the physical processes.

V_{linear} includes all the terms that linearly depend on the scalar fields,

$$\begin{aligned}V_{\text{linear}}=&\,S_1\left[u\mu_3^2+\lambda_3u^3+\frac{1}{2}\lambda_5uv^2+\frac{1}{2}\lambda_6u\omega^2+\frac{\sqrt{2}}{2}fv\omega+\frac{1}{2}\lambda_{12}u\Lambda^2\right]\\&+S_2\left[v\mu_1^2+\lambda_1v^3+\frac{1}{2}\lambda_4v\omega^2+\frac{1}{2}\lambda_5u^2v+\frac{\sqrt{2}}{2}fu\omega+\frac{\lambda_{10}}{2}v\Lambda^2\right]\\&+S_3\left[\omega\mu_2^2+\lambda_2\omega^3+\frac{\lambda_4}{2}\omega v^2+\frac{\lambda_6}{2}\omega u^2+\frac{\sqrt{2}}{2}fuv+\frac{\lambda_{11}}{2}\omega\Lambda^2\right]\\&+S_4\left[\mu^2\Lambda+\lambda\Lambda^3+\frac{1}{2}\lambda_{10}v^2\Lambda+\frac{1}{2}\lambda_{11}\Lambda\omega^2+\frac{1}{2}\lambda_{12}\Lambda u^2\right].\end{aligned}\tag{19}$$

Because of the gauge invariance, the coefficients vanish,

$$v\mu_1^2+\lambda_1v^3+\frac{1}{2}\lambda_4v\omega^2+\frac{1}{2}\lambda_5u^2v+\frac{\sqrt{2}}{2}fu\omega+\frac{\lambda_{10}}{2}v\Lambda^2=0,\tag{20}$$

$$\omega\mu_2^2+\lambda_2\omega^3+\frac{\lambda_4}{2}\omega v^2+\frac{\lambda_6}{2}\omega u^2+\frac{\sqrt{2}}{2}fuv+\frac{\lambda_{11}}{2}\omega\Lambda^2=0,\tag{21}$$

$$u\mu_3^2 + \lambda_3 u^3 + \frac{1}{2}\lambda_5 u v^2 + \frac{1}{2}\lambda_6 u\omega^2 + \frac{\sqrt{2}}{2} f v\omega + \frac{1}{2}\lambda_{12} u\Lambda^2 = 0, \tag{22}$$

$$\mu^2 + \lambda\Lambda^2 + \frac{1}{2}\lambda_{10} v^2 + \frac{1}{2}\lambda_{11}\omega^2 + \frac{1}{2}\lambda_{12} u^2 = 0, \tag{23}$$

which is also the condition of potential minimization,

$$\frac{\partial V}{\partial u} = \frac{\partial V}{\partial v} = \frac{\partial V}{\partial \omega} = \frac{\partial V}{\partial \Lambda} = 0. \tag{24}$$

The 3-3-1-1 gauge symmetry will be broken in the correct way and the potential will be bounded from below if we impose $\mu^2 < 0, \mu_{1,2,3}^2 < 0, \lambda > 0, \lambda_{1,2,3} > 0$, and other necessary conditions for $\lambda_{4,5,6,\ldots,12}$. In this case, the equations for the potential minimization above give a unique, nonzero solution for the VEVs (u, v, ω, Λ).

V_{mass} consists of all the terms in the potential that quadratically depend on the scalar fields. It can be decomposed into

$$V_{\text{mass}} = V_{\text{mass}}^{\text{charged}} + V_{\text{mass}}^{S} + V_{\text{mass}}^{A} + V_{\text{mass}}^{S'} + V_{\text{mass}}^{A'}, \tag{25}$$

where the first term includes all the mass terms of the charged scalars, while the remaining terms belong to the neutral scalars with each term for a distinct group of fields characterized by the W and CP parities, as mentioned before.

The mass spectrum of the charged scalars is given by

$$\begin{aligned} V_{\text{mass}}^{\text{charged}} = {} & \chi_2^+\chi_2^-\left(\mu_2^2 + \lambda_2\omega^2 + \frac{\lambda_4}{2}v^2 + \frac{\lambda_6}{2}u^2 + \frac{\lambda_{11}}{2}\Lambda^2\right) + \eta_2^+\eta_2^-\left(\mu_3^2 + \lambda_3 u^2 + \frac{1}{2}\lambda_5 v^2 + \frac{1}{2}\lambda_6\omega^2 + \frac{1}{2}\lambda_{12}\Lambda^2\right) \\ & + (\rho_1^+\rho_1^- + \rho_3^+\rho_3^-)\left(\mu_1^2 + \lambda_1 v^2 + \frac{1}{2}\lambda_4\omega^2 + \frac{1}{2}\lambda_5 u^2 + \frac{\lambda_{10}}{2}\Lambda^2\right) + \frac{\lambda_7}{2}(v\chi_2^- + \omega\rho_3^-)(v\chi_2^+ + \omega\rho_3^+) \\ & + \frac{\lambda_8}{2}(v\eta_2^- + u\rho_1^-)(u\rho_1^+ + v\eta_2^+) - f\frac{u}{\sqrt{2}}(\rho_3^+\chi_2^- + \rho_3^-\chi_2^+) - f\frac{\omega}{\sqrt{2}}(\eta_2^-\rho_1^+ + \eta_2^+\rho_1^-). \end{aligned} \tag{26}$$

From the potential-minimization conditions, we extract μ_1^2, μ_2^2, and μ_3^2 and substitute them into the above expression to yield

$$\begin{aligned} V_{\text{mass}}^{\text{charged}} &= \left(\frac{\lambda_7}{2} - \frac{fu}{\sqrt{2}v\omega}\right)(v\chi_2^- + \omega\rho_3^-)(v\chi_2^+ + \omega\rho_3^+) + \left(\frac{\lambda_8}{2} - \frac{f\omega}{\sqrt{2}uv}\right)(v\eta_2^- + u\rho_1^-)(v\eta_2^+ + u\rho_1^+) \\ &= \left(\frac{\lambda_7}{2} - \frac{fu}{\sqrt{2}v\omega}\right)(v^2 + \omega^2)H_4^- H_4^+ + \left(\frac{\lambda_8}{2} - \frac{f\omega}{\sqrt{2}vu}\right)(v^2 + u^2)H_5^- H_5^+, \end{aligned} \tag{27}$$

where we have defined

$$H_4^{\pm} \equiv \frac{v\chi_2^{\pm} + \omega\rho_3^{\pm}}{\sqrt{v^2 + \omega^2}}, \qquad H_5^{\pm} \equiv \frac{v\eta_2^{\pm} + u\rho_1^{\pm}}{\sqrt{u^2 + v^2}}. \tag{28}$$

The fields $H_4^{\pm}, H_5^{\pm}$ by themselves are physical charged scalars with masses given by, respectively,

$$m_{H_4}^2 = \left(\frac{\lambda_7}{2} - \frac{fu}{\sqrt{2}v\omega}\right)(v^2 + \omega^2), \qquad m_{H_5}^2 = \left(\frac{\lambda_8}{2} - \frac{f\omega}{\sqrt{2}vu}\right)(v^2 + u^2). \tag{29}$$

The field that is orthogonal to H_5, $G_W^{\pm} = \frac{u\eta_2^{\pm} - v\rho_1^{\pm}}{\sqrt{u^2+v^2}}$, has zero mass and can be identified as the Goldstone boson of the $W^{\pm}$ gauge boson. Similarly, the field that is orthogonal to H_4, $G_Y^{\pm} = \frac{\omega\chi_2^{\pm} - v\rho_3^{\pm}}{\sqrt{v^2+\omega^2}}$, is massless and can be identified as the Goldstone boson of the new $Y^{\pm}$ gauge boson.

For the neutral scalar fields, we start with the A group,

$$\begin{aligned}V^A_{\text{mass}} &= A_1^2\left(\frac{\mu_3^2}{2}+\frac{1}{2}\lambda_3u^2+\frac{1}{4}\lambda_5v^2+\frac{1}{4}\lambda_6\omega^2+\frac{1}{4}\lambda_{12}\Lambda^2\right)\\ &\quad +A_2^2\left(\frac{\mu_1^2}{2}+\frac{1}{2}\lambda_1v^2+\frac{1}{4}\lambda_4\omega^2+\frac{1}{4}\lambda_5u^2+\frac{\lambda_{10}}{4}v\Lambda^2\right)\\ &\quad +A_3^2\left(\frac{\mu_2^2}{2}+\frac{1}{2}\lambda_2\omega^2+\frac{\lambda_4}{4}v^2+\frac{\lambda_6}{4}u^2+\frac{\lambda_{11}}{4}\omega\Lambda^2\right) \qquad (30)\end{aligned}$$

$$\begin{aligned}&\quad +A_4^2\left(\frac{\mu^2}{2}+\frac{1}{2}\lambda\Lambda^2+\frac{1}{4}\lambda_{10}v^2+\frac{1}{4}\lambda_{11}\omega^2+\frac{1}{4}\lambda_{12}u^2\right)\\ &\quad -\frac{fu}{\sqrt{2}}A_2A_3-\frac{fv}{\sqrt{2}}A_1A_3-\frac{f\omega}{\sqrt{2}}A_1A_2\\ &= -\frac{f}{2\sqrt{2}}\left(\frac{v\omega}{u}+\frac{u\omega}{v}+\frac{uv}{\omega}\right)\left(\frac{v\omega A_1+u\omega A_2+uvA_3}{\sqrt{u^2v^2+v^2\omega^2+u^2\omega^2}}\right)^2, \qquad (31)\end{aligned}$$

with the help of the potential-minimization conditions. Therefore, we have a physical pseudoscalar field with a corresponding mass,

$$\mathcal{A}\equiv\frac{v\omega A_1+u\omega A_2+uvA_3}{\sqrt{u^2v^2+v^2\omega^2+u^2\omega^2}},\quad m^2_{\mathcal{A}}=-\frac{f}{\sqrt{2}}\left(\frac{v\omega}{u}+\frac{u\omega}{v}+\frac{uv}{\omega}\right). \qquad (32)$$

If $u, v, \omega > 0$, we have $f < 0$ so that the squared mass is always positive. We realize that A_4 is massless and can be identified as the Goldstone boson of the new neutral gauge boson C of $\mathrm{U}(1)_N$. The remaining massless fields are orthogonal to $\mathcal{A}$ as follows:

$$\begin{aligned}G_Z &= \frac{uA_1-vA_2}{\sqrt{u^2+v^2}},\\ G_{Z'} &= \frac{-uv(vA_1+uA_2)+\omega(u^2+v^2)A_3}{\sqrt{(u^2v^2+v^2\omega^2+u^2\omega^2)(u^2+v^2)}}. \qquad (33)\end{aligned}$$

They are the Goldstone bosons of the neutral gauge bosons Z and Z', respectively (where Z is standard model-like while Z' is 3-3-1 model-like).

For the A' group, we have

$$\begin{aligned}V^{A'}_{\text{mass}} &= A_1'^2\left(\frac{\mu_2^2}{2}+\frac{1}{2}\lambda_2\omega^2+\frac{\lambda_4}{4}v^2+\frac{\lambda_6}{4}u^2+\frac{\lambda_{11}}{4}\omega\Lambda^2\right)\\ &\quad +A_3'^2\left(\frac{\mu_3^2}{2}+\frac{1}{2}\lambda_2\omega^2+\frac{\lambda_4}{4}v^2+\frac{\lambda_6}{4}u^2+\frac{\lambda_{11}}{4}\omega\Lambda^2\right)\\ &\quad +\frac{fv}{\sqrt{2}}A_1'A_3'+\frac{\lambda_9}{4}(\omega A_3'-uA_1')^2\\ &= \frac{1}{2}\left(\frac{\lambda_9}{2}-\frac{1}{\sqrt{2}}\frac{fv}{u\omega}\right)(u^2+\omega^2)\left(\frac{\omega A_3'-uA_1'}{\sqrt{u^2+\omega^2}}\right)^2,\end{aligned}$$

by using the minimization conditions. Hence, a physical W-odd pseudoscalar and its mass have the form

$$A'\equiv\frac{\omega A_3'-uA_1'}{\sqrt{u^2+\omega^2}},\quad m^2_{A'}=\left(\frac{\lambda_9}{2}-\frac{1}{\sqrt{2}}\frac{fv}{u\omega}\right)(u^2+\omega^2). \qquad (34)$$

Similarly, for the S' group we obtain

$$V^{S'}_{\text{mass}}=\frac{1}{2}\left(\frac{\lambda_9}{2}-\frac{1}{\sqrt{2}}\frac{fv}{u\omega}\right)(u^2+\omega^2)\left(\frac{\omega S_3'+uS_1'}{\sqrt{u^2+\omega^2}}\right)^2, \qquad (35)$$

which yields a physical W-odd scalar with a corresponding mass,

$$S'\equiv\frac{\omega S_3'+uS_1'}{\sqrt{u^2+\omega^2}},\qquad m^2_{S'}=\left(\frac{\lambda_9}{2}-\frac{1}{\sqrt{2}}\frac{fv}{u\omega}\right)(u^2+\omega^2).$$

Some remarks are in order:

(1) We see that the scalar S' and pseudoscalar A' have the same mass. They can be identified as the real and imaginary components of a physical neutral complex field:

$$H'^0\equiv\frac{S'+iA'}{\sqrt{2}}=\frac{1}{\sqrt{u^2+\omega^2}}(u\chi_1^{0*}+\omega\eta_3^0),$$

with the mass

$$m^2_{H'}=\left(\frac{\lambda_9}{2}-\frac{1}{\sqrt{2}}\frac{fv}{u\omega}\right)(u^2+\omega^2). \qquad (36)$$

(2) The field that is orthogonal to H', $G_X^0=\frac{1}{\sqrt{u^2+\omega^2}}(\omega\chi_1^0-u\eta_3^{0*})$, is massless and can be identified as the Goldstone boson of the new neutral non-Hermitian gauge boson X^0.

Finally, there remains the S group of the W-even, real scalar fields. Using the potential-minimization conditions, we have

$$\begin{aligned}V^S_{\text{mass}} &= \left(\lambda_3u^2-\frac{1}{2\sqrt{2}}f\frac{v\omega}{u}\right)S_1^2+\left(\lambda_1v^2-\frac{1}{2\sqrt{2}}f\frac{u\omega}{v}\right)S_2^2\\ &\quad +\left(\lambda_2\omega^2-\frac{1}{2\sqrt{2}}f\frac{vu}{\omega}\right)S_3^2+\left(\lambda_5uv+\frac{1}{\sqrt{2}}f\omega\right)S_1S_2\\ &\quad +\left(\lambda_6u\omega+\frac{1}{\sqrt{2}}fv\right)S_1S_3+\left(\lambda_4\omega v+\frac{1}{\sqrt{2}}fu\right)S_2S_3\\ &\quad +\lambda\Lambda^2S_4^2+\lambda_{12}u\Lambda S_1S_4+\lambda_{10}v\Lambda S_2S_4+\lambda_{11}\omega\Lambda S_3S_4\\ &= \frac{1}{2}\begin{pmatrix}S_1 & S_2 & S_3 & S_4\end{pmatrix}M_S^2\begin{pmatrix}S_1\\ S_2\\ S_3\\ S_4\end{pmatrix}, \qquad (37)\end{aligned}$$

where

$$M_S^2 \equiv \begin{pmatrix} 2\lambda_3 u^2 - \frac{1}{\sqrt{2}} f \frac{v\omega}{u} & \lambda_5 uv + \frac{1}{\sqrt{2}} f\omega & \lambda_6 u\omega + \frac{1}{\sqrt{2}} fv & \lambda_{12} u\Lambda \\ \lambda_5 uv + \frac{1}{\sqrt{2}} f\omega & 2\lambda_1 v^2 - \frac{1}{\sqrt{2}} f \frac{u\omega}{v} & \lambda_4 \omega v + \frac{1}{\sqrt{2}} fu & \lambda_{10} v\Lambda \\ \lambda_6 u\omega + \frac{1}{\sqrt{2}} fv & \lambda_4 \omega v + \frac{1}{\sqrt{2}} fu & 2\lambda_2 \omega^2 - \frac{1}{\sqrt{2}} f \frac{vu}{\omega} & \lambda_{11} \omega\Lambda \\ \lambda_{12} u\Lambda & \lambda_{10} v\Lambda & \lambda_{11} \omega\Lambda & 2\lambda\Lambda^2 \end{pmatrix}. \quad (38)$$

In Ref. [2], the physical states were derived when the $B-L$-breaking scale is as large as the GUT scale, for example, so that S_4 is completely decoupled from the remaining three scalars of the 3-3-1 model. In this work we consider the possible $B-L$ interactions that might happen at the TeV scale, like those of the 3-3-1 model that are characterized by the ω and f scales. Therefore, let us assume that Λ is at the same order as f, ω and all are sufficiently large in comparison to the weak scales u, v so that the new physics is safe [2], i.e.,

$$-f \sim \omega \sim \Lambda \gg u \sim v. \quad (39)$$

Notice that all the physical scalar fields that have been found so far are new particles with corresponding masses given at the ω or $\sqrt{|f\omega|}$ scale.

The mass matrix (38) will provide a small eigenvalue for the mass of the standard model Higgs boson, whereas the remaining eigenvalues will be large enough to be identified as the corresponding masses of the new neutral scalars. To see this explicitly, it is appropriate to consider the leading-order contributions of the mass matrix (38). Imposing Eq. (39) and keeping only the terms that are proportional to $(\omega, \Lambda, f)^2$, we have the result

$$M_S^2|_{\mathrm{LO}} = \begin{pmatrix} -\frac{1}{\sqrt{2}} f \frac{v\omega}{u} & \frac{1}{\sqrt{2}} f\omega & 0 & 0 \\ \frac{1}{\sqrt{2}} f\omega & -\frac{1}{\sqrt{2}} f \frac{u\omega}{v} & 0 & 0 \\ 0 & 0 & 2\lambda_2\omega^2 & \lambda_{11}\omega\Lambda \\ 0 & 0 & \lambda_{11}\omega\Lambda & 2\lambda\Lambda^2 \end{pmatrix}. \quad (40)$$

The 2×2 matrix in the first diagonal box gives a zero eigenvalue with the corresponding eigenstate

$$m_H^2 = 0, \qquad H \equiv \frac{uS_1 + vS_2}{\sqrt{u^2+v^2}}. \quad (41)$$

This state is identified as the standard model Higgs boson. The remaining eigenvalue is

$$m_{H_1}^2 = -\frac{f\omega}{\sqrt{2}}\left(\frac{u}{v} + \frac{v}{u}\right) \sim \omega^2, \quad (42)$$

which corresponds to a new, heavy neutral scalar:

$$H_1 \equiv \frac{-vS_1 + uS_2}{\sqrt{u^2+v^2}}. \quad (43)$$

The 2×2 matrix in the second diagonal box provides two heavy eigenstates with masses at the ω scale given by, respectively,

$$\begin{aligned} H_2 &\equiv c_\varphi S_3 + s_\varphi S_4, \\ m_{H_2}^2 &= \lambda_2\omega^2 + \lambda\Lambda^2 \\ &\quad - \sqrt{\lambda_2^2\omega^4 + (\lambda_{11}^2 - 2\lambda\lambda_2)\omega^2\Lambda^2 + \lambda^2\Lambda^4} \sim \omega^2, \\ H_3 &\equiv -s_\varphi S_3 + c_\varphi S_4, \\ m_{H_3}^2 &= \lambda_2\omega^2 + \lambda\Lambda^2 \\ &\quad + \sqrt{\lambda_2^2\omega^4 + (\lambda_{11}^2 - 2\lambda\lambda_2)\omega^2\Lambda^2 + \lambda^2\Lambda^4} \sim \omega^2, \end{aligned}$$

where the mixing angle is given by

$$t_{2\varphi} = -\frac{\lambda_{11}\omega\Lambda}{\lambda\Lambda^2 - \lambda_2\omega^2}. \quad (44)$$

We have adopted the notations $s_x = \sin x, c_x = \cos x$, $t_x = \tan x$, and so forth, for any angle x (such as φ), which we use throughout this paper.

We see that at the leading order, the standard model-like Higgs boson has a vanishing mass. Hence, when considering the next-to-leading-order contribution, its generated mass is small due to the perturbative expansion. In fact, we can write the general mass matrix M_S^2 in a new basis of the states (H, H_1, H_2, H_3). Since the mass of the standard model-like Higgs boson is much smaller than those of the new particles, the resulting mass matrix will have a seesaw-like form [18] that can transparently be diagonalized. Indeed, by putting

$$\begin{pmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \end{pmatrix} = U \begin{pmatrix} H \\ H_1 \\ H_2 \\ H_3 \end{pmatrix}, \quad U \equiv \begin{pmatrix} \frac{u}{\sqrt{u^2+v^2}} & -\frac{v}{\sqrt{u^2+v^2}} & 0 & 0 \\ \frac{v}{\sqrt{u^2+v^2}} & \frac{u}{\sqrt{u^2+v^2}} & 0 & 0 \\ 0 & 0 & c_\varphi & -s_\varphi \\ 0 & 0 & s_\varphi & c_\varphi \end{pmatrix}, \quad (45)$$

the mass matrix (38) in the new basis is

$$M_S'^2 = U^T M_S^2 U = \begin{pmatrix} A_{1\times 1} & B_{1\times 3} \\ B_{1\times 3}^T & C_{3\times 3} \end{pmatrix}, \quad (46)$$

where

$$A \equiv 2\frac{v^4\lambda_1 + u^4\lambda_3 + u^2v^2\lambda_5}{u^2+v^2}, \qquad B^T \equiv \begin{pmatrix} \frac{uv[v^2(2\lambda_1-\lambda_5)+u^2(-2\lambda_3+\lambda_5)]}{u^2+v^2} \\ \frac{s_\varphi\Lambda(v^2\lambda_{10}+u^2\lambda_{12})+c_\varphi(\sqrt{2}fuv+v^2\omega\lambda_4+u^2\omega\lambda_6)}{\sqrt{u^2+v^2}} \\ \frac{-\sqrt{2}fs_\varphi uv+c_\varphi\Lambda(v^2\lambda_{10}+u^2\lambda_{12})-s_\varphi\omega(v^2\lambda_4+u^2\lambda_6)}{\sqrt{u^2+v^2}} \end{pmatrix}, \tag{47}$$

and C is a 3×3 matrix with corresponding components given by

$$\begin{aligned}
C_{11} &\equiv \frac{-\sqrt{2}f(u^2+v^2)^2\omega + 4u^3v^3(\lambda_1+\lambda_3-\lambda_5)}{2uv(u^2+v^2)},\\
C_{12} = C_{21} &\equiv \frac{2s_\varphi uv\Lambda(\lambda_{10}-\lambda_{12}) + c_\varphi[\sqrt{2}f(u^2-v^2)+2uv\omega(\lambda_4-\lambda_6)]}{2\sqrt{u^2+v^2}},\\
C_{13} = C_{31} &\equiv \frac{\sqrt{2}fs_\varphi(-u^2+v^2) + 2uv[c_\varphi\Lambda(\lambda_{10}-\lambda_{12}) + s_\varphi\omega(-\lambda_4+\lambda_6)]}{2\sqrt{u^2+v^2}},\\
C_{22} &\equiv 2s_\varphi^2\lambda\Lambda^2 + 2c_\varphi\left(-\frac{c_\varphi fuv}{2\sqrt{2}\omega} + s_\varphi\omega\Lambda\lambda_{11} + c_\varphi\omega^2\lambda_2\right),\\
C_{23} = C_{32} &\equiv (c_\varphi^2 - s_\varphi^2)\omega\Lambda\lambda_{11} + 2c_\varphi s_\varphi\left(\frac{fuv}{2\sqrt{2}\omega} + \lambda\Lambda^2 - \omega^2\lambda_2\right),\\
C_{33} &\equiv -\frac{fs_\varphi^2uv}{\sqrt{2}\omega} + 2c_\varphi\Lambda(c_\varphi\lambda\Lambda - s_\varphi\omega\lambda_{11}) + 2s_\varphi^2\omega^2\lambda_2.
\end{aligned} \tag{48}$$

Because $-f\sim\omega\sim\Lambda\gg u\sim v$, we achieve the seesaw form for M'^2_S, where $||C||\sim\omega^2\gg||B||\sim u\omega\gg||A||\sim u^2$, with $||A||\equiv\sqrt{\mathrm{Tr}(A^TA)}$, and so forth. Therefore, the standard model-like Higgs boson obtains a mass given by the seesaw formula [18],

$$\delta m_H^2 = A - BC^{-1}B^T \sim \mathcal{O}(u^2, v^2), \tag{49}$$

which is realized at the weak scales in spite of the large scales ω, Λ, and f (see below). The standard model-like Higgs boson is given by

$$H + \delta H = H - BC^{-1}\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix}. \tag{50}$$

The physical heavy scalars are orthogonal to this light state, and their masses change negligibly compared to their leading-order values.

The mass of the standard model-like Higgs boson can be approximated as

$$\delta m_H^2 = 2\left(\frac{\lambda_3u^4+\lambda_5u^2v^2+\lambda_1v^4}{u^2+v^2} + m_0^2 + m_1^2\frac{f}{\omega} + m_2^2\frac{f^2}{\omega^2}\right), \tag{51}$$

where the mass parameters m_0, m_1, and m_2 are given by

$$\begin{aligned}
m_0^2 \equiv &-\frac{1}{(\lambda_{11}^2-4\lambda\lambda_2)(v^2+u^2)}[-\lambda_{12}^2\lambda_2u^4 - \lambda(\lambda_6u^2+\lambda_4v^2)^2\\
&+\lambda_{12}u^2(\lambda_{11}\lambda_6u^2 - 2\lambda_{10}\lambda_2v^2 + \lambda_{11}\lambda_4v^2)\\
&+\lambda_{10}v^2(\lambda_{11}\lambda_6u^2 - \lambda_{10}\lambda_2v^2 + \lambda_{11}\lambda_4v^2)],
\end{aligned} \tag{52}$$

$$m_1^2 \equiv -\frac{\sqrt{2}uv[(\lambda_{11}\lambda_{12}-2\lambda\lambda_6)u^2 + (\lambda_{10}\lambda_{11}-2\lambda\lambda_4)v^2]}{(\lambda_{11}^2-4\lambda\lambda_2)(u^2+v^2)}, \tag{53}$$

$$m_2^2 \equiv \frac{2\lambda u^2v^2}{(\lambda_{11}^2-4\lambda\lambda_2)(u^2+v^2)}. \tag{54}$$

Because the quantity f/ω is finite, the Higgs mass δm_H^2 depends only on the weak scales u^2 and v^2, as stated. We will evaluate the Higgs mass and assign $\delta m_H^2 = (125\ \mathrm{GeV})^2$ as measured by the LHC [19,20]. For this purpose, let us assume $u=v$ and $\omega=-f$, which leads to

$$\delta m_H^2 = (\lambda_3+\lambda_5+\lambda_1)u^2 + 2m_0^2 - 2m_1^2 + 2m_2^2 \equiv \bar{\lambda}u^2. \tag{55}$$

Here, $\bar{\lambda}$ is a function of only the λ's, which can easily be achieved with the help of Eqs. (52), (53), and (54) for the respective $m_{0,1,2}^2$. In addition, we have

$u^2+v^2=(246\ \text{GeV})^2$, i.e., $u=\frac{246}{\sqrt{2}}$ GeV, which is given from the mass of the W boson, as shown below. Hence, we identify $\delta m_H^2=\bar{\lambda}(\frac{246}{\sqrt{2}}\ \text{GeV})^2=(125\ \text{GeV})^2$, which yields $\bar{\lambda}=(\frac{125\sqrt{2}}{246})^2\simeq 0.5$. This is an expected value for the effective self-interacting scalar coupling.

In summary, we have the 11 Higgs bosons (H^0, $\mathcal{A}^0$, $H^0_{1,2,3}$, $H^\pm_{4,5}$, $H'^{0,0*}$), as well as the nine Goldstone bosons corresponding to the nine massive gauge bosons ($G_W^\pm$, G_Z^0, $G_X^{0,0*}$, $G_Y^\pm$, $G_{Z'}^0$, G_C^0). Because of the constraints $u,v\ll\omega,\Lambda,-f$, the standard model-like Higgs boson ($\sim H$) is light, with a mass at the weak scales, whereas all the new Higgs bosons are heavy, with masses at the ω, Λ, or $-f$ scales. In the calculations below, we will ignore the mixing effects of the standard model Higgs boson H with the new particles $H_{1,2,3}$ (where the mixing angles defined by BC^{-1} are typically proportional to $\frac{u}{\omega}\ll 1$, which is actually small). Therefore, we have found the physical states H,H_1,H_2,H_3. Denoting $t_\beta=v/u$ and taking the effective limit $u/\omega, v/\omega\ll 1$, the physical scalar states are related to the gauge states as follows:

$$\begin{pmatrix} H \\ H_1 \end{pmatrix}\simeq\begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix}\begin{pmatrix} S_1 \\ S_2 \end{pmatrix},$$
$$\begin{pmatrix} \mathcal{A} \\ G_Z \end{pmatrix}\simeq\begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix}\begin{pmatrix} A_2 \\ A_1 \end{pmatrix},$$
$$\begin{pmatrix} H_2 \\ H_3 \end{pmatrix}\simeq\begin{pmatrix} c_\varphi & s_\varphi \\ -s_\varphi & c_\varphi \end{pmatrix}\begin{pmatrix} S_3 \\ S_4 \end{pmatrix},$$
$$\begin{pmatrix} H_5^- \\ G_W^- \end{pmatrix}=\begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix}\begin{pmatrix} \rho_1^- \\ \eta_2^- \end{pmatrix},$$
$$H_4\simeq\rho_3,\qquad G_Y\simeq\chi_2,\qquad G_X\simeq\chi_1,$$
$$H'\simeq\eta_3,\qquad G_{Z'}\simeq A_3, G_C=A_4. \tag{56}$$

As mentioned, the mixings of the standard model Higgs boson H with the new scalars $H_{1,2,3}$ are proportional to u/ω, where the proportional coefficients depend on the couplings of the scalar potential. Since the strengths of the scalar self-couplings are mostly unknown, these coefficients are undefined as well. Therefore, if the coefficients are small (as expected), the new physics effects via the mixings can be neglected, similar to the gauge-boson sector discussed below. Otherwise, it is important to note that the leading-order new physics effects must include the $\mathcal{O}(\{u,v\}/\{\omega,\Lambda,-f\})$ corrections to the couplings of the standard model Higgs boson due to the mixing with the new scalars, as well as the modifications of the H interactions to the new physics processes via the new scalars ($H_{1,2,3}$). In this case, the mixing parameters as determined by BC^{-1} have to be taken into account. However, it is also noted that even for the proportional coefficients of order unity (like a scalar self-coupling in the large strength regime), the modifications to the standard model Higgs couplings are around $|\Delta\kappa|\equiv u/\omega\sim 0.1$, which easily satisfies the κ_k bounds, as presented in Ref. [1].

We remind the reader that—apart from H', which will be identified as a viable dark matter candidate—the remaining scalars in this model should be sufficiently heavy in order to obey the bounds coming from the muon anomalous magnetic moment [21].

IV. GAUGE SECTOR

The gauge bosons obtain masses when the scalar fields develop VEVs. Therefore, their mass Lagrangian is given by

$$\mathcal{L}_{\text{mass}}^{\text{gauge}}=\sum_\Phi(D^\mu\langle\Phi\rangle)^\dagger(D_\mu\langle\Phi\rangle). \tag{57}$$

Substituting the scalar multiplets η, ρ, χ, and ϕ with their covariant derivatives, gauge charges, and VEVs (given above), we get

$$\begin{aligned}\mathcal{L}_{\text{mass}}^{\text{gauge}}&=\frac{g^2u^2}{8}\left[\left(A_{3\mu}+\frac{A_{8\mu}}{\sqrt{3}}-\frac{2}{3}t_XB_\mu+\frac{2}{3}t_NC_\mu\right)^2+2W_\mu^+W^{-\mu}+2X_\mu^{0*}X^{0\mu}\right]\\&+\frac{g^2v^2}{8}\left[\left(-A_{3\mu}+\frac{A_{8\mu}}{\sqrt{3}}+\frac{4}{3}t_XB_\mu+\frac{2}{3}t_NC_\mu\right)^2+2W_\mu^+W^{-\mu}+2Y_\mu^+Y^{-\mu}\right]\\&+\frac{g^2\omega^2}{8}\left[\left(-\frac{2A_{8\mu}}{\sqrt{3}}-\frac{2}{3}t_XB_\mu-\frac{4}{3}t_NC_\mu\right)^2+2Y_\mu^+Y^{-\mu}+2X_\mu^{0*}X^{0\mu}\right]+2g_N^2\Lambda^2C_\mu^2,\end{aligned} \tag{58}$$

where we have defined $t_X\equiv\frac{g_X}{g}$, $t_N\equiv\frac{g_N}{g}$, and

$$W_\mu^\pm=\frac{A_{1\mu}\mp iA_{2\mu}}{\sqrt{2}},\qquad X_\mu^{0,0*}=\frac{A_{4\mu}\mp iA_{5\mu}}{\sqrt{2}},\qquad Y_\mu^\mp=\frac{A_{6\mu}\mp iA_{7\mu}}{\sqrt{2}}. \tag{59}$$

The mass Lagrangian can be rewritten as

$$\mathcal{L}_{\text{mass}}^{\text{gauge}} = \frac{g^2}{4}(u^2+v^2)W^+W^- + \frac{g^2}{4}(v^2+\omega^2)Y^+Y^- + \frac{g^2}{4}(u^2+\omega^2)X^{0*}X^0 + \frac{1}{2}(A_3A_8BC)M^2\begin{pmatrix} A_3 \\ A_8 \\ B \\ C \end{pmatrix}, \quad (60)$$

where the Lorentz indices have been omitted and should be understood. The squared-mass matrix of the neutral gauge bosons is found to be

$$M^2 = \frac{g^2}{2}\begin{pmatrix} \frac{1}{2}(u^2+v^2) & \frac{u^2-v^2}{2\sqrt{3}} & -\frac{t_X(u^2+2v^2)}{3} & \frac{t_N(u^2-v^2)}{3} \\ \frac{u^2-v^2}{2\sqrt{3}} & \frac{1}{6}(u^2+v^2+4\omega^2) & -\frac{t_X(u^2-2(v^2+\omega^2))}{3\sqrt{3}} & \frac{t_N(u^2+v^2+4\omega^2)}{3\sqrt{3}} \\ -\frac{t_X(u^2+2v^2)}{3} & -\frac{t_X(u^2-2(v^2+\omega^2))}{3\sqrt{3}} & \frac{2}{9}t_X^2(u^2+4v^2+\omega^2) & -\frac{2}{9}t_Xt_N(u^2-2(v^2+\omega^2)) \\ \frac{t_N(u^2-v^2)}{3} & \frac{t_N(u^2+v^2+4\omega^2)}{3\sqrt{3}} & -\frac{2}{9}t_Xt_N(u^2-2(v^2+\omega^2)) & \frac{2}{9}t_N^2(u^2+v^2+4(\omega^2+9\Lambda^2)) \end{pmatrix}.$$

The non-Hermitian gauge bosons $W^\pm$, $X^{0,0*}$, and $Y^\pm$ by themselves are physical fields with the corresponding masses

$$m_W^2 = \frac{1}{4}g^2(u^2+v^2), \qquad m_X^2 = \frac{1}{4}g^2(u^2+\omega^2),$$
$$m_Y^2 = \frac{1}{4}g^2(v^2+\omega^2). \quad (61)$$

Because of the constraints $u, v \ll \omega$, we have $m_W \ll m_X \simeq m_Y$. W is identified as the standard model W boson, which implies

$$u^2+v^2 = (246\ \text{GeV})^2. \quad (62)$$

The X and Y fields are the new gauge bosons with large masses at the ω scale.

The neutral gauge bosons (A_3, A_8, B, C) mix via the mass matrix M^2. It is easily checked that M^2 has a zero eigenvalue with a corresponding eigenstate,

$$m_A^2 = 0, \qquad A_\mu = \frac{\sqrt{3}}{\sqrt{3+4t_X^2}}\left(t_XA_{3\mu} - \frac{t_X}{\sqrt{3}}A_{8\mu} + B_\mu\right), \quad (63)$$

which are independent of the VEVs and identified as those of the photon (notice that all the other eigenvalues of M^2 are nonzero). The independence of the VEVs for the photon field and its mass is a consequence of electric-charge conservation [22]. With this at hand, electromagnetic vertices can be calculated that result in the form $-eQ(f)\bar{f}\gamma^\mu fA_\mu$, where the electromagnetic coupling constant is identified as $e = gs_W$, with the sine of the Weinberg angle given by [22]

$$s_W = \frac{\sqrt{3}t_X}{\sqrt{3+4t_X^2}}. \quad (64)$$

The photon field can be rewritten as

$$\frac{A_\mu}{e} = \frac{A_{3\mu}}{g} - \frac{1}{\sqrt{3}}\frac{A_{8\mu}}{g} + \frac{B_\mu}{g_X}, \quad (65)$$

which is identical to the electric-charge operator expression in Eq. (2) if one replaces its generators by the corresponding gauge bosons over couplings (namely, Q is replaced by A_μ/e, T_i by $A_{i\mu}/g$, and X by B_μ/g_X). Hence, A_μ can be obtained from Q without using M^2. The mass eigenstate A_μ depends on just $A_{3\mu}$, $A_{8\mu}$, and B_μ, whereas the new gauge boson C_μ does not give any contribution, which stems from electric-charge conservation as well [22].

To identify the physical gauge bosons, we first rewrite the photon field in the form

$$A = s_WA_3 + c_W\left(-\frac{t_W}{\sqrt{3}}A_8 + \sqrt{1-\frac{t_W^2}{3}}B\right), \quad (66)$$

where we have used $t_X = \sqrt{3}s_W/\sqrt{3-4s_W^2}$. In the above expression, the combination in parentheses $(\cdots)$ is just the field that is associated with the weak hypercharge $Y = -\frac{1}{\sqrt{3}}T_8 + X$. The standard model Z boson is therefore identified as

$$Z = c_WA_3 - s_W\left(-\frac{t_W}{\sqrt{3}}A_8 + \sqrt{1-\frac{t_W^2}{3}}B\right), \quad (67)$$

which is orthogonal to A, as usual. The 3-3-1 model Z' boson, which is a new neutral boson, is orthogonal to the field that is coupled to the hypercharge Y (and thus it is orthogonal to both the A and Z bosons),

$$Z' = \sqrt{1-\frac{t_W^2}{3}}A_8 + \frac{t_W}{\sqrt{3}}B. \quad (68)$$

Hence, we can work in a new basis of the form (A, Z, Z', C), where the photon is a physical particle and is decoupled, while the other fields Z, Z', and C mix among themselves.

The mass matrix M^2 can be diagonalized via several steps. In the first step, we change the basis to $(A_3, A_8, B, C) \to (A, Z, Z', C)$,

$$\begin{pmatrix} A_3 \\ A_8 \\ B \\ C \end{pmatrix} = U_1 \begin{pmatrix} A \\ Z \\ Z' \\ C \end{pmatrix}, \qquad U_1 = \begin{pmatrix} s_W & c_W & 0 & 0 \\ -\frac{s_W}{\sqrt{3}} & \frac{s_W t_W}{\sqrt{3}} & \sqrt{1-\frac{t_W^2}{3}} & 0 \\ c_W\sqrt{1-\frac{t_W^2}{3}} & -s_W\sqrt{1-\frac{t_W^2}{3}} & \frac{t_W}{\sqrt{3}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (69)$$

In this new basis, the mass matrix M^2 becomes

$$M'^2 = U_1^T M^2 U_1 = \begin{pmatrix} 0 & 0 \\ 0 & M_s'^2 \end{pmatrix}, \quad (70)$$

where the 11 component is the zero mass of the photon (which is decoupled), while $M_s'^2$ is a 3×3 mass submatrix of Z, Z', and C,

$$M_s'^2 \equiv \begin{pmatrix} m_Z^2 & m_{ZZ'}^2 & m_{ZC}^2 \\ m_{ZZ'}^2 & m_{Z'}^2 & m_{Z'C}^2 \\ m_{ZC}^2 & m_{Z'C}^2 & m_C^2 \end{pmatrix}$$
$$= \frac{g^2}{2}\begin{pmatrix} \frac{(3+4t_X^2)(u^2+v^2)}{2(3+t_X^2)} & \frac{\sqrt{3+4t_X^2}((3-2t_X^2)u^2-(3+4t_X^2)v^2)}{6(3+t_X^2)} & \frac{\sqrt{3+4t_X^2}t_N(u^2-v^2)}{3\sqrt{3+t_X^2}} \\ \frac{\sqrt{3+4t_X^2}((3-2t_X^2)u^2-(3+4t_X^2)v^2)}{6(3+t_X^2)} & \frac{(3-2t_X^2)^2u^2+(3+4t_X^2)^2v^2+4(3+t_X^2)^2\omega^2}{18(3+t_X^2)} & \frac{t_N((3-2t_X^2)u^2+(3+4t_X^2)v^2+4(3+t_X^2)\omega^2)}{9\sqrt{3+t_X^2}} \\ \frac{\sqrt{3+4t_X^2}t_N(u^2-v^2)}{3\sqrt{3+t_X^2}} & \frac{t_N((3-2t_X^2)u^2+(3+4t_X^2)v^2+4(3+t_X^2)\omega^2)}{9\sqrt{3+t_X^2}} & \frac{2}{9}t_N^2(u^2+v^2+4(\omega^2+9\Lambda^2)) \end{pmatrix}.$$

Because of the conditions, $u, v \ll \omega, \Lambda$, we have $m_Z^2, m_{ZZ'}^2, m_{ZC}^2 \ll m_{Z'}^2, m_{Z'C}^2, m_C^2$. Hence, in the second step, the mass matrix M'^2 (or $M_s'^2$) can be diagonalized by using the seesaw formula [18] to separate the light state (Z) from the heavy states (Z', C). We denote the new basis as $(A, Z_1, \mathcal{Z}', \mathcal{C})$, so that A and Z_1 are physical fields and are decoupled while the rest mix,

$$\begin{pmatrix} A \\ Z \\ Z' \\ C \end{pmatrix} = U_2 \begin{pmatrix} A \\ Z_1 \\ \mathcal{Z}' \\ \mathcal{C} \end{pmatrix},$$

$$M''^2 = U_2^T M'^2 U_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & m_{Z_1}^2 & 0 \\ 0 & 0 & M_s''^2 \end{pmatrix}, \quad (71)$$

where $M_s''^2$ is a 2×2 mass submatrix of the $\mathcal{Z}', \mathcal{C}$ heavy states, while m_{Z_1} is the mass of the Z_1 light state. By virtue of the seesaw approximation, we have

$$U_2 \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \mathcal{E} \\ 0 & -\mathcal{E}^T & 1 \end{pmatrix},$$
$$\mathcal{E} \equiv (m_{ZZ'}^2 m_{ZC}^2)\begin{pmatrix} m_{Z'}^2 & m_{Z'C}^2 \\ m_{Z'C}^2 & m_C^2 \end{pmatrix}^{-1}, \quad (72)$$

$$m_{Z_1}^2 \simeq m_Z^2 - \mathcal{E}\begin{pmatrix} m_{ZZ'}^2 \\ m_{ZC}^2 \end{pmatrix}, \qquad M_s''^2 \simeq \begin{pmatrix} m_{Z'}^2 & m_{Z'C}^2 \\ m_{Z'C}^2 & m_C^2 \end{pmatrix}. \quad (73)$$

$\mathcal{E}$ is a two-component vector given by

$$\mathcal{E}_1 = -\frac{\sqrt{4t_X^2+3}\{3\Lambda^2[(2t_X^2-3)u^2+(4t_X^2+3)v^2]+t_X^2\omega^2(u^2+v^2)\}}{4\Lambda^2(t_X^2+3)^2\omega^2} \ll 1,$$

$$\mathcal{E}_2 = \frac{t_X^2\sqrt{4t_X^2+3}(u^2+v^2)}{8\Lambda^2(t_X^2+3)^{3/2}t_N} \ll 1,$$

which is suppressed at the leading order $u, v \ll \omega, \Lambda$. The Z_1, $\mathcal{Z}'$, and $\mathcal{C}$ fields are standard model-like, 3-3-1 model-like, and $U(1)_N$-like gauge bosons, respectively. To be concrete, we write $Z_1 \simeq Z - \mathcal{E}_1 Z' - \mathcal{E}_2 C$, $\mathcal{Z}' \simeq Z' + \mathcal{E}_1 Z$, and $\mathcal{C} \simeq C + \mathcal{E}_2 Z$, which differ from the Z, Z', and C fields by only small mixing terms, respectively.

Moreover, with the help of $t_X = \sqrt{3}s_W/\sqrt{3-4s_W^2}$, we have

$$\mathcal{E}_1 = -\frac{\sqrt{3-4s_W^2}}{4c_W^4}\left[\frac{v^2-c_{2W}u^2}{\omega^2}+\frac{s_W^2(u^2+v^2)}{9\Lambda^2}\right], \qquad \mathcal{E}_2 = \frac{s_W^2}{24c_W^3 t_N}\frac{u^2+v^2}{\Lambda^2}. \tag{74}$$

We realize that the first term in $\mathcal{E}_1$ is just the mixing angle of Z-Z' in the 3-3-1 model with right-handed neutrinos, $t_\theta \simeq \sqrt{3-4s_W^2}(c_{2W}u^2-v^2)/(4c_W^4\omega^2)$ [22], when $\Lambda \gg \omega$. Using $v_w^2 \equiv u^2+v^2 = (246\ \text{GeV})^2$ (which is the fixed weak scale) as well as $0 < u^2, v^2 < v_w^2$, the $\mathcal{E}_1$ parameter is bounded by

$$-\frac{\sqrt{3-4s_W^2}}{4c_W^4}\left[\left(\frac{v_w}{\omega}\right)^2+\frac{s_W^2}{9}\left(\frac{v_w}{\Lambda}\right)^2\right] < \mathcal{E}_1 < -\frac{\sqrt{3-4s_W^2}}{4c_W^4}\left[-c_{2W}\left(\frac{v_w}{\omega}\right)^2+\frac{s_W^2}{9}\left(\frac{v_w}{\Lambda}\right)^2\right], \tag{75}$$

where the second term in each set of brackets is negligible since $\Lambda \gtrsim \omega$. Therefore, the $\mathcal{E}_1$ bounds and the $\mathcal{E}_2$ parameter can be approximated as

$$-3.5\times10^{-3} < \mathcal{E}_1 < 3\times10^{-3},$$
$$\mathcal{E}_2 \simeq 0.014\left(\frac{1}{t_N}\right)\left(\frac{v_w}{\Lambda}\right)^2 \sim 10^{-4}, \tag{76}$$

provided that $s_W^2 \simeq 0.231$, $t_N \sim 1$, $\Lambda \sim \omega$, and $\omega > 3.198$ TeV, as given from the ρ parameter below. With such small values for the $\mathcal{E}_{1,2}$ mixing parameters, their corrections to the couplings of the Z boson—such as the well-measured $Zf\bar{f}$ ones (due to the mixing with the new Z', C gauge bosons)—can be neglected [1]. (However, notice that they can be changed due to the one-loop effects of Z', C as well as those of the non-Hermitian X, Y gauge bosons accompanied by the corresponding new fermions, which subsequently give the constraints on their masses and the g_N coupling. A detailed study of this matter is out of the scope of this work, and it should be addressed elsewhere). Even the modifications of the Z interactions (due to the mixings) to the new physics processes via the Z', C bosons are negligible, which will be explicitly shown when some of these processes are mentioned at the end of this work. Therefore, except for an evaluation of the mentioned ρ parameter, we will use only the leading-order terms below. In other words, the mixing of Z with the Z', C bosons can be neglected, so that $m_{Z_1} \simeq m_Z$, $Z_1 \simeq Z$, $\mathcal{Z}' \simeq Z'$, and $\mathcal{C} \simeq C$.

For the final step, it is easily to diagonalize M''^2 (or $M_s''^2$) to obtain the remaining two physical states, denoted by Z_2 and Z_N, such that

$$\begin{pmatrix} A \\ Z_1 \\ \mathcal{Z}' \\ \mathcal{C} \end{pmatrix} = U_3 \begin{pmatrix} A \\ Z_1 \\ Z_2 \\ Z_N \end{pmatrix}, \qquad U_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_\xi & -s_\xi \\ 0 & 0 & s_\xi & c_\xi \end{pmatrix},$$
$$M'''^2 = U_3^T M''^2 U_3 = \text{diag}(0, m_{Z_1}^2, m_{Z_2}^2, m_{Z_N}^2). \tag{77}$$

The mixing angle and new masses are given by

$$t_{2\xi} \simeq \frac{4\sqrt{3+t_X^2}t_N\omega^2}{(3+t_X^2)\omega^2-4t_N^2(\omega^2+9\Lambda^2)}, \tag{78}$$

$$m_{Z_N}^2 \simeq \frac{g^2}{18}\left((3+t_X^2)\omega^2+4t_N^2(\omega^2+9\Lambda^2)+\sqrt{((3+t_X^2)\omega^2-4t_N^2(\omega^2+9\Lambda^2))^2+16(3+t_X^2)t_N^2\omega^4}\right), \tag{79}$$

$$m_{Z_2}^2 \simeq \frac{g^2}{18}\left((3+t_X^2)\omega^2+4t_N^2(\omega^2+9\Lambda^2)-\sqrt{((3+t_X^2)\omega^2-4t_N^2(\omega^2+9\Lambda^2))^2+16(3+t_X^2)t_N^2\omega^4}\right). \tag{80}$$

It is noteworthy that the mixing of the 3-3-1 model Z' boson and the $\mathrm{U}(1)_N$ C boson is finite and may be large since $\omega\sim\Lambda$. Z_2 and Z_N are heavy particles with masses at the ω scale.

In summary, the physical fields are related to the gauge states as

$$\begin{pmatrix} A_3 \\ A_8 \\ B \\ C \end{pmatrix} = U \begin{pmatrix} A \\ Z_1 \\ Z_2 \\ Z_N \end{pmatrix}, \tag{81}$$

where

$$U = U_1U_2U_3 \simeq U_1U_3 = \begin{pmatrix} s_W & c_W & 0 & 0 \\ -\frac{s_W}{\sqrt{3}} & \frac{s_W t_W}{\sqrt{3}} & c_\xi\sqrt{1-\frac{t_W^2}{3}} & -s_\xi\sqrt{1-\frac{t_W^2}{3}} \\ c_W\sqrt{1-\frac{t_W^2}{3}} & -s_W\sqrt{1-\frac{t_W^2}{3}} & c_\xi\frac{t_W}{\sqrt{3}} & -s_\xi\frac{t_W}{\sqrt{3}} \\ 0 & 0 & s_\xi & c_\xi \end{pmatrix}. \tag{82}$$

The approximation above is given at the leading order $\{u^2, v^2\}/\{\omega^2, \Lambda^2\} \ll 1$, and this means that the standard model Z boson by itself is a physical field ($Z\simeq Z_1$) that does not mix with the new neutral gauge bosons, Z_2 and Z_N.

The next-to-leading-order term ($\mathcal{E}$) gives a contribution to the ρ parameter obtained by

$$\rho = \frac{m_W^2}{c_W^2 m_{Z_1}^2} = \frac{m_Z^2}{m_Z^2 - \mathcal{E}(m^2_{ZZ'}m^2_{ZC})^T} \simeq 1 + \mathcal{E}(m^2_{ZZ'}m^2_{ZC})^T/m_Z^2. \tag{83}$$

Here, notice that $m_W = c_W m_Z$ and $m_Z^2 \sim m^2_{ZZ'} \sim m^2_{ZC}$. To have a numerical value, let us put $u = v = (246/\sqrt{2})$ GeV and $\omega = \Lambda$. Hence, the deviation is

$$\Delta\rho \equiv \rho - 1 \simeq \frac{5s_W^2 t_W^4}{18\pi\alpha}\frac{u^2}{\omega^2} \simeq 0.236\frac{u^2}{\omega^2}, \tag{84}$$

where we have used $s_W^2 = 0.231$ and $\alpha = 1/128$ [1]. From the experimental data $\Delta\rho < 0.0007$ [1], we have $u/\omega < 0.0544$ or $\omega > 3.198$ TeV (provided that $u = 246/\sqrt{2}$ GeV, as mentioned). Therefore, the value of ω is on the TeV scale, as expected.

V. INTERACTIONS

A. Fermion–gauge boson interaction

The interactions of fermions with gauge bosons are derived from the Lagrangian

$$\mathcal{L}_{\text{fermion}} \equiv \bar{\Psi} i\gamma^\mu D_\mu \Psi, \tag{85}$$

where Ψ runs on all the fermion multiplets of the model. The covariant derivative as defined in Eq. (12) can be rewritten as $D_\mu = \partial_\mu + ig_s G_\mu + igP_\mu$, where $G_\mu \equiv t_i G_{i\mu}$ and $P_\mu \equiv T_i A_{i\mu} + t_X X B_\mu + t_N N C_\mu$ (note that $t_X = g_X/g$, $t_N = g_N/g$). Expanding the Lagrangian, we find

$$\mathcal{L}_{\text{fermion}} = \bar{\Psi} i\gamma^\mu \partial_\mu \Psi - g_s\bar{\Psi}\gamma^\mu G_\mu \Psi - g\bar{\Psi}\gamma^\mu P_\mu \Psi, \tag{86}$$

where the first term is kinematic whereas the last two terms give rise to the strong, electroweak, and $B-L$ interactions of the fermions.

Notice that the $\mathrm{SU}(3)_C$ generators, t_i, are equal to 0 for leptons and $\frac{\lambda_i}{2}$ for quarks q, where q indicates all the quarks of the model, such as $q = u, d, c, s, t, b, D_{1,2}, U$. Hence, the interactions of gluons with fermions as given by the second term of Eq. (86) yield

$$-g_s\bar{\Psi}\gamma^\mu G_\mu \Psi = -g_s\bar{q}_L\gamma^\mu \frac{\lambda_i}{2} q_L G_{i\mu} - g_s\bar{q}_R\gamma^\mu\frac{\lambda_i}{2} q_R G_{i\mu} = -g_s\bar{q}\gamma^\mu\frac{\lambda_i}{2} q G_{i\mu}, \tag{87}$$

which takes the usual form (i.e., only the colored particles have strong interactions).

Let us separate $P = P^{\text{CC}} + P^{\text{NC}}$, where

$$P^{\text{CC}} \equiv T_1A_1 + T_2A_2 + T_4A_4 + T_5A_5 + T_6A_6 + T_7A_7,$$
$$P^{\text{NC}} \equiv T_3A_3 + T_8A_8 + t_X XB + t_N \text{NC}. \tag{88}$$

Hence, the last term of Eq. (86) can be rewritten as

$$-g\bar{\Psi}\gamma^\mu P_\mu \Psi = -g\bar{\Psi}\gamma^\mu P^{\text{CC}}_\mu \Psi - g\bar{\Psi}\gamma^\mu P^{\text{NC}}_\mu\Psi. \tag{89}$$

Here, the first term provides the interactions of the non-Hermitian gauge bosons $W^\mp$, $X^{0,0*}$, and $Y^\pm$ with the fermions, while the last term leads to the interactions of the neutral gauge bosons A, Z_1, Z_2, and Z_N with the fermions.

Substituting the gauge states from Eq. (59) into P^{CC}, we get

$$P^{\text{CC}} = \frac{1}{\sqrt{2}}T^+W^+ + \frac{1}{\sqrt{2}}U^+X^0 + \frac{1}{\sqrt{2}}V^+Y^- + \text{H.c.}, \tag{90}$$

where the raising and lowering operators are defined as

$$T^\pm \equiv T_1 \pm iT_2, \quad U^\pm \equiv T_4 \pm iT_5, \quad V^\pm \equiv T_6 \pm iT_7. \tag{91}$$

Notice that $T^\pm$, $U^\pm$, and $V^\pm$ vanish for the right-handed fermion singlets. Therefore, the interactions of the non-Hermitian gauge bosons with fermions are obtained as

$$\begin{aligned}-g\bar{\Psi}\gamma^\mu P^{\mathrm{CC}}_\mu\Psi &= -\frac{g}{\sqrt{2}}\bar{\Psi}\gamma^\mu(T^+W^+_\mu + U^+X^0_\mu + V^+Y^-_\mu)\Psi + \mathrm{H.c.}\\ &= -\frac{g}{\sqrt{2}}\bar{\Psi}_L\gamma^\mu T^+\Psi_L W^+_\mu - \frac{g}{\sqrt{2}}\bar{\Psi}_L\gamma^\mu U^+\Psi_L X^0_\mu - \frac{g}{\sqrt{2}}\bar{\Psi}_L\gamma^\mu V^+\Psi_L Y^-_\mu + \mathrm{H.c.}\\ &= J^{-\mu}_W W^+_\mu + J^{0\mu}_X X^0_\mu + J^{+\mu}_Y Y^-_\mu + \mathrm{H.c.},\end{aligned} \tag{92}$$

where the currents associated with the corresponding non-Hermitian gauge bosons are given by

$$\begin{aligned}J^{-\mu}_W &\equiv -\frac{g}{\sqrt{2}}\bar{\Psi}_L\gamma^\mu T^+\Psi_L = -\frac{g}{\sqrt{2}}(\bar{\nu}_{aL}\gamma^\mu e_{aL} + \bar{u}_{aL}\gamma^\mu d_{aL}),\\ J^{0\mu}_X &\equiv -\frac{g}{\sqrt{2}}\bar{\Psi}_L\gamma^\mu U^+\Psi_L = -\frac{g}{\sqrt{2}}(\bar{\nu}_{aL}\gamma^\mu N^c_{aR} + \bar{u}_{3L}\gamma^\mu U_L - \bar{D}_{\alpha L}\gamma^\mu d_{\alpha L}),\\ J^{+\mu}_Y &\equiv -\frac{g}{\sqrt{2}}\bar{\Psi}_L\gamma^\mu V^+\Psi_L = -\frac{g}{\sqrt{2}}(\bar{e}_{aL}\gamma^\mu N^c_{aR} + \bar{d}_{3L}\gamma^\mu U_L + \bar{D}_{\alpha L}\gamma^\mu u_{\alpha L}).\end{aligned} \tag{93}$$

The interactions of the W boson are similar to those of the standard model, while the new interactions with the X and Y bosons are like those of the ordinary 3-3-1 model.

Substituting the gauge states given by Eq. (81) into P^{NC}, we have

$$\begin{aligned}P^{\mathrm{NC}}_\mu &= s_W Q A_\mu + \frac{1}{c_W}(T_3 - s^2_W Q)Z_\mu + \frac{1}{c_W}\left[c_\xi\left(\sqrt{\frac{3-4s^2_W}{3}}T_8 + \frac{s^2_W}{\sqrt{3-4s^2_W}}X\right) + s_\xi c_W t_N N\right]Z_{2\mu}\\ &\quad + \frac{1}{c_W}\left[-s_\xi\left(\sqrt{\frac{3-4s^2_W}{3}}T_8 + \frac{s^2_W}{\sqrt{3-4s^2_W}}X\right) + c_\xi c_W t_N N\right]Z_{N\mu}.\end{aligned} \tag{94}$$

For this expression, we have used $t_X = \sqrt{3}s_W/\sqrt{3-4s^2_W}$ and $Q = T_3 - T_8/\sqrt{3} + X$. The interactions of the neutral gauge bosons with fermions are given by

$$\begin{aligned}-g\bar{\Psi}\gamma^\mu P^{\mathrm{NC}}_\mu\Psi &= -gs_W\bar{\Psi}\gamma^\mu Q\Psi A_\mu - \frac{g}{c_W}\bar{\Psi}\gamma^\mu(T_3 - s^2_W Q)\Psi Z_\mu - \frac{g}{c_W}\bar{\Psi}\gamma^\mu\left[c_\xi\left(\sqrt{\frac{3-4s^2_W}{3}}T_8 + \frac{s^2_W}{\sqrt{3-4s^2_W}}X\right) + s_\xi c_W t_N N\right]\Psi Z_{2\mu}\\ &\quad - \frac{g}{c_W}\bar{\Psi}\gamma^\mu\left[-s_\xi\left(\sqrt{\frac{3-4s^2_W}{3}}T_8 + \frac{s^2_W}{\sqrt{3-4s^2_W}}X\right) + c_\xi c_W t_N N\right]\Psi Z_{N\mu}.\end{aligned} \tag{95}$$

Three remarks are in order.

(1) With the help of $e = gs_W$, the interactions of photons with fermions take the normal form,

$$-gs_W\bar{\Psi}\gamma^\mu Q\Psi A_\mu = -eQ(f)\bar{f}\gamma^\mu f A_\mu, \tag{96}$$

where f indicates any fermion of the model.

(2) The interactions of Z with fermions can be rewritten as

$$\begin{aligned}&-\frac{g}{c_W}\bar{\Psi}\gamma^\mu(T_3 - s^2_W Q)\Psi Z_\mu\\ &\quad = -\frac{g}{c_W}\{\bar{f}_L\gamma^\mu[T_3(f_L) - s^2_W Q(f_L)]f_L\\ &\qquad + \bar{f}_R\gamma^\mu[-s^2_W Q(f_R)]f_R\}Z_\mu,\\ &\quad = -\frac{g}{2c_W}\bar{f}\gamma^\mu[g^Z_V(f) - g^Z_A(f)\gamma_5]f Z_\mu,\end{aligned} \tag{97}$$

where

$$g^Z_V(f) \equiv T_3(f_L) - 2s^2_W Q(f), \qquad g^Z_A(f) \equiv T_3(f_L). \tag{98}$$

Therefore, the interactions of Z take the normal form. For convenience, the couplings of Z with fermions are given in Table II.

(3) It is noteworthy that the interactions of Z_2 with fermions are identical to those of Z_N if one makes the replacements $c_\xi \to -s_\xi$, $s_\xi \to c_\xi$ in the Z_2 interactions, and vice versa. Thus, we only need to obtain the interactions of either Z_2 or Z_N; the remainders are straightforward.

The interactions of Z_2 and Z_N with fermions can be rewritten in a common form similar to that of Z. Therefore, the last two terms of Eq. (95) yield

TABLE II. The couplings of Z with fermions.

f	$g^Z_V(f)$	$g^Z_A(f)$
ν_a	$\frac{1}{2}$	$\frac{1}{2}$
e_a	$-\frac{1}{2}+2s^2_W$	$-\frac{1}{2}$
N_a	0	0
u_a	$\frac{1}{2}-\frac{4}{3}s^2_W$	$\frac{1}{2}$
d_a	$-\frac{1}{2}+\frac{2}{3}s^2_W$	$-\frac{1}{2}$
U	$-\frac{4}{3}s^2_W$	0
D_α	$\frac{2}{3}s^2_W$	0

$$-\frac{g}{2c_W}\bar{f}\gamma^\mu[g^{Z_2}_V(f)-g^{Z_2}_A(f)\gamma_5]fZ_{2\mu}$$
$$-\frac{g}{2c_W}\bar{f}\gamma^\mu[g^{Z_N}_V(f)-g^{Z_N}_A(f)\gamma_5]fZ_{N\mu}, \tag{99}$$

where

$$g^{Z_2}_A(f)=-\frac{c_\xi s^2_W}{\sqrt{3-4s^2_W}}T_3(f_L)+\left(\frac{\sqrt{3}c_\xi c^2_W}{\sqrt{3-4s^2_W}}+\frac{2s_\xi c_W t_N}{\sqrt{3}}\right)T_8(f_L),$$
$$g^{Z_2}_V(f)=g^{Z_2}_A(f)+2\frac{c_\xi s^2_W}{\sqrt{3-4s^2_W}}Q(f)+2s_\xi c_W t_N(B-L)(f),$$
$$g^{Z_N}_{A,V}=g^{Z_2}_{A,V}(c_\xi\to -s_\xi, s_\xi\to c_\xi). \tag{100}$$

TABLE III. The couplings of Z_2 with fermions.

f	$g^{Z_2}_V(f)$	$g^{Z_2}_A(f)$
ν_a	$\frac{c_\xi c_{2W}}{2\sqrt{3-4s^2_W}}-\frac{5}{3}s_\xi c_W t_N$	$\frac{c_\xi c_{2W}}{2\sqrt{3-4s^2_W}}+\frac{1}{3}s_\xi c_W t_N$
e_a	$\frac{c_\xi(1-4s^2_W)}{2\sqrt{3-4s^2_W}}-\frac{5}{3}s_\xi c_W t_N$	$\frac{c_\xi}{2\sqrt{3-4s^2_W}}+\frac{1}{3}s_\xi c_W t_N$
N_a	$\frac{c_\xi c^2_W}{\sqrt{3-4s^2_W}}+\frac{2}{3}s_\xi c_W t_N$	$-\frac{c_\xi c^2_W}{\sqrt{3-4s^2_W}}-\frac{2}{3}s_\xi c_W t_N$
u_α	$-\frac{c_\xi(3-8s^2_W)}{6\sqrt{3-4s^2_W}}+\frac{1}{3}s_\xi c_W t_N$	$-\frac{c_\xi}{2\sqrt{3-4s^2_W}}-\frac{1}{3}s_\xi c_W t_N$
u_3	$\frac{c_\xi(3+2s^2_W)}{6\sqrt{3-4s^2_W}}+s_\xi c_W t_N$	$\frac{c_\xi c_{2W}}{2\sqrt{3-4s^2_W}}+\frac{1}{3}s_\xi c_W t_N$
d_α	$-\frac{c_\xi(3-2s^2_W)}{6\sqrt{3-4s^2_W}}+\frac{1}{3}s_\xi c_W t_N$	$-\frac{c_\xi c_{2W}}{2\sqrt{3-4s^2_W}}-\frac{1}{3}s_\xi c_W t_N$
d_3	$\frac{c_\xi\sqrt{3-4s^2_W}}{6}+s_\xi c_W t_N$	$\frac{c_\xi}{2\sqrt{3-4s^2_W}}+\frac{1}{3}s_\xi c_W t_N$
U	$-\frac{c_\xi(3-7s^2_W)}{3\sqrt{3-4s^2_W}}+2s_\xi c_W t_N$	$-\frac{c_\xi c^2_W}{\sqrt{3-4s^2_W}}-\frac{2}{3}s_\xi c_W t_N$
D_α	$\frac{c_\xi(3-5s^2_W)}{3\sqrt{3-4s^2_W}}-\frac{2}{3}s_\xi c_W t_N$	$\frac{c_\xi c^2_W}{\sqrt{3-4s^2_W}}+\frac{2}{3}s_\xi c_W t_N$

TABLE IV. The couplings of Z_N with fermions.

f	$g^{Z_N}_V(f)$	$g^{Z_N}_A(f)$
ν_a	$-\frac{s_\xi c_{2W}}{2\sqrt{3-4s^2_W}}-\frac{5}{3}c_\xi c_W t_N$	$-\frac{s_\xi c_{2W}}{2\sqrt{3-4s^2_W}}+\frac{1}{3}c_\xi c_W t_N$
e_a	$-\frac{s_\xi(1-4s^2_W)}{2\sqrt{3-4s^2_W}}-\frac{5}{3}c_\xi c_W t_N$	$-\frac{s_\xi}{2\sqrt{3-4s^2_W}}+\frac{1}{3}c_\xi c_W t_N$
N_a	$-\frac{s_\xi c^2_W}{\sqrt{3-4s^2_W}}+\frac{2}{3}c_\xi c_W t_N$	$\frac{s_\xi c^2_W}{\sqrt{3-4s^2_W}}-\frac{2}{3}c_\xi c_W t_N$
u_α	$\frac{s_\xi(3-8s^2_W)}{6\sqrt{3-4s^2_W}}+\frac{1}{3}c_\xi c_W t_N$	$\frac{s_\xi}{2\sqrt{3-4s^2_W}}-\frac{1}{3}c_\xi c_W t_N$
u_3	$-\frac{s_\xi(3+2s^2_W)}{6\sqrt{3-4s^2_W}}+c_\xi c_W t_N$	$-\frac{s_\xi c_{2W}}{2\sqrt{3-4s^2_W}}+\frac{1}{3}c_\xi c_W t_N$
d_α	$\frac{s_\xi(3-2s^2_W)}{6\sqrt{3-4s^2_W}}+\frac{1}{3}c_\xi c_W t_N$	$\frac{s_\xi c_{2W}}{2\sqrt{3-4s^2_W}}-\frac{1}{3}c_\xi c_W t_N$
d_3	$-\frac{s_\xi\sqrt{3-4s^2_W}}{6}+c_\xi c_W t_N$	$-\frac{s_\xi}{2\sqrt{3-4s^2_W}}+\frac{1}{3}c_\xi c_W t_N$
U	$\frac{s_\xi(3-7s^2_W)}{3\sqrt{3-4s^2_W}}+2c_\xi c_W t_N$	$\frac{s_\xi c^2_W}{\sqrt{3-4s^2_W}}-\frac{2}{3}c_\xi c_W t_N$
D_α	$-\frac{s_\xi(3-5s^2_W)}{3\sqrt{3-4s^2_W}}-\frac{2}{3}c_\xi c_W t_N$	$-\frac{s_\xi c^2_W}{\sqrt{3-4s^2_W}}+\frac{2}{3}c_\xi c_W t_N$

The interactions of Z_2 and Z_N with fermions are listed in Tables III and IV, respectively.

B. Scalar–gauge boson interaction

The interactions of gauge bosons with scalars arise from

$$\mathcal{L}_{\text{scalar}}\equiv(D^\mu\Phi)^\dagger(D_\mu\Phi), \tag{101}$$

where Φ runs on all the scalar multiplets of the model. From Eqs. (16) and (17), Φ has the common form $\Phi=\langle\Phi\rangle+\Phi'$. Moreover, the covariant derivative has the form $D_\mu=\partial_\mu+igP_\mu=\partial_\mu+ig(P^{\text{CC}}_\mu+P^{\text{NC}}_\mu)$ (see the previous subsection for details). Notice that the strong interaction vanishes because the scalars are colorless. Substituting all of these into the Lagrangian, we have

TABLE V. The interactions of a non-Hermitian gauge boson with two scalars.

Vertex	Coupling	Vertex	Coupling
$W^+_\mu H^-_5\overleftrightarrow{\partial}^\mu H_1$	$-\frac{ig}{2}$	$W^+_\mu H^-_5\overleftrightarrow{\partial}^\mu\mathcal{A}$	$\frac{g}{2}$
$Y^+_\mu H'^*\overleftrightarrow{\partial}^\mu H^-_5$	$-\frac{igs_\beta}{\sqrt{2}}$	$Y^+_\mu H^-_4\overleftrightarrow{\partial}^\mu H$	$-\frac{igs_\beta}{2}$
$Y^+_\mu H^-_4\overleftrightarrow{\partial}^\mu H_1$	$-\frac{igc_\beta}{2}$	$Y^+_\mu H^-_4\overleftrightarrow{\partial}^\mu\mathcal{A}$	$\frac{gc_\beta}{2}$
$X^0_\mu H^+_4\overleftrightarrow{\partial}^\mu H^-_5$	$\frac{igc_\beta}{\sqrt{2}}$	$X^0_\mu H'\overleftrightarrow{\partial}^\mu H$	$\frac{igc_\beta}{2}$
$X^0_\mu H'\overleftrightarrow{\partial}^\mu H_1$	$-\frac{igs_\beta}{2}$	$X^0_\mu H'\overleftrightarrow{\partial}^\mu\mathcal{A}$	$\frac{gs_\beta}{2}$

TABLE VI. The interactions of a neutral gauge boson with two scalars.

Vertex	Coupling	Vertex	Coupling
$A_\mu H_5^+ \overleftrightarrow{\partial}^\mu H_5^-$	ie	$A_\mu H_4^+ \overleftrightarrow{\partial}^\mu H_4^-$	ie
$Z_\mu H_4^+ \overleftrightarrow{\partial}^\mu H_4^-$	$-\frac{igs_W^2}{c_W}$	$Z_\mu H_5^+ \overleftrightarrow{\partial}^\mu H_5^-$	$\frac{igc_{2W}}{2c_W}$
$Z_\mu \mathcal{A} \overleftrightarrow{\partial}^\mu H_1$	$\frac{g}{2c_W}$	$Z_{2\mu} H_1 \overleftrightarrow{\partial}^\mu \mathcal{A}$	$g[\frac{c_\xi(c_\beta^2-c_{2W}s_\beta^2)}{2c_W\sqrt{3-4s_W^2}}+\frac{t_N s_\xi c_{2\beta}}{3}]$
$Z_{2\mu} H_4^+ \overleftrightarrow{\partial}^\mu H_4^-$	$ig(\frac{-c_{2W}c_\xi}{c_W\sqrt{3-4s_W^2}}+\frac{t_N s_\xi}{3})$	$Z_{2\mu} H_5^- \overleftrightarrow{\partial}^\mu H_5^+$	$ig[\frac{c_\xi(c_\beta^2-c_{2W}s_\beta^2)}{2c_W\sqrt{3-4s_W^2}}+\frac{t_N s_\xi c_{2\beta}}{3}]$
$Z_{2\mu} H' \overleftrightarrow{\partial}^\mu H'^*$	$-ig(\frac{c_W c_\xi}{\sqrt{3-4s_W^2}}-\frac{t_N s_\xi}{3})$	$Z_{2\mu} H \overleftrightarrow{\partial}^\mu \mathcal{A}$	$\frac{gs_{2\beta}}{2}(\frac{c_W c_\xi}{\sqrt{3-4s_W^2}}+\frac{2t_N s_\xi}{3})$
$Z_{N\mu} H_4^+ \overleftrightarrow{\partial}^\mu H_4^-$	$ig(\frac{c_{2W}s_\xi}{c_W\sqrt{3-4s_W^2}}+\frac{t_N c_\xi}{3})$	$Z_{N\mu} H_5^- \overleftrightarrow{\partial}^\mu H_5^+$	$ig[\frac{-s_\xi(c_\beta^2-c_{2W}s_\beta^2)}{2c_W\sqrt{3-4s_W^2}}+\frac{t_N c_\xi c_{2\beta}}{3}]$
$Z_{N\mu} H' \overleftrightarrow{\partial}^\mu H'^*$	$ig(\frac{c_W s_\xi}{\sqrt{3-4s_W^2}}+\frac{t_N c_\xi}{3})$	$Z_{N\mu} H \overleftrightarrow{\partial}^\mu \mathcal{A}$	$\frac{gs_{2\beta}}{2}(\frac{-c_W s_\xi}{\sqrt{3-4s_W^2}}+\frac{2t_N c_\xi}{3})$
$Z_{N\mu} H_1 \overleftrightarrow{\partial}^\mu \mathcal{A}$	$g[\frac{-s_\xi(c_\beta^2-c_{2W}s_\beta^2)}{2c_W\sqrt{3-4s_W^2}}+\frac{t_N c_\xi c_{2\beta}}{3}]$		

$$\begin{aligned}\mathcal{L}_{\text{scalar}} = {} & (\partial^\mu\Phi')^\dagger(\partial_\mu\Phi') + [ig(\partial^\mu\Phi')^\dagger(P_\mu\langle\Phi\rangle) + \text{H.c.}] \\ & + g^2\langle\Phi\rangle^\dagger P^\mu P_\mu\langle\Phi\rangle \\ & + [ig(\partial^\mu\Phi')^\dagger(P_\mu\Phi') + \text{H.c.}] \\ & + [g^2\langle\Phi\rangle P^\mu P_\mu \Phi' + \text{H.c.}] + g^2\Phi'^\dagger P^\mu P_\mu \Phi'. \end{aligned} \quad (102)$$

The terms in the first and second lines are, respectively, the kinematic, scalar-gauge mixing, and mass terms, which are not relevant to this analysis. The third and fourth lines includes all the interactions of three and four fields among the scalars and gauge bosons that we are interested in.

To calculate the interactions, we need to present Φ and P_μ in terms of the physical fields. Indeed, the gauge part takes the form $P_\mu = P_\mu^{\text{CC}} + P_\mu^{\text{NC}}$, where its terms have already been obtained in Eqs. (90) and (94), respectively. On the other hand, the physical scalars are related to the gauge states by Eq. (56). Let us work in a basis where all the Goldstone bosons are gauged away. In this unitary gauge, the scalar multiplets are given by

$$\eta = \begin{pmatrix} \frac{u}{\sqrt{2}} \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{1}{\sqrt{2}}(c_\beta H - s_\beta H_1 + i s_\beta \mathcal{A}) \\ s_\beta H_5^- \\ H' \end{pmatrix},$$

$$\rho = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \\ 0 \end{pmatrix} + \begin{pmatrix} c_\beta H_5^+ \\ \frac{1}{\sqrt{2}}(s_\beta H + c_\beta H_1 + i c_\beta \mathcal{A}) \\ H_4^+ \end{pmatrix},$$

$$\chi = \begin{pmatrix} 0 \\ 0 \\ \frac{\omega}{\sqrt{2}} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{\sqrt{2}}(c_\varphi H_2 - s_\varphi H_3) \end{pmatrix},$$

$$\phi = \frac{\Lambda}{\sqrt{2}} + \frac{s_\varphi H_2 + c_\varphi H_3}{\sqrt{2}}. \quad (103)$$

TABLE VII. The interactions of a scalar with two non-Hermitian gauge bosons.

Vertex	Coupling	Vertex	Coupling
$H_2 X^0 X^{0*}$	$\frac{g^2\omega}{2}c_\varphi$	$H_3 X^0 X^{0*}$	$-\frac{g^2\omega}{2}s_\varphi$
$H_2 Y^+ Y^-$	$\frac{g^2\omega}{2}c_\varphi$	$H_3 Y^+ Y^-$	$-\frac{g^2\omega}{2}s_\varphi$
$H W^+ W^-$	$\frac{g^2\sqrt{u^2+v^2}}{2}$	$H X^0 X^{0*}$	$\frac{g^2 u}{2}c_\beta$
$H_4^- W^+ X^{0*}$	$\frac{g^2 v}{2\sqrt{2}}$	$H_1 X^0 X^{0*}$	$-\frac{g^2 u}{2}s_\beta$
$H Y^+ Y^-$	$\frac{g^2 v}{2}s_\beta$	$H_1 Y^+ Y^-$	$\frac{g^2 v}{2}c_\beta$
$H_5^- X^0 Y^+$	$\frac{g^2\sqrt{u^2+v^2}}{2\sqrt{2}}s_{2\beta}$	$H'^* W^- Y^+$	$\frac{g^2 u}{2\sqrt{2}}$

TABLE VIII. The interactions of a scalar with a non-Hermitian gauge boson and a neutral gauge boson.

Vertex	Coupling
$H_5^+ W^- Z_2$	$g^2 u s_\beta(\frac{c_W c_\xi}{\sqrt{3-4s_W^2}}+\frac{2t_N s_\xi}{3})$
$H_5^+ W^- Z_N$	$g^2 u s_\beta(-\frac{c_W s_\xi}{\sqrt{3-4s_W^2}}+\frac{2t_N c_\xi}{3})$
$H' X^0 Z$	$\frac{g^2 u}{4c_W}$
$H' X^0 Z_2$	$\frac{g^2 u}{2}(-\frac{c_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{2t_N}{3}s_\xi)$
$H' X^0 Z_N$	$\frac{g^2 u}{2}(\frac{s_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{2t_N}{3}c_\xi)$
$H_4^- Y^+ A$	$\frac{gve}{2}$
$H_4^- Y^+ Z$	$-\frac{g^2 v}{4c_W}(1+2s_W^2)$
$H_4^- Y^+ Z_2$	$\frac{g^2 v}{2}[\frac{(1-2c_{2W})c_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{2t_N s_\xi}{3}]$
$H_4^- Y^+ Z_N$	$\frac{g^2 v}{2}[-\frac{(1-2c_{2W})s_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{2t_N c_\xi}{3}]$

TABLE IX. The interactions of a scalar with two neutral gauge bosons.

Vertex	Coupling
$H_2Z_2Z_2$	$4\Lambda g^2 t_N^2 s_\varphi s_\xi^2 + \omega c_\varphi g^2 (\frac{c_W c_\xi}{\sqrt{3-4s_W^2}} + \frac{2t_N}{3} s_\xi)^2$
$H_2Z_NZ_N$	$4\Lambda g^2 t_N^2 s_\varphi c_\xi^2 + \omega c_\varphi g^2 (-\frac{c_W s_\xi}{\sqrt{3-4s_W^2}} + \frac{2t_N}{3} c_\xi)^2$
$H_2Z_2Z_N$	$4\Lambda g^2 t_N^2 s_\varphi s_{2\xi} + 2\omega c_\varphi g^2 (\frac{c_W c_\xi}{\sqrt{3-4s_W^2}} + \frac{2t_N}{3} s_\xi)(-\frac{c_W s_\xi}{\sqrt{3-4s_W^2}} + \frac{2t_N}{3} c_\xi)$
$H_3Z_2Z_2$	$4\Lambda g^2 t_N^2 c_\varphi s_\xi^2 - \omega s_\varphi g^2 (\frac{c_W c_\xi}{\sqrt{3-4s_W^2}} + \frac{2t_N}{3} s_\xi)^2$
$H_3Z_NZ_N$	$4\Lambda g^2 t_N^2 c_\varphi c_\xi^2 - \omega s_\varphi g^2 (-\frac{c_W s_\xi}{\sqrt{3-4s_W^2}} + \frac{2t_N}{3} c_\xi)^2$
$H_3Z_2Z_N$	$4\Lambda g^2 t_N^2 c_\varphi s_{2\xi} - 2\omega s_\varphi g^2 (\frac{c_W c_\xi}{\sqrt{3-4s_W^2}} + \frac{2t_N}{3} s_\xi)(-\frac{c_W s_\xi}{\sqrt{3-4s_W^2}} + \frac{2t_N}{3} c_\xi)$
HZZ	$\frac{g^2}{4c_W^2}\sqrt{u^2+v^2}$
HZ_2Z_2	$g^2[uc_\beta(\frac{c_{2W}c_\xi}{2c_W\sqrt{3-4s_W^2}} + \frac{t_N}{3}s_\xi)^2 + vs_\beta(\frac{c_\xi}{2c_W\sqrt{3-4s_W^2}} + \frac{t_N}{3}s_\xi)^2]$
HZ_NZ_N	$g^2[uc_\beta(\frac{-c_{2W}s_\xi}{2c_W\sqrt{3-4s_W^2}} + \frac{t_N}{3}c_\xi)^2 + vs_\beta(\frac{-s_\xi}{2c_W\sqrt{3-4s_W^2}} + \frac{t_N}{3}c_\xi)^2]$
HZZ_2	$\frac{g^2}{c_W}[uc_\beta(\frac{c_{2W}c_\xi}{2c_W\sqrt{3-4s_W^2}} + \frac{t_N}{3}s_\xi) - vs_\beta(\frac{c_\xi}{2c_W\sqrt{3-4s_W^2}} + \frac{t_N}{3}s_\xi)]$
HZZ_N	$\frac{g^2}{c_W}[uc_\beta(\frac{-c_{2W}s_\xi}{2c_W\sqrt{3-4s_W^2}} + \frac{t_N}{3}c_\xi) - vs_\beta(\frac{-s_\xi}{2c_W\sqrt{3-4s_W^2}} + \frac{t_N}{3}c_\xi]$
HZ_2Z_N	$2g^2[uc_\beta(\frac{c_{2W}c_\xi}{2c_W\sqrt{3-4s_W^2}} + \frac{t_N}{3}s_\xi)(\frac{-c_{2W}s_\xi}{2c_W\sqrt{3-4s_W^2}} + \frac{t_N}{3}c_\xi) + vs_\beta(\frac{c_\xi}{2c_W\sqrt{3-4s_W^2}} + \frac{t_N}{3}s_\xi)(\frac{-s_\xi}{2c_W\sqrt{3-4s_W^2}} + \frac{t_N}{3}c_\xi)]$
$H_1Z_2Z_2$	$g^2[-us_\beta(\frac{c_{2W}c_\xi}{2c_W\sqrt{3-4s_W^2}} + \frac{t_N}{3}s_\xi)^2 + vc_\beta(\frac{c_\xi}{2c_W\sqrt{3-4s_W^2}} + \frac{t_N}{3}s_\xi)^2]$
$H_1Z_NZ_N$	$g^2[-us_\beta(\frac{-c_{2W}s_\xi}{2c_W\sqrt{3-4s_W^2}} + \frac{t_N}{3}c_\xi)^2 + vc_\beta(\frac{-s_\xi}{2c_W\sqrt{3-4s_W^2}} + \frac{t_N}{3}c_\xi)^2]$
H_1ZZ_2	$-\frac{g^2}{c_W}[us_\beta(\frac{c_{2W}c_\xi}{2c_W\sqrt{3-4s_W^2}} + \frac{t_N}{3}s_\xi) + vc_\beta(\frac{c_\xi}{2c_W\sqrt{3-4s_W^2}} + \frac{t_N}{3}s_\xi)]$
H_1ZZ_N	$-\frac{g^2}{c_W}[us_\beta(\frac{-c_{2W}s_\xi}{2c_W\sqrt{3-4s_W^2}} + \frac{t_N}{3}c_\xi) + vc_\beta(\frac{-s_\xi}{2c_W\sqrt{3-4s_W^2}} + \frac{t_N}{3}c_\xi)]$
$H_1Z_2Z_N$	$2g^2[-us_\beta(\frac{c_{2W}c_\xi}{2c_W\sqrt{3-4s_W^2}} + \frac{t_N}{3}s_\xi)(\frac{-c_{2W}s_\xi}{2c_W\sqrt{3-4s_W^2}} + \frac{t_N}{3}c_\xi) + vc_\beta(\frac{c_\xi}{2c_W\sqrt{3-4s_W^2}} + \frac{t_N}{3}s_\xi)(\frac{-s_\xi}{2c_W\sqrt{3-4s_W^2}} + \frac{t_N}{3}c_\xi)]$

Notice that in each expansion above for the multiplet $\Phi = \eta, \rho, \chi, \phi$, the first term is identified as $\langle\Phi\rangle$, while the second term is Φ' with the physical fields explicitly displayed. The notations for the scalar multiplets including the gauge bosons in this unitary gauge have conveniently been kept unchanged.

The interactions of one gauge boson with two scalars arise from

$$ig(\partial^\mu\Phi')^\dagger(P_\mu\Phi') + \text{H.c.} = ig(\partial^\mu\Phi')^\dagger(P_\mu^{\text{CC}}\Phi') + ig(\partial^\mu\Phi')^\dagger(P_\mu^{\text{NC}}\Phi') + \text{H.c.}. \quad (104)$$

Substituting all the known multiplets into this expression gives us the values in Tables V and VI. We note that $A\overleftrightarrow{\partial}B \equiv A(\partial B) - (\partial A)B$ is frequently used.

The interactions of one scalar with two gauge bosons are given by

$$g^2\langle\Phi\rangle P^\mu P_\mu\Phi' + \text{H.c.} = g^2\langle\Phi\rangle P^{\text{CC}\mu}P_\mu^{\text{CC}}\Phi' + g^2\langle\Phi\rangle \times (P^{\text{CC}\mu}P_\mu^{\text{NC}} + P^{\text{NC}\mu}P_\mu^{\text{CC}})\Phi' + g^2\langle\Phi\rangle P^{\text{NC}\mu}P_\mu^{\text{NC}}\Phi' + \text{H.c.}. \quad (105)$$

These interactions are listed in Tables VII, VIII, and IX, corresponding to the terms on the rhs, respectively.

The interactions of two scalars and two gauge bosons are derived from

$$g^2\Phi'^\dagger P^\mu P_\mu\Phi' = g^2\Phi'^\dagger P^{\text{CC}\mu}P_\mu^{\text{CC}}\Phi' + g^2\Phi'^\dagger(P^{\text{CC}\mu}P_\mu^{\text{NC}} + P^{\text{NC}\mu}P_\mu^{\text{CC}})\Phi' + g^2\Phi'^\dagger P^{\text{NC}\mu}P_\mu^{\text{NC}}\Phi', \quad (106)$$

which give us the values in Tables X, XI, and XII, respectively.

TABLE X. The interactions of two non-Hermitian gauge bosons and two scalars.

Vertex	Coupling	Vertex	Coupling
$X^0X^{0*}H_2H_2$	$\frac{g^2}{4}c_\varphi^2$	$X^0X^{0*}H_3H_3$	$\frac{g^2}{4}s_\varphi^2$
$X^0X^{0*}H_2H_3$	$-\frac{g^2}{4}s_{2\varphi}$	$Y^+Y^-H_2H_2$	$\frac{g^2}{4}c_\varphi^2$
$Y^+Y^-H_3H_3$	$\frac{g^2}{4}s_\varphi^2$	$Y^+Y^-H_2H_3$	$-\frac{g^2}{4}s_{2\varphi}$
$W^+W^-H_5^+H_5^-$	$\frac{g^2}{2}$	$X^0X^{0*}H_5^+H_5^-$	$\frac{g^2}{2}c_\beta^2$
$X^0X^{0*}H_4^+H_4^-$	$\frac{g^2}{2}$	$Y^+Y^-H_4^+H_4^-$	$\frac{g^2}{2}$
W^+W^-HH	$\frac{g^2}{4}$	$W^+W^-H_1H_1$	$\frac{g^2}{4}$
$W^+W^-\mathcal{A}\mathcal{A}$	$\frac{g^2}{4}$	Y^+Y^-HH	$\frac{g^2}{4}s_\beta^2$
$Y^+Y^-H_1H_1$	$\frac{g^2}{4}c_\beta^2$	$Y^+Y^-HH_1$	$\frac{g^2}{4}s_{2\beta}$
$Y^+Y^-\mathcal{A}\mathcal{A}$	$\frac{g^2}{4}c_\beta^2$	$X^0Y^+H_5^-H$	$\frac{g^2}{2\sqrt{2}}s_{2\beta}$
$X^0Y^+H_5^-H_1$	$\frac{g^2}{2\sqrt{2}}c_{2\beta}$	$X^0Y^+H_5^-\mathcal{A}$	$i\frac{g^2}{2\sqrt{2}}c_{2\beta}$
$W^+Y^-H_4^+H_5^-$	$\frac{g^2}{2}c_\beta$	$W^-X^0HH_4^+$	$\frac{g^2}{2\sqrt{2}}s_\beta$
$W^-X^0H_1H_4^+$	$\frac{g^2}{2\sqrt{2}}c_\beta$	$W^-X^0\mathcal{A}H_4^+$	$\frac{-ig^2}{2\sqrt{2}}c_\beta$
$X^{0*}X^0HH$	$\frac{g^2}{4}c_\beta^2$	$X^{0*}X^0H_1H_1$	$\frac{g^2}{4}s_\beta^2$
$X^{0*}X^0HH_1$	$-\frac{g^2}{4}s_{2\beta}$	$X^{0*}X^0\mathcal{A}\mathcal{A}$	$\frac{g^2}{4}s_\beta^2$
$Y^+Y^-H_5^+H_5^-$	$\frac{g^2}{2}s_\beta^2$	$X^{0*}X^0H'^*H'$	$\frac{g^2}{2}$
$Y^+Y^-H'^*H'$	$\frac{g^2}{2}$	W^+Y^-HH'	$\frac{g^2}{2\sqrt{2}}c_\beta$
$W^+Y^-H_1H'$	$-\frac{g^2}{2\sqrt{2}}s_\beta$	$W^+Y^-\mathcal{A}H'$	$\frac{-ig^2}{2\sqrt{2}}s_\beta$
$W^-X^0H_5^+H'$	$\frac{g^2s_\beta}{2}$		

VI. NEW PHYSICS EFFECTS AND CONSTRAINTS

A. Dark matter: complex scalar H'

The spectrum of scalar particles in the model contains an electrically neutral particle H' that is odd under W parity. Because the W parity symmetry is exact and unbroken by the VEVs, H' is stabilized and thus cannot decay if it is the lightest of the W particles. In this regime we obtain the relic density of H' at the present time and derive some constraints on its mass. Such a scalar is within the context of the so-called Higgs portal, which has been intensively exploited in the literature [23,24] due to its interaction with the standard model Higgs boson via the scalar-potential regime. We will show that H' can be a viable dark matter candidate that yields the correct abundance ($\Omega h^2 = 0.11$–0.12) and obeys the direct-detection bounds [25].

In the early Universe, H' was in thermal equilibrium with the standard model particles. As the Universe expanded and cooled down, it reached a point where the temperature was roughly equal to the H' mass, which prevented the production of H' particles via the annihilation of the standard model particles; only annihilations between H' particles occurred. However, as the Universe keeps expanding, there is a point where the H' particles can no longer annihilate themselves into the standard model particles—the so-called freeze-out. In this way, the H' particles leftover from the freeze-out populate the Universe today. In order to find the relic density of a dark matter particle one would need to solve the Boltzmann equation [26], which we will do for the fermion dark matter case. However, since H' is a scalar dark matter particle there are only s-wave contributions to the annihilation cross section, and thus the abundance can be approximated as

$$\Omega_{H'}h^2 \simeq \frac{0.1\ \mathrm{pb}}{\langle\sigma v_{\mathrm{rel}}\rangle}. \qquad (107)$$

Here, $\langle\sigma v_{\mathrm{rel}}\rangle$ is the thermal average over the cross section for two H''s annihilating into standard model particles multiplied by the relative velocity between the two H' particles.

For dark matter masses below $m_H/2$ the Higgs portal is quite constrained, as discussed in Refs. [23,24]. For dark matter masses larger than the Higgs mass the annihilation channel $H'H' \to HH$ plays a major role in determining the abundance. Therefore, we will focus on the Higgs portal in order to estimate the abundance and derive a bound on the scalar dark matter candidate. That being said, the interaction of H' with H is obtained as follows:

$$\mathcal{L}_{H'-H} = \left(\frac{\lambda_5}{2} + \lambda_3\right)H^2H'^*H'. \qquad (108)$$

The scattering amplitude for $H'H' \to HH$ is

$$iM(H'H' \to HH) = i(\lambda_5 + 2\lambda_3) \equiv i\lambda'. \qquad (109)$$

It is also noted that there may be other contributions to λ' mediated by the Higgs H, the new scalars, and the new gauge bosons. However, such corrections are subleading, with the assumptions that the λ' coupling is of order unity and that H' is heavy enough. Therefore, the differential cross section in the center-of-mass frame is given by

$$\frac{d\sigma}{d\Omega} = \frac{|M(H'H' \to HH)|^2|\vec{k}|}{64\pi^2 s|\vec{p}|}\cdot\frac{1}{2}, \qquad (110)$$

where H' has energy and momentum $H'(E,\vec{p})$, and thus $H'^*(E,-\vec{p})$. Also, the two outgoing Higgs bosons possess $H(E,\vec{k})$ and $H(E,-\vec{k})$. The coefficient $\frac{1}{2}$ is due to the creation of the two identical particles. Thus, we have $\sqrt{s} = 2E$.

From the experimental side, the dark matter is nonrelativistic $(v \sim 10^{-3}c)$. We approximate

TABLE XI. The interactions of two scalars with a non-Hermitian gauge boson and a neutral gauge boson.

Vertex	Coupling	Vertex	Coupling
$H_1H_5^-W^+A$	$ge/2$	$\mathcal{A}H_5^-W^+A$	$ige/2$
$H_1H_5^-W^+Z$	$\frac{g^2}{4c_W}(c_{2W}-1)$	$\mathcal{A}H_5^-W^+Z$	$\frac{ig^2}{4c_W}(c_{2W}-1)$
$HH_5^-W^+Z_2$	$\frac{1}{2}g^2s_{2\beta}(\frac{c_\xi c_W}{\sqrt{3-4s_W^2}}+\frac{2t_Ns_\xi}{3})$	$HH_5^-W^+Z_N$	$\frac{1}{2}g^2s_{2\beta}(\frac{-s_\xi c_W}{\sqrt{3-4s_W^2}}+\frac{2t_Nc_\xi}{3})$
$H_1H_5^-W^+Z_2$	$g^2[\frac{c_\xi(c_\beta^2-s_\beta^2c_{2W})}{2c_W\sqrt{3-4s_W^2}}+\frac{t_N}{3}s_\xi c_{2\beta}]$	$H_1H_5^-W^+Z_N$	$g^2[\frac{-s_\xi(c_\beta^2-s_\beta^2c_{2W})}{2c_W\sqrt{3-4s_W^2}}+\frac{t_N}{3}c_\xi c_{2\beta}]$
$\mathcal{A}H_5^-W^+Z_2$	$ig^2[\frac{c_\xi(c_\beta^2-s_\beta^2c_{2W})}{2c_W\sqrt{3-4s_W^2}}+\frac{t_N}{3}s_\xi c_{2\beta}]$	$\mathcal{A}H_5^-W^+Z_N$	$ig^2[\frac{-s_\xi(c_\beta^2-s_\beta^2c_{2W})}{2c_W\sqrt{3-4s_W^2}}+\frac{t_N}{3}c_\xi c_{2\beta}]$
$H_5^-H_4^+X^0A$	$\sqrt{2}gec_\beta$	$H_5^-H_4^+X^0Z$	$\frac{g^2c_\beta}{2\sqrt{2}c_W}(4c_W^2-3)$
$H_5^-H_4^+X^0Z_2$	$\frac{g^2c_\beta}{\sqrt{2}}[\frac{c_\xi(3-4c_W^2)}{2c_W\sqrt{3-4s_W^2}}+\frac{2t_Ns_\xi}{3}]$	$H_5^-H_4^+X^0Z_N$	$\frac{g^2c_\beta}{\sqrt{2}}[\frac{-s_\xi(3-4c_W^2)}{2c_W\sqrt{3-4s_W^2}}+\frac{2t_Nc_\xi}{3}]$
$HH_4^+Y^-A$	$\frac{ges_\beta}{2}$	$H_1H_4^+Y^-A$	$\frac{gec_\beta}{2}$
$\mathcal{A}H_4^+Y^-A$	$\frac{-igec_\beta}{2}$	$HH_4^+Y^-Z$	$\frac{-g^2s_\beta(2-c_{2W})}{4c_W}$
$H_1H_4^+Y^-Z$	$\frac{-g^2c_\beta(2-c_{2W})}{4c_W}$	$\mathcal{A}H_4^+Y^-Z$	$\frac{ig^2c_\beta(2-c_{2W})}{4c_W}$
$HH_4^+Y^-Z_2$	$\frac{g^2s_\beta}{2}[\frac{c_\xi(1-2c_{2W})}{2c_W\sqrt{3-4s_W^2}}+\frac{2t_Ns_\xi}{3}]$	$H_1H_4^+Y^-Z_2$	$\frac{g^2c_\beta}{2}[\frac{c_\xi(1-2c_{2W})}{2c_W\sqrt{3-4s_W^2}}+\frac{2t_Ns_\xi}{3}]$
$\mathcal{A}H_4^+Y^-Z_2$	$\frac{-ig^2c_\beta}{2}[\frac{c_\xi(1-2c_{2W})}{2c_W\sqrt{3-4s_W^2}}+\frac{2t_Ns_\xi}{3}]$	$HH_4^+Y^-Z_N$	$\frac{g^2s_\beta}{2}[\frac{-s_\xi(1-2c_{2W})}{2c_W\sqrt{3-4s_W^2}}+\frac{2t_Nc_\xi}{3}]$
$H_1H_4^+Y^-Z_N$	$\frac{g^2c_\beta}{2}[\frac{-s_\xi(1-2c_{2W})}{2c_W\sqrt{3-4s_W^2}}+\frac{2t_Nc_\xi}{3}]$	$\mathcal{A}H_4^+Y^-Z_N$	$\frac{-ig^2c_\beta}{2}[\frac{-s_\xi(1-2c_{2W})}{2c_W\sqrt{3-4s_W^2}}+\frac{2t_Nc_\xi}{3}]$
$HH'X^0Z$	$\frac{g^2c_\beta}{4c_W}$	$H_1H'X^0Z$	$\frac{-g^2s_\beta}{4c_W}$
$\mathcal{A}H'X^0Z$	$\frac{-ig^2s_\beta}{4c_W}$	$HH'X^0Z_2$	$\frac{g^2c_\beta}{2}(\frac{-c_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{2t_Ns_\xi}{3})$
$H_1H'X^0Z_2$	$\frac{-g^2s_\beta}{2}(\frac{-c_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{2t_Ns_\xi}{3})$	$\mathcal{A}H'X^0Z_2$	$-\frac{ig^2s_\beta}{2}(\frac{-c_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{2t_Ns_\xi}{3})$
$HH'X^0Z_N$	$\frac{g^2c_\beta}{2}(\frac{s_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{2t_Nc_\xi}{3})$	$H_1H'X^0Z_N$	$\frac{-g^2s_\beta}{2}(\frac{s_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{2t_Nc_\xi}{3})$
$\mathcal{A}H'X^0Z_N$	$-\frac{ig^2s_\beta}{2}(\frac{s_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{2t_Nc_\xi}{3})$	$H_5^+H'Y^-A$	$\frac{-ges_\beta}{\sqrt{2}}$
$H_5^+H'Y^-Z$	$\frac{-g^2s_\beta c_{2W}}{2\sqrt{2}c_W}$	$H_5^+H'Y^-Z_2$	$\frac{g^2s_\beta}{\sqrt{2}}(\frac{-c_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{2t_Ns_\xi}{3})$
$H_5^+H'Y^-Z_N$	$\frac{g^2s_\beta}{\sqrt{2}}(\frac{s_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{2t_Nc_\xi}{3})$		

$$E=\frac{m_{H'}}{\sqrt{1-v^2}}\simeq m_{H'}\left(1+\frac{1}{2}v^2\right), \tag{111}$$

where v is the velocity of the dark matter given in natural units, $v\ll 1$. We also have

$$s=4E^2\simeq 4m_{H'}^2(1+v^2),$$
$$|\vec{p}|=\frac{m_{H'}v}{\sqrt{1-v^2}}\simeq m_{H'}v\left(1+\frac{1}{2}v^2\right)\simeq m_{H'}v. \tag{112}$$

The Einstein relation implies

$$|\vec{k}|=\sqrt{E^2-m_H^2}\simeq\sqrt{m_{H'}^2(1+v^2)-m_H^2}$$
$$\simeq m_{H'}\sqrt{1+v^2-\frac{m_H^2}{m_{H'}^2}}\simeq m_{H'}\left(1+\frac{v^2}{2}-\frac{m_H^2}{2m_{H'}^2}\right). \tag{113}$$

Therefore, the differential cross section takes the form

$$\frac{d\sigma}{d\Omega}\simeq\frac{\lambda'^2m_{H'}(1+\frac{v^2}{2}-\frac{m_H^2}{2m_{H'}^2})}{64\pi^2 4m_{H'}^2(1+v^2)m_{H'}2v}. \tag{114}$$

TABLE XII. The interactions of two scalars with two neutral gauge bosons.

Vertex	Coupling
$H_2H_2Z_2Z_2$	$g^2[2t_N^2s_\varphi^2s_\xi^2+\frac{1}{2}c_\varphi^2(\frac{c_Wc_\xi}{\sqrt{3-4s_W^2}}+\frac{2t_Ns_\xi}{3})^2]$
$H_2H_2Z_NZ_N$	$g^2[2t_N^2s_\varphi^2c_\xi^2+\frac{1}{2}c_\varphi^2(\frac{-c_Ws_\xi}{\sqrt{3-4s_W^2}}+\frac{2t_Nc_\xi}{3})^2]$
$H_2H_2Z_2Z_N$	$g^2[2t_N^2s_\varphi^2s_{2\xi}+c_\varphi^2(\frac{c_Wc_\xi}{\sqrt{3-4s_W^2}}+\frac{2t_Ns_\xi}{3})(\frac{-c_Ws_\xi}{\sqrt{3-4s_W^2}}+\frac{2t_Nc_\xi}{3})]$
$H_3H_3Z_2Z_2$	$g^2[2t_N^2c_\varphi^2s_\xi^2+\frac{1}{2}s_\varphi^2(\frac{c_Wc_\xi}{\sqrt{3-4s_W^2}}+\frac{2t_Ns_\xi}{3})^2]$
$H_3H_3Z_NZ_N$	$g^2[2t_N^2c_\varphi^2c_\xi^2+\frac{1}{2}s_\varphi^2(\frac{-c_Ws_\xi}{\sqrt{3-4s_W^2}}+\frac{2t_Nc_\xi}{3})^2]$
$H_3H_3Z_2Z_N$	$g^2[2t_N^2s_{2\xi}c_\varphi^2+s_\varphi^2(\frac{c_Wc_\xi}{\sqrt{3-4s_W^2}}+\frac{2t_Ns_\xi}{3})(\frac{-c_Ws_\xi}{\sqrt{3-4s_W^2}}+\frac{2t_Nc_\xi}{3})]$
$H_2H_3Z_2Z_2$	$g^2[2t_N^2s_{2\varphi}s_\xi^2-\frac{s_{2\varphi}}{2}(\frac{c_Wc_\xi}{\sqrt{3-4s_W^2}}+\frac{2t_Ns_\xi}{3})^2]$
$H_2H_3Z_NZ_N$	$g^2[2t_N^2s_{2\varphi}c_\xi^2-\frac{s_{2\varphi}}{2}(\frac{-c_Ws_\xi}{\sqrt{3-4s_W^2}}+\frac{2t_Nc_\xi}{3})^2]$
$H_2H_3Z_2Z_N$	$g^2[2t_N^2s_{2\varphi}s_{2\xi}-s_{2\varphi}(\frac{c_Wc_\xi}{\sqrt{3-4s_W^2}}+\frac{2t_Ns_\xi}{3})(\frac{-c_Ws_\xi}{\sqrt{3-4s_W^2}}+\frac{2t_Nc_\xi}{3})]$
$H_5^+H_5^-AA$	e^2
$H_5^+H_5^-ZZ$	$\frac{g^2c_{2W}^2}{4c_W^2}$
$H_5^+H_5^-Z_2Z_2$	$g^2[c_\beta^2(\frac{c_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Ns_\xi}{3})^2+s_\beta^2(\frac{c_\xi c_{2W}}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Ns_\xi}{3})^2]$
$H_5^+H_5^-Z_NZ_N$	$g^2[c_\beta^2(\frac{-s_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Nc_\xi}{3})^2+s_\beta^2(\frac{-s_\xi c_{2W}}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Nc_\xi}{3})^2]$
$H_5^+H_5^-AZ$	$\frac{egc_{2W}}{c_W}$
$H_5^+H_5^-ZZ_2$	$\frac{g^2c_{2W}}{c_W}[c_\beta^2(\frac{c_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Ns_\xi}{3})-s_\beta^2(\frac{c_\xi c_{2W}}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Ns_\xi}{3})]$
$H_5^+H_5^-ZZ_N$	$\frac{g^2c_{2W}}{c_W}[c_\beta^2(\frac{-s_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Nc_\xi}{3})-s_\beta^2(\frac{-s_\xi c_{2W}}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Nc_\xi}{3})]$
$H_5^+H_5^-Z_2Z_N$	$2g^2[c_\beta^2(\frac{c_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Ns_\xi}{3})(\frac{-s_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Nc_\xi}{3})+s_\beta^2(\frac{c_\xi c_{2W}}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Ns_\xi}{3})(\frac{-s_\xi c_{2W}}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Nc_\xi}{3})]$
$H_4^+H_4^-AA$	e^2
$H_4^+H_4^-ZZ$	$\frac{g^2s_W^4}{c_W^2}$
$H_4^+H_4^-Z_2Z_2$	$g^2(\frac{-c_{2W}c_\xi}{c_W\sqrt{3-4s_W^2}}+\frac{t_Ns_\xi}{3})^2$
$H_4^+H_4^-Z_NZ_N$	$g^2(\frac{c_{2W}s_\xi}{c_W\sqrt{3-4s_W^2}}+\frac{t_Nc_\xi}{3})^2$
$H_4^+H_4^-AZ$	$\frac{-2egs_W^2}{c_W}$
$H_4^+H_4^-AZ_2$	$2eg(\frac{-c_{2W}c_\xi}{c_W\sqrt{3-4s_W^2}}+\frac{t_Ns_\xi}{3})$
$H_4^+H_4^-AZ_N$	$2eg(\frac{c_{2W}s_\xi}{c_W\sqrt{3-4s_W^2}}+\frac{t_Nc_\xi}{3})$
$H_4^+H_4^-ZZ_2$	$\frac{-2g^2s_W^2}{c_W}(\frac{-c_{2W}c_\xi}{c_W\sqrt{3-4s_W^2}}+\frac{t_Ns_\xi}{3})$
$H_4^+H_4^-ZZ_N$	$\frac{-2g^2s_W^2}{c_W}(\frac{c_{2W}s_\xi}{c_W\sqrt{3-4s_W^2}}+\frac{t_Nc_\xi}{3})$

(Table continued)

TABLE XII. (*Continued*)

Vertex	Coupling
$H_4^+H_4^-Z_2Z_N$	$2g^2(\frac{-c_{2W}c_\xi}{c_W\sqrt{3-4s_W^2}}+\frac{t_Ns_\xi}{3})(\frac{c_{2W}s_\xi}{c_W\sqrt{3-4s_W^2}}+\frac{t_Nc_\xi}{3})$
$HHZZ$	$\frac{g^2}{8c_W^2}$
HHZ_NZ_N	$\frac{g^2}{2}[s_\beta^2(\frac{-s_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Nc_\xi}{3})^2+c_\beta^2(\frac{-s_\xi c_{2W}}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Nc_\xi}{3})^2]$
H_1H_1ZZ	$\frac{g^2}{8c_W^2}$
$H_1H_1Z_2Z_2$	$\frac{g^2}{2}[c_\beta^2(\frac{c_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Ns_\xi}{3})^2+s_\beta^2(\frac{c_\xi c_{2W}}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Ns_\xi}{3})^2]$
$H_1H_1Z_NZ_N$	$\frac{g^2}{2}[c_\beta^2(\frac{-s_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Nc_\xi}{3})^2+s_\beta^2(\frac{-s_\xi c_{2W}}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Nc_\xi}{3})^2]$
$H_1HZ_2Z_2$	$\frac{1}{2}g^2s_{2\beta}(\frac{c_\xi^2s_{2W}^2}{4c_W^2(3-4s_W^2)}+\frac{t_Ns_{2\xi}s_W^2}{3c_W\sqrt{3-4s_W^2}})$
$H_1HZ_NZ_N$	$\frac{1}{2}g^2s_{2\beta}(\frac{s_\xi^2s_{2W}^2}{4c_W^2(3-4s_W^2)}-\frac{t_Ns_{2\xi}s_W^2}{3c_W\sqrt{3-4s_W^2}})$
$HHZZ_2$	$\frac{g^2}{2c_W}[c_\beta^2(\frac{c_\xi c_{2W}}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Ns_\xi}{3})-s_\beta^2(\frac{c_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Ns_\xi}{3})]$
$HHZZ_N$	$\frac{g^2}{2c_W}[c_\beta^2(\frac{-s_\xi c_{2W}}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Nc_\xi}{3})-s_\beta^2(\frac{-s_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Nc_\xi}{3})]$
HHZ_2Z_N	$g^2[s_\beta^2(\frac{c_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Ns_\xi}{3})(\frac{-s_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Nc_\xi}{3})+c_\beta^2(\frac{c_\xi c_{2W}}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Ns_\xi}{3})(\frac{-s_\xi c_{2W}}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Nc_\xi}{3})]$
$H_1H_1ZZ_2$	$\frac{g^2}{2c_W}[s_\beta^2(\frac{c_\xi c_{2W}}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Ns_\xi}{3})-c_\beta^2(\frac{c_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Ns_\xi}{3})]$
$H_1H_1ZZ_N$	$\frac{g^2}{2c_W}[s_\beta^2(\frac{-s_\xi c_{2W}}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Nc_\xi}{3})-c_\beta^2(\frac{-s_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Nc_\xi}{3})]$
$H_1H_1Z_2Z_N$	$g^2[c_\beta^2(\frac{c_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Ns_\xi}{3})(\frac{-s_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Nc_\xi}{3})+s_\beta^2(\frac{c_\xi c_{2W}}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Ns_\xi}{3})(\frac{-s_\xi c_{2W}}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Nc_\xi}{3})]$
H_1HZZ_2	$\frac{-g^2s_{2\beta}}{2c_W}(\frac{c_\xi c_W}{\sqrt{3-4s_W^2}}+\frac{2t_Ns_\xi}{3})$
H_1HZZ_N	$\frac{-g^2s_{2\beta}}{2c_W}(\frac{-s_\xi c_W}{\sqrt{3-4s_W^2}}+\frac{2t_Nc_\xi}{3})$
$H_1HZ_2Z_N$	$g^2s_{2\beta}[(\frac{c_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Ns_\xi}{3})(\frac{-s_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Nc_\xi}{3})-(\frac{c_\xi c_{2W}}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Ns_\xi}{3})(\frac{-s_\xi c_{2W}}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Nc_\xi}{3})]$
$H'^*H'Z_2Z_2$	$g^2(\frac{c_Wc_\xi}{\sqrt{3-4s_W^2}}-\frac{t_Ns_\xi}{3})^2$
$H'^*H'Z_NZ_N$	$g^2(\frac{-c_Ws_\xi}{\sqrt{3-4s_W^2}}-\frac{t_Nc_\xi}{3})^2$
$H'^*H'Z_2Z_N$	$2g^2(\frac{c_Wc_\xi}{\sqrt{3-4s_W^2}}-\frac{t_Ns_\xi}{3})(\frac{-c_Ws_\xi}{\sqrt{3-4s_W^2}}-\frac{t_Nc_\xi}{3})$
$\mathcal{A}\mathcal{A}ZZ$	$\frac{g^2}{8c_W^2}$
$\mathcal{A}\mathcal{A}Z_2Z_2$	$\frac{g^2}{2}[s_\beta^2(\frac{c_\xi c_{2W}}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Ns_\xi}{3})^2+c_\beta^2(\frac{c_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Ns_\xi}{3})^2]$
$\mathcal{A}\mathcal{A}Z_NZ_N$	$\frac{g^2}{2}[s_\beta^2(\frac{-s_\xi c_{2W}}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Nc_\xi}{3})^2+c_\beta^2(\frac{-s_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Nc_\xi}{3})^2]$
$\mathcal{A}\mathcal{A}ZZ_2$	$\frac{g^2}{4c_W}[\frac{c_\xi(c_\beta^2-c_{2W}s_\beta^2)}{c_W\sqrt{3-4s_W^2}}+\frac{2t_Ns_\xi c_{2\beta}}{3}]$
$\mathcal{A}\mathcal{A}ZZ_N$	$\frac{g^2}{4c_W}[\frac{-s_\xi(c_\beta^2-c_{2W}s_\beta^2)}{c_W\sqrt{3-4s_W^2}}+\frac{2t_Nc_\xi c_{2\beta}}{3}]$
$\mathcal{A}\mathcal{A}Z_2Z_N$	$\frac{g^2}{2}[s_\beta^2(\frac{c_\xi c_{2W}}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Ns_\xi}{3})(\frac{-s_\xi c_{2W}}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Nc_\xi}{3})+c_\beta^2(\frac{c_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Ns_\xi}{3})(\frac{-s_\xi}{2c_W\sqrt{3-4s_W^2}}+\frac{t_Nc_\xi}{3})]$

It is clear that the rhs is independent of the solid angle, where $d\Omega = d\varphi \sin\theta d\theta$. Hence, integrating over the total space is the same as multiplying by 4π, $\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = 4\pi \frac{d\sigma}{d\Omega}$. Because the relative velocity between the two dark matter particles is $v_{\mathrm{rel}} = 2v$, we find out

$$\sigma v_{\mathrm{rel}} \simeq 4\pi .2v \frac{\lambda'^2 m_{H'}(1 + \frac{v^2}{2} - \frac{m_H^2}{2m_{H'}^2})}{64\pi^2 4m_{H'}^2(1+v^2)m_{H'}2v}$$
$$\simeq \frac{\lambda'^2}{64\pi}\frac{1}{m_{H'}^2}\left(1 - \frac{v^2}{2} - \frac{m_H^2}{2m_{H'}^2}\right). \quad (115)$$

Taking the thermal average over both sides, we get

$$\langle \sigma v_{\mathrm{rel}} \rangle \simeq \frac{\lambda'^2}{64\pi}\frac{1}{m_{H'}^2}\left(1 - \frac{\langle v^2\rangle}{2} - \frac{m_H^2}{2m_{H'}^2}\right). \quad (116)$$

Notice that $\langle v^2 \rangle = \frac{3}{2x_F}$ (where $x_F = \frac{m_{H'}}{T_F} \simeq 20$) is given at the freeze-out temperature [26]. As mentioned before, we are in the regime $m_H^2 \ll m_{H'}^2$; thus,

$$\langle \sigma v_{\mathrm{rel}} \rangle \simeq \left(\frac{\alpha}{150\ \mathrm{GeV}}\right)^2 \lambda'^2 \left(\frac{1.328\ \mathrm{TeV}}{m_{H'}}\right)^2. \quad (117)$$

The relic density of the dark matter (H') satisfies the Boltzmann equation with the solution given by $\Omega_{H'}h^2 \simeq \frac{0.1\ \mathrm{pb}}{\langle \sigma v_{\mathrm{rel}}\rangle} \simeq 0.11$. It follows that $\langle \sigma v_{\mathrm{rel}} \rangle \simeq 1$ pb. Since $(\frac{\alpha}{150\ \mathrm{GeV}})^2 \simeq 1$ pb, we get

$$\lambda'^2 \left(\frac{1.328\ \mathrm{TeV}}{m_{H'}}\right)^2 \simeq 1, \quad (118)$$

which leads to the condition for the mass of the dark matter H',

$$m_{H'} \simeq \lambda' \times 1.328\ \mathrm{TeV}. \quad (119)$$

To conclude, H' is a dark matter particle if it has a mass $m_{H'} \simeq 1.328$ TeV, provided that $\lambda' \simeq 1$. In the context of the Higgs portal, for the couplings of order unity the direct-detection bounds demand dark matter masses at the TeV order (see Refs. [23,24]). Therefore, this scalar is a viable dark matter candidate that provides the right abundance and obeys the direct-detection bounds. Hereafter, we will focus our attention on the neutral fermion of the model, which is a natural dark matter candidate because it can be easily chosen to be the lightest particle among the W-odd particles under the parity symmetry discussed previously.

B. Dark matter: Dirac vs Majorana fermions

Among the neutral fermions N_a, the lightest one will be denoted as N, which should not be confused with the $\mathrm{U}(1)_N$ charge or the subscripts of the Z_N gauge boson, the g_N gauge coupling, and the t_N parameter. The neutrino and charged lepton that directly couple to this neutral fermion (N) via the X and Y gauge bosons are denoted as ν and l, respectively. There remain two other flavors of neutrinos and charged leptons, which are denoted as ν_α and l_α, respectively. In this section we will not dwell on unnecessary details regarding the abundance and direct-detection computation. In Fig. 1 we show the diagrams that contribute to the abundance and direct-detection signals of the fermion candidate N. Surely, the diagram that contributes to the direct-detection signal is actually the t-channel diagram on the right side of Fig. 1.

As will be explicitly shown at the end of Sec. VI E, the modifications to the couplings of the Z and $Z_{2,N}$ gauge bosons with fermions due to the mixing effects (Z with $Z_{2,N}$) are so small that they can be neglected in this analysis. Similarly, the modifications to the $Z_{2,N}ZH$ couplings due to these mixings as well as the neutral scalar mixings (H with $H_{1,2,3}$) are negligible.

In addition, it is well known that the interactions of Z_2 and Z_N are exchangeable and only differ by the replacement $(c_\xi \to -s_\xi; s_\xi \to c_\xi)$, respectively. Therefore, given that these massive gauge bosons ($Z_{2,N}$) are active particles (i.e., their scales and couplings are equivalent), they play similar roles in new physical processes (some of these can also be seen in subsequent sections). Hence, for simplicity we might consider one particle (Z_2) to be active (which dominantly sets the dark matter observables), while the other one (Z_N) almost decouples (which gives negligible contributions). For this aim, we first assume that $\Lambda > \omega$ but not too large, so that our postulate regarding the Λ scale, (i.e., that it is comparable to ω) still holds. Hence, we

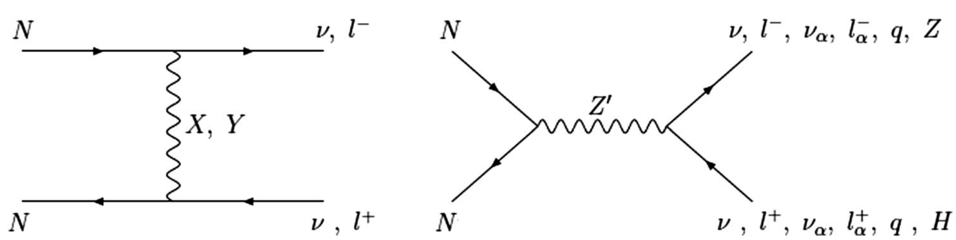

FIG. 1. Diagrams that contribute to the abundance of the neutral fermion. The diagram depicting a neutral fermion scattering off nuclei can be immediately found because it is just the t channel of the right panel, mediated by the Z'-type gauge bosons (Z_2 and Z_N). The Z_2-mediated processes are the most relevant ones though, as we shall see later.

choose $\Lambda = 10$ TeV and vary ω below this value so that $0.1 < \omega/\Lambda < 1$ (shown in detail in the cases below). In addition to ω and Λ, the $Z_{2,N}$ masses as well as their mixing angle (ξ) still depend on their respective gauge couplings. g and g_X were fixed via the electromagnetic coupling e and the Weinberg angle, whereas g_N is unknown. But, we could demand $\alpha_N \equiv \frac{g_N^2}{4\pi} < 1$ or $|g_N| < 2\sqrt{\pi}$ so that this interaction is perturbative. Without loss of generality, we set $0 < t_N < 2\sqrt{\pi}/g = \frac{s_W}{\sqrt{\alpha}} \simeq 5.43$. When t_N is large, $t_N \lesssim 5.43$, we have $m_{Z_N} \gg m_{Z_2}$ and the mixing is small, $t_{2\xi} \simeq -\frac{c_W}{3\sqrt{3-4s_W^2}t_N}\frac{\omega^2}{\Lambda^2} \simeq -\frac{0.146}{t_N}\frac{\omega^2}{\Lambda^2} \ll 1$, as given from Eq. (78). This is the case considered for the relic density of the fermion candidate as a function of its mass (m_f), and $t_N = 5.43$ is taken into account. Notice that the dark matter annihilation is via s channels mediated by $Z_{2,N}$. The contribution of Z_2 is like $\frac{g^2}{s-m_{Z_2}^2}$, while that of Z_N is $\frac{g_N^2}{s-m_{Z_N}^2} \simeq -\frac{g_N^2}{m_{Z_N}^2} \sim -\frac{1}{\Lambda^2}$ where $s \equiv 4m_f^2 \sim m_{Z_2}^2 \ll m_{Z_N}^2$. Therefore, Z_N gives a smaller contribution of order ω^2/Λ^2 which almost vanishes, whereas the relic density is sensitive to Z_2.

Provided that the relic density of the dark matter attains the right value, we consider the contributions of both Z_2 and Z_N. This is done by varying $0 < t_N < 5.43$ and $-\pi/2 < \xi < 0$ [from Eq. (78)]. When $t_N \lesssim 5.43$, Z_2 dominates the annihilation, as given above. But when t_N is decreased to $t_N \simeq \frac{c_W}{2\sqrt{3-4s_W^2}}\frac{\omega}{\Lambda} \simeq 0.219\frac{\omega}{\Lambda}$ or $\xi \simeq -\pi/4$ [which is the pole of $t_{2\xi}$ as obtained from Eq. (78)], m_{Z_N} becomes comparable to m_{Z_2}, and Z_2 and Z_N possess equivalent gauge couplings due to the large mixing. In this case, the Z_2 and Z_N bosons give simultaneous dominant contributions to the dark matter annihilation despite the fact that $\omega \ll \Lambda$. Finally, when t_N is approximately zero, $t_N \approx 0$, the Z_N boson governs the annihilation cross section, while the contribution of Z_2 is negligible. The regime where Z_N dominantly contributes to dark matter annihilation is very narrow since it is bounded by the maximal mixing value at $t_N \simeq 0.219\omega/\Lambda$, which is close to zero due to $\omega < \Lambda$. On the other hand, the regime where Z_2 dominates dark matter annihilation covers most of the range of t_N. This is the reason why Z_2 was predicted to govern dark matter observables while Z_N was almost neglected, provided that $\omega < \Lambda$. It is also clear from the above analysis that Z_2 and Z_N can have a large mixing in spite of a small ω/Λ, given that $t_N \simeq 0.219\omega/\Lambda$. On the other hand, the large regime $t_N \lesssim 5.43$ implies that these gauge bosons can slightly mix, $t_{2\xi} \simeq -\frac{0.146}{t_N}\frac{\omega^2}{\Lambda^2} \ll 1$, even if ω/Λ is close to one. Below, we will display detailed computations for all the cases mentioned.

If the candidate N is a Dirac fermion, it has both vector and axial-vector couplings with the neutral gauge bosons. The abundance is shown in Fig. 2. (In this figure and the following ones, ω is sometimes denoted as w.) It is clear from Fig. 2 that the gauge boson Z_2 overwhelms the remaining annihilation channels, in agreement with Ref. [10], and the resonance at $m_{Z_2}/2$ is crucial in determining the abundance. Moreover, we see that the mass regions 100–200 GeV for $\omega = 3$ TeV, 100–500 GeV for $\omega = 5$ TeV, or 100–1000 GeV for $\omega = 7$ TeV provide the right abundance. Additionally, in the left panel of Fig. 3 we show the region of the parameter space $\cos(\xi) \times$ the neutral fermion mass that yields the right abundance, where ξ is the Z_2 and Z_N mixing angle. When this angle goes to zero the coupling Z_2-quarks decreases, which causes the scattering cross section to rapidly decrease, as shown in the right panel of Fig. 3. There and throughout this work we let the cosine of this mixing angle run from zero to unity. [Correspondingly, ξ (t_N) runs from $-\pi/2$ (0) to 0 (5.43)]. As for the Majorana case, the overall abundance is enhanced and hence we find a larger region of the parameter space that yields the right abundance, as can seen in Fig. 4.

As for the direct-detection signal, the Dirac fermion dark matter candidates give rise to spin-independent (vector) and spin-dependent (axial-vector) scattering cross sections. But, due to the A^2 enhancement that is typical of heavy targets used in direct-detection experiments, the

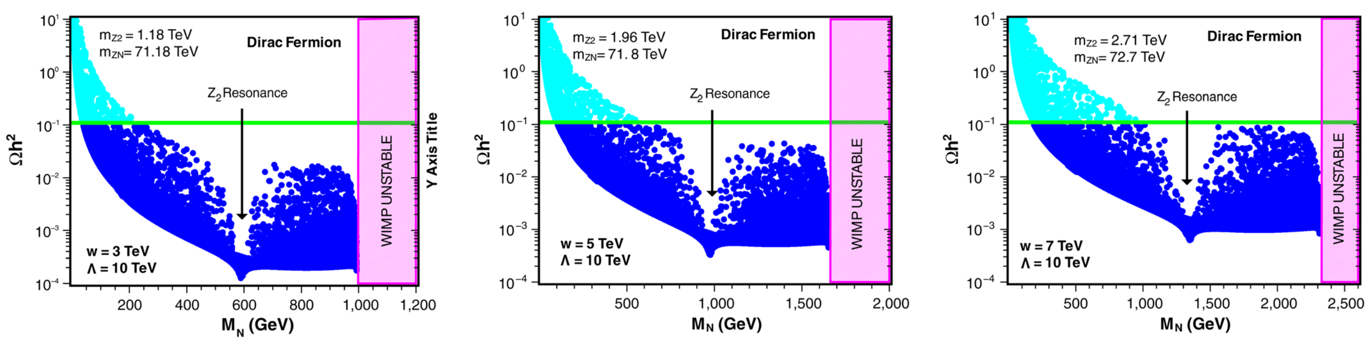

FIG. 2 (color online). Abundance of the Dirac fermion N as a function of its mass for different scales of the symmetry breaking. The shaded region is excluded for inducing weakly interacting massive particle (WIMP) decay such as $N \to X\nu$. One can clearly see that the Z_2 resonance plays a major role in the annihilation computation. See text for more details.

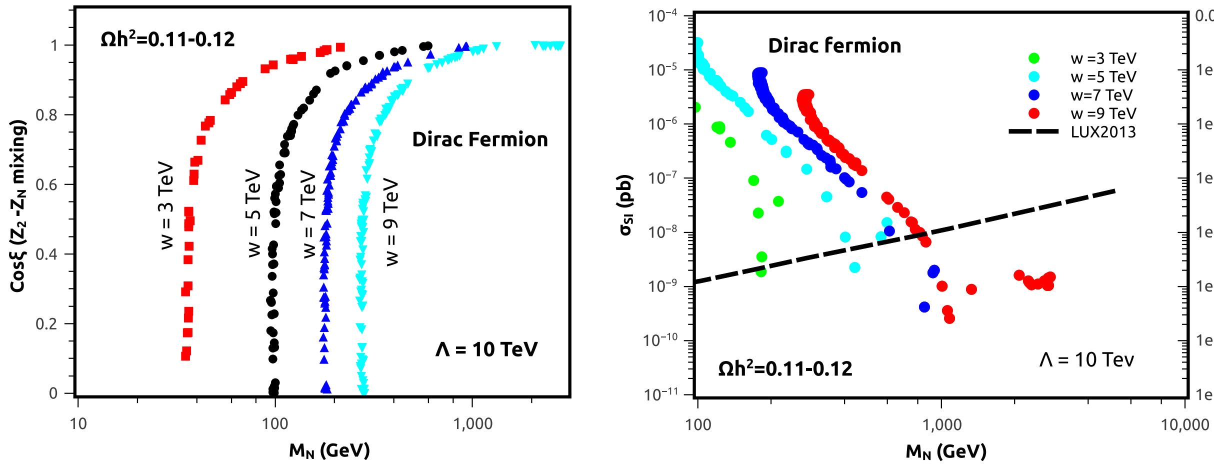

FIG. 3 (color online). Left: The mixing angle × fermion mass plane that yields the right abundance for a Dirac fermion. The discontinuity in the plots is due to the Z_2 resonance which pushes down the overall abundance. Right: Spin-independent scattering cross section in terms of the Dirac fermion mass for different values of the symmetry breaking. One can easily conclude that the current LUX bounds require $\omega \gtrsim 5$ TeV. The mixing angle ξ is free to float in our analyses. As the mixing angle goes to zero ($\cos\xi \to 1$) the coupling Z_2-quarks decreases, as seen in Table IV.

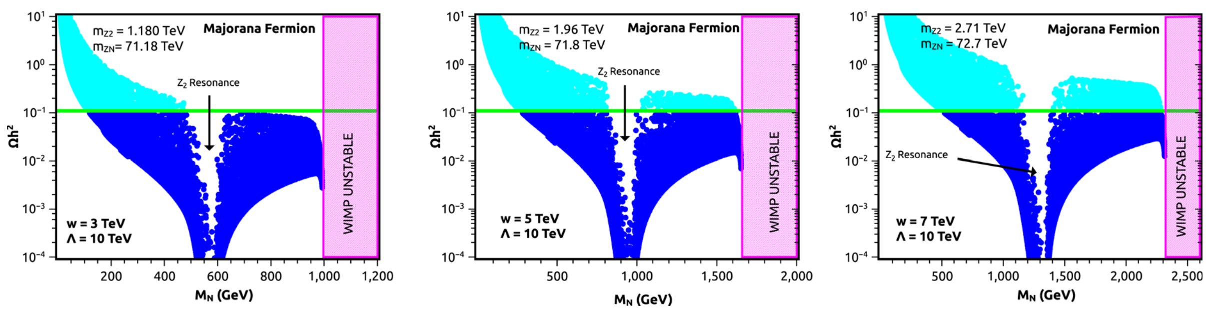

FIG. 4 (color online). Abundance of the Majorana fermion N as a function of its mass for different scales of the symmetry breaking. The shaded region is excluded from inducing WIMP decay such as $N \to X\nu$. One can clearly see that the Z_2 resonance plays a major role in the annihilation computation. See text for more details.

spin-independent bounds are the most stringent ones. This can be seen in Fig. 3. On the other hand, the Majorana fermions have zero vector current. This is because the current of a fermion is equal to the current of an antifermion, but if one applies the Majorana condition ($\psi = \psi^c$) one finds that the vector current must vanish (which has also been used for the abundance computation mentioned above). Therefore, only the spin-dependent bounds apply, which we show in Fig. 5. The LUX Collaboration has not reported their spin-dependent bounds yet, so the strongest constraints come from XENON100 [27]. One should conclude from Fig. 5 that the XENON100 bounds are quite loose for the Majorana fermion.

C. Monojet and dijet bounds

Monojet and dijet resonances have been searched for at the Tevatron, ATLAS, and CMS, with null results so far. Such signals have been intensively exploited in the literature. In particular, the dijet bounds are not sensitive to either the dark matter mass or to the Z_2-dark matter couplings, but they are quite sensitive to the Z_2-quarks couplings. In Ref. [28] lower bounds (namely, $M_{Z'} \sim 1.7$ TeV) were found for dark matter masses smaller than 500 GeV and under the assumption that the Z' boson couples similarly to the standard model Z boson. One might notice that the Z_2 gauge boson couples similarly to the Z boson. Therefore, the bounds found in Ref. [28] apply here to some extent since the couplings are not precisely identical. That being said, the result shown in the leftmost panel of Fig. 2 might be in tension with the existing dijet bounds. The remaining plots do obey the dijet bounds since they are obtained at Z_2 masses greater than 1.7 TeV. It is important to keep in mind that the collider bounds derived from simplified models are more comprehensive than the ones that use an effective-operator approach, because the production cross sections that use an effective operator either overestimate or underestimate the collider bounds, as discussed in Refs. [29,30]. Concerning the monojet

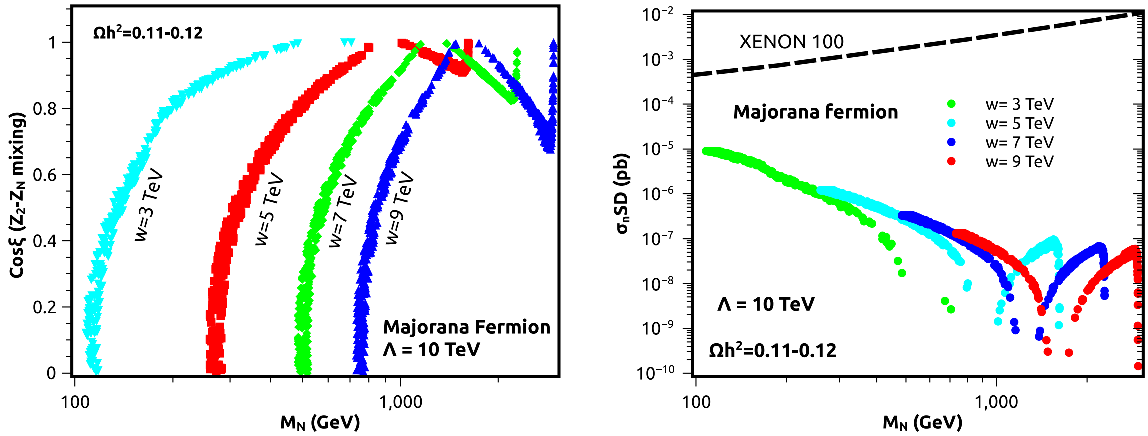

FIG. 5 (color online). Left: The mixing angle × fermion mass plane that yields the right abundance for a Majorana fermion. Right: Spin-dependent scattering cross section in terms of the Majorana fermion mass for different values of the symmetry breaking. One can easily conclude that the current XENON100 bounds are rather loose.

bounds, it has been shown that the current direct-detection limits coming from LUX are typically more stringent. Therefore, we will not refer to the monojet bounds hereafter.

D. FCNCs

The fermions get masses from the Yukawa interactions when the scalar fields develop VEVs, as presented in Ref. [2]. Due to W-parity conservation, the up quarks (u_a) do not mix with U and the down quarks (d_a) do not mix with D_α (recall that the exotic quarks are W odd while the ordinary quarks are W even). The exotic quarks gain large masses at the ω scale and become decoupled, whereas the ordinary quarks mix among themselves via a mass Lagrangian of the form

$$\mathcal{L}^{u,d}_{\text{mass}} = -\bar{u}_{aL} m^u_{ab} u_{bR} - \bar{d}_{aL} m^d_{ab} d_{bR} + \text{H.c.}, \tag{120}$$

where

$$m^u_{\alpha a} = \frac{1}{\sqrt{2}} h^u_{\alpha a} v, \qquad m^u_{3a} = -\frac{1}{\sqrt{2}} h^u_a u,$$
$$m^d_{\alpha a} = -\frac{1}{\sqrt{2}} h^d_{\alpha a} u, \qquad m^d_{3a} = -\frac{1}{\sqrt{2}} h^d_a v. \tag{121}$$

The mass matrices $m^u = \{m^u_{ab}\}$ and $m^d = \{m^d_{ab}\}$ can be diagonalized to yield physical states and masses,

$$u_L = V_{uL}(u\, c\, t)^T_L, \qquad u_R = V_{uR}(u\, c\, t)^T_R,$$
$$d_L = V_{dL}(d\, s\, b)^T_L, \qquad d_R = V_{dR}(d\, s\, b)^T_R, \tag{122}$$

$$V^\dagger_{uL} m^u V_{uR} = \text{diag}(m_u, m_c, m_t),$$
$$V^\dagger_{dL} m^d V_{dR} = \text{diag}(m_d, m_s, m_b), \tag{123}$$

where $u = \{u_a\}$ and $d = \{d_a\}$. The CKM matrix [31] is defined as $V_{\text{CKM}} = V^\dagger_{uL} V_{dL}$.

All the mixing matrices $V_{uL}, V_{dL}, V_{uR}, V_{dR}$, and V_{CKM} are unitary. The Glashow-Iliopoulos-Maiani (GIM) mechanism [32] of the standard model works in this model, which is a consequence of W-parity conservation. Let us note that in the 3-3-1 model with right-handed neutrinos, the ordinary quarks and exotic quarks that have different T_3 weak isospins mix (which is due to the unwanted nonzero VEVs of η^0_3 and χ^0_1, as well as the lepton-number-violating interactions $\bar{Q}_{3L}\chi u_{aR}$, $\bar{Q}_{3L}\eta U_R$, $\bar{Q}_{3L}\rho D_{\alpha R}$, $\bar{Q}_{\alpha L}\chi^* d_{aR}$, $\bar{Q}_{\alpha L}\eta^* D_{\beta R}$, and $\bar{Q}_{\alpha L}\rho^* U_R$ and their Hermitian conjugation, which directly couple ordinary quarks to exotic quarks via mass terms [33]). Hence, in that model the dangerous tree-level FCNCs of the Z boson are due to the nonunitarity of the mixing matrices listed above ($V_{uL}, V_{dL}, V_{uR}, V_{dR}$). Even the dangerous FCNCs come from the one-loop contributions of the W boson due to the nonunitarity of the CKM matrix (V_{CKM}). Therefore, the standard model GIM mechanism does not work. This will be analyzed at the end of this subsection.

In this model, the tree-level FCNCs happen only with the new gauge bosons Z_2 and Z_N (notice that there is a negligible contribution coming from the Z boson due to the mixing with $Z_{2,N}$, as explicitly shown below). This is due to the nonuniversal property of quark representations under $\text{SU}(3)_L$, i.e., the third quark generation differs from the first two generations. Indeed, from Eq. (95) for the interactions of $Z_{2,N}$, the right-handed flavors (Ψ_R) are conserved since $T_8 = 0$, $X = Q$, and $N = B - L$, which are universal for ordinary up and down quarks. But the left-handed flavors (Ψ_L) change due to the fact that T_8 differs for quark triplets and antitriplets. [Note that X and N are related to T_8 by Eq. (2); the source for the FCNCs is due to T_8 only since T_3 is also universal for ordinary up and down quarks for the same reason as the flavor-conserved Z

current.] The interactions that lead to flavor changing can be derived from Eq. (95) as

$$\begin{aligned}\mathcal{L}_{T_8} &= \bar{\Psi}_L\gamma^\mu T_8\Psi_L(g_2Z_{2\mu}+g_NZ_{N\mu}),\\ g_2 &\equiv -g\left(c_\xi\frac{1}{\sqrt{1-t_W^2/3}}+s_\xi\frac{2t_N}{\sqrt{3}}\right),\\ g_N &\equiv g_2(c_\xi\to -s_\xi; s_\xi\to c_\xi),\end{aligned}\tag{124}$$

where Ψ_L indicates all ordinary left-handed quarks. We can rewrite

$$\begin{aligned}\mathcal{L}_{T_8} &= (\bar{u}_L\gamma^\mu T_u u_L+\bar{d}_L\gamma^\mu T_d d_L)(g_2Z_{2\mu}+g_NZ_{N\mu})\\ &= [\bar{u}'_L\gamma^\mu(V_{uL}^\dagger T_u V_{uL})u'_L+\bar{d}'_L\gamma^\mu(V_{dL}^\dagger T_d V_{dL})d'_L]\\ &\quad\times(g_2Z_{2\mu}+g_NZ_{N\mu}),\end{aligned}\tag{125}$$

where $u'=(u,c,t)$, $d'=(d,s,b)$, and $T_u=T_d=\frac{1}{2\sqrt{3}}\mathrm{diag}(-1,-1,1)$. Hence, the tree-level FCNCs are described by the Lagrangian

$$\mathcal{L}_{\mathrm{FCNC}} = \bar{q}'_{iL}\gamma^\mu q'_{jL}\frac{1}{\sqrt{3}}(V_{qL}^*)_{3i}(V_{qL})_{3j}(g_2Z_{2\mu}+g_NZ_{N\mu})\quad (i\neq j),\tag{126}$$

where we have denoted q as either u or d.

The FCNCs lead to hadronic mixings, such as $K^0-\bar{K}^0$, $D^0-\bar{D}^0$, $B^0-\bar{B}^0$, and $B_s^0-\bar{B}_s^0$, caused by the pairs $(q'_i,q'_j)=(d,s)$, (u,c), (d,b), and (s,b), respectively. These mixings are described by the effective interactions obtained from the above Lagrangian via $Z_{2,N}$ exchanges,

$$\mathcal{L}_{\mathrm{FCNC}}^{\mathrm{eff}} = (\bar{q}'_{iL}\gamma^\mu q'_{jL})^2\frac{1}{3}[(V_{qL}^*)_{3i}(V_{qL})_{3j}]^2\left(\frac{g_2^2}{m_{Z_2}^2}+\frac{g_N^2}{m_{Z_N}^2}\right).\tag{127}$$

The strongest constraint comes from the $K^0-\bar{K}^0$ mixing [1],

$$\frac{1}{3}[(V_{dL}^*)_{31}(V_{dL})_{32}]^2\left(\frac{g_2^2}{m_{Z_2}^2}+\frac{g_N^2}{m_{Z_N}^2}\right)<\frac{1}{(10^4\ \mathrm{TeV})^2}.\tag{128}$$

Assuming that u_a is flavor diagonal, the CKM matrix is just V_{dL} (i.e., $V_{\mathrm{CKM}}=V_{dL}$). Therefore, $|(V_{dL}^*)_{31}(V_{dL})_{32}|\simeq 3.6\times 10^{-4}$ [1] and we have

$$\sqrt{\frac{g_2^2}{m_{Z_2}^2}+\frac{g_N^2}{m_{Z_N}^2}}<\frac{1}{2\ \mathrm{TeV}}.\tag{129}$$

This gives constraints on the mass and coupling of the new neutral gauge bosons,

$$m_{Z_{2,N}}>g_{2,N}\times 2\ \mathrm{TeV}.\tag{130}$$

There is another bound coming from the $B_s^0-\bar{B}_s^0$ mixing that is given by [1]

$$\frac{1}{3}[(V_{dL}^*)_{32}(V_{dL})_{33}]^2\left(\frac{g_2^2}{m_{Z_2}^2}+\frac{g_N^2}{m_{Z_N}^2}\right)<\frac{1}{(100\ \mathrm{TeV})^2}.\tag{131}$$

In this case, the CKM factor is $|(V_{dL}^*)_{32}(V_{dL})_{33}|\simeq 3.9\times 10^{-2}$ [1]. Therefore, we have

$$\sqrt{\frac{g_2^2}{m_{Z_2}^2}+\frac{g_N^2}{m_{Z_N}^2}}<\frac{1}{2.25\ \mathrm{TeV}},\tag{132}$$

which implies

$$m_{Z_{2,N}}>g_{2,N}\times 2.25\ \mathrm{TeV}.\tag{133}$$

To be concrete, suppose that Z_2 and Z_N have approximately equal masses and $t_N=g_N/g=1$ so that the $B-L$ interaction strength is equivalent to that of the weak interaction. From Eq. (129), we get

$$m_{Z_2}\approx m_{Z_N}>2.037\ \mathrm{TeV},\tag{134}$$

while the relation (132) yields

$$m_{Z_2}\approx m_{Z_N}>2.291\ \mathrm{TeV}.\tag{135}$$

Here, we have used $g^2=4\pi\alpha/s_W^2$, with $s_W^2=0.231$ and $\alpha=1/128$. This is in good agreement with the recent bound reported in Ref. [34]. Notice that we have used $m_{Z_N}\gg m_{Z_2}$ in the dark matter subsections, which translates to $m_{Z_2}\gtrsim 1$ TeV.

Finally, let us give some remarks on the FCNCs due to the mixing effect of the neutral gauge bosons. In this case, the Lagrangian (124) is changed by the replacement

$$g_2Z_{2\mu}+g_NZ_{N\mu}\to g_1Z_{1\mu}+g_2Z_{2\mu}+g_NZ_{N\mu},\tag{136}$$

where

$$g_1\equiv g_2(c_\xi\to-\mathcal{E}_1; s_\xi\to-\mathcal{E}_2)=-\frac{\sqrt{3}g}{4c_W^3}\frac{v^2-c_{2W}u^2}{\omega^2}.\tag{137}$$

Correspondingly, the effective interactions for the FCNCs given by Eq. (127) are also changed by the replacement

$$\frac{g_2^2}{m_{Z_2}^2}+\frac{g_N^2}{m_{Z_N}^2}\to\frac{g_1^2}{m_{Z_1}^2}+\frac{g_2^2}{m_{Z_2}^2}+\frac{g_N^2}{m_{Z_N}^2}.\tag{138}$$

Let us compare the new contribution with the existing one,

$$R \equiv \frac{g_1^2/m_{Z_1}^2}{(g_2^2/m_{Z_2}^2) + (g_N^2/m_{Z_N}^2)}. \tag{139}$$

It is sufficient to consider two cases: $\Lambda \gg \omega$ and $\Lambda \sim \omega$. For the first case, R is similar to (becomes) the 3-3-1 model with right-handed neutrinos,

$$R \simeq \frac{g_1^2/m_{Z_1}^2}{g_2^2/m_{Z_2}^2} \simeq \frac{1}{4c_W^4}\frac{(v^2 - c_{2W}u^2)^2}{\omega^2(u^2+v^2)} < \frac{1}{4c_W^4}\left(\frac{v_{\mathrm{w}}}{\omega}\right)^2 < 0.0025, \tag{140}$$

which is very small. Above, we have used $m_{Z_1}^2 \simeq g^2(u^2+v^2)/(4c_W^2)$, $m_{Z_2}^2 \simeq g^2 c_W^2 \omega^2/(3-4s_W^2)$, $v_{\mathrm{w}}^2 = u^2 + v^2 = (246\ \mathrm{GeV})^2$, and $\omega > 3.198$ TeV as derived from the ρ parameter. For the second case, the contributions of Z_2 and Z_N are equivalent. So, the first remark is that $R \sim (g_1^2/g_{2,N}^2)(m_{Z_{2,N}}^2/m_{Z_1}^2) \sim \mathcal{E}_{1,2}^2(m_{Z_{2,N}}^2/m_{Z_1}^2) \sim (u^4/\omega^4)\times(\omega^2/u^2) = u^2/\omega^2$, which starts at the $(u/\omega)^2$ order and must be small too. Indeed, let us show this explicitly:

$$\begin{aligned} R &\leq \frac{g_1^2/m_{Z_1}^2}{2|g_2 g_N|/(m_{Z_2}m_{Z_N})} \\ &= \frac{1}{8c_W^3 t_N |s_{2\xi}|\sqrt{3-4s_W^2}}\frac{(v^2 - c_{2W}u^2)^2}{\omega\Lambda(u^2+v^2)} \\ &< \frac{1}{8c_W^3 t_N |s_{2\xi}|\sqrt{3-4s_W^2}}\frac{v_{\mathrm{w}}^2}{\omega\Lambda} \\ &\simeq 0.00076, \end{aligned} \tag{141}$$

provided that $t_N = 1$, $\xi = -\pi/4$ ($s_{2\xi}$ is finite due to the large mixing of Z_2 and Z_N, and thus such a value could be chosen), and $\Lambda = \omega = 3.198$ TeV. Above, we have also used $m_{Z_2}m_{Z_N} = 2g^2 c_W t_N \omega\Lambda/\sqrt{3-4s_W^2}$, which can be derived from Eqs. (79) and (80), the expression (78) for the ξ mixing angle, and $m_{Z_1}^2$ as approximated before. In summary, the mixing effects with the Z boson do not affect the FCNCs.

For the sake of completeness, let us point out the dangerous FCNCs of the Z boson due to the mixing of the ordinary quarks and exotic quarks that happens in the 3-3-1 model with right-handed neutrinos, which should be suppressed. The mixing matrices are redefined as $(u_1 u_2 u_3 U)_{L,R}^T = V_{uL,R}(uctT)_{L,R}^T$ and $(d_1 d_2 d_3 D_1 D_2)_{L,R}^T = V_{dL,R}(dsbDS)_{L,R}^T$, so that the 4×4 mass matrix of up quarks (u_a, U) and the 5×5 mass matrix of down quarks (d_a, D_α) are diagonalized [33]. The Lagrangian that describes the FCNCs of the Z boson is given by $(\pm)\frac{g}{2c_W}\bar{q}'_{iL}\gamma^\mu q'_{jL}(V_{qL}^*)_{Ii}(V_{qL})_{Ij}Z_\mu$, where $I = 4$ for V_u and a plus sign is applied, but $I = 4$, 5 for V_d and a minus sign is applied. (Note, however, that the right chiral currents of Z_μ do not flavor change since $T_3 = 0$ for any right-handed fermion.) All of these lead to the effective interactions for the hadronic mixings due to the exchange of the Z boson,

$$(\bar{q}'_{iL}\gamma^\mu q'_{jL})^2[(V_{qL}^*)_{Ii}(V_{qL})_{Ij}]^2\frac{1}{u^2+v^2}, \tag{142}$$

where we have used $m_Z^2 = g^2(u^2+v^2)/(4c_W^2)$, and we note that $v_{\mathrm{w}}^2 \equiv u^2 + v^2 = (246\ \mathrm{GeV})^2$. In the 3-3-1 model with right-handed neutrinos, the Lagrangian for the FCNCs of the Z' boson is easily obtained as $\frac{-g}{\sqrt{1-t_W^2/3}}\bar{q}'_{iL}\gamma^\mu q'_{jL}\frac{1}{\sqrt{3}}[V_{qL}^\dagger V_{qL}]_{ij}Z'_\mu$, where $[V_{uL}^\dagger V_{uL}]_{ij} \equiv (V_{uL}^*)_{3i}(V_{uL})_{3j} - \frac{1}{2}(V_{uL}^*)_{4i}(V_{uL})_{4j}$ and $[V_{dL}^\dagger V_{dL}]_{ij} \equiv (V_{dL}^*)_{3i}(V_{dL})_{3j} + \frac{3}{2}(V_{dL}^*)_{Ii}(V_{dL})_{Ij}$. Hence, the effective interactions for the hadronic mixings due to the Z' contribution are given by

$$(\bar{q}'_{iL}\gamma^\mu q'_{jL})^2[V_{qL}^\dagger V_{qL}]_{ij}^2\frac{1}{\omega^2}, \tag{143}$$

where we have adopted $m_{Z'}^2 \simeq \frac{g^2 c_W^2}{3-4s_W^2}\omega^2$ [22]. Since the weak scale v_{w} in Eq. (142) is too low in comparison to the 3-3-1 scale ω in Eq. (143), it is clear that if the mixing of the ordinary quarks and exotic quarks is similar in size to that of the ordinary quarks, $(V_{qL}^*)_{Ii}(V_{qL})_{Ij} \sim (V_{qL}^*)_{3i}(V_{qL})_{3j}$, the FCNCs due to the Z boson (142) are too large ($\sim\omega^2/v_{\mathrm{w}}^2 \sim 10^2$ times the one coming from Z' or the bound for the $K^0 - \bar{K}^0$ mixing); as such, the theory is invalid. Hence, the FCNCs due to the ordinary and exotic quark mixing are more dangerous than those coming from the nonuniversal interactions of the Z' boson. To avoid the large FCNCs, one must assume $|(V_{qL}^*)_{Ii}(V_{qL})_{Ij}| \ll |(V_{qL}^*)_{3i}(V_{qL})_{3j}|$ [and that the FCNCs of Z' are dominated by the ordinary quark mixing, $[V_{qL}^\dagger V_{qL}]_{ij} \simeq (V_{qL}^*)_{3i}(V_{qL})_{3j}$]. Indeed, the $K^0 - \bar{K}^0$ mixing constrains Eq. (142) to be

$$|(V_{dL}^*)_{I1}(V_{dL})_{I2}| \lesssim 10^{-5}. \tag{144}$$

This mixing of the exotic and ordinary quarks is much smaller than the smallest mixing element (about 5×10^{-3}) of the ordinary quark flavors from the CKM matrix [1]. Therefore, the 3-3-1-1 gauge symmetry as well as the resulting W parity provide a more natural framework that not only solves these problems (including the large FCNCs, the unitarity of the CKM matrix, the lepton and baryon number symmetries, and the CPT theorem, which have strictly been proven by experiment [1]), but it also gives the small neutrino masses and dark matter candidates.

E. LEPII searches for Z_2 and Z_N

LEPII searches for new neutral gauge bosons via the channel $e^+e^- \to f\bar{f}$, where f is any ordinary fermion [35]. In this model, the new physics effect in such a process is due to the dominant contribution of the Z_2 and Z_N gauge bosons, which are s-channel exchanges for $f \neq e$. The effective interaction for these contributions can be derived [with the help of Eq. (99)] as

$$\mathcal{L}^{\mathrm{eff}}_{\mathrm{LEP2}} = \frac{g^2}{c_W^2 m_I^2}[\bar{e}\gamma^\mu(a_L^I(e)P_L + a_R^I(e)P_R)e][\bar{f}\gamma_\mu(a_L^I(f)P_L + a_R^I(f)P_R)f] \quad (I = Z_2, Z_N), \tag{145}$$

where the chiral couplings are given by

$$a_L^I(f) = \frac{g_V^I(f) + g_A^I(f)}{2}, \qquad a_R^I(f) = \frac{g_V^I(f) - g_A^I(f)}{2}. \tag{146}$$

Let us study a particular process for $f = \mu$, $e^+e^- \to \mu^+\mu^-$. The chiral couplings can be obtained from Tables III and IV as

$$\begin{aligned} a_L^{Z_2}(e_a) &= \frac{c_\xi c_{2W}}{2\sqrt{3 - 4s_W^2}} - \frac{2}{3}s_\xi c_W t_N, \\ a_R^{Z_2}(e_a) &= -\frac{c_\xi s_W^2}{\sqrt{3 - 4s_W^2}} - s_\xi c_W t_N, \\ a_{L,R}^{Z_N} &= a_{L,R}^{Z_2}(c_\xi \to -s_\xi; s_\xi \to c_\xi). \end{aligned} \tag{147}$$

The effective interaction can be rewritten as

$$\mathcal{L}^{\mathrm{eff}}_{\mathrm{LEP2}} = \frac{g^2}{c_W^2}\left(\frac{[a_L^{Z_2}(e)]^2}{m_{Z_2}^2} + \frac{[a_L^{Z_N}(e)]^2}{m_{Z_N}^2}\right)(\bar{e}\gamma^\mu P_L e)(\bar{\mu}\gamma_\mu P_L \mu) + (LR) + (RL) + (RR), \tag{148}$$

where the last three terms differ from the first one only in chiral structures.

Notice that LEPII searches for such chiral interactions and gives several constraints on the respective couplings, which are commonly given at the order of a few TeV [35]. Therefore, let us choose a typical value,

$$\frac{g^2}{c_W^2}\left(\frac{[a_L^{Z_2}(e)]^2}{m_{Z_2}^2} + \frac{[a_L^{Z_N}(e)]^2}{m_{Z_N}^2}\right) < \frac{1}{(6\ \mathrm{TeV})^2}. \tag{149}$$

It is noted that this value, 6 TeV, is also a bound derived for the case of the $U(1)_{B-L}$ gauge boson [36].

Similar to the previous subsection, we suppose that Z_2 and Z_N have approximately equal masses ($m_{Z_2} \approx m_{Z_N}$) and $t_N = 1$. The above constraint leads to

$$m_{Z_2} \approx m_{Z_N} > 2.737\ \mathrm{TeV}. \tag{150}$$

This bound is in good agreement with the limit in the previous subsection via the FCNCs and the ones given in the literature [34]. As we previously emphasized, in the dark matter subsections we have adopted $m_{Z_N} \gg m_{Z_2}$, and therefore in this regime a bound on $m_{Z_2} \sim$ TeV arises.

Finally, let us discuss the contribution of the mixing effects of the neutral gauge bosons to the above process. When the mixing is turned on, the interacting Lagrangian of the neutral gauge bosons takes the form $-\frac{g}{c_W}\bar{f}\gamma^\mu[\tilde{a}_L^{Z_i}(f)P_L + \tilde{a}_R^{Z_i}(f)P_R]fZ_{i\mu}$, where $i = 1, 2, N$ and the (chiral) couplings of the neutral gauge bosons are correspondingly changed as follows:

$$\begin{aligned} a_{L,R}^{Z}(f) &\to \tilde{a}_{L,R}^{Z_1}(f) \equiv a_{L,R}^{Z}(f) + a_{L,R}^{Z_2}(f)(c_\xi \to -\mathcal{E}_1; s_\xi \to -\mathcal{E}_2), \\ a_{L,R}^{Z_2}(f) &\to \tilde{a}_{L,R}^{Z_2}(f) \equiv a_{L,R}^{Z_2}(f) + a_{L,R}^{Z}(f) \times (\mathcal{E}_1 c_\xi + \mathcal{E}_2 s_\xi), \\ a_{L,R}^{Z_N}(f) &\to \tilde{a}_{L,R}^{Z_N}(f) \equiv a_{L,R}^{Z_N}(f) + a_{L,R}^{Z}(f) \times (-\mathcal{E}_1 s_\xi + \mathcal{E}_2 c_\xi). \end{aligned} \tag{151}$$

We realize that the second term in each expression is the $\mathcal{E}_{1,2}$ correction corresponding to the existing couplings due to the mixing, which can be neglected because of the small values of $\mathcal{E}_{1,2}$, as given in Eq. (76). Indeed, for the concerned process $e^+e^- \to \mu^+\mu^-$, let us consider the ratios of the corrections to the respective existing couplings for $f = e_a$ (the charged leptons). With the Z_1 couplings, we have

$$\left|\frac{a_L^{Z_2}(e_a)(c_\xi \to -\mathcal{E}_1; s_\xi \to -\mathcal{E}_2)}{a_L^Z(e_a)}\right| = \left|\frac{\mathcal{E}_1}{\sqrt{3 - 4s_W^2}} - \frac{4c_W t_N}{3c_{2W}}\mathcal{E}_2\right| < 2.43 \times 10^{-3}, \tag{152}$$

$$\left|\frac{a_R^{Z_2}(e_a)(c_\xi \to -\mathcal{E}_1; s_\xi \to -\mathcal{E}_2)}{a_R^Z(e_a)}\right| = \left|\frac{\mathcal{E}_1}{\sqrt{3 - 4s_W^2}} + \frac{c_W t_N}{s_W^2}\mathcal{E}_2\right| < 2.43 \times 10^{-3}, \tag{153}$$

which are easily obtained with the help of Eq. (76), $s_W^2 = 0.231$, and $\Lambda \sim \omega > 3.198$ TeV. Similarly, for the Z_2 couplings, we have

$$\left|\frac{a_L^Z(e_a)\times(\mathcal{E}_1 c_\xi + \mathcal{E}_2 s_\xi)}{a_L^{Z_2}(e_a)}\right| = \left|\frac{\mathcal{E}_1 c_\xi + \mathcal{E}_2 s_\xi}{\frac{c_\xi}{\sqrt{3-4s_W^2}} - \frac{4c_W}{3c_{2W}} t_N s_\xi}\right| < 5.04\times 10^{-3}, \tag{154}$$

$$\left|\frac{a_R^Z(e_a)\times(\mathcal{E}_1 c_\xi + \mathcal{E}_2 s_\xi)}{a_R^{Z_2}(e_a)}\right| = \left|\frac{\mathcal{E}_1 c_\xi + \mathcal{E}_2 s_\xi}{\frac{c_\xi}{\sqrt{3-4s_W^2}} + \frac{c_W}{s_W^2} t_N s_\xi}\right| < 5.04\times 10^{-3}, \tag{155}$$

where we notice that the mixing angle of the Z' and C gauge bosons is bounded by $-\pi/4 < \xi < 0$ if $t_N > 0$, or by $0 < \xi < \pi/4$ if $t_N < 0$. The corrections to the Z_N couplings are small as well. Therefore, the mixing effects of the neutral gauge bosons do not affect the standard model $e^+e^- \to \mu^+\mu^-$ process or our results given above with the $Z_{2,N}$ exchanges in the absence of the mixing.

F. Radiative β decays involving $Z_{2,N}$ and the violation of CKM unitarity

CKM unitarity implies $\sum_{d'=d,s,b} V^*_{u'd'} V_{u''d'} = \delta_{u'u''}$ and $\sum_{u'=u,c,t} V^*_{u'd'} V_{u'd''} = \delta_{d'd''}$, where the elements of the CKM matrix $V_{u'd'} \equiv (V^\dagger_{uL} V_{dL})_{u'd'}$ ($u' = u,\ c,\ t$ and $d' = d,\ s,\ b$) are defined as before. The standard model calculations have provided a very good agreement with the above relations [1]. However, if there is a possible deviation, it is the sign for the violation of CKM unitarity.

Focusing on the first row, the experimental bound yields [1]

$$\Delta_{\mathrm{CKM}} = 1 - \sum_{d'=d,s,b} |V_{ud'}|^2 < 10^{-3}. \tag{156}$$

This violation can give the constraints on the new neutral $Z_{2,N}$ gauge bosons as a result of their loop effects that contribute to Δ_{CKM}.

Indeed, the Δ_{CKM} deviation is derived from the one-loop radiative corrections via the new $Z_{2,N}$ and W bosons to quark β-decay amplitudes from which the V_{ud}, V_{us}, and V_{ub} elements are extracted, including muon decay which normalizes the quark β-decay amplitudes. These have previously been studied in other theories (such as in Ref. [37], where similar respective diagrams for quark and muon β decays were displayed). Generalizing the results in Ref. [37], the deviation is obtained as

$$\Delta_{\mathrm{CKM}} \simeq -\frac{3}{4\pi^2}\sum_{I=Z_2,Z_N}\frac{m_W^2}{m_I^2}\ln\left(\frac{m_W^2}{m_I^2}\right)(\mathcal{G}^I_{e_L})_{11}\left[(\mathcal{G}^I_{e_L})_{11} - \frac{(\mathcal{G}^I_{d_L})_{11} + (\mathcal{G}^I_{u_L})_{11}}{2}\right], \tag{157}$$

where the lepton and quark couplings are given in the physical basis of the left chiral fields when coupled to $Z_{2,N}$, i.e., $\bar{f}'_L \gamma^\mu \mathcal{G}^I_{f_L} f'_L I_\mu$, with $\mathcal{G}^I_{f_L} \equiv -\frac{g}{c_W} V^\dagger_{fL} a^I_L(f) V_{fL}$, which gives

$$(\mathcal{G}^I_{e_L})_{11} = (\mathcal{G}^I_{\nu_L})_{11} = -\frac{g}{c_W} a^I_L(e_a), \qquad (\mathcal{G}^{Z_2}_{u_L})_{11} = \frac{g c_\xi\sqrt{3-4s_W^2}}{6c_W}, \qquad (\mathcal{G}^{Z_N}_{u_L})_{11} = \frac{-g s_\xi\sqrt{3-4s_W^2}}{6c_W},$$
$$(\mathcal{G}^{Z_2}_{d_L})_{11} = \frac{g c_\xi\sqrt{3-4s_W^2}}{6c_W} - \frac{g}{c_W}\left(\frac{c_\xi c_W^2}{\sqrt{3-4s_W^2}} + \frac{2}{3}s_\xi c_W t_N\right)|(V_{dL})_{31}|^2,$$
$$(\mathcal{G}^{Z_N}_{d_L})_{11} = (\mathcal{G}^{Z_2}_{d_L})_{11}(c_\xi \to -s_\xi; s_\xi \to c_\xi). \tag{158}$$

Notice that the mixing effect of the neutral gauge bosons (Z with $Z_{2,N}$) do not affect these processes, as was explicitly pointed out in the previous subsection.

Therefore, we have

$$\Delta_{\mathrm{CKM}} \simeq -\frac{3g^2}{4\pi^2}\frac{m_W^2}{m_{Z_2}^2}\ln\left(\frac{m_W^2}{m_{Z_2}^2}\right)\left[\frac{2}{3}s_\xi t_N - \frac{c_\xi c_{2W}}{2c_W\sqrt{3-4s_W^2}}\right]\left[\frac{2}{3}s_\xi t_N - \frac{c_\xi(3-5s_W^2)}{3c_W\sqrt{3-4s_W^2}}\right] + (Z_2 \to Z_N; c_\xi \to -s_\xi; s_\xi \to c_\xi). \tag{159}$$

We consider two typical cases: $\Lambda \gg \omega$ and $\Lambda \sim \omega$. In the first case, Z_N does not contribute, i.e., the second term above vanishes, and $\xi = 0$. Therefore, this is the case of the 3-3-1 model with right-handed neutrinos. We have

$$\Delta_{\mathrm{CKM}} \simeq -0.0033 \frac{m_W^2}{m_{Z_2}^2} \ln\left(\frac{m_W^2}{m_{Z_2}^2}\right). \quad (160)$$

Using the bound (156) and $m_W = 80.4$ GeV, the Z_2 mass is constrained by $m_{Z_2} > 200$ GeV. In fact, the Z_2 mass should be in the TeV range due to the other constraints given above. For example, by taking $m_{Z_2} > 1$ TeV, we get $\Delta_{\mathrm{CKM}} < 10^{-4}$. Consequently, this case gives a very small contribution to the violation of CKM unitarity and thus the model easily evades the experimental bound. In the second case, assuming that the new neutral gauge bosons have approximately equal masses ($m_{Z_2} \simeq m_{Z_N}$) and $t_N = 1$, we derive

$$\Delta_{\mathrm{CKM}} \simeq -0.0143 \frac{m_W^2}{m_{Z_{2,N}}^2} \ln\left(\frac{m_W^2}{m_{Z_{2,N}}^2}\right). \quad (161)$$

Using the bound (156) we have $m_{Z_2}^2 \simeq m_{Z_N}^2 > 600$ GeV. The model in this case easily evades the experimental bound too. To conclude, the new neutral gauge bosons $Z_{2,N}$ give a negligible contribution to the violation of CKM unitarity.

VII. DISCUSSION AND CONCLUSION

In the standard model, the fermions come in generations, with each subsequent generation being a replication of the former. The gauge anomaly is cancelled out over every generation. Thus, on this theoretical ground the number of generations can be arbitrary. This may be due to the fact that the $\mathrm{SU}(2)_L$ anomaly trivially vanishes for any chiral fermion representation. If the $\mathrm{SU}(2)_L$ is minimally extended to $\mathrm{SU}(3)_L$ with a corresponding enlargement of the lepton and quark representations (i.e., the doublets are enlarged to triplets/antitriplets while the singlets remain the same, but for some cases the lepton singlets are put into the corresponding triplets/antitriplets as well), the new $\mathrm{SU}(3)_L$ anomaly generally does not vanish for each nontrivial representation. Subsequently, this constrains the generation number to be an integer multiple of three—the fundamental color number—in order to cancel the anomaly over the total fermion content, which provides a partial solution to the number of fermion generations. Aside from this feature, some very fundamental aspects of the standard model can also be understood by the presence of the $\mathrm{SU}(3)_L$ that causes the electric-charge quantization [9], the Peccei-Quinn-like symmetry for the strong CP problem [8], and the oddly heavy top quark [7]. On the other hand, the $B-L$ number and electric charge Q operators do not commute, and they are also algebraically nonclosed with respect to the $\mathrm{SU}(3)_L$ generators. If we suppose that $B-L$ is conserved similarly to Q, such a $\mathrm{SU}(3)_L$ theory is only manifest if it includes two extra Abelian factors so that all the algebras are closed, and the resulting gauge symmetry $\mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X \otimes \mathrm{U}(1)_N$ yields a unification of the weak, electromagnetic, and $B-L$ interactions [apart from the strong interaction from the $\mathrm{SU}(3)_C$ gauge group]. Besides the B and L symmetries, some very fundamental matters of the 3-3-1 model can also be understood by this setup.

Firstly, the breakdown of the 3-3-1-1 gauge symmetry produces a conserved Z_2 subgroup (as a remnant) called W parity—similar to R parity in supersymmetry—that plays an important role and provides insights into the present model. The lightest wrong-lepton particle is stabilized due to W-parity conservation, which is responsible for dark matter. Two dark matter particles have been recognized: a neutral complex scalar H' and a neutral fermion N of either Dirac or Majorana nature. The GIM mechanism for the standard model currents works in this model due to W-parity conservation, while the new FCNCs are strictly suppressed. In fact, the experimental bounds can be easily evaded, with the expected masses for the new neutral gauge bosons $Z_{2,N}$ being a few TeV. Because of W-parity conservation, the new neutral non-Hermitian gauge boson X does not mix with the neutral $Z_{1,2,N}$ gauge bosons. Hence, there is no mass splitting within the real and imaginary components of X that ensures the conservation of CPT symmetry. These problems of the 3-3-1 model with right-handed neutrinos have been solved.

We have shown that the $B-L$ interactions can coexist with the new 3-3-1 interactions at the TeV scale. To realize this, the scales of the 3-3-1-1 and 3-3-1 breakings are taken to be at the same energy scale $\Lambda \sim \omega$. In this regime, the scalar potential has been diagonalized. The number of Goldstone bosons matches the number of massive gauge bosons. There are 11 physical scalar fields, one of which is identified as the standard model Higgs boson. The new physical scalar fields $H^0_{1,2,3}$, $\mathcal{A}^0$, $H^\pm_{4,5}$, and $H'^{0,0*}$ are heavy, with masses at the ω, Λ, or $\sqrt{|\omega f|}$ scale. There is a finite mixing between the Higgs scalars—S_4 for the $\mathrm{U}(1)_N$ breaking and S_3 for the 3-3-1 breaking—that gives two physical fields, $H_{2,3}$. The standard model Higgs boson is light with a mass at the weak scale due to the seesaw-type mechanism associated with the little hierarchy $u, v \ll \omega, \Lambda, -f$. The Higgs mass gets the correct value of 125 GeV provided that the effective coupling $\bar{\lambda} \simeq 0.5$, with the assumption $u = v$, $\omega = -f$. All of the physical scalar fields are W even, except for H' and H_4 (the W particles), which are W odd.

In the proposed regime $\Lambda \sim \omega$ the gauge sector has been diagonalized, and we recognize the standard model gauge bosons $W^\pm$, A, and Z. Moreover, we have six new gauge bosons: $X^{0,0*}$, $Y^\pm$, and $Z_{2,N}$. Although the Z boson mixes with the new neutral gauge bosons, it is light due to a seesaw-type mechanism in the gauge sector. In order to reproduce the standard model W-boson mass, we have constrained $u^2 + v^2 = (246\ \mathrm{GeV})^2$. From the experimental bound on the ρ parameter, we get $\omega > 3.198$ TeV provided that $\Lambda \simeq \omega$ and $u \simeq v$. There is a finite mixing between the $\mathrm{U}(1)_N$ gauge boson and the Z' of the 3-3-1 model that produces two physical states by diagonalization:

the 3-3-1-like gauge boson Z_2 and the $U(1)_N$-like gauge boson Z_N. All the gauge bosons are W even except for X and Y, which are the W particles. The new neutral complex gauge boson X cannot be a dark matter candidate because it annihilates into standard model particles before the thermal equilibrium process has ended [2].

All the interactions of the gauge bosons with the fermions and scalars have been obtained. The result shows that every interaction conserves W parity. The corresponding standard model interactions are recovered. The new interactions as well as their implications for phenomenological processes of new physics are rich and warrant further studies. In this paper, some of the new interactions have been used for analyzing the new FCNCs, the LEPII Collider, the violation of CKM unitarity, and fermionic dark matter observables. Because of the seesaw-type mixing suppression between the light and heavy states —namely between the Z and new $Z_{2,N}$ gauge bosons, as well as between the H and new $H_{1,2,3}$ Higgs bosons—the mixing effects are radically small. It has been explicitly pointed out that the new physics effects via these mixings in the gauge sector can to be safely neglected. For the scalar sector, the new physics effects via these mixings are also negligible, and were disregarded for most cases involving small scalar self-couplings (see the main text for more details). The scalar self-couplings would give considerable contributions if they were stronger, but they are still within the current bounds. The accuracy of the standard model Higgs mechanism (if it is the case) could give some constraints on these mixing effects.

Supposing that the scalar dark matter H' dominantly annihilates into the standard model Higgs boson H via the Higgs portal, the relic density of H' has been calculated. The correct experimental value is obtained if $m_{H'} = 1.328$ TeV, where it is assumed that the $H'^*H' \to HH$ coupling is equal to unity, $\lambda' = 1$. When the neutral fermion is a Dirac particle, we conclude that a ω scale of the symmetry breaking greater than ~ 5 TeV is required in order to obey the LUX2013 bounds. On the other hand, when the neutral fermion is a Majorana particle the direct-detection bounds are quite loose and a larger region of the parameter space has been found that yields the right abundance. The fermion dark matter observables are governed by the Z_2 gauge boson provided that $\Lambda > \omega$. Only if $g_N \ll g$ with $\Lambda \sim \omega$ either the Λ is smaller than the ω (which is hardly occurred) with $g_N \sim g$, the Z_N contribution becomes comparable to that of the Z_2 boson.

We have shown that the CKM matrix is unitary and the ordinary GIM mechanism of the standard model works in this model due to W-parity conservation. We have also discussed the fact that this mechanism does not work in the 3-3-1 model with right-handed neutrinos, and in such a case the tree-level FCNCs due to the ordinary and exotic quark mixing are more dangerous than those coming from the nonuniversal couplings of the $Z_{2,N}$ gauge bosons. All the FCNCs associated with the Z boson due to the above fermion mixing are prevented because of W-parity conservation. The new FCNCs coupled to $Z_{2,N}$ are highly suppressed as well. In fact, the FCNCs due to $Z_{2,N}$ can be present, but they can be easily evaded by the new physics in the TeV range. Using the current bound on the $K^0 - \bar{K}^0$ system, we have shown that $m_{Z_{2,N}} > 2.037$ TeV under the assumption that Z_2 and Z_N have approximately equal masses and that $t_N = 1$ (i.e., the $B - L$ interaction strength is equal to that of the weak interaction). For the $B_s^0 - \bar{B}_s^0$ system, the bound is $m_{Z_{2,N}} > 2.291$ TeV, under the same assumptions as in the previous case. For the hierarchical masses of Z_2 and Z_N, the smaller mass will take a smaller bound, e.g., $m_{Z_2} > g_2 \times 2$ TeV, corresponding to the $K^0 - \bar{K}^0$ system, where g_2 is the reduced gauge coupling that has a natural value smaller than unity.

The new neutral currents in the model are now detected by the experiments. We have calculated the contributions of Z_2 and Z_N—which dominate the corrections of the new physics—to the process $e^+e^- \to \mu^+\mu^-$ at the LEPII Collider. From the experimental bounds, we have shown that $m_{Z_{2,N}} > 2.737$ TeV provided that these gauge bosons have approximately equal masses and that $t_N = 1$. Similarly, for the hierarchal Z_2 and Z_N masses, the smaller mass will possess a smaller bound than the above result. Moreover, we have also indicated that the violation of CKM unitarity due to the one-loop effects of the new neutral gauge bosons $Z_{2,N}$ are negligible if the $Z_{2,N}$ masses are in the TeV range, which is expected.

Finally, the 3-3-1-1 model—which unifies the electroweak and $B - L$ interactions along with the strong interaction—is a self-consistent extension of the standard model that solves the potential problems of the 3-3-1 model, namely, the consistency of the B, L, and CPT symmetries, and the large FCNCs. The new physics of the 3-3-1-1 model is interesting, possibly appearing in the TeV region. For all of these reasons, we believe that the 3-3-1-1 model is a compelling theory that warrants much experimental attention.

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