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Neutrino mixing with nonzero θ_{13} in Zee–Babu model

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The exact solution for the neutrino mass matrix of the Zee–Babu model is derived. Tribimaximal mixing imposes conditions on the Yukawa couplings, from which the normal mass hierarchy is preferred. The derived conditions give a possibility of Majorana maximal CP violation in the neutrino sector. We have shown that nonzero θ_{13} is generated if Yukawa couplings between leptons almost equal to each other. The model gives some regions of the parameters where neutrino mixing angles and the normal neutrino mass hierarchy obtained are consistent with the recent experimental data.

Keywords: Neutrino mass and mixing; nonstandard-model neutrinos; Zee–Babu model.

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1. Introduction

Nowadays, particle physicists are attracted by two exciting subjects: Higgs and neutrino physics. The neutrino mass and mixing are the first evidence of beyond Standard Model (SM) physics. Many experiments show that neutrinos have tiny masses and their mixing is sill mysterious.[1,2] Recent data are a clear sign of rather large value θ_{13}.[3]

The tribimaximal (TBM) form for explaining the lepton mixing scheme was first proposed by Harrison–Perkins–Scott (HPS), which apart from the phase

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redefinitions, is given by[4,5]

$$U_{\mathrm{HPS}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{1}$$

can be considered as a good approximation for the recent neutrino experimental data, where the large mixing angles are completely different from the quark mixing ones defined by the Cabibbo–Kobayashi–Maskawa (CKM) matrix.[6–8]

The most recent fits suggest that one of the mixing angles is approximately zero and another has a value that implies a mass eigenstate that is nearly an equal mixture of ν_μ and ν_τ. The parameters of neutrino oscillations such as the squared mass differences and mixing angles are now very constrained. The data in PDG2010[9] imply

$$\begin{gathered} \sin^2(2\theta_{12}) = 0.87 \pm 0.03\,, \quad \sin^2(2\theta_{23}) > 0.92\,, \quad \sin^2(2\theta_{13}) < 0.15\,, \\ \Delta m^2_{21} = (7.59 \pm 0.20) \times 10^{-5}\ \mathrm{eV}^2\,, \quad \Delta m^2_{32} = (2.43 \pm 0.13) \times 10^{-3}\ \mathrm{eV}^2\,, \end{gathered} \tag{2}$$

where (and hereafter) the best fits are taken into account. Whereas, the new data[10–19] have been given to be slightly modified from the old fits (2):

$$\begin{gathered} \sin^2(2\theta_{12}) = 0.857 \pm 0.024\,, \\ \sin^2(2\theta_{13}) = 0.098 \pm 0.013\,, \quad \sin^2(2\theta_{23}) > 0.95\,, \\ \Delta m^2_{21} = (7.50 \pm 0.20) \times 10^{-5}\ \mathrm{eV}^2\,, \quad \Delta m^2_{32} = (2.32^{+0.12}_{-0.08}) \times 10^{-3}\ \mathrm{eV}^2\,. \end{gathered} \tag{3}$$

On the other hand, the discovery of the long-awaited Higgs boson at around 125 GeV (Refs. 20 and 21) opened a new chapter in particle physics. It is essential for us to determine which model the discovered Higgs boson belongs to? For this aim, the diphoton decay of the Higgs boson plays a very important role. It is expected that new physics might enter here to modify the SM Higgs property.

For the above-mentioned reasons, the search for an extended model coinciding with the current data on neutrino and Higgs physics is one of our top priorities. In our opinion, the model with the simplest particle content is preferred. In the SM, neutrinos are strictly massless. For neutrino mass, an original model pointed out by Zee in Ref. 22 in which new scalars are added in the Higgs sector with neutrino masses induced at the one-loop level. After that a two-loop scenario called the Zee–Babu model[23] was proposed. The Zee–Babu model[22–24] with just two additional charged Higgs bosons (h^-, k^{--}) carrying lepton number 2, is very attractive.[a]

[a]In the recent paper,[30] the parameter space of the model under consideration has been reanalyzed, and the lower bounds for masses of the singly and doubly charged Higgses lie between 1 TeV to 2 TeV.

In this model, neutrinos get mass from two-loop radiative corrections, which can fit current neutrino data. Moreover, the singly and doubly charged scalars that are new in the model can explain the large annihilation cross-section of a dark matter pair into two photons as hinted at by the recent analysis of the Fermi γ-ray space telescope data,[25] if the new charged scalars are relatively light and have large couplings to a pair of dark matter particles. These new scalars can also enhance the $B(H \to \gamma\gamma)$, as the recent LHC results may suggest.

The Zee–Babu model contains the Yukawa couplings which are specific for lepton number violating processes. There has been much work[24,27–31] constraining the parameter space of the model, however the explicit values of neutrino masses and mixings have not been considered.

In this paper, starting from the neutrino mass matrix, we get the exact solution, i.e. the eigenstates and the eigenvalues. As a consequence, the neutrino mixing matrix follows. With this exact solution, we can fit current data and get constraint on the couplings. We hope that experiments in the near future will approve or rule out the model.

2. Neutrino Mass Matrix in the Zee–Babu Model

The Zee–Babu model[23] includes two $\mathrm{SU}(2)_L$ singlet Higgs fields, a singly charged field h^- and a doubly charged field k^{--}. Moreover, right-handed neutrinos are not introduced. The addition of these singlets give rise to the Yukawa couplings:

$$\mathcal{L}_Y = f_{ab}\,\overline{(\psi_{aL})^C}\psi_{bL}h^+ + h'_{ab}\,\overline{(l_{aR})^C}\,l_{bR}k^{++} + \mathrm{H.c.}\,, \tag{4}$$

where ψ_L stands for the left-handed lepton doublet, l_R for the right-handed charged lepton singlet and $(a, b = e, \mu, \tau)$ being the generation indices, a superscript ^{C} indicating charge conjugation. Here $\psi^C = C\bar{\psi}^T$ with C being the charge-conjugation matrix. The coupling constant f_{ab} is antisymmetric ($f_{ab} = -f_{ba}$), whereas h_{ab} is symmetric ($h_{ab} = h_{ba}$). Gauge invariance precludes the singlet Higgs fields from coupling to the quarks. In terms of the component fields, the interaction Lagrangian is given by

$$\begin{aligned}\mathcal{L}_Y = {}& 2\big[f_{e\mu}\big(\bar{\nu_e^c}\mu_L - \bar{\nu_\mu^c}e_L\big) + f_{e\tau}\big(\bar{\nu_e^c}\tau_L - \bar{\nu_\tau^c}e_L\big) + f_{\mu\tau}\big(\bar{\nu_\mu^c}\tau_L - \bar{\nu_\tau^c}\mu_L\big)\big]h^+ \\ &+ \big[h_{ee}\bar{e^c}e_R + h_{\mu\mu}\bar{\mu^c}\mu_R + h_{\tau\tau}\bar{\tau^c}\tau_R \\ &+ h_{e\mu}\bar{e^c}\mu_R + h_{e\tau}\bar{e^c}\tau_R + h_{\mu\tau}\bar{\mu^c}\tau_R\big]k^{++} + \mathrm{H.c.}\,,\end{aligned} \tag{5}$$

where we have used $h_{aa} = h'_{aa}$, $h_{ab} = 2h'_{ab}$ for $a \neq b$. Equation (4) conserves lepton number, therefore, itself cannot be responsible for neutrino mass generation.

The Higgs potential contains the terms:

$$V(\phi, h^+, k^{++}) = \mu(h^-h^-k^{++} + h^+h^+k^{--}) + \cdots\,, \tag{6}$$

which violate lepton number by two units. They are expected to cause Majorana neutrino masses.

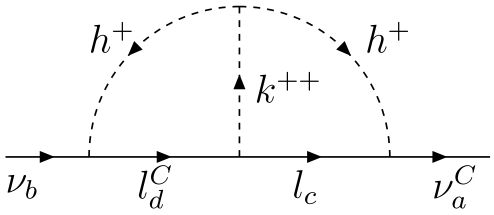

Fig. 1. The two-loop diagram in the Zee–Babu model.

In the literature, Majorana neutrino masses are generated at the two-loop level via the diagram shown in Ref. 24 and again depicted in Fig. 1. The corresponding mass matrix for Majorana neutrinos is as follows:

$$M_{ab} = 8\mu f_{ac} h^*_{cd} m_c m_d I_{cd} (f^+)_{db} \,. \tag{7}$$

The integral I_{cd} is given by[32]

$$I_{cd} = \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \frac{1}{k^2 - m_c^2} \frac{1}{k^2 - M_h^2} \frac{1}{q^2 - m_d^2} \times \frac{1}{q^2 - M_h^2} \frac{1}{(k-q)^2 - M_k^2} \,. \tag{8}$$

Note that Eq. (8) can be simplified by neglecting the charged lepton masses m_c and m_d.[26]

To evaluate the integral above, one neglects the charged lepton masses in the denominator, since these masses are much smaller than the charged scalar masses M_h and M_k. Then

$$I_{cd} \simeq I = \frac{1}{(16\pi^2)^2} \frac{1}{M^2} \frac{\pi^2}{3} \tilde{I}(r) \,, \quad M \equiv \max(M_k, M_h) \,, \tag{9}$$

which does not depend on lepton masses. Here $\tilde{I}(r)$ is a function of the ratio of the masses of the charged Higgses $r \equiv M_k^2/M_h^2$,

$$\tilde{I}(r) = \begin{cases} 1 + \dfrac{3}{\pi^2}(\log^2 r - 1) & \text{for } r \gg 1 \,, \\ 1 & \text{for } r \to 0 \,, \end{cases} \tag{10}$$

which is close to 1 for a wide range of scalar masses.

The neutrino mass matrix arising from (7) is symmetric and given by

$$\mathcal{M}_\nu = -I\mu f_{\mu\tau}^2 \begin{pmatrix} \epsilon^2\omega_{\tau\tau} + 2\epsilon\epsilon'\omega_{\mu\tau} + \epsilon'^2\omega_{\mu\mu} & \epsilon\omega_{\tau\tau} + \epsilon'(\omega_{\mu\tau} - \epsilon\omega_{e\tau} - \epsilon'\omega_{e\mu}) & -\epsilon'\omega_{\mu\mu} - \epsilon(\omega_{\mu\tau} + \epsilon\omega_{e\tau} + \epsilon'\omega_{e\mu}) \\ \star & \omega_{\tau\tau} + \epsilon'^2\omega_{ee} - 2\epsilon'\omega_{e\tau} & \epsilon\epsilon'\omega_{ee} - \omega_{\mu\tau} - \epsilon\omega_{e\tau} + \epsilon'\omega_{e\mu} \\ \star & \star & \omega_{\mu\mu} + 2\epsilon\omega_{e\mu} + \epsilon^2\omega_{ee} \end{pmatrix}, \tag{11}$$

where we have redefined parameters:

$$\epsilon \equiv \frac{f_{e\tau}}{f_{\mu\tau}}\,, \quad \epsilon' \equiv \frac{f_{e\mu}}{f_{\mu\tau}}\,, \quad \omega_{ab} \equiv m_a h^*_{ab} m_b\,. \tag{12}$$

Let us denote

$$\begin{aligned} \omega'_{\tau\tau} &\equiv \omega_{\tau\tau} + \epsilon'^{\,2}\omega_{ee} - 2\epsilon'\omega_{e\tau}\,, \\ \omega'_{\mu\tau} &\equiv \omega_{\mu\tau} + \epsilon\omega_{e\tau} - \epsilon'\omega_{e\mu} - \epsilon\epsilon'\omega_{ee}\,, \\ \omega'_{\mu\mu} &\equiv \omega_{\mu\mu} + 2\epsilon\omega_{e\mu} + \epsilon^2\omega_{ee}\,. \end{aligned} \tag{13}$$

Then the neutrino mass matrix can be rewritten in the compact form

$$\mathcal{M}_\nu = -I\mu f^2_{\mu\tau}\begin{pmatrix} \epsilon^2\omega'_{\tau\tau} + 2\epsilon\epsilon'\omega'_{\mu\tau} + \epsilon'^{\,2}\omega'_{\mu\mu} & \epsilon\omega'_{\tau\tau} + \epsilon'\omega'_{\mu\tau} & -\epsilon\omega'_{\mu\tau} - \epsilon'\omega'_{\mu\mu} \\ \star & \omega'_{\tau\tau} & -\omega'_{\mu\tau} \\ \star & \star & \omega'_{\mu\mu} \end{pmatrix}. \tag{14}$$

The above matrix has three exact eigenvalues given by

$$m_1 = 0\,, \quad m_{2,3} = \frac{1}{2}\left(-kF \pm \sqrt{k^2\left[F^2 + 4(1+\epsilon^2+\epsilon'^{\,2})(\omega'^{\,2}_{\mu\tau} - \omega'_{\mu\mu}\omega'_{\tau\tau})\right]}\right), \tag{15}$$

where we have denoted

$$k = \mu I f^2_{\mu\tau}\,, \quad F = (1+\epsilon'^{\,2})\omega'_{\mu\mu} + 2\epsilon\epsilon'\omega'_{\mu\tau} + (1+\epsilon^2)\omega'_{\tau\tau}\,. \tag{16}$$

The massless eigenstate is given by

$$\nu_1 = \frac{1}{\sqrt{f^2_{e\mu} + f^2_{e\tau} + f^2_{\mu\tau}}}(f_{\mu\tau}\nu_e - f_{e\tau}\nu_\mu + f_{e\mu}\nu_\tau)\,. \tag{17}$$

The mass matrix (14) is diagonalized as

$$U^T\mathcal{M}_\nu U = \mathrm{diag}(0, m_2, m_3)\,,$$

where

$$U = \begin{pmatrix} \frac{1}{\sqrt{1+\epsilon^2+\epsilon'^{\,2}}} & -\frac{A_1}{\sqrt{1+A_1^2+B_1^2}} & \frac{A_2}{\sqrt{1+A_2^2+B_2^2}} \\ -\frac{\epsilon}{\sqrt{1+\epsilon^2+\epsilon'^{\,2}}} & -\frac{B_1}{\sqrt{1+A_1^2+B_1^2}} & \frac{B_2}{\sqrt{1+A_2^2+B_2^2}} \\ \frac{\epsilon'}{\sqrt{1+\epsilon^2+\epsilon'^{\,2}}} & -\frac{1}{\sqrt{1+A_1^2+B_1^2}} & \frac{1}{\sqrt{1+A_2^2+B_2^2}} \end{pmatrix} \tag{18}$$

with the new notations

$$A_{1,2} = \frac{-k\left[\epsilon(\epsilon'^{\,2}-1)\omega'_{\mu\mu} + 2\epsilon'(1+\epsilon^2)\omega'_{\mu\tau} + \epsilon(1+\epsilon^2)\omega'_{\tau\tau}\right] \pm \epsilon\sqrt{k^2F'}}{2k\left[\epsilon\epsilon'\omega'_{\mu\mu} + (1+\epsilon^2)\omega'_{\mu\tau}\right]}\,, \tag{19}$$

$$B_{1,2} \equiv \frac{k(1+\epsilon'^{\,2})\omega'_{\mu\mu} - k(1+\epsilon^2)\omega'_{\tau\tau} \pm \sqrt{k^2F'}}{2k\left[\epsilon\epsilon'\omega'_{\mu\mu} + (1+\epsilon^2)\omega'_{\mu\tau}\right]} \tag{20}$$

and

$$F' = F^2 + 4(1+\epsilon^2+\epsilon'^2)(\omega'^2_{\mu\tau} - \omega'_{\mu\mu}\omega'_{\tau\tau})\,. \tag{21}$$

The eigenstates ν_i corresponding to the eigenvalues m_i $(i = 1, 2, 3)$ are found to be

$$\begin{aligned}
\nu_1 &= \frac{1}{\sqrt{f^2_{e\mu} + f^2_{e\tau} + f^2_{\mu\tau}}}(f_{\mu\tau}\nu_e - f_{e\tau}\nu_\mu + f_{e\mu}\nu_\tau)\,,\\
\nu_2 &= -\frac{A_1}{\sqrt{1+A_1^2+B_1^2}}\nu_e - \frac{B_1}{\sqrt{1+A_1^2+B_1^2}}\nu_\mu - \frac{1}{\sqrt{1+A_1^2+B_1^2}}\nu_\tau\,,\\
\nu_3 &= \frac{A_2}{\sqrt{1+A_2^2+B_2^2}}\nu_e + \frac{B_2}{\sqrt{1+A_2^2+B_2^2}}\nu_\mu + \frac{1}{\sqrt{1+A_2^2+B_2^2}}\nu_\tau\,.
\end{aligned} \tag{22}$$

From the explicit expressions of $A_{1,2}$ and $B_{1,2}$ in (19) and (20), some useful relations are in order

$$\begin{aligned}
A_1A_2 + B_1B_2 + 1 &= 0\,,\\
A_1 - \epsilon B_1 + \epsilon' &= 0\,,\\
A_2 - \epsilon B_2 + \epsilon' &= 0\,,\\
\frac{(A_1 - A_2)}{(B_1 - B_2)} &= \epsilon\,.
\end{aligned} \tag{23}$$

One also has

$$\begin{aligned}
A_1A_2 &= \frac{(\epsilon'^2 - \epsilon^2)\omega'_{\mu\tau} + \epsilon\epsilon'(\omega'_{\tau\tau} - \omega'_{\mu\mu})}{\epsilon\epsilon'\omega'_{\mu\mu} + (1+\epsilon^2)\omega'_{\mu\tau}}\,,\\
B_1B_2 &= -\frac{(1+\epsilon'^2)\omega'_{\mu\tau} + \epsilon\epsilon'\omega'_{\tau\tau}}{\epsilon\epsilon'\omega'_{\mu\mu} + (1+\epsilon^2)\omega'_{\mu\tau}}\,.
\end{aligned}$$

3. Constraints from the Tribimaximal Mixing

The current data on neutrino mass and mixing show that TBM mixing[4,5] as displayed in (1) is very specific. Comparing (18) with (1) yields the following conditions

$$\epsilon = \epsilon' = \frac{1}{2}\,, \tag{24}$$

$$A_2 = 0\,, \quad A_1 = B_1 = -1\,, \quad B_2 = 1\,. \tag{25}$$

Equations (24) and (12) lead to

$$f_{e\mu} = f_{e\tau} = \frac{1}{2}f_{\mu\tau}\,. \tag{26}$$

Substitution of (24) into expressions of $A_{1,2}$, $B_{1,2}$ in (19) and (20) yields

$$A_{1,2} = \frac{k(3\omega'_{\mu\mu} - 10\omega'_{\mu\tau} - 5\omega'_{\tau\tau}) \pm \sqrt{k^2F_0}}{4k(\omega'_{\mu\mu} + 5\omega'_{\mu\tau})}\,, \tag{27}$$

$$B_{1,2} = \frac{5k(\omega'_{\mu\mu} - \omega'_{\tau\tau}) \pm \sqrt{k^2F_0}}{2k(\omega'_{\mu\mu} + 5\omega'_{\mu\tau})}\,, \tag{28}$$

with

$$F_0 = 4(\omega'_{\mu\mu} + 5\omega'_{\mu\tau})^2 + (\omega'_{\mu\mu} - \omega'_{\mu\tau})(21\omega'_{\mu\mu} - 20\omega'_{\mu\tau} - 25\omega'_{\tau\tau})\,. \tag{29}$$

If $\omega'_{\mu\mu} = \omega'_{\tau\tau} = \omega'$ we have:

$$A_{1,2} = -\frac{1}{2}\left(1 \mp \frac{k(\omega' + 5\omega'_{\mu\tau})}{\sqrt{k^2(\omega' + 5\omega'_{\mu\tau})^2}}\right), \tag{30}$$

$$B_{1,2} = \pm\frac{k(\omega' + 5\omega'_{\mu\tau})}{\sqrt{k^2(\omega' + 5\omega'_{\mu\tau})^2}}\,. \tag{31}$$

It can be checked that with the help of (24), all remaining conditions in (25) are satisfied if

$$\omega'_{\mu\mu} = \omega'_{\tau\tau} \equiv \omega' \tag{32}$$

and $k(\omega' + 5\omega'_{\mu\tau})$ are negative real numbers. This can be equivalently converted into a relation among the Yukawa couplings

$$\omega_{\mu\mu} + \omega_{e\mu} = \omega_{\tau\tau} - \omega_{e\tau}\,. \tag{33}$$

Note that our derived constraints are somewhat different from those given in Ref. 28.

From the conditions (24) and (32) we obtain[b]

$$m_1 = 0\,, \quad m_{2,3} = -\frac{1}{4}\left[k(5\omega' + \omega'_{\mu\tau}) \mp \sqrt{k^2(\omega' + 5\omega'_{\mu\tau})^2}\,\right]. \tag{34}$$

The complex phases which can arise when diagonalizing the neutrino mass matrix (14) can be absorbed by the redefinition of the mass matrix eigenvectors, as it should be given that both $m_{2,3}$ are physical observables. Hence, in this work we assume m_2 and m_3 to be real.

Depending on the sign of the function in the square root, we have two cases in which $k(5\omega' + \omega'_{\mu\tau})$ being either positive or negative. To fit the experimental data in Ref. 9 the following condition must be satisfied

$$k(\omega' + 5\omega'_{\mu\tau}) < 0\,. \tag{35}$$

[b] The integration in Fig. 1 is linear divergent and has a surface term,[33] which give a similar form of mass matrix.

The neutrino masses in (15) becomes

$$m_1 = 0\,, \quad m_2 = -\frac{3k}{2}(\omega' + \omega'_{\mu\tau})\,, \quad m_3 = k(-\omega' + \omega'_{\mu\tau})\,. \tag{36}$$

Taking the central values from the data[9] as displayed in (2), we have the two following solutions:

(1) $m_1 = 0$, $m_2 = 0.008712$ eV, $m_3 = -0.050059$ eV and then

$$U = \begin{pmatrix} \frac{2}{\sqrt{6}} & 0.57735 & 4.17428 \times 10^{-17} \\ -\frac{1}{\sqrt{6}} & 0.57735 & 0.707107 \\ \frac{1}{\sqrt{6}} & -0.57735 & 0.707107 \end{pmatrix}. \tag{37}$$

In this case, $\omega'_{\mu\tau}$ and ω' depend only on k due to the following relations:

$$\omega'_{\mu\tau} = -\frac{0.0279335}{k}\,, \qquad \omega' = \frac{0.0221255}{k}\,, \tag{38}$$

$$\frac{\omega'_{\mu\tau}}{\omega'} = -1.2625\,, \qquad k(\omega' + 5\omega'_{\mu\tau}) = -0.117542 < 0\,, \tag{39}$$

(2) $m_1 = 0$, $m_2 = -0.00871206$ eV, $m_3 = -0.050059$ eV and

$$U = \begin{pmatrix} \frac{2}{\sqrt{6}} & 0.57735 & -5.93338 \times 10^{-17} \\ -\frac{1}{\sqrt{6}} & 0.57735 & -0.707107 \\ -\frac{1}{\sqrt{6}} & 0.57735 & 0.707107 \end{pmatrix}. \tag{40}$$

In this case, $\omega'_{\mu\tau}$ and ω' depend only on k according to the following relations:

$$\omega'_{\mu\tau} = -\frac{0.0221255}{k}\,, \qquad \omega' = \frac{0.0279335}{k}\,, \tag{41}$$

$$\frac{\omega'_{\mu\tau}}{\omega'} = -0.792076\,, \qquad k(\omega' - 5\omega'_{\mu\tau}) = -0.0826938 < 0\,. \tag{42}$$

The expressions (39) and (42) show that $\omega'_{\mu\mu}$, $\omega'_{\tau\tau}$ and $\omega'_{\mu\tau}$ are of the same order, and the normal neutrino mass hierarchy was used.[c]

[c]Here, we have assumed a normal neutrino mass hierarchy in which $m_1 = \lambda_1 = 0$, $m_2 = \lambda_2$, $m_3 = \lambda_3$ where λ_i $(i = 1, 2, 3)$ are eigenvalues of M_ν in (14). A spectrum with inverted ordering can be obtained by using the notation $m'_3 = \lambda_1 = 0$, $m'_2 = \lambda_3 \equiv m_3$ and $m'_1 = \lambda_2 \equiv m_2$.

In terms of the usual neutrino-oscillation parameters, the matrices (37) and (40) mean that

$$\sin^2_{23} = \frac{1}{2}\,, \quad \sin^2_{12} = \frac{1}{3}\,, \quad \sin^2_{13} = 0\,, \tag{43}$$

which are in good agreement with the TBM form.[9] However, with a vanishing θ_{13} now excluded at more than 10σ (Ref. 17) the situation has changed somewhat and the result in (2) should be considered just as a good approximation.

Using the standard parametrization of the neutrino mixing matrix (the PMNS matrix) in terms of three angles and CP violating phases[6–8]

$$\begin{aligned}
U &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}\, e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\, e^{i\delta} & 0 & c_{13} \end{pmatrix} \\
&\quad \times \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\gamma/2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \\
&\quad \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\gamma/2} & 0 \\ 0 & 0 & 1 \end{pmatrix},
\end{aligned} \tag{44}$$

where δ and γ are the Dirac and Majorana CP phase, respectively, and $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$ $(ij = 12, 23, 13)$. The above Majorana mass matrix is diagonalized by the PMNS matrix

$$U^T \mathcal{M}_\nu U = \mathcal{M}_{\text{diag}} = \text{diag}(m_1, m_2, m_3)\,.$$

In the case of the normal mass hierarchy, the four parameters are described as[26,28]

$$\epsilon = \tan\theta_{12}\frac{s_{23}}{c_{13}} + \tan\theta_{13}\, e^{i\delta}\,, \quad \epsilon' = \tan\theta_{12}\frac{s_{23}}{c_{13}} - \tan\theta_{13}\, e^{i\delta}\,, \tag{45}$$

$$\begin{aligned}
\frac{\omega'_{\mu\tau}}{\omega'_{\mu\mu}} &= -\frac{c_{13}^2 s_{23}c_{23}}{c_{13}^2c_{23}^2 + r_{2/3}(s_{12}s_{13}c_{23}\, e^{-i\delta} + c_{12}s_{23})^2 e^{-i\gamma}} \\
&\quad - \frac{r_{2/3}(s_{12}s_{13}c_{23}\, e^{-i\delta} + c_{12}s_{23})(s_{12}s_{13}s_{23}\, e^{-i\delta} - c_{12}c_{23})e^{-i\gamma}}{c_{13}^2c_{23}^2 + r_{2/3}(s_{12}s_{13}c_{23}\, e^{-i\delta} + c_{12}s_{23})^2 e^{-i\gamma}}\,, \\
\frac{\omega'_{\tau\tau}}{\omega'_{\mu\mu}} &= \frac{c_{13}^2 s_{23}^2 + r_{2/3}(s_{12}s_{13}s_{23}\, e^{-i\delta} - c_{12}c_{23})^2 e^{-i\gamma}}{c_{13}^2c_{23}^2 + r_{2/3}(s_{12}s_{13}c_{23}\, e^{-i\delta} + c_{12}s_{23})^2 e^{-i\gamma}}\,,
\end{aligned} \tag{46}$$

with $r_{2/3} = m_2/m_3$.

Table 1. The values of γ corresponding to m_2, m_3.

m_2 [eV]	m_3 [eV]	$e^{-i\gamma}$	γ [rad]
0.008712	−0.0500591	1.00002	$\pi/2$
0.00871	−0.0500591	1.00001	$\pi/2$

We can easily see that with the help of (32), Eq. (46) is automatically satisfied. On the other hand, from (45) one can find the values of γ corresponding to those of m_2 and m_3 as shown in Table 1, in which the values of γ is approximately equal to $\frac{\pi}{2}$. So the condition (32) leads to Majorana maximal CP violation: $\sin\gamma_{CP} \simeq 1$, as mentioned in Ref. 35.

The recent considerations have implied $\theta_{13} \neq 0$, but small as given in Ref. 10. A deviation from the TBM form would be achieved with a nonzero value of A_2 and a small difference of ϵ and ϵ' as shown in Sec. 4.

4. Experimental Constraints with Nonzero θ_{13}

The realistic neutrino mixing will be slightly deviated from the TBM form. This will be achieved with a very small value of A_2 and $\epsilon' \simeq \epsilon \simeq \frac{1}{2}$. With the help of (23), the matrix U in (18) becomes

$$U = \begin{pmatrix} \frac{1}{\sqrt{1+\epsilon^2+\epsilon'^2}} & \frac{\epsilon^2+\epsilon'(\epsilon'+A_2)}{\sqrt{(1+\epsilon^2+\epsilon'^2)[\epsilon^2+(1+\epsilon^2)A_2^2+2A_2\epsilon'+\epsilon'^2]}} & \frac{A_2\epsilon}{\sqrt{(1+A_2^2)\epsilon^2+(A_2+\epsilon')^2}} \\ -\frac{\epsilon}{\sqrt{1+\epsilon^2+\epsilon'^2}} & \frac{\epsilon(1-A_2\epsilon')}{\sqrt{(1+\epsilon^2+\epsilon'^2)[\epsilon^2+(1+\epsilon^2)A_2^2+2A_2\epsilon'+\epsilon'^2]}} & \frac{A_2+\epsilon'}{\sqrt{(1+A_2^2)\epsilon^2+(A_2+\epsilon')^2}} \\ \frac{\epsilon'}{\sqrt{1+\epsilon^2+\epsilon'^2}} & -\frac{A_2(1+\epsilon^2)+\epsilon'}{\sqrt{(1+\epsilon^2+\epsilon'^2)[\epsilon^2+(1+\epsilon^2)A_2^2+2A_2\epsilon'+\epsilon'^2]}} & \frac{\epsilon}{\sqrt{(1+A_2^2)\epsilon^2+(A_2+\epsilon')^2}} \end{pmatrix}. \tag{47}$$

Combining (47) and (44) we obtain:

$$t_{12} = \frac{U_{12}}{U_{11}} = \frac{\epsilon^2 + \epsilon'(A_2 + \epsilon')}{\sqrt{\epsilon^2 + (1+\epsilon^2)A_2^2 + 2A_2\epsilon' + \epsilon'^2}}\,, \tag{48}$$

$$t_{23} = \frac{U_{23}}{U_{33}} = \frac{A_2 + \epsilon'}{\epsilon} \tag{49}$$

with $t_{ij} = \tan\theta_{ij}$ $(ij = 12, 23, 13)$.

Since ϵ and ϵ' are close to each other, it can be assumed that

$$\epsilon' = \alpha\epsilon\,, \tag{50}$$

where α is a constant close to 1.

From the expressions (48)–(50), we obtain the following relations:

$$t_{23} = -\frac{\alpha\epsilon^3(1+t_{12}^2) + \sqrt{\Gamma}}{\alpha^2\epsilon^3 - \epsilon(1+\epsilon^2)t_{12}^2}\,, \tag{51}$$

$$A_2 = \frac{\epsilon^3\alpha(1+\alpha^2) - \alpha\epsilon t_{12}^2 + \sqrt{\Gamma}}{t_{12}^2(1+\epsilon^2) - \alpha^2\epsilon^2}\,, \tag{52}$$

or

$$t_{23} = \frac{-\alpha\epsilon^3(1+t_{12}^2)+\sqrt{\Gamma}}{\alpha^2\epsilon^3-\epsilon(1+\epsilon^2)t_{12}^2}\,, \tag{53}$$

$$A_2 = \frac{\epsilon^3\alpha(1+\alpha^2)-\alpha\epsilon t_{12}^2-\sqrt{\Gamma}}{t_{12}^2(1+\epsilon^2)-\alpha^2\epsilon^2}\,, \tag{54}$$

where

$$\Gamma = \epsilon^2 t_{12}^2\big[1+(1+\alpha^2)\epsilon^2\big]\big[(1+\alpha^2)\epsilon^2-t_{12}^2\big]\,. \tag{55}$$

Substituting A_2 from (52) into (47) yields

$$U = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix}, \tag{56}$$

with

$$\begin{aligned}
U_{11} &= \frac{1}{\sqrt{1+(1+\alpha^2)\epsilon^2}}\,,\\
U_{21} &= -\frac{\epsilon}{\sqrt{1+(1+\alpha^2)\epsilon^2}}\,,\\
U_{31} &= \frac{\alpha\epsilon}{\sqrt{1+(1+\alpha^2)\epsilon^2}}\,,\\
U_{12} &= -\frac{\epsilon\big[\epsilon t_{12}^2+(1+\alpha^2)\epsilon^3 t_{12}^2+\alpha\sqrt{\Gamma}\big]}{\sqrt{\epsilon^3[1+(1+\alpha^2)\epsilon^2]^2\Gamma'}}\,,\\
U_{22} &= \frac{\epsilon\big[\alpha^4\epsilon^4-(1+\epsilon^2)t_{12}^2+\alpha^2\epsilon^2(1+\epsilon^2-t_{12}^2)+\alpha\epsilon\sqrt{\Gamma}\big]}{\sqrt{\epsilon^3[1+(1+\alpha^2)\epsilon^2]^2\Gamma'}}\,,\\
U_{32} &= \frac{\alpha(1+\alpha^2)\epsilon^5+\alpha\epsilon^3+(1+\epsilon^2)\sqrt{\Gamma}}{\sqrt{\epsilon^3[1+(1+\alpha^2)\epsilon^2]^2\Gamma'}}\,,\\
U_{13} &= \frac{\epsilon\big[\alpha(1+\alpha^2)\epsilon^3-\alpha\epsilon t_{12}^2+\sqrt{\Gamma}\big]}{\sqrt{\epsilon^3[1+(1+\alpha^2)\epsilon^2]\Gamma'}}\,,\\
U_{23} &= -\frac{\alpha\epsilon^3(1+t_{12}^2)+\sqrt{\Gamma}}{\sqrt{\epsilon^3[1+(1+\alpha^2)\epsilon^2]\Gamma'}}\,,\\
U_{33} &= \frac{\epsilon[\alpha^2\epsilon^2-t_{12}^2(1+\epsilon^2)]}{\sqrt{\epsilon^3[1+(1+\alpha^2)\epsilon^2]\Gamma'}}\,,
\end{aligned} \tag{57}$$

where

$$\Gamma' = (1-\alpha^2)\epsilon t_{12}^2+\epsilon^3(1+\alpha^2)(\alpha^2+t_{12}^2)+2\alpha\sqrt{\Gamma}\,. \tag{58}$$

We see that the neutrino mixing matrix in (56) with the elements given in (57) depends only on three parameters α, ϵ and t_{12}. It is easy to show that the model

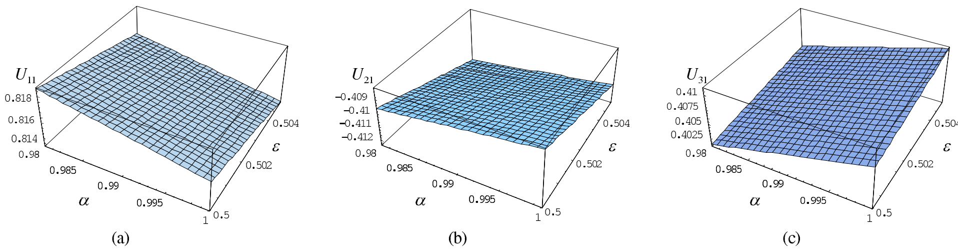

Fig. 2. U_{11}, U_{21}, U_{31} as functions of α and ϵ with $\alpha \in (0.98, 1.00)$ and $\epsilon \in (0.50, 0.505)$.

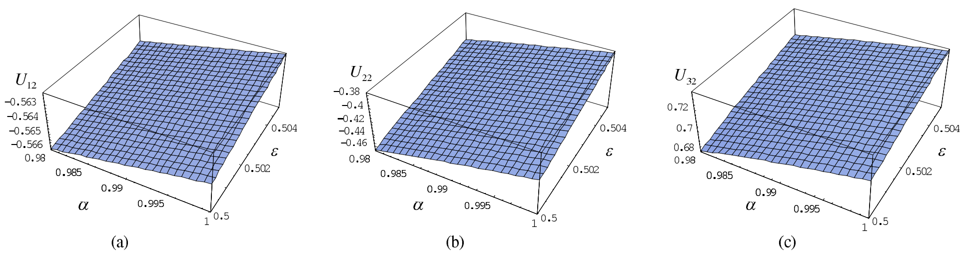

Fig. 3. U_{12}, U_{22}, U_{32} as functions of α and ϵ with $\alpha \in (0.98, 1.00)$ and $\epsilon \in (0.50, 0.505)$.

can fit the recent experimental constraints on the neutrino mixing angles. Indeed, by choosing $\alpha \in (0.98, 1.00)$, $\epsilon \in (0.50, 0.505)$ and taking the new data given in Ref. 10 with $t_{12} = 0.691$, we obtain

$$\begin{aligned} &U_{11} \in (0.814 \div 0.818)\,, \quad &&U_{12} \in -(0.563 \div 0.566)\,, \quad &&U_{13} \in (0.010 \div 0.140),\\ &U_{21} \in -(0.409 \div 0.412)\,, \quad &&U_{22} \in -(0.380 \div 0.460)\,, \quad &&U_{23} \in -(0.790 \div 0.830)\,,\\ &U_{31} \in (0.4025 \div 0.410)\,, \quad &&U_{32} \in (0.680 \div 0.720)\,, \quad &&U_{33} \in -(0.540 \div 0.600)\,, \end{aligned} \tag{59}$$

or

$$U = \begin{pmatrix} 0.814 \div 0.818 & -(0.563 \div 0.566) & 0.010 \div 0.140 \\ -(0.409 \div 0.412) & -(0.380 \div 0.460) & -(0.790 \div 0.830) \\ 0.4025 \div 0.410 & 0.680 \div 0.720 & -(0.540 \div 0.600) \end{pmatrix}. \tag{60}$$

It is interesting to note that the model-independent parametrization of non-TBM structures based on deviations from the reactor, solar and atmospheric angles[36] and on small perturbations of the TBM mixing eigenvectors[37] is similar to our approach here. Our set ϵ, α, t_{12} is equivalent to the set ϵ_{12}, ϵ_{23}, ϵ_{13} in Ref. 37.

Figures 2(a)–2(c), Figs. 3(a)–3(c) and Figs. 4(a)–4(c) give the dependence of the elements of U matrix on α and ϵ with $t_{12} = 0.691$.

With $\alpha \in (0.98, 1.00)$ and $\epsilon \in (0.50, 0.505)$, from (51) we obtain $t_{23} \in (1.3, 1.59)$ or $\theta_{23} \in (52.43^\circ, 56.31^\circ)$, and $A_2 \in (0.15, 0.25)$ which are shown in Figs. 5(a) and 5(b), respectively. In this case, $s_{13} \in (0.1, 0.14)$ or $\theta_{13} \in (5.74^\circ, 8.05^\circ)$.

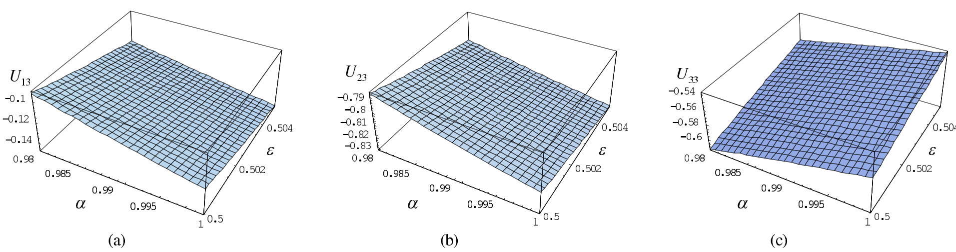

Fig. 4. U_{13}, U_{23}, U_{33} as functions of α and ϵ with $\alpha \in (0.98, 1.00)$ and $\epsilon \in (0.50, 0.505)$.

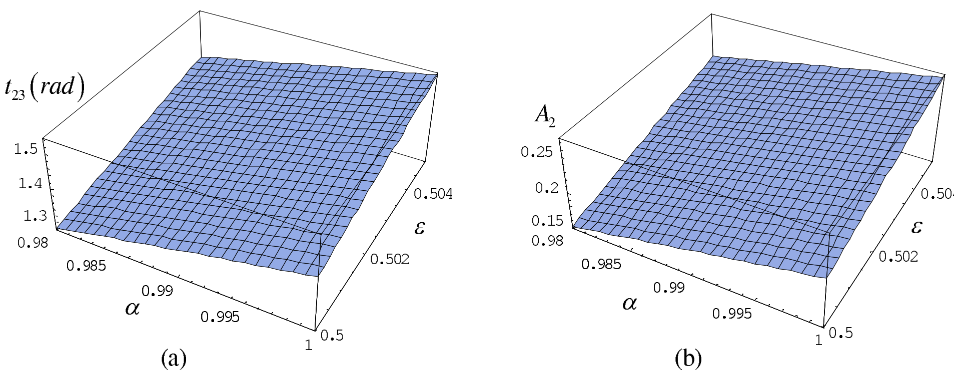

Fig. 5. (a) t_{23} as a function of α and ϵ with $\alpha \in (0.98, 1.00)$ and $\epsilon \in (0.50, 0.505)$; (b) A_2 as a function of α and ϵ with $\alpha \in (0.98, 1.00)$ and $\epsilon \in (0.50, 0.505)$.

Similarly, substituting A_2 from (54) into (47) yields

$$U = \begin{pmatrix} 0.814 \div 0.818 & 0.563 \div 0.566 & -(0.010 \div 0.140) \\ -(0.409 \div 0.412) & 0.69 \div 0.73 & 0.54 \div 0.58 \\ 0.4025 \div 0.410 & -(0.38 \div 0.44) & 0.8 \div 0.83 \end{pmatrix} \tag{61}$$

provided that $\alpha \in (0.98, 1.00)$ and $\epsilon \in (0.50, 0.505)$. In this case, $t_{23} \in (0.65, 0.75)$ or $\theta_{23} \in (45°, 50.19°)$, $s_{13} \in (0.02, 0.08)$ or $\theta_{13} \in (1.15°, 4.6°)$ and $A_2 \in (0.05, 0.15)$. We note that in these regions of the values of α and ϵ, θ_{13} is smaller than that given in Ref. 10, but the other regions of these parameters will provide a consistent range of θ_{13}, such as, when $\alpha \in (0.98, 1.00)$ and $\epsilon \in (0.50, 0.51)$ then $|s_{13}| \in (0.1, 0.16)$ or $\theta_{13} \in (5.74°, 9.21°)$. This range of θ_{13} satisfies the recent experimental data in Ref. 10.

From (51) and (53) we can have the relations of t_{23} and t_{12}, α, ϵ as shown in Figs. 6(a)–6(c) and Figs. 7(a)–7(c), respectively, in which the values of θ_{23} obtained encompass the best fit values in Ref. 10.

With the help of (50), F in (16) becomes:

$$F = (1 + \alpha^2\epsilon^2)\omega'_{\mu\mu} + 2\alpha\epsilon^2\omega'_{\mu\tau} + (1 + \epsilon^2)\omega'_{\tau\tau} \tag{62}$$

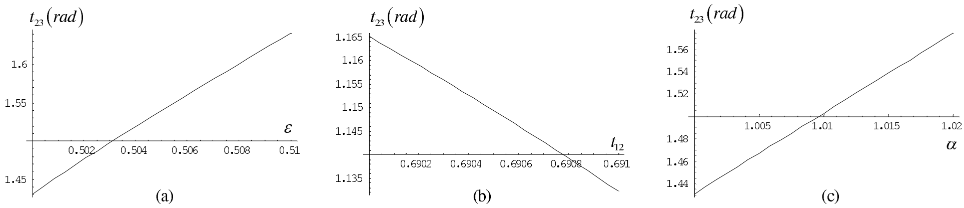

Fig. 6. (a) t_{23} as a function of ϵ with $\epsilon \in (0.50, 0.51)$ and $\alpha = 1$, $t_{12} = 0.691$; (b) t_{23} as a function of t_{12} with $t_{12} \in (0.690, 0.691)$ and $\alpha = 1$, $\epsilon = 0.49$; (c) t_{23} as a function of α with $\alpha \in (1.0, 1.02)$, $t_{12} = 0.691$, $\epsilon = 0.50$ from (51).

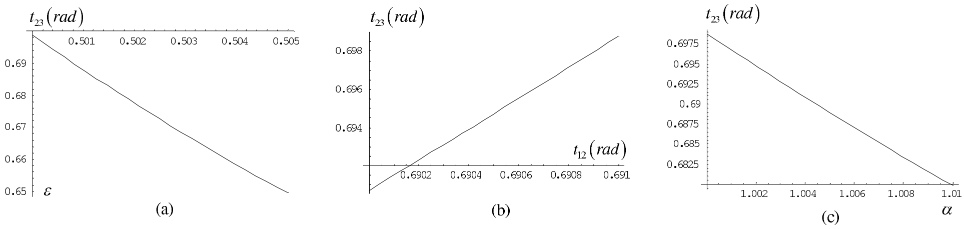

Fig. 7. (a) t_{23} as a function of ϵ with $\epsilon \in (0.50, 0.505)$ and $\alpha = 1$, $t_{12} = 0.691$; (b) t_{23} as a function of t_{12} with $t_{12} \in (0.690, 0.691)$ and $\alpha = 1$, $\epsilon = 0.50$; (c) t_{23} as a function of α with $\alpha \in (1.00, 1.01)$, $t_{12} = 0.691$, $\epsilon = 0.50$ from (53).

and the physical neutrino masses from (15) is defined

$$m_1 = 0\,, \quad m_{2,3} = \frac{1}{2}\left(-kF \pm \sqrt{k^2(F^2+B)}\right), \tag{63}$$

with

$$B = 4[1 + (1+\alpha^2)\epsilon^2](\omega'^2_{\mu\tau} - \omega'_{\mu\mu}\omega'_{\tau\tau})\,. \tag{64}$$

Taking the central values from the data[10] on neutrino mass square differences

$$\Delta m^2_{21} = 7.50 \times 10^{-5}\ \mathrm{eV}^2\,, \quad \Delta m^2_{32} = 2.32 \times 10^{-3}\ \mathrm{eV}^2\,, \tag{65}$$

we obtain

$$k = \frac{0.0402359}{F}\,. \tag{66}$$

The neutrino masses are explicitly given as

$$m_1 = 0\,, \quad m_2 = 0.00871206\ \mathrm{eV}\,, \quad m_3 = -0.048948\ \mathrm{eV}\,, \tag{67}$$

which are in a normal ordering.

The ratio of m_2 to m_3 is given

$$\frac{|m_2|}{|m_3|} = 0.177986\,, \tag{68}$$

which is the same order as in Ref. 28.

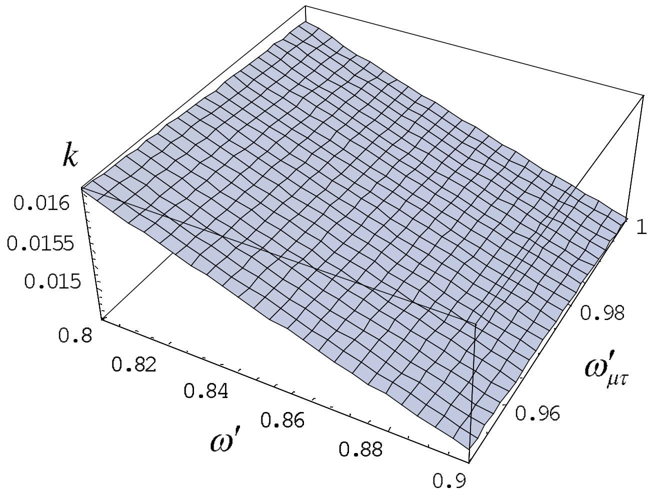

Fig. 8. k as a function of α, ϵ, ω' and ω'_{23} with $\omega'_{\mu,\tau} \in (0.95, 1.0)$, $\omega' \in (0.80, 0.9)$ and $\alpha = 1.00$, $\epsilon = 0.5$.

Without loss of generality, we assume $\omega'_{\mu\mu} = \omega'_{\tau\tau} = \omega'$. From (16), (50) and (66), we obtain the dependence of k on α, ϵ and ω and ω'_{23}:

$$k = \frac{0.0402359}{[(2 + (1 + \alpha^2)\epsilon^2)]\omega' + 2\alpha\epsilon^2\omega'_{\mu\tau}} \,. \tag{69}$$

In the case $\alpha = 1.00$ and $\epsilon = 0.5$, one has

$$k = -\frac{0.0402359}{2.5\omega' + 0.5\omega'_{\mu\tau}} \,.$$

Figure 8 gives the dependence of k on ω', $\omega'_{\mu\tau}$ with $\omega'_{\mu,\tau} \in (0.95, 1.0)$ and $\omega' \in (0.80, 0.9)$.

5. Summary

In this paper, we have derived the exact eigenvalues and eigenstates of the neutrino mass matrix in the Zee–Babu model. Tribimaximal mixing imposes some conditions on the Yukawa couplings. The constraints derived in this work slightly differ from other ones given in the literature, and the normal mass hierarchy is preferred. The derived conditions give a possibility of Majorana maximal CP violation in the neutrino sector. We have shown that nonzero θ_{13} is generated, if Yukawa couplings between leptons almost equal to each other. We have analyzed behaviors of the mixing angles as functions of the Yukawa couplings, and the model parameter space has been derived.

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