

Higgs revised in supersymmetric economical 3-3-1 model with B/μ -type terms

D. T. Binh, L. T. Hue^a, D. T. Huong, H. N. Long

Institute of Physics, Vietnam Academy of Science and Technology, 10 Dao Tan, Ba Dinh, Hanoi, Vietnam

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Abstract We re-investigate the scalar potential and the Higgs sector of the supersymmetric economical 3-3-1 model (SUSYE331) in the presence of the B/μ -type terms, which has many important consequences. First, the model contains no massless Higgs fields. Second, we prove that the soft mass parameters of the Higgses must be at the $SU(3)_L$ scale. As a result, the masses of the Higgses drift toward this scale except one light real neutral Higgs with the mass of $m_Z|c_{2\gamma}|$ at the tree level. We also show that there are some Higgses containing many properties of the Higgses in the minimal supersymmetric standard model (MSSM), especially in the neutral Higgs sector. One exact relation in the MSSM, $m_{H^\pm}^2 = m_A^2 + m_W^2$, is still true in the SUSYE331. Based on this result we make some comments on the lepton flavor violating decays of these Higgses as one of signatures of new physics in the SUSYE331 model which may be detected by present colliders.

1 Introduction

The discovery of a new particle by LHC experiments is the most intriguing event in both theoretical and experimental current physics. As found by both ATLAS and CMS [1, 2] this new particle, with mass around 125.5 GeV, carries many properties of the Higgs boson predicted by the Standard Model (SM). On the other hand, many works tried to determine whether this Higgs is really the SM Higgs or some new Higgs in models beyond the SM [3–7]. Many properties of this new Higgs are available in [8]. Some very helpful discussions on which models are excluded or still acceptable by the existence of the new Higgs found are for example in [9]. At this time apart from the SM, the MSSM is the most attractive model which both experimental and theoretical physics focus on. For the review of SM Higgs see [10]. A review of

MSSM Higgses is in [11, 12]. For the MSSM there are five physical Higgses, including one CP-odd neutral Higgs and two CP-even neutral Higgses. The mass of the lighter neutral Higgs is shown to be smaller than $m_Z|\cos(2\beta)|$ at tree level. Here β is determined by the relation $t_\beta = v_2/v_1$, the ratio of the two Higgs vacuum expectation values (VEVs) in the MSSM. The mass of this light Higgs can increase up to 135 GeV after including loop corrections [13]. Of course, the value of 125.5 GeV still satisfies this constraint but the mass spectrum of supersymmetric particles has drifted to the TeV scale [3, 4, 6, 7].

There is another class of supersymmetric (SUSY) models, called SUSY 3-3-1 models, which is not mentioned above. The SUSY 3-3-1 models are SUSY versions of the 3-3-1 models [14–22] constructed in order to explain some issues as the so-called family replication, the electric charged quantization [23–27], the large difference between masses of quarks in different families [28],...The greatest disadvantage of these models is the complication in the Higgs sector, namely: these models need many Higgs multiplets to generate the masses of the fermions. Some models with the simplest Higgs sector, such as [29–32] need only two $SU(3)_L$ Higgs triplets. But some fermions in these models get zero masses at the tree level and they need to get non-vanishing masses from loop corrections [29, 30] or effective non-renormalizable operators [31]. To solve this problem as well as the problem of dark matter in these models, some supersymmetric versions of these 3-3-1 models were introduced [33–37]. These models, of course, keep the interesting properties of the 3-3-1 as well as SUSY models. But the needed Higgs multiplets are doubled compared with the non-SUSY version to cancel the gauge anomaly caused by Higgsinos. The Higgs sectors are now much more complicated. Anyway, they were investigated in detail for the supersymmetric economical 3-3-1 (SUSYE331) model [33, 38] and the supersymmetric reduced minimal 3-3-1 (SUSYRM331) model [36, 37]. In this work we will concentrate on the SUSYE331 Higgs sector for two important reasons:

^a e-mail: lthue@iop.vast.ac.vn

- First, the SUSYE331 has the simplest Higgs sector in SUSY331 models and it was widely investigated as regards phenomenology such as Higgs sector [33,38], inflation scenarios [39–41], the mass spectrum of SUSY particles [42,43], and lepton flavor violating (LFV) decays [44,45]. One problem of this model is the absence of B/μ -type terms in the scalar potential. These terms are very important for the vacuum stability of general SUSY models. They were first addressed in [45], but the consequences of their presence were not shown in detail.
- Second, as mentioned above, the presence of the 125.5 GeV Higgs strongly affects the parameter space of all present models including SUSY331 models. It is indeed necessary to consider the reality of the SUSYE331 under the impact the appearance of this Higgs.

Comparing the Higgs sectors of the SUSY331 models with that of the MSSM is the straightforward way to estimate the compatibility of them with Higgs experiments at this time. For the SUSYE331, we try to identify some Higgses as “like-MSSM” Higgses and the others as being really $SU(3)_L$ Higgses. While finding the exact physical solutions for Higgses in the presence of the B/μ -type terms is almost impossible, we can calculate them in an approximate way with high accuracy, based on the presence of the $SU(3)_L$ scale itself. In the SUSY versions such as SUSYE331 this scale corresponds to two Higgses χ and χ' with two VEVs, which is assumed to be much larger than the SM symmetry breaking scale w , $w' \gg 246$ GeV. Constraints from the heavy neutral Z' of the model predict that this scale is of the order of the TeV scale [46]. Combining with the conditions of the minimum of the scalar potential, it can be deduced that both B/μ -terms and soft parameters should be of the same order, the electroweak $\mathcal{O}(m_W^2)$ or the $SU(3)_L$ scale. We show this conclusion in detail in Sect. 3. In that section, we also construct all squared Higgs mass matrices of the model, find exact solutions for physical CP-odd neutral Higgses, and establish two equations determining the mass eigenvalues of CP-even neutral and charged Higgses. Approximate solutions of these Higgs masses will be discussed in Sect. 4 after we prove that the B/μ -terms and soft parameters favor the $SU(3)_L$ scale. With this condition, the Higgs spectrum of the SUSYE331 is split into two parts, in which the first part contains Higgses with properties being similar to MSSM Higgses. Some other Higgs properties are also mentioned in this section. Furthermore, in Sect. 5, like-MSSM Higgses are discussed in more detail by comparing them with MSSM Higgses in coupling with the standard particles. In Sect. 6, we discuss the LFV decay of the neutral Higgs, $H^0 \rightarrow \mu\tau$, in the SUSYE331 model. This kind of decay was investigated in [44] without the appearance of B/μ -type terms. It is noted that detecting LFV decay at TeVatron and LHC was discussed in [47], and the sensitivity of the LHC for these decays has been dis-

cussed [48]. In the revised SUSYE331 version, only MSSM-like Higgses can have a large LFV decay branching ratio for $H^0 \rightarrow \mu\tau$. This result is easily obtained based on many previous works on this kind of decay for the MSSM and extended versions of the MSSM [49–55]. First of all, we start our work by reviewing the SUSYE331 particle content in Sect. 2.

2 A review of the model SUSYE331

In this section we only list the particle content of the SUSYE331 which we consider in this work. The details were thoroughly investigated for example in [33,38].

The superfield content is defined in a standard way as follows:

$$\widehat{F} = (\widetilde{F}, F), \quad \widehat{S} = (S, \widetilde{S}), \quad \widehat{V} = (\lambda, V), \quad (1)$$

where the components F , S , and V stand for the fermion, scalar, and vector fields, while their superpartners are denoted \widetilde{F} , \widetilde{S} , and λ , respectively [34–37].

The superfields containing leptons under the 3-3-1 gauge group transform as

$$\widehat{L}_{aL} = (\widehat{\nu}_a, \widehat{l}_a, \widehat{\nu}_a^c)^T \sim (1, 3, -1/3), \quad \widehat{l}_{aL}^c \sim (1, 1, 1), \quad (2)$$

where $\widehat{\nu}_L^c = (\widehat{\nu}_R)^c$ and $a = 1, 2, 3$ is a generation index.

The superfields for the left-handed quarks of the first generation are in triplets,

$$\widehat{Q}_{1L} = (\widehat{u}_1, \widehat{d}_1, \widehat{u}'_1)^T \sim (3, 3, 1/3). \quad (3)$$

We omit the color index of the quarks. The right-handed singlet counterparts of these superfields are denoted

$$\widehat{u}_{1L}^c, \widehat{u}'_{1L}^c \sim (3^*, 1, -2/3), \quad \widehat{d}_{1L}^c \sim (3^*, 1, 1/3). \quad (4)$$

Conversely, the last two generations are contained in superfields which transform as antitriplets of the $SU(3)_L$,

$$\widehat{Q}_{\alpha L} = (\widehat{d}_\alpha, -\widehat{u}_\alpha, \widehat{d}'_\alpha)^T \sim (3, 3^*, 0), \quad \alpha = 2, 3, \quad (5)$$

while the right-handed counterparts are in singlets,

$$\widehat{u}_{\alpha L}^c \sim (3^*, 1, -2/3), \quad \widehat{d}_{\alpha L}^c, \widehat{d}'_{\alpha L}^c \sim (3^*, 1, 1/3). \quad (6)$$

The prime superscript is used to distinguish exotic quarks and SM quarks having the same electric charges. The mentioned fermion content is originally from the 3-3-1 model with right-handed neutrinos [17–22,29,30], so it is anomaly-free.

The two superfields $\widehat{\chi}$ and $\widehat{\rho}$ contain the scalar sector of the economical 3-3-1 model (E331) [32]:

$$\begin{aligned} \widehat{\chi} &= (\widehat{\chi}_1^0, \widehat{\chi}^-, \widehat{\chi}_2^0)^T \sim (1, 3, -1/3), \\ \widehat{\rho} &= (\widehat{\rho}_1^+, \widehat{\rho}^0, \widehat{\rho}_2^+)^T \sim (1, 3, 2/3). \end{aligned} \tag{7}$$

To cancel the chiral anomalies of the Higgsino sector, two extra superfields, $\widehat{\chi}'$ and $\widehat{\rho}'$, are added as follows:

$$\begin{aligned} \widehat{\chi}' &= (\widehat{\chi}_1^0, \widehat{\chi}'^+, \widehat{\chi}_2^0)^T \sim (1, 3^*, 1/3), \\ \widehat{\rho}' &= (\widehat{\rho}_1^-, \widehat{\rho}'^0, \widehat{\rho}_2^-)^T \sim (1, 3^*, -2/3). \end{aligned} \tag{8}$$

According to the analysis in [33], at the tree level, ρ' is enough to generate masses for all charged leptons, while it contributes in part to the down-quark masses. Also, the ρ generates masses to the neutral leptons and contributes in part to the up-quark masses. On the other hand, both χ and χ' only contribute to masses of both the usual and exotic quarks. It can be supposed that ρ and ρ' may play similar roles as Higgses in the MSSM. It is recalled that the above Higgs sector does not generate masses for all quarks of the model. Therefore corrections from the loop levels are needed.

As normal 3-3-1 models, the $SU(3)_L \otimes U(1)_X$ gauge group is broken via two steps:

$$SU(3)_L \otimes U(1)_X \xrightarrow{w, w'} SU(2)_L \otimes U(1)_Y \xrightarrow{v, v', u, u'} U(1)_Q, \tag{9}$$

where the VEVs are defined by

$$\sqrt{2}\langle \chi \rangle^T = (u, 0, w), \quad \sqrt{2}\langle \chi' \rangle^T = (u', 0, w'), \tag{10}$$

$$\sqrt{2}\langle \rho \rangle^T = (0, v, 0), \quad \sqrt{2}\langle \rho' \rangle^T = (0, v', 0). \tag{11}$$

The vector superfields \widehat{V}_c^a , \widehat{V}^a , and \widehat{V}' containing the usual gauge bosons are related with the gauge groups $SU(3)_C$, $SU(3)_L$, and $U(1)_X$.

The VEVs w and w' are responsible for first stage of symmetry breaking, $SU(3)_L \times U(1)_X \rightarrow SU(2)_L \times U(1)_Y$, and this provides the mass for the new particles, namely

$$\begin{aligned} T_i \left(0, 0, \frac{w}{\sqrt{2}} \right)^T &\neq 0, \quad T_i \left(0, 0, \frac{w'}{\sqrt{2}} \right)^T \\ &\neq 0, \quad i = 4, 5, 6, 7, 8, \\ X \left(0, 0, \frac{w}{\sqrt{2}} \right)^T &\neq 0, \quad X \left(0, 0, \frac{w'}{\sqrt{2}} \right)^T \neq 0. \end{aligned} \tag{12}$$

In the gauge boson sector, only the new gauge bosons Y^\pm , X , X^* and Z' gain masses at this stage of the symmetry breaking. In contrast, the three other generators T_1 , T_2 , and T_3 characterizing the $SU(2)_L$ group are conserved. Also, the generator of the $U(1)_Y$, defined as

$$\frac{Y}{2} = -\frac{1}{\sqrt{3}}T_8 + X,$$

is also conserved. We would like to emphasize that at the first stage of breaking, there is no mixture between the Z and the Z' . In the second stage the standard model electroweak symmetry is broken down to $U(1)_Q$ by u , u' and v , v' and this is responsible for the masses of the ordinary particles. To keep consistency with the MSSM, we should suppose

$$u, u', v, v' \ll w, w'. \tag{13}$$

For more details, the reader is referred to [33]. After the first step of symmetry breaking, we can obtain the effective Lagrangian for the Higgs fields. From the effective Higgs potential, we can proceed with the discussion by comparison with the MSSM Higgs sector.

The full Lagrangian of the model has the form $\mathcal{L}_{\text{susy}} + \mathcal{L}_{\text{soft}}$, where the first term is the supersymmetric part and the last term explicitly breaks the supersymmetry. More details of this Lagrangian are discussed in [33]. Our work mainly focuses on the Higgs sector of the model.

3 Revised scalar potential for Higgses and Higgs sector

In the soft term involving the scalar potential, we add a new term,

$$(b_\rho \rho \rho' + b_\chi \chi \chi' + \text{H.c.}),$$

to the original supersymmetric Higgs potential constructed in [33]. The revised potential now is

$$\begin{aligned} V_{\text{SUSYE331}} &\equiv V_{\text{scalar}} + V_{\text{soft}} \\ &= \frac{\mu_\chi^2}{4} (\chi^\dagger \chi + \chi'^\dagger \chi') + \frac{\mu_\rho^2}{4} (\rho^\dagger \rho + \rho'^\dagger \rho') \\ &\quad + \frac{g'^2}{12} \left(-\frac{1}{3} \chi^\dagger \chi + \frac{1}{3} \chi'^\dagger \chi' + \frac{2}{3} \rho^\dagger \rho - \frac{2}{3} \rho'^\dagger \rho' \right)^2 \\ &\quad + \frac{g^2}{8} \sum_{b=1}^8 (\chi_i^\dagger \lambda_{ij}^b \chi_j - \chi_i'^\dagger \lambda_{ij}^{*b} \chi_j' + \rho_i^\dagger \lambda_{ij}^b \rho_j \\ &\quad - \rho_i'^\dagger \lambda_{ij}^{*b} \rho_j')^2 \\ &\quad + m_\rho^2 \rho^\dagger \rho + m_\chi^2 \chi^\dagger \chi + m_{\rho'}^2 \rho'^\dagger \rho' + m_{\chi'}^2 \chi'^\dagger \chi' \\ &\quad - (b_\rho \rho \rho' + b_\chi \chi \chi' + \text{H.c.}). \end{aligned} \tag{14}$$

As discussed in the MSSM, we can redefine the phases of the Higgs fields in order to get real values of both b_χ and b_ρ . In addition, these parameters must be positive to avoid the minimum value of the potential corresponding to the zero values of the neutral Higgses. It implies that electroweak symmetric breaking does not occur.

Assuming that the VEVs of the neutral components u , u' , v , v' , w and w' are real, we expand all Higgs fields around the VEVs as follows:

$$\begin{aligned} \chi^T &= \left(\frac{u+S_1+iA_1}{\sqrt{2}}, \chi^-, \frac{w+S_2+iA_2}{\sqrt{2}} \right), \\ \rho^T &= \left(\rho_1^+, \frac{v+S_5+iA_5}{\sqrt{2}}, \rho_2^+ \right), \\ \chi'^T &= \left(\frac{u'+S_3+iA_3}{\sqrt{2}}, \chi'^+, \frac{w'+S_4+iA_4}{\sqrt{2}} \right), \\ \rho'^T &= \left(\rho_1'^-, \frac{v'+S_6+iA_6}{\sqrt{2}}, \rho_2'^- \right). \end{aligned} \tag{15}$$

The minimum of the V_{SUSYE331} is equivalent to the canceling of five linear neutral Higgs terms, as listed below:

$$\begin{aligned} \mu_\rho^2 + 4m_\rho^2 &= 4\frac{v'}{v}b_\rho \\ &\quad - \frac{2g^{2'} + 9g^2}{27} \left[2(v^2 - v'^2) + w'^2 - w^2 + u'^2 - u^2 \right], \\ \mu_\chi^2 + 4m_\chi^2 &= 4\frac{u'}{u}b_\chi \\ &\quad - \frac{g'^2}{27} \left[w^2 - w'^2 + u^2 - u'^2 + 2(v'^2 - v^2) \right] \\ &\quad - \frac{g^2}{3} \left[2(u^2 - u'^2 + w^2 - w'^2) + v'^2 - v^2 \right], \end{aligned} \tag{16}$$

$$m_\rho^2 + m_{\rho'}^2 + \frac{1}{2}\mu_\rho^2 = b_\rho \frac{v^2 + v'^2}{vv'}, \tag{17}$$

$$m_\chi^2 + m_{\chi'}^2 + \frac{1}{2}\mu_\chi^2 = b_\chi \frac{u^2 + u'^2}{uu'}, \tag{18}$$

$$(-u'w + uw') \left[b_\chi + \frac{g^2}{4}(uu' + ww') \right] = 0. \tag{19}$$

From condition (19), it is easy to see that we have the equality $u/u' = w/w'$, the same as shown in [33]. The formulas in (16) are obtained when this equality is inserted in four other independent linear vanishing conditions. As our convention we will use the notation defined in previous works,

$$\begin{aligned} \tan \beta = t_\beta = \frac{u}{u'}, \quad \tan \gamma = t_\gamma = \frac{v}{v'}, \quad t = \frac{g'}{g}, \\ m_W^2 = \frac{g^2}{4}(v^2 + v'^2), \quad m_X^2 = \frac{g^2}{4}(u^2 + u'^2)(t_\beta^2 + 1), \end{aligned} \tag{20}$$

where m_X and m_W are the masses of the non-Hermitian boson X and W boson, respectively.

The four equations (16)–(18) now can be rewritten in the form

$$\begin{aligned} \frac{1}{4}\mu_\rho^2 + m_\rho^2 &= \frac{b_\rho}{t_\gamma} + \frac{2t^2 + 9}{27} \\ &\quad \times \left[-m_X^2 \cos 2\beta + 2m_W^2 \cos 2\gamma \right], \end{aligned} \tag{21}$$

$$\begin{aligned} \frac{1}{4}\mu_\chi^2 + m_\chi^2 &= \frac{b_\chi}{t_\beta} + \frac{t^2 + 18}{27}m_X^2 \cos 2\beta \\ &\quad - \frac{(2t^2 + 9)}{27}m_W^2 \cos 2\gamma, \end{aligned} \tag{22}$$

$$\begin{aligned} s_{2\gamma} \equiv \sin 2\gamma &= \frac{2b_\rho}{m_\rho^2 + m_{\rho'}^2 + \frac{1}{2}\mu_\rho^2}, \\ s_{2\beta} \equiv \sin 2\beta &= \frac{2b_\chi}{m_\chi^2 + m_{\chi'}^2 + \frac{1}{2}\mu_\chi^2}. \end{aligned} \tag{23}$$

The two equations in (23) directly tell us two separated constraints for b_ρ and b_χ :

$$2b_\rho \leq m_\rho^2 + m_{\rho'}^2 + \frac{1}{2}\mu_\rho^2 \quad \text{and} \quad 2b_\chi \leq m_\chi^2 + m_{\chi'}^2 + \frac{1}{2}\mu_\chi^2. \tag{24}$$

These two conditions are similar to the constraint to the b -term in the D -flat directions of the MSSM. They guarantee that the scalar potential has a lower bound. So it will have a minimum.

Using the results in (23) to solve the series of two equations (21) and (22) we can determine $\cos 2\gamma$ and $\cos 2\beta$ as functions of the soft parameters. But it will be more convenient to estimate the order of the soft parameters by writing $\cos 2\gamma$ and $\cos 2\beta$ as follows:

$$\begin{aligned} c_{2\gamma} \equiv \cos 2\gamma &= \frac{2c_W^2 \left(\frac{1}{4}\mu_\rho^2 + m_\rho^2 - \frac{b_\rho}{t_\gamma} \right) + \left(\frac{1}{4}\mu_\chi^2 + m_\chi^2 - \frac{b_\chi}{t_\beta} \right)}{m_W^2}, \\ c_{2\beta} \equiv \cos 2\beta &= \frac{\left(\frac{1}{4}\mu_\rho^2 + m_\rho^2 - \frac{b_\rho}{t_\gamma} \right) + 2 \left(\frac{1}{4}\mu_\chi^2 + m_\chi^2 - \frac{b_\chi}{t_\beta} \right)}{m_X^2} \\ &= \frac{2m_W^2 c_{2\gamma}}{m_X^2} - \frac{(3 - 4s_W^2) \left(\frac{1}{4}\mu_\rho^2 + m_\rho^2 - \frac{b_\rho}{t_\gamma} \right)}{m_X^2}. \end{aligned} \tag{25}$$

It is very important to note that the two equations in (25) have upper bounds: $|c_{2\gamma}|, |c_{2\beta}| \leq 1$. Combined with the property $m_W \ll m_X$ of the SUSYE331, the parameters on the right hand side of (25) must be on the same scale of $\mathcal{O}(m_W^2)$ or $\mathcal{O}(m_X^2)$. It means that we have only two cases,

$$\left| \frac{1}{4}\mu_\rho^2 + m_\rho^2 - \frac{b_\rho}{t_\gamma} \right| \sim \left| \frac{1}{4}\mu_\chi^2 + m_\chi^2 - \frac{b_\chi}{t_\beta} \right| \sim \mathcal{O}(m_W^2), \tag{26}$$

$$\text{or} \quad \left| \frac{1}{4}\mu_\rho^2 + m_\rho^2 - \frac{b_\rho}{t_\gamma} \right| \sim \left| \frac{1}{4}\mu_\chi^2 + m_\chi^2 - \frac{b_\chi}{t_\beta} \right| \sim \mathcal{O}(m_X^2). \tag{27}$$

If there is not much hierarchy among the soft and $\mu_{\rho,\chi}$ parameters, they all should be of the same scale. In addition, the case of (27) appears when the two quantities $2c_W^2 \left(\frac{1}{4}\mu_\rho^2 + m_\rho^2 - \frac{b_\rho}{t_\gamma} \right)$ and $\left(\frac{1}{4}\mu_\chi^2 + m_\chi^2 - \frac{b_\chi}{t_\beta} \right)$ have opposite signs, so that they cancel each other to result in the total being of the $\mathcal{O}(m_W^2)$ scale. The degeneration among the supersymmetric parameters characterized for a large break-

ing scale also happens in the normal $SU(2)_L \times U(1)_L$ supersymmetric model.

Because the Higgs sector in this model is very complicated, it is not easy to find the exact solutions for the mass spectrum as well as the mass eigenstates of the Higgses. Instead, we will use some appropriate approximations to solve the problems. In the next section we will use the parameter $\epsilon = m_W^2/m_X^2$, which satisfies $\epsilon \ll 1$, as the perturbative variable to do approximate calculations.

We firstly determine the mass eigenvalues of the pseudo-scalar neutral Higgses because they are calculated exactly. We will use them as independent parameters in formulas representing the Higgs mass spectra.

3.1 Pseudo-scalar or CP-odd neutral Higgses

The mass Lagrangian of the pseudo-scalar Higgses is split into two parts,

$$-\mathcal{L}_A^{\text{mass}} = \frac{1}{2}(A_1, A_2, A_3, A_4) \times M_{A\chi}^2(A_1, A_2, A_3, A_4)^T + \frac{1}{2}(A_5, A_6)M_{A\rho}^2(A_5, A_6)^T \tag{28}$$

with

$$M_{A\chi}^2 = \frac{g^2}{4} \begin{pmatrix} w'^2 + \frac{4b_\chi}{g^2 t_\beta^2}, & -u'w', & ww' + \frac{4b_\chi t_\beta}{g^2}, & -uw't_\beta, \\ -u'w', & u'^2 + \frac{4b_\chi}{g^2 t_\beta^2}, & -uw't_\beta & uu' + \frac{4b_\chi t_\beta}{g^2} \\ ww't_\beta + \frac{4b_\chi}{g^2} & -uw't_\beta & w^2 t_\beta + \frac{4b_\chi t_\beta^2}{g^2} & -uw't_\beta \\ -uw't_\beta & uu't_\beta + \frac{4b_\chi}{g^2} & -uw't_\beta^2 & u^2 t_\beta^2 + \frac{4b_\chi t_\beta}{g^2} \end{pmatrix},$$

and

$$M_{A\rho}^2 = \frac{4b_\rho}{t_\gamma} \times \begin{pmatrix} 1 & t_\gamma \\ t_\gamma & t_\gamma^2 \end{pmatrix}.$$

This leads to the result that there are three massless solutions and three massive ones, defined as

$$\begin{aligned} m_{A_1}^2 &\equiv m_{H_{A_1}}^2 = \frac{2b_\rho}{s_{2\gamma}} = \frac{1}{2}\mu_\rho^2 + m_\rho^2 + m_{\rho'}^2, \\ m_{A_2}^2 &\equiv m_{H_{A_2}}^2 = \frac{2b_\chi}{s_{2\beta}} = \frac{1}{2}\mu_\chi^2 + m_\chi^2 + m_{\chi'}^2, \\ m_{A_3}^2 &\equiv m_{H_{A_3}}^2 = m_{A_2}^2 + m_X^2. \end{aligned} \tag{29}$$

Because ρ and ρ' play the roles of MSSM Higgses, H_{A_1} seems to be the same as the CP-odd Higgs in the MSSM. To compare the Higgs mass spectrum with the $SU(3)_L$ scale in the following calculations, we will use some new notation, defined by

$$k_1 = \frac{m_{A_1}^2}{m_X^2}, \quad k_2 = \frac{m_{A_2}^2}{m_X^2}, \quad h_W = \sqrt{\frac{1}{3 - 4s_W^2}}. \tag{30}$$

It is easy to write the three massive eigenstates as

$$\begin{aligned} H_{A_1} &= A_5 c_\gamma + A_6 s_\gamma, \\ H_{A_2} &= A_1 c_\beta s_\zeta + A_2 c_\beta c_\zeta + A_3 s_\beta s_\zeta + A_4 s_\beta c_\zeta, \\ H_{A_3} &= -A_1 c_\beta c_\zeta + A_2 c_\beta s_\zeta - A_3 s_\beta c_\zeta + A_4 s_\beta s_\zeta, \end{aligned} \tag{31}$$

where $\tan \zeta = u'/w'$, $\cos \zeta = c_\zeta$, $\sin \zeta = s_\zeta$, $\cos \beta = c_\beta$, $\sin \beta = s_\beta$, $\cos \gamma = c_\gamma$, $\sin \gamma = s_\gamma$. Three massless eigenstates are

$$\begin{aligned} H_{A_4} &= -A_5 s_\gamma + A_6 c_\gamma, \\ H_{A_5} &= -A_2 s_\beta + A_4 c_\beta, \\ H_{A_6} &= -A_1 s_\beta + A_3 c_\beta. \end{aligned} \tag{32}$$

They are Goldstone bosons eaten by neutral gauge bosons Z , Z' , and X^0 . There do not exist any physical massless CP-odd neutral Higgses in the model.

3.2 Neutral scalar Higgs

In the basis of $(S_1, S_2, S_3, S_4, S_5, S_6)$ the squared mass matrix of the real scalar neutral Higgses can be written in the form of

$$M_{6S}^2 = \begin{pmatrix} m_{S_{11}}^2 & m_{S_{12}}^2 & m_{S_{13}}^2 & m_{S_{14}}^2 & m_{S_{15}}^2 & m_{S_{16}}^2 \\ & m_{S_{22}}^2 & m_{S_{23}}^2 & m_{S_{24}}^2 & m_{S_{25}}^2 & m_{S_{26}}^2 \\ & & m_{S_{33}}^2 & m_{S_{34}}^2 & m_{S_{35}}^2 & m_{S_{36}}^2 \\ & & & m_{S_{44}}^2 & m_{S_{45}}^2 & m_{S_{46}}^2 \\ & & & & m_{S_{55}}^2 & m_{S_{56}}^2 \\ & & & & & m_{S_{66}}^2 \end{pmatrix}, \tag{33}$$

where the precise formulas of the elements are listed in Appendix A. The squared mass matrices of both neutral and charged Higgses are different from those in [33] by B/μ -type terms.

The eigenvalues of this matrix are squared masses of the physical CP-even neutral Higgses at tree level, denoted $\lambda = m_{H^0}^2$. They must satisfy the equation $\det(M_{6S}^2 - \lambda I_6) = 0$, or equivalently

$$\lambda \left[\lambda - \left(1 + t_\beta^2\right) \left(\frac{b_\chi}{t_\beta} + \frac{g^2}{4}(u'^2 + w'^2)\right) \right] f(\lambda) = 0 \tag{34}$$

$$\text{with } f(\lambda) = a\lambda^4 + b\lambda^3 + c\lambda^2 + d\lambda + e. \tag{35}$$

Equation (34) has one massless solution and one exact massive solution $\lambda = m_{A_3}^2$. The massless Higgs is eaten by the X boson. The function $f(\lambda)$ can be reduced to a simpler form by defining a new variable as follows:

$$\lambda = X \times m_X^2. \tag{36}$$

From (23) and (30) we get

$$\begin{aligned}
 b_\chi &= \frac{1}{2}m_{A_2}^2 s_{2\beta} = \frac{1}{2}m_X^2 k_1 s_{2\beta}, \\
 b_\rho &= \frac{1}{2}m_{A_1}^2 s_{2\gamma} = \frac{1}{2}m_X^2 k_2 s_{2\gamma}, \\
 g' &= t \times g
 \end{aligned}
 \tag{37}$$

with

$$t^2 = \frac{18s_W^2}{3 - 4s_W^2}.$$

We define the quantity

$$\epsilon = \frac{m_W^2}{m_X^2} = \frac{v^2 + v'^2}{(u^2 + w'^2)(1 + t_\beta^2)}, \tag{38}$$

which measures the ratio of two spontaneous breaking scales $SU(2)_L$ and $SU(3)_L$. Based on the calculation in [29,30,33] we get a relation

$$\epsilon \simeq \frac{m_W^2}{m_{Z'}^2} \times \frac{4c_W^2}{4c_W^2 - 1}, \tag{39}$$

where $m_{Z'}$ is the mass of the heavy neutral Hermitian boson Z' and θ_W is the Weinberg angle, $c_W = \cos \theta_W$. The current bound of $m_{Z'}$ is $m_{Z'} > 2500 \text{ GeV}$ [46], leading to the result $\epsilon < 2.0 \times 10^{-3}$, which can be used to find solutions of Eq. (35) approximately.

The equation $f(\lambda) = 0$ now can be written in the form of

$$g(X) = AX^4 + BX^3 + CX^2 + DX + E = 0, \tag{40}$$

where

$$\begin{aligned}
 A &= 1, \\
 B &= -\left(4c_W^2 h_W^2 + k_1 + k_2 + 4h_W^2 \times \epsilon\right), \\
 C &= 4c_W^2 h_W^2 \left(k_1 + k_2 c_{2\beta}^2\right) \\
 &\quad + k_1 k_2 + h_W^2 \left(1 + k_1 c_{2\gamma}^2 + k_2\right) \times \epsilon, \\
 D &= -4c_W^2 h_W^2 k_1 k_2 c_{2\beta}^2 \\
 &\quad - 4h_W^2 \left[k_2 c_{2\beta}^2 + c_{2\gamma}^2 k_1 (1 + k_2)\right] \times \epsilon, \\
 E &= 4h_W^2 k_1 k_2 c_{2\gamma}^2 c_{2\beta}^2 \times \epsilon.
 \end{aligned}
 \tag{41}$$

The function $g(X)$ will be used to estimate the approximate mass eigenvalues of the real neutral Higgses in the following section. We will study in more detail the mass spectrum of neutral Higgs with some assumptions on the soft parameters. Now let us consider the charged Higgs mass spectrum.

3.3 Charged Higgs

In the basis of $(\chi^+, \chi'^+, \rho_1^+, \rho_2^+, \rho_1'^+, \rho_2'^+)$, the squared mass matrix can be written as

$$\begin{aligned}
 &M_{6\text{charged}}^2 \\
 &= \frac{g^2}{4} \begin{pmatrix} m_{\chi^-\chi^+}^2 & m_{\chi^-\chi'^+}^2 & uv & vw & -uv' & -v'w \\ & m_{\chi'^-\chi'^+}^2 & -vu' & -w'v & v'u' & v'w' \\ & & m_{\rho_1^-\rho_1^+}^2 & m_{\rho_1^-\rho_2^+}^2 & -\frac{4b_\rho}{g^2} - vv' & 0 \\ & & & -m_{\rho_2^-\rho_2^+}^2 & 0 & -\frac{4b_\rho}{g^2} - vv' \\ & & & & m_{\rho_1^-\rho_1'^+}^2 & m_{\rho_1^-\rho_2'^+}^2 \\ & & & & & m_{\rho_2^-\rho_2'^+}^2 \end{pmatrix}.
 \end{aligned}
 \tag{42}$$

Detailed formulas for the elements of the matrix are shown in Appendix B.

The masses of the charged Higgses are solutions of the equation $\text{Det}(M_{6\text{charged}}^2 - \lambda I_6) = 0$. Each solution $\lambda = m_{H^\pm}^2$ corresponds to one mass eigenvalue of $M_{6\text{charged}}^2$, and I_6 is the 6×6 unit matrix. Changing variables as in the case of the neutral Higgses, we obtain the equation

$$X^2 \left[\lambda - \left(m_{A_1}^2 + m_W^2\right) \right] \times f(X) = 0 \tag{43}$$

with $X = m_{H^\pm}^2/m_X^2$ and where m_W is the mass of the W boson. The function $f(X)$ is a polynomial of degree 3, represented as

$$\begin{aligned}
 f(X) &= X^3 + AX^2 + BX + C, \quad \text{where} \\
 A &= -(1 + k_1 + k_2 + \epsilon), \\
 B &= -c_{2\beta}^2 + k_1(1 + c_{2\gamma}c_{2\beta} + k_2) \\
 &\quad + [k_2 + c_{2\gamma}c_{2\beta}(2 + k_2)] \times \epsilon - c_\gamma^2 \times \epsilon^2, \\
 C &= (1 + \epsilon) [c_{2\beta} - c_{2\gamma}(\epsilon + k_1)] [c_{2\beta}(1 + k_2) - c_{2\gamma}\epsilon].
 \end{aligned}
 \tag{44}$$

For the charged Higgs sector, there are two Goldstone bosons eaten by the W^\pm and Y^\pm bosons. There is an exact value of the mass, $m_{H_4^\pm}^2 = m_W^2 + m_{A_1}^2$. The three other values will be investigated in the following section.

4 Constraint to Higgs masses

As stated above, in this section we will investigate in more detail the mass spectrum of the Higgses. We will see that there exist many relations among Higgs masses, soft parameters, and $\mu_{\rho,\chi}$ terms in the scalar potential. First, from (23), (25), (29), and the lower constraint to the CP-odd neutral Higgs masses from a recent experiment, we conclude that all parameters of the model must be above the electroweak breaking scale. Furthermore, the equations in (25) indicate that the soft-breaking parameters must be smaller than the $SU(3)_L$ breaking scale, and $c_{2\gamma}$ should not be too small.

To continue, we will investigate the masses of the neutral and charged Higgses in the two cases listed in (26) and (27). From these two cases and (29), it is easy to prove that $m_{A_1}^2$ and

$m_{A_2}^2$ have the same order as the parameters in (26). We will concentrate on the values of $m_{A_1}^2$ and $m_{A_2}^2$ in the following sections.

4.1 Case 1: Soft parameters in the electroweak breaking scale

This case is expressed in (26). The result is that k_1, k_2 , and $c_{2\beta}$ are of the order $\mathcal{O}(\epsilon)$. So we define

$$k_1 = k'_1 \times \epsilon, \quad k_2 = k'_2 \times \epsilon \tag{46}$$

with $k'_1, k'_2 \sim \mathcal{O}(1)$. The factor $c_{2\beta}$ will be considered later. Based on the Viet theorem, the equations given in Eq. (41) show that Eq. (40) produces four positive solutions related to the physical squared masses of the Higgses. Without loss of generality, we denote these four solutions as $X_1 \leq X_2 \leq X_3 \leq X_4$. The Viet theorem gives the four solutions satisfying the conditions

$$\begin{aligned} \sum_{i=1}^4 X_i &= 4c_W^2 h_W^2 + k_1 + k_2 + 4h_W^2 \times \epsilon, \\ \sum_{i < j; i, j=1}^4 X_i X_j &= 4c_W^2 h_W^2 (k_1 + k_2 c_{2\beta}^2) \\ &\quad + k_1 k_2 + h_W^2 (1 + k_1 c_{2\gamma}^2 + k_2) \times \epsilon, \\ \sum_{i < j < k} X_i X_j X_k &= 4c_W^2 h_W^2 k_1 k_2 c_{2\beta}^2 \\ &\quad + 4h_W^2 [k_2 c_{2\beta}^2 + c_{2\gamma}^2 k_1 (1 + k_2)] \times \epsilon, \\ X_1 X_2 X_3 X_4 &= 4h_W^2 k_1 k_2 c_{2\gamma}^2 c_{2\beta}^2 \times \epsilon. \end{aligned} \tag{47}$$

Because of the existence of the $4c_W^2 h_W^2$ term in the first equation of (47), there must be at least one heavy Higgs which is equivalent to $\mathcal{O}(m_X^2)$. The fourth equation shows that $X_1 X_2 X_3 X_4 \leq \mathcal{O}(\epsilon)$ in this case. So there is at least one light Higgs having a mass related with $X_i \leq \mathcal{O}(\epsilon)$. We first estimate this mass by assigning $X_1 = X'_1 \times \epsilon$ with $X'_1 \leq \mathcal{O}(1)$. Inserting this X_1 into Eq. (40) then setting the factor of the lowest order of ϵ to vanish, we have

$$(X'_1 - k'_1 c_{2\beta}^2) [c_W^2 X_1'^2 - (1 + c_W^2 k'_1) X'_1 + c_{2\gamma}^2 k'_1] = 0. \tag{48}$$

Equation (48) indicates that there are three light Higgses. But one of them relates with X'_1 such that

$$X'_1 = k'_1 c_{2\beta}^2 \sim \frac{m_{A_1}^2}{m_W^2} \times |c_{2\beta}|^2. \tag{49}$$

This value is too small because of the factor $c_{2\beta}^2 \sim \mathcal{O}(\epsilon^2)$ given in Eq. (25) and the soft parameter scale is the same as that of the $SU(2)_L$ breaking. Then we have

$$X'_1 = \frac{m_{H_1^0}^2}{m_W^2} \sim \frac{m_{A_1}^2}{m_W^2} \times \left(\frac{m_W^2}{m_X^2} \right)^2 \rightarrow m_{H_1^0} \sim m_{A_1} \times \mathcal{O}(10^{-3}).$$

Because $m_{A_1} \sim \mathcal{O}(m_W)$ in this case, if this is SM Higgs $m_{H_1^0}$ is too small compared with the recent experimental bound from LEP [57]. If not, one of two remaining solutions in (48) will be identified with the value around 125.5 GeV. The formula representing these two values is

$$m_{H_{2,3}^0}^2 \simeq \frac{1}{2} \left(m_Z^2 + m_{A_1}^2 \mp \sqrt{(m_{A_1}^2 - m_Z^2)^2 + 4s_{2\gamma}^2 m_Z^2 m_{A_1}^2} \right). \tag{50}$$

Equation (50) is of exactly the same form as that presented for the neutral Higgs masses in the MSSM. From previous work for the MSSM we immediately obtain some interesting consequences. At tree level the lighter Higgs gets a mass which is smaller than $m_Z |c_{2\gamma}|$. This Higgs is normally identified with the like-SM Higgs discovered at the LHC [1,2] because its mass can increase after including loop corrections. On the other hand, some recent works also were concerned with a case named the “*low- M_H scenario*” where the heavier Higgs corresponds to the discovered state [3,4]. Although this case predicts light charged Higgses, the parameter space is very small. This is because it requires all of these light Higgses to have heavily suppressed couplings to the gauge bosons to escape the search of LEP.

From the above investigation, the SUSYE331 soft parameters considered at the $SU(2)_L$ symmetry breaking are not the favorite choice. They should be of the $SU(3)_L$ breaking scale. It is case 2 that we concentrate on in this work.

4.2 Case 2: Soft parameters of the $SU(3)_L$ breaking scale

4.2.1 CP-even neutral Higgses

The Higgs sector in this case is very complicated. Mathematically, exact solutions of the polynomial equations (40) and (44) can be determined, but they are too long; also it is very hard to see any physics in these expressions. Instead, we firstly find approximate solutions of the mass eigenvalues based on the very small values of ϵ .

For light neutral Higgses, the last equation in (41) shows that there is only one light neutral Higgs. Being of the order of $\mathcal{O}(\epsilon)$, the squared mass of this Higgs is given as $X_1 = X'_1 \times \epsilon + \mathcal{O}(\epsilon^2)$ where $X'_1 \sim \mathcal{O}(1)$. Inserting this value into (40) and then forcing the factor of the lowest order of ϵ to be zero, we have

$$X'_1 \simeq \frac{c_{2\gamma}^2}{c_W^2} \text{ equivalent } m_{H_1^0}^2 \simeq M_Z^2 c_{2\gamma}^2. \tag{51}$$

This formula for the neutral Higgs mass is completely the same as that in the case of the MSSM. Furthermore, the contribution from the next leading order is proportional to $(\frac{1}{2}m_W \times \epsilon) \sim 0.08$ GeV. So the mass of the light Higgs needs to get major corrections from the loop contributions.

For the three heavy neutral Higgses, we denote their masses as $X_i = X'_i + X''_i \times \epsilon$, where both $X'_i, X''_i \sim \mathcal{O}(1)$ and $i = 2, 3, 4$. Then these masses can be written in the form

$$m_{H_i^0}^2 = X'_i m_X^2 + X''_i \times m_W^2 + \mathcal{O}(\epsilon) \times m_W^2. \tag{52}$$

The main contributions to the heavy Higgs masses come from $X_i \times m_X^2 \sim m_{H_i^0}^2$, namely

$$m_{H_2^0}^2 \simeq X'_2 m_X^2 = m_{A_1}^2, \tag{53}$$

$$m_{H_{3,4}^0}^2 \simeq X'_{3,4} \times m_X^2 = \frac{1}{2} \left(m_{A_2}^2 + m_{Z'}^2 \mp \sqrt{(m_{A_2}^2 - m_{Z'}^2)^2 + 4m_{Z'}^2 m_{A_2}^2 s_{2\beta}^2} \right), \tag{54}$$

where $m_{Z'}$ is the mass of the neutral Z' boson [38], $m_{Z'}^2 = 4m_X^2 c_W^2 / (4c_W^2 - 1)$. The values of X'_i are computed from X'_i based on the following formula:

$$X''_i = \frac{A_0}{B_0} \text{ where } A_0 = 4h_W^2 (X'_i - k_1 c_{2\gamma}^2) (X_i'^2 - (k_2 + 1)X'_i + k_2 c_{2\beta}^2), B_0 = 4c_W^2 k_2 (2X'_i - k_1) c_{2\beta}^2 + X'_i [4h_W^2 c_W^2 (2k_1 - 3X'_i) + 2k_1 k_2 - 3(k_1 + k_2)X'_i + 4X_i'^2]. \tag{55}$$

It is noted that X''_i is the correction to the squared Higgs masses. For the correction of the Higgs masses, using Eq. (53), we can get approximate values of the Higgs mass:

$$m_{H_i^0} = \sqrt{X'_i m_X^2 + X''_i \times m_W^2 + \mathcal{O}(\epsilon) \times m_W^2} \simeq m_X \times \sqrt{X'_i} + \frac{X''_i}{\sqrt{X'_i}} \times \frac{m_W^2}{m_X}. \tag{56}$$

If we assume that the scale $m_X \simeq \mathcal{O}(\text{TeV})$, the correction to the Higgs mass at the next leading order is $X''_i / \sqrt{X'_i} \times 2.4$ GeV. This correction is too small compared with the heavy Higgs mass of the TeV scale. So, in our calculation, this correction can be ignored. For more details, the analytic formulas of neutral Higgs masses can be found in Appendix A.

For illustration of our results, all analytic formulas of the neutral Higgs masses can be compared with the numerical investigation shown in Fig. 1. In this figure, we use Mathematica 7.0 directly to find the eigenvalues of the squared mass matrix (33). It is easy to see that the four blue curves represent four heavy Higgs masses, while the lightest Higgs

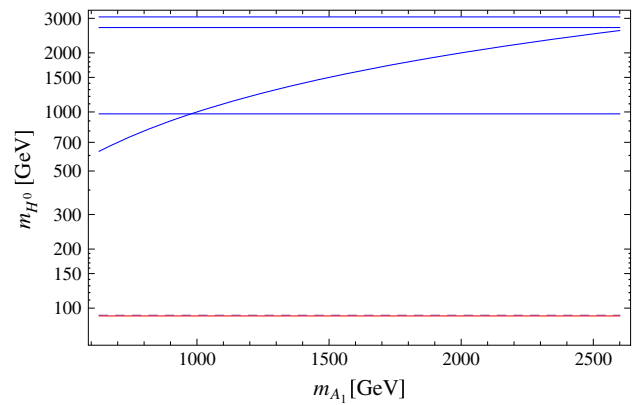


Fig. 1 Plots of $m_{H_j^0}$ ($j = 1, 2, \dots, 5$) as functions of m_{A_1} . The parameters are fixed as $m_X = 2.5$ TeV, $m_{A_2} = 1.0$ TeV, $\frac{u^2 + u'^2}{v^2 + v'^2} = 10^{-4}$, and $m_W = 80.4$ GeV, $t_\gamma = 50$, $t_\beta = 10$. The red line represents the mass of the lightest neutral Higgs. The dashed line fixes the values of $m_Z \simeq 92.0$ GeV

has a mass $m_{H_1^0} \simeq m_Z$ when $t_\gamma \gg 1$. All of these masses are consistent with those shown by our analytic results. This will be helpful to estimate the mass eigenstates of these Higgses in Appendix A.

In conclusion, the SUSYE331 model has five physical CP-even neutral Higgses, including one light Higgs and four other heavy Higgses. The light Higgs can be identified to the Standard Model-like Higgs. One of the heavy Higgses has exactly a mass $m_{H_5^0}$ at the tree level which obeys $m_{H_5^0}^2 = m_{A_2}^2 + m_X^2$. The squared masses of the three other Higgses can be approximately computed up to $\mathcal{O}(\epsilon) \times m_W^2$. The above analysis makes some interesting properties of the SUSYE331 clear. Although the model has four Higgs multiplets, they separate into two pairs having different absolute $U(1)_X$ charges. Two Higgses in each pair have opposite signs in order to cancel the gauge anomaly. The appearance of the Higgses in pairs makes the SUSYE331 have many similar properties to the MSSM. In particular, while ρ and ρ' couple with all leptons and quarks, χ and χ' do not couple with the leptons. So ρ and ρ' play the same role as Higgses in the MSSM. Furthermore, if the CP-odd neutral Higgs is very heavy, the light CP-even neutral Higgs mass at the tree level has exactly the form as in (51) where t_γ in the SUSYE331 plays the same role as t_β in the MSSM. This value is smaller than the mass of the Z boson, $m_Z \simeq 92$ GeV. Compared with the 125.5 GeV value of the Higgs mass discovered recently in the LHC, the MSSM needs large values of $|c_{2\beta}|$. t_β should also be large, corresponding to large corrections from the squark loops for the Higgs mass in order to get a consistent light Higgs mass. The case of SUSYE331 is a bit different. Apart from t_γ there appears a new parameter t_β defined as the ratio of w and w' , which are the two VEVs of χ and χ' . One can see that the light Higgs state is a mixing of all

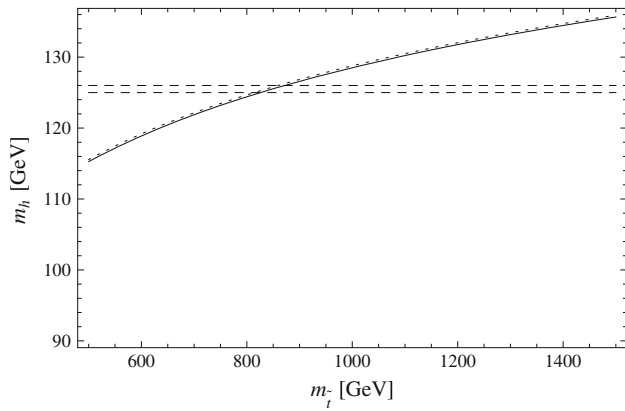


Fig. 2 The mass of the lightest CP-even neutral Higgs including one-loop correction of the top and stop quark. The *black (dotted)* curves present the mass in the SUSYE331 (MSSM) as a function of stop quark. Two *dashed lines* correspond to 125 and 126 GeV. In the case of the SUSYE331 $m_X = 2.0$ TeV is chosen

neutral components of the four Higgs multiplets. As a result, corrections to this Higgs mass will come from squark loops related with both t_γ and t_β .

The mass of the lightest Higgs will easily and naturally reach the value of a recent experimental result if loop corrections are included. This can be realized through the well-known results calculated for the MSSM [12, 59–61], where the largest one-loop corrections to m_h^2 arise from the top quark and the stop scalar. In the SUSYE331 model, choosing a simplifying case based on [59] we can show that the lightest Higgs mass can get a contribution of a one-loop correction similar to those of the MSSM. The details are presented in Appendix C, and Fig. 2 presents the mass of the lightest Higgs according to (98). In a more accurate calculation, the mixing between left and right stops should be included; then the case will be the same as that called the decoupling limit, indicated in [12] (section 7). Apart from this, we believe that one-loop corrections from the very heavy exotic quarks and their superpartners may also increase the mass of this lightest neutral Higgs. This topic is out of the scope of this work. The simple estimation in this work is as an illustration enough to show that the CP-even neutral Higgs spectrum of the SUSYE331 is consistent with present experimental results. Because t_γ is larger than 1 ($\frac{\pi}{4} < \gamma < \frac{\pi}{2}$) we get the constraint $c_{2\gamma} < 0$.

One more comment needs to be added here. At this scale of the soft parameters, light Higgs $m_{H_1^0}^2$ in Eq. (51) and a heavy Higgs in Eq. (52) contain many similar properties to those in the MSSM, while other Higgses are characterized for $SU(3)_L$ scale. So we can use many known properties of the MSSM to study these like-MSSM Higgses. Also, the CP-odd neutral Higgs H_{A_1} in (31) carries properties of that in the MSSM. As we will show in the next section, the Higgs sector in the SUSYE331 is separated into two parts. The first part is closely related to MSSMs while the second part is related to the $SU(3)_L \times U(1)_X$ properties.

4.2.2 Charged Higgs

If all soft parameters live on the $SU(3)_L$ scale, the second formula given in (25) shows that the values of $c_{2\beta}$ should not be too small. Applying this constraint to Eq. (45), one can prove that all solutions of (43) correspond to very large values of the charged Higgs masses. Similar to the case of the neutral Higgs, if we denote $X_i = X'_i + X''_i \times \epsilon$ ($i = 1, 2, 3$), then

$$m_{H_i^\pm}^2 = X_i \times m_X^2 = X'_i \times m_X^2 + X''_i \times m_W^2 + \mathcal{O}(\epsilon) \times m_W^2, \tag{57}$$

where the main contributions to the three charged Higgs masses are

$$\begin{aligned} m_{H_1^\pm}^2 &\simeq X'_1 \times m_X^2 = m_X^2 + m_{A_2}^2, \\ m_{H_{2,3}^\pm}^2 &\simeq X'_{2,3} \times m_X^2 = \frac{1}{2} \\ &\times \left(m_{A_1}^2 \mp \sqrt{\left(m_{A_1}^2 - 2m_X^2 c_{2\beta} c_{2\gamma} \right)^2 + 4m_X^4 c_{2\beta}^2 s_{2\gamma}^2} \right), \end{aligned} \tag{58}$$

and $X''_i \equiv a_x/b_x$ depends on X'_i according to the following formula:

$$\begin{aligned} a_x &= -c_{2\beta} c_{2\gamma} \left[1 + (k_1 + 1)(k_2 + 1) + (k_2 + 2)X'_i \right] \\ &\quad + c_{2\gamma}^2 k_1 + c_{2\beta}^2 (1 + k_2) + k_2 X'_i - X_i'^2, \\ b_x &= c_{2\beta}^2 - c_{2\beta} c_{2\gamma} k_1 - k_1(k_1 + k_2) \\ &\quad + 2(1 + k_1 + k_2)X'_i - 3X_i'^2. \end{aligned} \tag{60}$$

We need to emphasize that the masses of the Higgses in (59) must be positive. This corresponds to the condition

$$c_{2\beta} (c_{2\beta} - k_1 c_{2\gamma}) < 0. \tag{61}$$

If so then $k_1 c_{2\gamma} < c_{2\beta} < 0$ because $c_{2\gamma} < 0$. From this we have $\pi/4 < \beta < \pi/2$ and $t_\beta > 1$.

There is another way to deduce an exact constraint, which is stricter than the constraint given in Eq. (61). By applying the Viet theorem to Eq. (45) with three charged Higgs masses X_1, X_2 , and X_3 , we have $X_1 X_2 X_3 = -(1 + \epsilon) [c_{2\beta} - c_{2\gamma}(\epsilon + k_1)] \times [c_{2\beta}(1 + k_2) - c_{2\gamma}\epsilon] > 0$. In the case of $\epsilon \ll 1$ it leads to the consequence that $(c_{2\beta} - c_{2\gamma} k_1) c_{2\beta} (1 + k_2) < 0$, the same result as shown in Eq. (61). Combining with the condition of $c_{2\gamma} < 0$, we get an exact condition for the positivity of all charged Higgs masses: $(k_1 + \epsilon) c_{2\gamma} < c_{2\beta} < \frac{c_{2\gamma} \epsilon}{1 + k_2} < 0$, which implies that

$$\frac{(m_{A_1}^2 + m_W^2) c_{2\gamma}}{m_X^2} < c_{2\beta} < \frac{c_{2\gamma} m_W^2}{m_X^2 + m_{A_2}^2} < 0. \tag{62}$$

If this condition is satisfied, then all charged Higgs masses in the SUSYE331 are of the order of the $SU(3)_L$ scale. Of

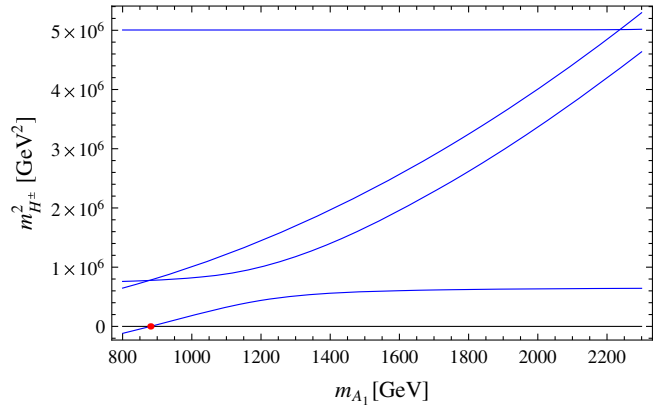
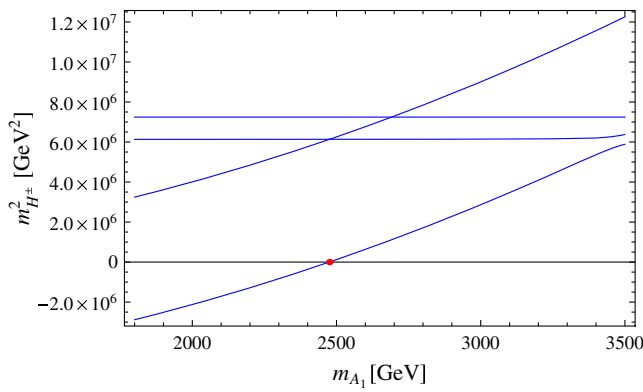


Fig. 3 Plots of $m_{H_i^\pm}^2$ as functions of m_{A_1} . The parameters are fixed as $m_X = 2.5$ TeV (left panel) and $m_X = 2.0$ TeV (right panel), $m_{A_2} = 1.0$ TeV, $\frac{u^2 + u'^2}{v^2 + v'^2} = 10^{-4}$, and $m_W = 80.4$ GeV. The left panel corresponds to large values of t_γ and t_β : $t_\gamma = 50.$, $t_\beta = 10$. The right

panel corresponds to smaller values of t_γ and t_β : $t_\gamma = 5.0$, $t_\beta = 1.2$. The red points imply the values of $m_{A_1}^2 = \frac{m_X^2 c_{2\beta}}{c_{2\gamma}} - m_W^2$ giving the squared mass of the lightest charged Higgs $m_{H_2^\pm}^2 \simeq 0$

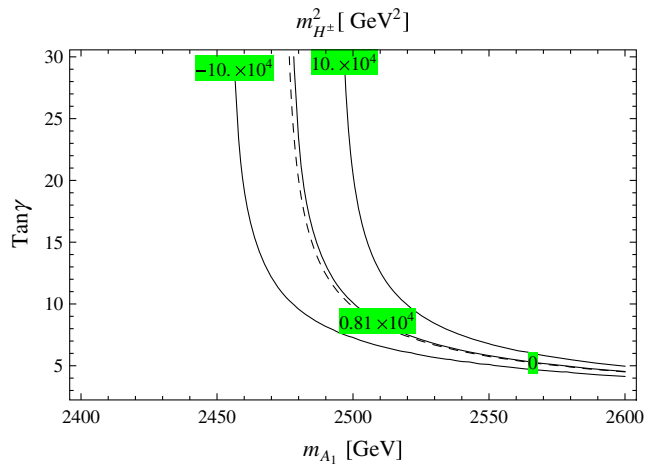
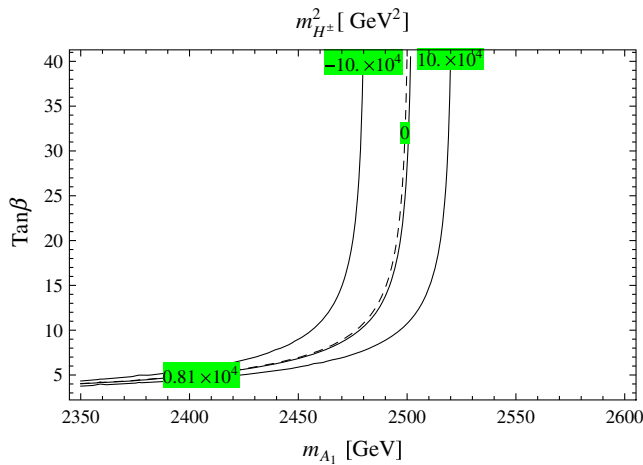


Fig. 4 Contours of the lightest values of $m_{H_i^\pm}^2$ as functions of two variables: (m_{A_1}, t_β) (left panel) or (m_{A_1}, t_γ) (right panel). The parameters are fixed as $m_X = 2.5$ TeV, $m_{A_2} = 1.0$ TeV, $\frac{u^2 + u'^2}{v^2 + v'^2} = 10^{-4}$, and

$m_W^2 = 80.2$. In addition $t_\gamma = 30$ for the left panel and $t_\beta = 10$ in the right panel. The dashed line corresponds to $m_{H_2^\pm}^2 = 0$

course, on this scale, there is no massless charged Higgs in this model and all of these masses are much larger than the current experimental bound at LEP [56].

Finally, as an illustration for our qualitative estimations we will numerically investigate some cases of charged Higgs masses. The results are shown in Figs. 3 and 4. The left panel of Fig. 3 shows the case of large t_γ and t_β where we can fix $c_{2\gamma} \simeq c_{2\beta} = -1$. Inserting these values into (59) we have two values $m_{H^\pm}^2 = \{m_X^2, m_{A_1}^2 - m_X^2\}$. This means that in order to cancel the tachyon Higgs m_{A_1} must be larger than m_X . A strict constraint of m_{A_1} comes from (62): $m_{A_1}^2 > \left| \frac{c_{2\beta}}{c_{2\gamma}} \right| m_X^2 - m_W^2$. This limit value of m_{A_1} is represented by the red points in Fig. 3.

It is easy to see that the two constant lines in the left panel represent two values $m_{H^\pm}^2 = \{m_X^2 + m_{A_2}^2, m_X^2\}$, while the two

other curves show values of $m_{H^\pm}^2 = \{m_{A_1}^2 + m_W^2, m_{A_1}^2 - m_X^2\}$. These two curves are parallel because they are different from each other at constant values $m_X^2 + m_W^2$. This property does not occur in the case of small t_γ , as shown in the right panel of Fig. 3. In all cases, there always exists a lower constraint of m_{A_1} to cancel the tachyon charged Higgs. This value lies at the $SU(3)_L$ scale unless $|c_{2\beta}|$ (t_β) is small, as we illustrate in the left panel of Fig. 4. This also shows the consequence that the SUSYE331 still contains a light charged Higgs if the value of $m_{A_1}^2$ is very close to the values of $\left| \frac{c_{2\beta}}{c_{2\gamma}} \right| m_X^2 - m_W^2$.

There is an interesting consistence of the model that can be seen in Fig. 4. It shows the contours of the lightest mass of the charged Higgs $m_{H^\pm}^2$ as functions of $m_{A_1}^2$ and t_β (t_γ). The allowed regions correspond to the condition $m_{H^\pm}^2 > 90^2$ [GeV] at tree level. As we have discussed, the model

requires a large t_γ to get the consistent lightest neutral Higgs. Fortunately, the allowed region on the right panel favors both large t_γ and m_{A_1} . The small values of t_γ require very large values of m_{A_1} . On the other hand, the allowed region with large m_{A_1} in the left panel also supports large values of t_β . We can see that in the limit of large t_γ (t_β) the lightest charged Higgs mass almost does not depend on the values of t_γ (t_β), while it is very sensitive to the variance of m_{A_1} .

5 MSSM Higgses vs. SUSYE331 Higgses

To compare more precisely the properties of the MSSM Higgs spectrum with some Higgses in the model under consideration we will investigate the couplings of the Higgs particles. In this part, we concentrate on the couplings of the Higgses in the SUSYE331.

Let us briefly review the Higgs spectrum in the MSSM. In this model, in order to provide a mass for up and down fermions as well as to cancel the anomaly, two doublet Higgses, H_u, H_d , are introduced. After the symmetry breaking $SU(2)_L \times U(1)_Y \rightarrow U(1)_Q$, the gauge bosons W^\pm, Z become massive particles and the physical Higgs spectrum contains two CP-even neutral, H, h , one odd-CP neutral, A , and two singly charged Higgses, H^\pm .

In the SUSYE331 the electroweak symmetry is broken by VEVs: u, u', v, v' , where u, u' are the VEVs of the first components of χ, χ' and the residual values are the VEVs of ρ, ρ' . Because the u, u' carry lepton number, they break the lepton number symmetry. Hence they must be small and we can ignore them when we estimate the effect of electroweak breaking. It means that the main contributions to the mass of the SM particles are obtained by the VEVs of ρ, ρ' . In other words, these two Higgses have the same roles as the two Higgs doublets H_u and H_d in the MSSM. Therefore, to find the similarity between the Higgs spectrum in the MSSM and the SUSYE331, we will concentrate on studying the five particular Higgses of the SUSYE331, H_1^0, H_2^0, H_{A_1} , and H_4^\pm , where all of them are related with the ρ, ρ' , and B_ρ -term.

Let us consider the couplings of H_1^0, H_2^0, H_{A_1} , and H_4^\pm with the SM fermions and gauge bosons. In the limit of large t_γ and $u, u' = 0$, the soft as well as $SU(3)_L$ parameters are assumed to be much larger than the $SU(2)_L$ breaking scale, in the sense that $m_{A_{1,2}}^2 \gg m_Z^2$. The physical states H_1^0, H_2^0, H_{A_1} , and H_4^\pm have the following form:

$$\begin{pmatrix} H_{A_4} \\ H_{A_1} \end{pmatrix} = \begin{pmatrix} c_\gamma & -s_\gamma \\ s_\gamma & c_\gamma \end{pmatrix} \begin{pmatrix} A_6 \\ A_5 \end{pmatrix}, \quad \begin{pmatrix} H_5^\pm \\ H_4^\pm \end{pmatrix} = \begin{pmatrix} c_\gamma & s_\gamma \\ -s_\gamma & c_\gamma \end{pmatrix} \begin{pmatrix} \rho_{1^\pm} \\ \rho_{1^\pm} \end{pmatrix} \quad (63)$$

Table 1 Higgs-gauge boson couplings

Vertex	Factor	Vertex	Factor
$H_1^0 W^+ W^-$	$igm_W s_{2\gamma}$	$H_2^0 W^+ W^-$	$-igm_W c_{2\gamma}$
$H_{A_1} H_2^0 Z_\mu$	$\frac{-g}{2c_W} (p + p')^\mu$	$H_4^+ H_4^- Z_\mu$	$\frac{-gc_{2W}}{2c_W} (p + p')^\mu$
$H_4^+ H_4^- A_\mu$	$-ie(p + p')^\mu$	$H_2^0 H_4^\pm W_\mu^\mp$	$\pm \frac{ig}{2} (p + p')^\mu$
$H_{A_1} H_4^\pm W_\mu^\mp$	$\frac{g}{2} (p + p')^\mu$		
$H_1^0 H_1^0 W_\mu^+ W_\nu^-$	$\frac{ig^2}{2} g^{\mu\nu}$	$H_2^0 H_2^0 W_\mu^+ W_\nu^-$	$\frac{ig^2}{2} g^{\mu\nu}$
$H_{A_1} H_{A_1} W_\mu^+ W_\nu^-$	$\frac{ig^2}{2} g^{\mu\nu}$	$H_1^0 H_1^0 Z_\mu Z_\nu$	$\frac{ig^2}{2c_W^2} g^{\mu\nu}$
$H_2^0 H_2^0 Z_\mu Z_\nu$	$\frac{ig^2}{2c_W^2} g^{\mu\nu}$	$H_{A_1} H_{A_1} Z_\mu Z_\nu$	$\frac{ig^2}{2c_W^2} g^{\mu\nu}$
$H_4^+ H_4^- Z_\mu W_\nu$	$\frac{ig^2 c_{2W}}{2c_W^2} g^{\mu\nu}$	$H_4^+ H_4^- Z_\mu A_\nu$	$\frac{ieg_{2W}}{c_W} g^{\mu\nu}$
$H_2^0 H_4^\pm W_\mu^\mp Z_\nu$	$\frac{ig^2 s_W}{2c_W} g^{\mu\nu}$	$H_{A_1} H_4^\pm W_\mu^\mp Z_\nu$	$\mp \frac{g^2 s_W}{2c_W} g^{\mu\nu}$
$H_2^0 H_4^\pm W_\mu^\mp A_\nu$	$\frac{-ieg}{2} g^{\mu\nu}$	$H_{A_1} H_4^\pm W_\mu^\mp Z_\nu$	$\pm \frac{eg}{2} g^{\mu\nu}$

and

$$\begin{pmatrix} H_2^0 \\ H_1^0 \end{pmatrix} = \begin{pmatrix} s_\gamma & -c_\gamma \\ c_\gamma & s_\gamma \end{pmatrix} \begin{pmatrix} S_6 \\ S_5 \end{pmatrix}. \quad (64)$$

The non-zero masses of these particles are given by

$$m_{A_1}^2 = \frac{b_\rho}{s_{2\gamma}}, \quad m_{H_4^\pm} = m_{A_1}^2 + m_W^2, \quad m_{H_2^0}^2 = m_{A_1}^2 + O(m_W^2), \quad m_{H_1^0} = m_Z |c_{2\gamma}|. \quad (65)$$

The other particles are massless and identified with the Goldstone bosons. Based on the physical states, we can find the couplings of the Higgses H_1^0, H_2^0, H_{A_1} , and H_4^\pm with the SM particles. The couplings of them with the SM gauge bosons are listed in Table 1.

From Eq. (63), it can be realized that the equivalent role of the two parameters β and γ in the two models.¹ The formula (64) shows that the case we are working in, the SUSYE331, is similar to that of the decoupling regime in the MSSM where $\alpha \rightarrow \beta - \pi/2$. In this limit, the couplings of the considered Higgses with the SM gauge bosons given in Table 1 are consistent with those of the Higgses in the MSSM shown in [11].

The couplings of the considered Higgses in the SUSYE331 with the fermions are listed in Table 2. The results show that the couplings among these Higgses are the same as those in the MSSM. Finally, we will investigate the LFV of the Higgses decaying to leptons in the SUSYE331 model in the following section.

¹ In fact, the signs of some elements in the transformation matrices in the SUSYE331 may be different from the MSSM; see for example [11, 12]. This also happens in the two definitions of [11] and [12] in the MSSM. The reason is the difference in signs of the two definitions: (i) the B/μ term in the Lagrangian; (ii) the mass eigenstates of the Higgses. These mathematical differences do not affect the final physical results.

6 Lepton flavor violating decay of Higgs to muon and tauon

The LFV decays of neutral Higgses in the SUSYE331 were studied in [44] based on the parametrization of slepton mixing in [53, 54] and the model constructed in [33] without the presence of B/μ -type terms. In this work, we use the revised model where these B/μ -type terms are added to guarantee the stability of the vacuum of the model. As a result, the mass eigenstates of all Higgses in general are different from those in [33, 38]. The Higgs sector becomes more complicated and it is not easy to represent analytically the masses as well as the mass eigenstates of the real neutral Higgses in terms of the original parameters. In the limit of large t_γ we can use the LFV Lagrangian established in [44],

$$-\mathcal{L}_{H\mu\tau}^{FV} \simeq Y_\tau(\Delta_R^\rho \mu^c \tau + \Delta_L^\rho \tau^c \mu)(\rho^{0*} - t_\gamma \rho^0) + \text{H.c.}, \tag{66}$$

which is not affected by the diagonalization of the neutral Higgs mass matrix. As noted in [44] we recall that ρ^0, ρ'^0 are neutral Higgses which generate masses for the lepton after spontaneous breaking; Δ_R^ρ and Δ_L^ρ are one-loop contributions to the LFV Lagrangian. We emphasize that the presence of the B/μ -type terms in the model under consideration does not modify the analytic formulas of the effective couplings $\Delta_R^\rho, \Delta_L^\rho$ given in [44].

Unlike the previous version, one of the many features of the SUSYE331 in this work is the presence of massive pseudo-scalar Higgses. Especially, the formulas in (31) and (32) imply that only H_{A_1} can decay to leptons. Furthermore, it is easy to prove that

$$\text{Im}(\rho^{0*} - t_\gamma \rho^0) = \frac{-i}{\sqrt{2}c_\gamma} \times H_{A_1}. \tag{67}$$

For the real neutral Higgses, we cannot find the exact mass eigenvalues or mass eigenstates of all these Higgses. The approximate estimation presented above just only helps to understand some qualitative aspects of them and also shows that the Higgs sector of the model is consistent with recent results of experiments. A detailed analysis to estimate the mass eigenstates of the neutral Higgses is presented in Appendix A. In this work, the real parts of ρ^0 and ρ'^0 can be estimated as $S_5 = H_1^0 s_\gamma - H_2^0 c_\gamma$ and $S_6 = H_1^0 c_\gamma + H_2^0 s_\gamma$. The effective Lagrangian for the LFV decays of the neutral Higgses are

$$\mathcal{L}_{H^0\mu\tau}^{FV} = \frac{Y_\tau}{\sqrt{2}c_\gamma} \times (\Delta_R^\rho \mu^c \tau + \Delta_L^\rho \tau^c \mu) (H_2^0 + iH_{A_1}) + \text{H.c.} \tag{68}$$

This Lagrangian has the same form as that of the MSSM in the limit of the CP-odd neutral Higgs having a heavy mass. The lepton flavor conserving (LFC) part of the Lagrangian

Table 2 Coupling of the neutral Higgs bosons to fermions.

Particles	Up fermion	Down fermion	Exotic up quark	Exotic down quark
SM Higgs	1	1	0	0
H_1^0	1	1	$\mathcal{O}\left(\frac{m_W}{m_X}\right)$	$\mathcal{O}\left(\frac{m_W}{m_X}\right)$
H_2^0	$-\frac{1}{t_\gamma}$	t_γ	$\mathcal{O}\left(\frac{m_W}{m_X}\right)$	$\mathcal{O}\left(\frac{m_W}{m_X}\right)$
H_{A_1}	$\frac{i}{t_\gamma}$	it_γ	$\mathcal{O}\left(\frac{m_W}{m_X}\right)$	$\mathcal{O}\left(\frac{m_W}{m_X}\right)$

at tree level can be deduced from [38]. Using the notation in [44], this part has the form

$$\mathcal{L}_{H^0\mu\tau}^{FC} = -(Y_\mu \mu^c \mu + Y_\tau \tau^c \tau) \rho'^0 + \text{H.c.} \tag{69}$$

We note that the light Higgs H_1^0 has very suppressed LFV effective couplings in this case. At the tree level, the charged leptons only couple to the Higgs ρ' and $\sqrt{2}\rho'^0 = (H_1^0 c_\gamma + H_2^0 s_\gamma) + i(c_\gamma H_{A_1} + s_\gamma H_{A_1})$. The LFV branching ratio of the neutral Higgses H^0 can be calculated through the branching ratios $\text{BR}(H^0 \rightarrow \tau^+ \tau^-)$, namely,

$$\begin{aligned} \text{BR}(H^0 \rightarrow \tau^+ \mu^-) &= \text{BR}(H^0 \rightarrow \tau^- \mu^+) \\ &= \frac{1}{c_\gamma^2 s_\gamma^2} \times (|\Delta_L^\rho|^2 + |\Delta_R^\rho|^2) \text{BR}(H^0 \rightarrow \tau^+ \tau^-) \\ &= \frac{(t_\gamma^2 + 1)^2}{t_\gamma^2} (|\Delta_L^\rho|^2 + |\Delta_R^\rho|^2) \text{BR}(H^0 \rightarrow \tau^+ \tau^-), \end{aligned} \tag{70}$$

where $H^0 = H_{A_1}, H_2^0$.

In the case of $t_\gamma \gg 1$, obtaining the Lagrangian (66), we obtain a result that is the same as that indicated in the MSSM for heavy neutral Higgses. We have

$$\begin{aligned} \text{BR}(H^0 \rightarrow \tau^+ \mu^-) \\ &= t_\gamma^2 (|\Delta_L^\rho|^2 + |\Delta_R^\rho|^2) \text{BR}(H^0 \rightarrow \tau^+ \tau^-). \end{aligned} \tag{71}$$

The neutral Higgs–fermion–fermion couplings in our work are different from [38]. They are listed in Table 2. We just consider H_1^0, H_2^0 , and H_{A_1} .

Following this table, the couplings of the light neutral Higgs to fermions are the same as those in the SM. While the CP-even and CP-odd neutral Higgses are different, they strongly couple with the down fermion with large t_γ . Furthermore, these two Higgses do weakly couple with exotic quarks of the model. They carry the properties of the neutral Higgses in the MSSM and the ν MSSM shown in [55]. As mentioned in [55] and as detailed for example in [11], the coupling of these Higgses to W^+W^- and Z^0Z^0 are very suppressed if their masses are very heavy. For the SUSYE331, a similar case occurs for the vertex type of H^0VV where V denotes any gauge bosons Z, Z', W^\pm, Y^\pm or X^0 . The couplings are deduced from the following term:

$$\sum_{H^0} (D^\mu H^0)^\dagger (D_\mu H^0) \rightarrow i g_V^2 V_\mu V^{\mu\dagger} \sum_{H^0} (\langle H^0 \rangle H^{0\dagger}) + \text{H.c.}, \tag{72}$$

where g_V is defined from the covariant derivative $D_\mu = \partial_\mu + i \sum_V g_V V_\mu$. As shown in Appendix A, the leading contributions to the Higgses H_1^0 , H_2^0 , and H_{A_1} come only from the two Higgses ρ^0 and ρ'^0 , and the couplings with all gauge bosons are proportional to $g_V^2 m_W/g$. The next leading contributions are related with χ^0 and χ'^0 by a factor of $\sqrt{\epsilon} = m_W/m_X$. Because these two Higgses contain real components having VEVs $w, w' \sim \frac{2m_X}{g}$, the value of the coupling of $H_1^0 VV$ is proportional to $g_V^2 m_W/g^2$. In contrast, the coupling of $H_2^0 VV$ is still suppressed because of a factor $s_{2\gamma} < \frac{1}{t_\gamma}$. So in the case of our work the leading and next to leading contributions to the HVV couplings of $\{H_1^0, H_2^0, H_{A_1}\}$ to the gauge bosons are $g_V^2 m_W/g \times \{\sin \gamma, 0, 0\}$ and $g_V^2 m_W/g^2 \times \{\mathcal{O}(1), s_{2\gamma}, 0\}$, respectively. It means that the coupling of $H_2^0 VV$ is very suppressed and H_{A_1} does not couple to the gauge boson pairs. This is the same case as in the MSSM and the ν MSSM. Therefore, H_2^0 and H_{A_1} decay mainly to down fermions such as $b\bar{b}$ and $\tau\bar{\tau}$ [58]. This will lead to large LFV branching ratios of neutral heavy Higgses which can be detected by the LHC. A detailed investigation can be found in [55] for example.

7 Conclusion

In this work we have concentrated on the Higgs sector of the SUSYE331 model. Unlike the previous work [33,38], by adding two B/μ -type terms in the soft term of the SUSYE331 model we have shown that these terms not only guarantee the vacuum stability but also cancel all of the tachyon Higgses appearing in the previous version. Especially, from the conditions of the minimum of the scalar potential we indicated that the soft parameters and the B/μ -terms in this model naturally favor the order of $SU(3)_L$. This is the property of the SUSYE331 model which does not occur in supersymmetric versions of the $SU(2)_L \times U(1)_Y$. Because of this, all of three CP-odd neutral Higgses will get masses at least around 1 TeV. They are denoted $m_{A_1}^2, m_{A_2}^2$, and $m_{A_3}^2 = m_{A_2}^2 + m_X^2$. These four Higgs states are found exactly according to the original Higgs basis. For the neutral Higgs sector, there are four massive Higgses in which there is one light Higgs with squared mass $m_{H_1^0}^2 \simeq m_Z^2 c_{2\gamma}^2$, the same as in the MSSM. Among the three other CP-even neutral heavy Higgses, there is one exact value, $m_{H_4^0}^2 = m_{A_3}^2$. In the charged Higgs sector, there is also one exact value of the charged Higgs mass, $m_{H_{\pm}}^2 = m_{A_1}^2 + m_W^2$. This formula suggests the similarity of the Higgses H_{\pm}^{\pm} and H_{A_1} to those in the MSSM. In summary, in the limit of large values of the soft param-

eters, B/μ -type terms, t_γ , and t_β , the Higgs spectrum of the SUSYE331 contains all Higgses carrying many properties of the MSSM Higgs spectrum. The remaining ones characterize the SUSYE331 because they almost relate with the $SU(3)_L$ Higgses χ and χ' . Among these Higgses, there maybe exists a charged Higgs tachyon, unless the conditions $\frac{(m_{A_1}^2 + m_W^2)c_{2\gamma}}{m_X^2} < c_{2\beta} < \frac{c_{2\gamma} m_W^2}{m_X^2 + m_{A_2}^2} < 0$ are satisfied. They give two important consequences: i) for $t_\gamma > 1$ ($c_{2\gamma} < 0$) t_β is larger than 1 too, ii) if the value of $m_{A_1}^2$ is very close to the value of $(\frac{c_{2\beta}}{c_{2\gamma}} m_X^2 - m_W^2)$, there will appear a light charged Higgs characteristic for the existence of $SU(3)_L$ itself, which supports the charged Higgs searches at LHC and other colliders.

It is emphasized that the above classification helps us to exploit many known results for the MSSM to estimate the properties of the first class of Higgses in the SUSYE331, although they seem to be only true at the tree level. For completeness it is really necessary to study in detail the effect from loop corrections because new particles will generate new diagrams in higher order calculations. As an illustration, we consider the LFV decays of neutral Higgs bosons to leptons in the SUSYE331. The loop contributions to these decays were indicated in [44]. This result does not depend on the appearance of B/μ -type terms. The calculation in this work shows that the LFV decays of the three neutral Higgses H_{A_1}, H_1^0 , and H_2^0 are consistent with the conclusions for the MSSM neutral Higgses shown in [55]. Here the H_1^0 is the lighter CP-even neutral Higgs. It is normally identified with SM-like Higgs. The two Higgses H_{A_1} and H_2^0 are very heavy Higgses with degenerate masses. Furthermore they decay mainly to down fermions such as $b\bar{b}$ and $\tau\bar{\tau}$, leading to the enhancement of LFV branching ratios up to $\mathcal{O}(10^{-4})$ for the MSSM and the SUSYE331. This is really new and of significance for the heavy neutral Higgses which can be checked by experiments.

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Appendix A: CP-even neutral Higgs squared mass matrix

We list precisely all of the elements of the CP-even neutral Higgs squared mass matrix as follows:

$$m_{S_{11}}^2 = \frac{1}{2} \frac{b_X}{t_\beta} + \frac{g^2}{8} w'^2 + \frac{g^2}{108} (18 + t^2) u^2,$$

$$\begin{aligned}
 m_{S12}^2 &= -\frac{g^2}{8}u'w' + \frac{g^2}{108}(18+t^2)uw, \\
 m_{S13}^2 &= -\frac{1}{2}b_\chi - \frac{g^2}{8}ww' - \frac{g^2}{108}(18+t^2)uu', \\
 m_{S14}^2 &= \frac{g^2}{8}u'w - \frac{g^2}{108}(18+t^2)uw', \\
 m_{S15}^2 &= -\frac{g^2}{108}(9+2t^2)uv, \\
 m_{S16}^2 &= \frac{g^2}{108}(9+2t^2)uv' \\
 m_{S22}^2 &= \frac{b_\chi}{2t_\beta} + \frac{g^2}{8}u'^2 + \frac{g^2}{108}(18+t^2)w^2, \\
 m_{S23}^2 &= \frac{g^2}{8}u'w - \frac{g^2}{108}(18+t^2)u'w, \\
 m_{S24}^2 &= -\frac{b_\chi}{2} - \frac{g^2}{8}uu' - \frac{g^2}{108}(18+t^2)ww', \\
 m_{S25}^2 &= -\frac{g^2}{108}(9+2t^2)wv, \\
 m_{S26}^2 &= \frac{g^2}{108}(9+2t^2)wv', \\
 m_{S33}^2 &= \frac{b_\chi}{2t_\beta} + \frac{g^2}{108}(18+t^2)u'^2 + \frac{g^2}{8}w^2 \\
 m_{S34}^2 &= \frac{g^2}{108}(18+t^2)u'w' - \frac{g^2}{8}uw, \\
 m_{S35}^2 &= \frac{g^2}{108}(9+2t^2)u'v, \\
 m_{S36}^2 &= -\frac{g^2}{108}(9+2t^2)u'v', \\
 m_{S44}^2 &= \frac{b_\chi}{2t_\beta} + \frac{g^2}{108}(18+t^2)w'^2 + \frac{g^2}{8}u^2,
 \end{aligned}$$

$$\begin{aligned}
 m_{S56}^2 &= -\frac{g^2}{54}(9+2t^2)vv', \\
 m_{S66}^2 &= \frac{b_\rho}{2t_\gamma} + \frac{g^2}{54}(9+2t^2)v'^2.
 \end{aligned}$$

To estimate the contributions from the original Higgs basis S_i to the physical Higgs basis we do a rotation of the squared mass matrix (33) with the rotation C represented as follows:

$$C = \begin{pmatrix} s_\beta & 0 & c_\beta & 0 & 0 & 0 \\ -c_\beta & 0 & s_\beta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -c_\gamma & s_\gamma \\ 0 & 0 & 0 & 0 & s_\gamma & c_\gamma \\ 0 & s_\alpha & 0 & c_\alpha & 0 & 0 \\ 0 & c_\alpha & 0 & -s_\alpha & 0 & 0 \end{pmatrix}, \tag{73}$$

where

$$\begin{aligned}
 t_{2\alpha} &= t_{2\beta} \times \frac{m_{A_2}^2 + m_{Z'}^2}{m_{A_2}^2 - m_{Z'}^2}, \quad \text{and} \\
 \frac{s_{2\alpha}}{s_{2\beta}} &= -\frac{m_{A_2}^2 + m_{Z'}^2}{\sqrt{c_{2\beta}^2(m_{A_2}^2 - m_{Z'}^2)^2 + s_{2\beta}^2(m_{A_2}^2 + m_{Z'}^2)^2}} \\
 &\simeq -\frac{m_{H_3}^2 + m_{H_4}^2}{-m_{H_3}^2 + m_{H_4}^2}. \tag{74}
 \end{aligned}$$

Because $s_{2\beta} > 0$ we have $s_{2\alpha} < 0$. The sign of $c_{2\alpha}$ depends on the quantity $m_{A_2}^2 - m_{Z'}^2$. Because of this we have $\pi/2 < \alpha < 3\pi/2$.

After this rotation, we keep only large contributions to the squared mass matrix which are proportional to m_X^2 , $m_W m_X$, and m_W^2 in the non-diagonal elements of the matrix. Then we have

$$\begin{aligned}
 M_{H^0}^2 &= C.M_{6S}^2.C^T \\
 &= m_X^2 \times \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0, & k_2 + 1, & 0 & 0 & 0 & 0 \\ 0 & 0, & k_1 + \frac{s_{2\gamma}^2 \epsilon}{4c_W^2 - 1}, & \frac{2\epsilon \sin 4\gamma}{4c_W^2 - 1}, & -\frac{2\sqrt{\epsilon} s_{2\gamma} \cos(\beta + \alpha)}{4c_W^2 - 1}, & \frac{2\sqrt{\epsilon} s_{2\gamma} \sin(\beta + \alpha)}{4c_W^2 - 1} \\ 0 & 0 & \frac{2\epsilon \sin 4\gamma}{4c_W^2 - 1}, & \frac{4c_{2\gamma}^2 \epsilon}{4c_W^2 - 1}, & -\frac{2\sqrt{\epsilon} c_{2\gamma} \cos(\beta + \alpha)}{4c_W^2 - 1}, & \frac{2\sqrt{\epsilon} c_{2\gamma} \sin(\beta + \alpha)}{4c_W^2 - 1} \\ 0 & 0 & -\frac{2\sqrt{\epsilon} c_{2\gamma} \cos(\beta + \alpha)}{4c_W^2 - 1}, & -\frac{2\sqrt{\epsilon} s_{2\gamma} \cos(\beta + \alpha)}{4c_W^2 - 1}, & M_{55}^2, & 0 \\ 0 & 0 & \frac{2\sqrt{\epsilon} c_{2\gamma} \sin(\beta + \alpha)}{4c_W^2 - 1}, & \frac{2\sqrt{\epsilon} s_{2\gamma} \sin(\beta + \alpha)}{4c_W^2 - 1}, & 0, & M_{66}^2 \end{pmatrix} \tag{75}
 \end{aligned}$$

where

$$\begin{aligned}
 M_{55}^2 \times m_X^2 &= m_{Z'}^2 \cos^2(\beta + \alpha) + m_{A_2}^2 \sin^2(\beta - \alpha), \\
 M_{66}^2 \times m_X^2 &= m_{Z'}^2 \sin^2(\beta + \alpha) + m_{A_2}^2 \cos^2(\beta - \alpha). \tag{76}
 \end{aligned}$$

In this new basis, all non-diagonal elements of the squared mass matrix are of the order $\mathcal{O}(\sqrt{\epsilon})$ or $\mathcal{O}(\epsilon)$. So we can use this basis of the Higgses to represent the mass eigenstates of

the heavy Higgses. In particular, these states are related with the originals by

$$\begin{aligned}
 m_{H_5^0}^2 &= m_{A_2}^2 + m_X^2; & H_5^0 &= -c_\beta S_1 + s_\beta S_3, \\
 m_{H_2^0}^2 &= m_{A_1}^2 + \mathcal{O}(m_W^2); & H_2^0 &= -c_\gamma S_5 + s_\gamma S_6, \\
 m_{H_3^0}^2 &= m_{Z'}^2 \cos^2(\beta + \alpha) \\
 &\quad + m_{A_2}^2 \sin^2(\beta - \alpha) + \mathcal{O}(m_W^2); & H_3^0 &= s_\alpha S_2 + c_\alpha S_4 \\
 m_{H_4^0}^2 &= m_{Z'}^2 \sin^2(\beta + \alpha) \\
 &\quad + m_{A_2}^2 \cos^2(\beta - \alpha) + \mathcal{O}(m_W^2); & H_4^0 &= c_\alpha S_2 - s_\alpha S_4.
 \end{aligned}
 \tag{77}$$

In addition, we have a massless state $H' = s_\beta S_1 + c_\beta S_3$ eaten by the X^0 boson. For the light Higgs we can see from the matrix (75) that the diagonal element $(M_{H^0}^2)_{44} = \frac{4m_W^2 c_{2\gamma}^2}{4c_W^2 - 1} = \frac{4c_W^2}{4c_W^2 - 1} m_Z^2 c_{2\gamma}^2$ is different from the eigenvalue of $m_Z^2 c_{2\gamma}^2$ predicted in (51). This is because of the non-diagonal elements in the matrix (75), which are proportional to $\sqrt{\epsilon}$. They can cause corrections of the order of $\epsilon \times m_X^2 \simeq m_W^2$ to all Higgs masses and affect directly the mass of the light Higgs. For example, we consider the case of large t_γ and t_β . This means that $\gamma, \beta \rightarrow \pi/2$ and $\sin 4\gamma = s_{2\gamma} \rightarrow 0, c_{2\gamma} \rightarrow -1$. Furthermore, because α is defined in (74) and $m_{A_2}^2 < m_X^2$ as chosen in a numerical investigation we obtain $\pi/2 < \alpha < \pi$ and $\alpha \rightarrow \pi/2$. Inserting these values into (76) we have $m_{55}^2 \rightarrow m_{Z'}^2 \cos^2(\beta + \alpha)$, and the largest contributions to the Higgs masses from the non-zero diagonal elements are only $(M_{H^0}^2)_{45} = (M_{H^0}^2)_{54} = 2\sqrt{\epsilon} c_{2\gamma} \cos(\beta + \alpha) / (4c_W^2 - 1)$. We then take a rotation with a tiny angle η defined by

$$\tan 2\eta = -\frac{4\sqrt{\epsilon} \cos 2\gamma \cos(\beta + \alpha)}{(4c_W^2 - 1)M_{55}^2 - 4c_{2\gamma}^2 \epsilon}.$$

The light Higgs mass now is

$$\begin{aligned}
 m_{H_1^0}^2 &\simeq m_X^2 \times \left[\frac{c_{2\gamma}^2 \epsilon}{4c_W^2 - 1} - \frac{1}{m_{55}^2} \right. \\
 &\quad \left. \times \left(\frac{2\sqrt{\epsilon} c_{2\gamma} \cos(\beta + \alpha)}{4c_W^2 - 1} \right)^2 \right] \simeq m_{Z'}^2,
 \end{aligned}$$

as predicted. In this case the mass eigenvalue of the light Higgs has the form $H_1^0 = s_\gamma S_5 + c_\gamma S_6 + \mathcal{O}(\frac{m_W}{m_X}) \times (s_\alpha S_2 + c_\alpha S_4)$.

In general, the dominant contributions to the mass eigenstate of the light Higgs is $H_1^0 = s_\gamma S_5 + c_\gamma S_6$. The next contributions to this eigenstate and other heavy Higgses, H_2^0, H_3^0 , and H_4^0 , are all proportional to a factor of $m_W/m_X \simeq 0.03$. This contribution to H_2^0 is more suppressed because of a factor $s_{2\gamma} \sim \frac{1}{t_\gamma}$. So these contributions can be ignored in many investigations such as the LFV decays of the neutral Higgses.

Appendix B: Charged Higgs squared mass matrix

The non-zero elements of charged Higgs squared mass matrix are listed as follows:

$$\begin{aligned}
 m_{\chi^-\chi^+}^2 &= \frac{4b_\chi}{g^2} + (w^{2'} + u^{2'}) + (v^2 - v'^2), \\
 m_{\chi^-\chi'^+}^2 &= -\frac{4b_\chi}{g^2} - t_\beta (w^{2'} + u^{2'}), \\
 m_{\chi'^-\chi'^+}^2 &= \frac{4b_\chi}{g^2} + (u^2 + w^2) - (v^2 - v'^2), \\
 m_{\rho_1^-\rho_1^+}^2 &= \frac{4b_\rho}{g^2 t_\gamma} + (u^2 - u'^2) + v'^2, \\
 m_{\rho_1^-\rho_2^+}^2 &= uw - u'w', \quad m_{\rho_2^-\rho_2^+}^2 = \frac{4b_\rho}{g^2 t_\gamma} \\
 &\quad + v'^2 + (w^2 - w'^2), \\
 m_{\rho_1'^-\rho_1'^+}^2 &= \frac{4b_\rho t_\gamma}{g^2} + v^2 + (u'^2 - u^2), \quad m_{\rho_1'^-\rho_2'^+}^2 \\
 &= -m_{\rho_1^-\rho_2^+}^2, \\
 m_{\rho_2'^-\rho_2'^+}^2 &= \frac{4b_\rho t_\gamma}{g^2} + v^2 + (w^{2'} - w^2).
 \end{aligned}
 \tag{78}$$

In the limit of $u, u' \rightarrow 0$, the matrix has a simpler form, and after taking a rotation this matrix by a transformation C_{H^\pm} with

$$C_{H^\pm} = \begin{pmatrix} -\frac{s_\beta m_X}{\sqrt{m_X^2 + m_W^2}} & -\frac{c_\beta m_X}{\sqrt{m_X^2 + m_W^2}} & 0 & \frac{s_\gamma m_W}{\sqrt{m_X^2 + m_W^2}} & 0 & \frac{c_\gamma m_W}{\sqrt{m_X^2 + m_W^2}} \\ 0 & 0 & s_\gamma & 0 & c_\gamma & 0 \\ 0 & 0 & c_\gamma & 0 & -s_\gamma & 0 \\ 0 & 0 & 0 & c_\gamma & 0 & -s_\gamma \\ \frac{c_\beta}{\sqrt{m_X^2 + m_W^2}} & 0 & -s_\beta & 0 & 0 & 0 \\ \frac{s_\beta m_X}{\sqrt{m_X^2 + m_W^2}} & \frac{c_\beta m_X}{\sqrt{m_X^2 + m_W^2}} & 0 & \frac{s_\gamma m_W}{\sqrt{m_X^2 + m_W^2}} & 0 & \frac{c_\gamma m_W}{\sqrt{m_X^2 + m_W^2}} \end{pmatrix},
 \tag{79}$$

we get

$$C_{H^\pm} M_{6\text{charged}}^2 C_{H^\pm}^\dagger = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_{A_1}^2 + m_W^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{A_1}^2 + m_W^2 - c_{2\beta} c_{2\gamma} m_X^2 & s_{2\gamma} s_{2\beta} m_W m_X & -s_{2\gamma} c_{2\beta} m_X m_U \\ 0 & 0 & 0 & s_{2\gamma} s_{2\beta} m_W m_X & m_{A_2}^2 - c_{2\beta} c_{2\gamma} m_W^2 + m_X^2 & -c_{2\gamma} s_{2\beta} m_W m_U \\ 0 & 0 & 0 & -s_{2\gamma} c_{2\beta} m_X m_U & -c_{2\gamma} s_{2\beta} m_W m_U & c_{2\beta} c_{2\gamma} (m_W^2 + m_X^2) \end{pmatrix}. \tag{80}$$

From this and $(H_6^+, H_5^+, H_4^+, H_3^+, H_2^+, H_1^+)^T = C_{H^\pm} (\chi^+, \chi'^+, \rho_1^+, \rho_2^+, \rho_1'^+, \rho_2'^+)^T$ we easily see that there are two Goldstone bosons H_5^\pm , which are eaten by W^\pm and the two massive Higgses, H_4^\pm , with masses arising mainly from ρ and ρ' .

Appendix C: Corrections to lightest neutral Higgs mass

To illustrate the contribution from the loop corrections to the lightest neutral Higgs mass, we use the same simplest estimation as done in the MSSM [59], namely as follows.

- We choose $\beta \rightarrow \frac{\pi}{2}$, $\gamma \rightarrow \frac{\pi}{2}$ and $u \rightarrow 0$. This limit leads to $w' \rightarrow 0$, $w \rightarrow W = 2m_X/g$, $v' \rightarrow 0$, and $v \rightarrow V = 2m_W/g$. This choice is consistent with $b_\rho \rightarrow 0$, $b_\chi \rightarrow 0$, $\frac{1}{4}\mu_\rho + m_\rho' \rightarrow \infty$, and $\frac{1}{4}\mu_\chi + m_\chi' \rightarrow \infty$. Hence the antitriplets χ', ρ' can be integrated out when we consider the symmetry breaking of $SU(3)_L \times U(1)_X$. For convenience we define the new parameters such as

$$m_1^2 = \frac{1}{4}\mu_\chi^2 + m_\chi^2, \quad m_2^2 = \frac{1}{4}\mu_\rho^2 + m_\rho^2, \tag{81}$$

$$\frac{S_5 + v}{\sqrt{2}} \rightarrow \phi_2, \quad \text{and} \quad \frac{S_2 + W}{\sqrt{2}} \rightarrow \phi_1. \tag{82}$$

With these conventions, the superpotential at the tree level can be written as

$$V_{\text{SUSYE331}} \rightarrow V_0 = m_2^2 \phi_2^2 + m_1^2 \phi_1^2 + \frac{9g^2 + 2g'^2}{54} [k\phi_1^4 - \phi_1^2 \phi_2^2 + \phi_2^4], \tag{83}$$

where $t^2 \equiv (g'/g)^2 = 18s_W^2/(3 - 4s_W^2)$ and $k = (18 + t^2)/[2(9 + 2t^2)] = c_W^2$. The tree level minimization gives

$$\begin{aligned} \left. \frac{\partial V_0}{\partial \phi_1} \right|_{\phi_1=W/\sqrt{2}, \phi_2=V/\sqrt{2}} &= 0 \rightarrow m_1^2 \\ &= -\frac{9 + 2t^2}{27} (2km_X^2 - m_W^2), \\ \left. \frac{\partial V_0}{\partial \phi_2} \right|_{\phi_1=W/\sqrt{2}, \phi_2=V/\sqrt{2}} &= 0 \rightarrow m_2^2 \\ &= \frac{9 + 2t^2}{27} (m_X^2 - 2m_W^2). \end{aligned} \tag{84}$$

The mass Lagrangian at tree level related with the term $(\frac{\partial^2 V_0}{\partial \phi_i \partial \phi_i})$ can be written as

$$\mathcal{L}_{\text{mass}} = -\frac{4(9 + 2t^2)m_X^2}{27} (\phi_1, \phi_2) \begin{pmatrix} 2k & -\epsilon' \\ -\epsilon' & 2\epsilon'^2 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \tag{85}$$

with $\epsilon' = \sqrt{\epsilon} = \frac{m_W}{m_X} \ll 1$, which we can use as a perturbative parameter. The lightest mass eigenvalue is

$$m_{0h}^2 = \frac{2(9 + 2t^2)m_X^2}{27} \left(k + \epsilon'^2 - \sqrt{(k - \epsilon'^2)^2 + \epsilon'^2} \right). \tag{86}$$

Using the approximation $\sqrt{(k - \epsilon'^2)^2 + \epsilon'^2} \simeq k - \epsilon'^2 + \frac{\epsilon'^2}{2k}$ we obtain $m_{0h}^2 \simeq m^2/c_W^2 \simeq m_Z^2$, this being consistent with the result shown in Appendix A.

This result confirms that the VEV of χ gives a tiny contribution to the lightest neutral Higgs mass. Now we construct the effective potential for the neutral Higgs at the one-loop level. We concentrate on terms related with only ϕ_2 which give the largest contribution to the mass of the lightest CP-even neutral Higgs. Let us remind the reader of the role of the triplets χ and ρ in generating mass for quarks. The Yukawa interactions containing χ, ρ are given by

$$\mathcal{L}_u^Y = -\frac{1}{3} \left[\kappa_{4\alpha i} Q_{\alpha L} d_{iL}^c \chi + \kappa'_{4\alpha\beta} Q_{\alpha L} d_{\beta L}^c \chi + \kappa_{3\alpha i} Q_{\alpha L} u_{iL}^c \rho + \kappa'_{3\alpha i} Q_{\alpha L} u_{iL}^c \rho \right]. \tag{87}$$

We choose $\kappa_{4\alpha i} \rightarrow 3\kappa_4 \delta_{\alpha i}$, $\kappa_{3\alpha i} \rightarrow -3y_{3\alpha} \delta_{\alpha i}$ ($y_c \equiv y_{32}$, $y_t \equiv y_{33}$) and ignore the mixing of top and exotic u-quarks. Therefore the mass of the top quark is $m_t = y_t v/\sqrt{2}$.

The masses of sfermions in the SUSYE331 were analyzed in [43]. In this work with the assumption of the Yukawa term the largest supersymmetric contributions to the masses of the two left and right stops are the same and equal to $y_t \phi_2$. For the simplest case, we also assume that the contribution from the soft term for each left or right stop is m_q^2 . All contributions for the stop quark coming

from the D-term is ignored. This assumption is similar to that given in [59]. Hence, the squared masses of the top quark and stop have the form

$$m_t^2 = y_t^2 \phi_2^2, \quad m_{\tilde{t}}^2 = y_t^2 \phi_2^2 + m_{\tilde{q}}^2. \tag{88}$$

The full one-loop potential now is

$$V(Q) = V_0(Q) + \Delta V_1(Q), \tag{89}$$

where

$$\Delta V_1(Q) = \frac{1}{64\pi^2} \text{Str} \left[\mathcal{M}^4 \left(\ln \frac{\mathcal{M}^2}{Q^2} - c \right) \right]. \tag{90}$$

Here \mathcal{M}^2 is the field-dependent generalized squared mass matrix and the supertrace is defined as

$$\text{Str} f(\mathcal{M}^2) = \sum_i (-1)^{2J_i} (2J_i + 1) f(m_i^2) \tag{91}$$

with J_i is the spin of the field having mass m_i . We take the contribution only from the top quarks and stops, namely

$$\begin{aligned} \Delta V_1(Q) = \frac{3}{16\pi^2} & \left[2m_t^4 \ln \left(\frac{m_t^2}{m_{\tilde{t}}^2} \right) \right. \\ & \left. + (m_t^4 - m_{\tilde{t}}^4) \left(\ln \frac{m_t^2}{Q^2} - c \right) \right]. \end{aligned} \tag{92}$$

From Eq. (88) we have

$$\frac{\partial(m_t^2)}{\partial(\phi_1)} = \frac{\partial(m_{\tilde{t}}^2)}{\partial(\phi_1)} = 0, \tag{93}$$

$$\frac{\partial(m_t^2)}{\partial(\phi_2)} = \frac{\partial(m_{\tilde{t}}^2)}{\partial(\phi_2)} = 2y_t^2 \phi_1. \tag{94}$$

This leads to the consequence that $\frac{\partial \Delta V_1}{\partial \phi_1} = 0$. The minimal condition is equivalent to the following equation:

$$\begin{aligned} \left. \frac{\partial (V_0 + \Delta V_1)}{\partial \phi_2} \right|_{\phi_1=W/\sqrt{2}, \phi_2=V/\sqrt{2}} &= 0 \\ \rightarrow (m_t^2 - m_{\tilde{t}}^2) \left(\ln \frac{m_t^2}{\hat{Q}} - c \right) & \\ = - (m_t^2 - m_{\tilde{t}}^2) - 2m_t^2 \ln \frac{m_t^2}{m_{\tilde{t}}^2}. & \end{aligned} \tag{95}$$

Because of Eq. (95) one can obtain

$$\frac{\partial^2 V(\phi_1, \phi_2)}{\partial \phi_2^2} = \frac{\partial^2 V_0}{\partial \phi_2^2} + \frac{3g^2}{8\pi^2} \frac{m_t^4}{m_{\tilde{t}}^2} \ln \left(\frac{m_t^2}{m_{\tilde{t}}^2} \right). \tag{96}$$

The last term in Eq. (96) is the correction from the one-loop effective potential. The squared mass matrix for the neutral Higgs is obtained as follows:

$$\frac{1}{2} \left[\frac{\partial^2 V(\phi_1, \phi_2)}{\partial \phi_i \partial \phi_j} \right]_{\phi_1=W/\sqrt{2}, \phi_2=V/\sqrt{2}}. \tag{97}$$

Diagonalizing the matrix found from Eq. (97) we find the formula of the lightest squared mass as follows:

$$\begin{aligned} m_h^2 = \frac{2}{3 - 4s_W^2} & \left(c_W^2 m_X^2 + \Delta + m_W^2 \right. \\ & \left. - \sqrt{(c_W^2 m_X^2 - \Delta - m_W^2)^2 + m_X^2 m_W^2} \right) \end{aligned} \tag{98}$$

with $\Delta = \frac{3g^2(3-4s_W^2)}{64\pi^2} \frac{m_t^4}{m_{\tilde{t}}^2} \ln \left(\frac{m_t^2}{m_{\tilde{t}}^2} \right)$. In the case of $\Delta \sim \mathcal{O}(m_W^2) \ll m_X^2$ we obtain $m_h^2 \simeq m_Z^2 + \frac{3g^2}{16\pi^2} \frac{m_t^4}{m_{\tilde{t}}^2} \ln \left(\frac{m_t^2}{m_{\tilde{t}}^2} \right)$. This result is the same as that in the MSSM.

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