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TOTAL CROSS-SECTION FOR PHOTON–AXION CONVERSIONS IN EXTERNAL ELECTROMAGNETIC FIELD

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We reconsider the conversion of the photon into axion in the external electromagnetic fields, namely in the static fields and in a periodic field of the wave guide. The total cross-sections for the conversion are evaluated in detail. The result shows that with strong strength of external electromagnetic fields, the cross-sections are large enough to measure the axion production. In the wave guide, there exists the resonant conversion at the low energies, in which the value of cross-sections is much enhanced.

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1. Introduction

The Peccei–Quinn (PQ) mechanism[1,2] provides a simple explanation for the strong CP problem by the introduction of a light pseudoscalar particle, called the axion,[3,4] which receives a small coupling to electromagnetism (similar to neutral pion in QCD). At present, the axion mass is constrained by laboratory searches[5–7] and by astrophysical and cosmological considerations[8–10] to between 10^{-6} eV and 10^{-3} eV. If the axion has a mass near the low limit of order 10^{-5} eV, it is a good candidate for the dark matter of the Universe. It was also argued that an axion–photon oscillation can explain the observed dimming supernovas if it has a rather small order of 10^{-16} eV.[11]

Neutral pions, gravitons, hypothetical axions, or other light particles with a two-photon interaction can transform into photons in external electromagnetic (EM) fields, an effect first discussed by Primakoff.[12] This effect is the basis of Sikivie's methods for the detection of axions in a resonant cavity.[13] The experiment CAST (Cern Axion Solar Telescope)[14–16] at CERN searches for axions from the sun or other sources in the Universe. The experiment uses a large magnet from LHC to convert solar axions into detectable X-ray photons. The potential of the CAST experiment for exotic particles was discussed in Refs. 17 and 18.

In our previous works,[19,20] we have calculated the different cross-sections for the photon–axion conversion in external EM fields in detail. However, some numerical evaluations in comparison with the experiments are not realistic. The purpose of this paper is to evaluate *the total cross-sections* for the photon–axion conversion in external EM fields, including the static fields as in Ref. 19 and also the periodic field of the wave guide.[20]

Consider the conversion of the photon γ with momentum q into the axion a with momentum p in an external electromagnetic field. The matrix element is given by[19,20]

$$\langle p|\mathcal{M}|q\rangle = -\frac{g_{a\gamma}}{2(2\pi)^2\sqrt{q_0 p_0}}\varepsilon_\mu(\mathbf{q},\sigma)\varepsilon^{\mu\nu\alpha\beta}q_\nu \int_V e^{i\mathbf{kr}}F^{\text{class}}_{\alpha\beta}\,d\mathbf{r}\,, \tag{1}$$

where $\mathbf{k}\equiv\mathbf{q}-\mathbf{p}$ is the momentum transfer to the EM field, $\varepsilon^\mu(\mathbf{q},\sigma)$ represents the polarization vector of the photon, and $g_{a\gamma}\equiv g_\gamma\frac{\alpha}{\pi f_a} = g_\gamma\alpha m_a(m_u + m_d)(\pi f_\pi m_\pi\sqrt{m_u m_d})^{-1}$. In particular, in the Dine–Fischler–Srednicki–Zhitnitskii model:[21,22] $g_\gamma(\text{DFSZ})\simeq 0.36$, and in the Kim–Shifman–Vainshtein–Zakharov model[23,24] (where the axions do not couple to light quarks and leptons): $g_\gamma(\text{KSVZ})\simeq -0.97$.

2. Conversions in the Electric Field

We reconsider the photon–axion conversion in the homogeneous electric field of the flat condenser of size $l_x\times l_y\times l_z$ (instead of $a\times b\times c$ in Ref. 19). Using the coordinate system with the x-axis parallel to the direction of the field, i.e. $F^{10}=-F^{01}=E$, where E is the strength of electric field. From Eq. (1) we obtained the differential cross-section for the conversion[19]

$$\frac{d\sigma^e(\gamma\to a)}{d\Omega} = \frac{g^2_{a\gamma}E^2}{2(2\pi)^2}\left[\frac{\sin(\frac{1}{2}l_x k_x)\sin(\frac{1}{2}l_y k_y)\sin(\frac{1}{2}l_z k_z)}{k_x k_y k_z}\right]^2(q_y^2+q_z^2)\,. \tag{2}$$

From Eq. (2) we see that if the photon moves in the direction of the electric field, i.e. $q^\mu=(q,q,0,0)$ then the different cross-section vanishes. If the momentum of the photon is parallel to the y-axis (the orthogonal direction), i.e. $q^\mu=(q,0,q,0)$ then Eq. (2) becomes

$$\frac{d\sigma^e(\gamma\to a)}{d\Omega'} = \frac{32g^2_{a\gamma}E^2q^2}{(2\pi)^2}\left[\sin\left(\frac{l_x}{2}p\sin\theta\sin\varphi'\right)\sin\left(\frac{l_y}{2}(q-p\cos\theta)\right)\right.$$
$$\left.\times\sin\left(\frac{l_z}{2}p\sin\theta\cos\varphi'\right)\right]^2(p^2\sin^2\theta\sin\varphi'\cos\varphi'(q-p\cos\theta))^{-2}\,, \tag{3}$$

where φ' is the angle between the z-axis and the projection of $\mathbf{p}$ on the xz-plane. For the forward scattering case ($\theta \approx 0$), we have

$$\frac{d\sigma^e(\gamma \to a)}{d\Omega'} = \frac{2g_{a\gamma}^2 E^2 l_x^2 l_z^2}{(2\pi)^2 \left(1 - \sqrt{1 - \frac{m_a^2}{q^2}}\right)^2} \sin^2 \left[\frac{q l_y}{2} \left(1 - \sqrt{1 - \frac{m_a^2}{q^2}}\right)\right]. \tag{4}$$

In the limit $m_a^2 \ll q^2$ and $l_y \sim m_a^{-1}$, from Eq. (4) we have

$$\frac{d\sigma^e(\gamma \to a)}{d\Omega'} \simeq \frac{g_{a\gamma}^2 E^2 l_x^2 l_z^2}{16\pi^2}. \tag{5}$$

We can see from Eq. (5) that in this case *the cross-section does not depend on the given photon energies*. It is noticed that in our previous work[19] we have obtained Eq. (3) and evaluated the different cross-sections in detail, however some numerical evaluations in comparison with the experiments are not realistic.

Now we are mainly interested in *the total cross-section* $\sigma^e(q) = \int d\Omega(d\sigma^e/d\Omega)$ from the general formula (3). For this purpose, we note that the integrand as well as the total cross-section depend on the given photon momentum q (at least larger than the axion mass) are very rapidly oscillated with q. To overcome this, we plot a large spectrum of points $(q, \sigma^e(q))$ corresponding to a large number of values of q in the interested domain. The orientation of the spectrum will reflect the correct variation of the cross-section. The parameters are chosen as follows,[19] $l_x = l_y = l_z = 1$ m, the intensity of the electric field $E = \frac{100\ \text{kV}}{\text{m}}$ and the axion mass $m_a = 10^{-5}$ eV (near the low limit of mass window[8]). The total cross-section on the selected range of the provided momenta, $q = 10^{-4}$–10^{-3} eV, are given in Fig. 1. The upper plot is depicted as 300 points, and the lower one is for 3000 points. As demonstrated in the two plots, when the number of points is increased, the resonances become shaper, in which the cross-sections are quite large ($\sigma \sim 10^{-29}$ cm^2). The numerical evaluation for Eq. (5), the cross-section is given by $\frac{d\sigma^e(\gamma \to a)}{d\Omega'} \simeq 1.8 \times 10^{-37}$ cm^2.

It is noticed that this parameter is the derivative one, not concerning as any characteristic scales of the model. Its value depends only on a choice of the parameters such as the axion mass, the size of condenser, the field strength, and so on. Let us remark that when the momentum of photon is perpendicular to the electric field E, we have then the most optimal condition for the experiments.

3. Conversions in the Magnetic Field

We move on to conversions in the strong magnetic field of the solenoid with a radius R and a length h. If the momentum of the photon is parallel to the x-axis, the different cross-section is given by[19]

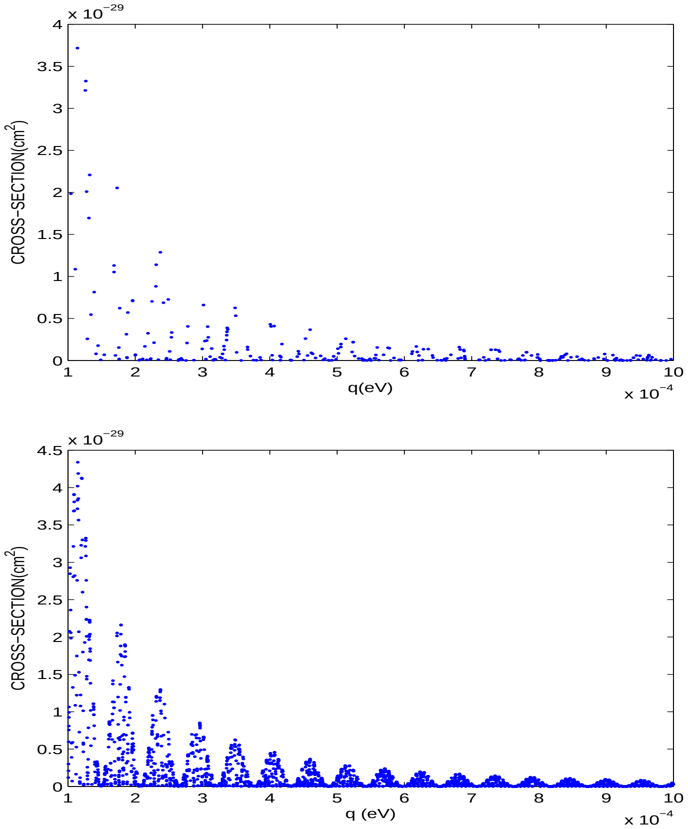

Fig. 1. The total cross-section for the photon–axion conversion in an electric field as a function of provided momentum, $q = 10^{-4}$–10^{-3} eV. The upper plot is depicted as 300 points, and the lower one is for 3000 points.

$$
\begin{aligned}
\frac{d\sigma^m(\gamma \to a)}{d\Omega'} & \\
&= 2g_{a\gamma}^2 R^2 B^2 J_1^2 \left(Rq \sqrt{\left(1 - \cos\theta\sqrt{1 - \frac{m_a^2}{q^2}}\right)^2 + \left(1 - \frac{m_a^2}{q^2}\right)\sin^2\theta\cos^2\varphi'} \right) \\
&\quad \times \left[\left(1 - \cos\theta\sqrt{1 - \frac{m_a^2}{q^2}}\right)^2 + \left(1 - \frac{m_a^2}{q^2}\right)\sin^2\theta\cos^2\varphi'\right]^{-1} q^{-2} \\
&\quad \times \sin^2\left(\frac{hq}{2}\sqrt{1 - \frac{m_a^2}{q^2}}\sin\theta\sin\varphi'\right)\left[\left(1 - \frac{m_a^2}{q^2}\right)\sin^2\theta\sin^2\varphi'^2\right]^{-1}, \qquad (6)
\end{aligned}
$$

where B is the strength of magnetic field and J_1 is the spherical Bessel function of the first kind. For the forward scattering case and in the limit $m_a^2 \ll q^2$, $R \leq m_a^{-1}$,

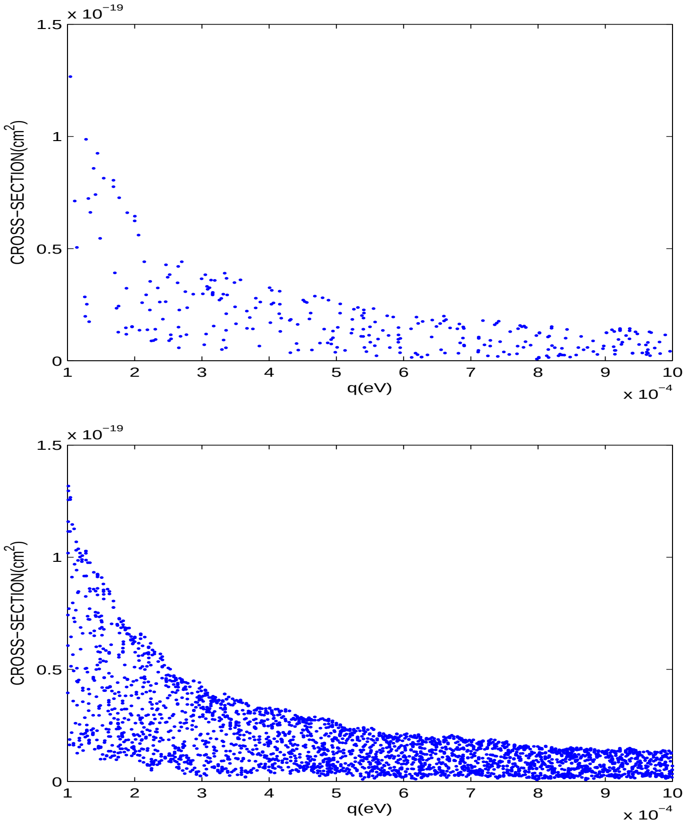

Fig. 2. The total cross-section for the photon–axion conversion in the magnetic field as a function of provided momentum. The upper plot is depicted as 300 points, and the lower one is for 3000 points.

we have

$$\frac{d\sigma^m(\gamma \to a)}{d\Omega'} \simeq \frac{1}{2\pi^2} g_{a\gamma}^2 V h B^2 \,, \tag{7}$$

where V is the volume of the solenoid. From (7) we see that *the conversion probability is proportional to the square of the field strength, the active length and the volume of the solenoid*.

To evaluate the total cross-section from the general formula (6), the parameter values are given as before and the remaining ones are chosen as follows: $R = l = 1$ m $= 5.07 \times 10^6$ eV^{-1} and $B = 9$ Tesla $= 9 \times 195.35$ eV^2.[15] The total cross-sections on the selected range of momenta q is presented in Fig. 2. The upper plot is depicted as 300 points, and the lower one is for 3000 points. From the figures we see that the total cross-sections for the axion production in the strong magnetic field ($\sigma \sim 10^{-19}$ cm^2) are much larger than that in the electric field. This is due to $B \gg E$. By numerical evaluation for (7), the cross-section is given by $\frac{d\sigma^m(\gamma\to a)}{d\Omega'} \simeq 1.1 \times 10^{-32}$ cm^2.

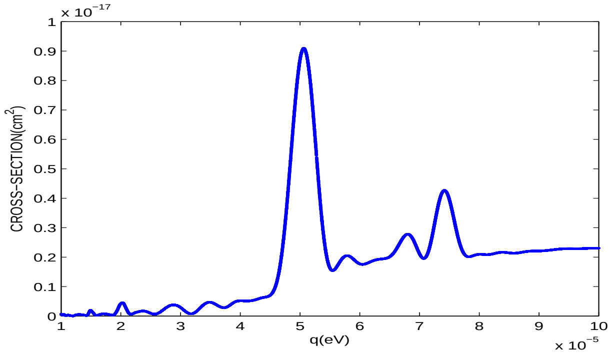

Fig. 3. The total cross-section for the photon–axion conversion in the wave guide as a function of provided momentum. The moment range is chosen at the low values, $q = 10^{-5}$–10^{-4} eV.

4. Conversions in the Wave Guide

Now we are interested in conversions in the external EM field of the TE_{10} mode of the wave guide with frequency equal to the axion mass.[20] The nontrivial solution of the TE_{10} mode is given by[25]

$$\begin{aligned} H_z &= H_0 \cos\left(\frac{\pi x}{l_x}\right) e^{ikz - i\omega t}\,, \\ H_x &= -\frac{ikl_x}{\pi} H_0 \sin\left(\frac{\pi x}{l_x}\right) e^{ikz - i\omega t}\,, \\ E_y &= i\frac{\omega a \mu}{\pi} H_0 \sin\left(\frac{\pi x}{l_x}\right) e^{ikz - i\omega t}\,. \end{aligned} \tag{8}$$

Here the propagation of the EM wave is in the z-axis. If the momentum of the photon is parallel to the x-axis, then the different cross-section is given by the x-axis, then

$$\begin{aligned} \frac{d\sigma(\gamma \to a)}{d\Omega'} &= \frac{8 g_{a\gamma}^2 H_0^2 l_x^2 q^2}{\pi^4}\left(1 + \frac{\omega}{q}\right)\left[\omega(q - p\cos\theta) - \frac{\pi^2}{l_x^2}\right]^2 \\ &\quad \times \left[\frac{\cos\frac{l_x}{2}(q - p\cos\theta)\sin\frac{l_y}{2}(p\sin\theta\cos\varphi')\sin\frac{l_z}{2}(-p\sin\theta\sin\varphi' + k)}{[(q - p\cos\theta)^2 - \frac{\pi^2}{l_x^2}]\cdot p\sin\theta\cos\varphi'(-p\sin\theta\sin\varphi' + k)}\right]^2 . \end{aligned} \tag{9}$$

To evaluate the total cross-section for formula (9), we take $H_0 = B$, $\omega = m_a = 10^{-5}$ eV. The remaining parameters are chosen as before. Figure 3 shows the dependence of the total cross-section σ as a function of momentum q. The moment range is chosen at the lower values, $q = 10^{-5}$–10^{-4} eV. We can see from the figure that there exists *a main resonant conversion* at the value $q = 5.1 \times 10^{-5}$ eV, the cross-section is given by $\sigma \simeq 10^{-17}$ cm^2, which is much larger than those in the static fields. *This is the best case for photon–axion conversions*. We note that the inverse process of the axion–photon conversion ($a \to \gamma$) is also important for the axion detection in experiments, which was calculated in detail in Ref. 26.

5. Conclusion

In this paper we have reconsidered the conversion of the photon into axion in the external electromagnetic fields, namely in the static fields and in a periodic field of the wave guide. The numerical evaluations of the total cross-section are also given in detail. Our result shows that with the strong strength of external electromagnetic fields, the cross-sections are large enough to measure the axion production. In the wave guide there exists the resonant conversion at the low energies, in which the value of cross-sections is much enhanced.

Finally, in this work we have considered only a theoretical basis for the experiments, other techniques concerning construction and particle detection can be found in Refs. 15, 16 and 27.

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