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3-3-1 model with inert scalar triplet

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We show that the typical 3-3-1 models are only self-consistent if they contain interactions that explicitly violate the lepton number. The 3-3-1 model with right-handed neutrinos can by itself work as an economical 3-3-1 model as a natural recognition of the above criteria, while it also produces an inert scalar triplet (η) responsible for dark matter. This is ensured by a Z_2 symmetry (assigned so that only η is odd while all other multiplets which perform the economical 3-3-1 model are even), which is not broken by the vacuum. The minimal 3-3-1 model can provide dark matter by a similar realization. Taking the former into account, we show that the dark matter candidate (H_η) contained in η transforms as a singlet in the effective limit under the standard model symmetry and is naturally heavy. The H_η relic density and direct detection cross section will have the correct values when the mass of H_η is in TeV range, as expected. The model predicts the H_η mass $m_{H_\eta} = \lambda_5 \times 2$ TeV and the H_η-nucleon scattering cross section $\sigma_{H_\eta - N} = 1.56 \times 10^{-44}$ cm^2, provided that the new neutral Higgs boson is much heavier than the dark matter.

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I. INTRODUCTION

The standard model has been very successful in describing the world of fundamental particles and the interactions among them [1]. Notably, the Higgs particle—a long-standing hypothesized scalar that consequently provides the masses for all other particles—has finally been discovered by the recent CERN-LHC experiments, where the new discovered resonance is standard model-like [2,3]. However, the standard model fails to explain a large portion of the total mass-energy of the Universe—such as dark matter (>20%) and dark energy (>70%)—which lies beyond the standard model particle content [1].

The most well-motivated theories that by themselves produce dark matter are supersymmetry [4], extradimensions [5], or the little Higgs model [6]. In a recent article, we added to this list by showing that dark matter can also naturally arise from the 3-3-1-1 gauge theory by itself [7] (a theory that originally provides the potential explanations for the fermion generation number [8], the uncharacteristically heavy top quark [9], strong *CP* [10], and the electric charge quantization [11]). Indeed, this 3-3-1-1 gauge symmetry—which includes *B-L* (baryon minus lepton number) as its residual and noncommuting gauge charge—is the necessary extension of 3-3-1 models [12–14] that respect the conservation of lepton and baryon numbers, similar to the case of the electric charge operator. In other words, this new theory of strong, electroweak, and *B-L* interactions is a direct consequence of a nonclosed algebra between *B-L* and the 3-3-1 symmetry [7]. Consequently, the conserved and unbroken *W* parity (similar to *R* parity in supersymmetry) can arise as a residual symmetry of the broken 3-3-1-1 gauge symmetry or a more detailed *B-L*. (This breaking possibly happens at a scale matching the 3-3-1 breaking scale of TeV order that makes the model consistent without the necessity of a large desert, as in grand unified theories [15,16].) Among the existing 3-3-1 models, we have found that most of the new particles of the 3-3-1 model with neutral fermions [14], the so-called *W* particles, transform nontrivially (that is, oddly) under the *W* parity, which is responsible for dark matter [7].

By contrast, all the new particles in the 3-3-1 model with right-handed neutrinos [12] as well as those of the minimal 3-3-1 model [13] are even under the *W* parity. Therefore, the *W* parity transforms trivially, which is useless for these models with regards to the problem of dark matter [7]. On the other hand, it is well-known that the 3-3-1 model with right-handed neutrinos might actually accommodate potential candidates for dark matter [17]. However, all the extra symmetries studied therein (which had existed before the *W* parity)—such as the Z_2, lepton charge, or even a generic continuous symmetry—are subsequently violated or broken if they are imposed for their stability, which leads to the fast decay of dark matter, as was explicitly shown in Ref. [7] (this will also be extensively analyzed

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below before concluding this work). Hence, it is necessary to find a new mechanism, other than the useless W parity and the mentioned extra symmetries, that is responsible for the dark matter stability in the 3-3-1 model with right-handed neutrinos. This mechanism should also be applicable to the minimal 3-3-1 model for a realization of dark matter (notice that this model has previously been predicted containing no dark matter, by contrast).

To proceed further, we first suppose that the lepton number in the 3-3-1 model with right-handed neutrinos is an approximate symmetry, which avoids the gauged symmetry of the lepton number and the 3-3-1-1 extension [7]. This proposal realizes a theory that explicitly violates the W parity or lepton-number symmetry in order to make it (our 3-3-1 model) self-consistent. In particular, the 3-3-1 model with right-handed neutrinos often works with three scalar triplets ρ, η, and χ, where η and χ transform identically under the 3-3-1 gauge symmetry. However, η and χ have different lepton charges [18]. Since the lepton-charge symmetry is already violated, these two scalars can act as equivalent representations under any group that operates on the model. We could therefore remove one of them from the theory (such as η). The result is a new, consistent model working with only two scalar triplets ρ, χ which explicitly recognizes the violation of W parity or lepton-number symmetry. This theory has been extensively studied over the last decade and named the economical 3-3-1 model [19]. However, the economical 3-3-1 model does not contain any dark matter either, which is unlike the conclusion of Ref. [20].

In this work, by contrast to the approach in Ref. [19], we will retain η in the theory, and study how it is hidden (instead of removing it). For this aim, we first assume that η transforms oddly under a Z_2 symmetry, whereas χ, ρ, and all other fields are even (notice that this Z_2 differs from the one mentioned above). We then prove that the vacuum can be stabilized, conserving the Z_2 symmetry. The lightest particle residing in the "inert" scalar triplet η is thus stabilized and is responsible for dark matter, while the remaining scalars ρ, χ develop the vacuum expectation values (VEVs) for breaking the gauge symmetry and for mass generation in the correct way, like the economical 3-3-1 model. This approach is completely distinguished from the previous studies [7,17] because it is based on the economical 3-3-1 model (with lepton-number violation being responsible for neutrino masses) instead of the 3-3-1 model with right-handed neutrinos (with the lepton number conserved, which is unrealistic). Also, its results—namely, the dark matter candidate and phenomenology—are entirely different from that of the inert doublet model [21], as well as those in Refs. [7,17]. In the same way, the minimal 3-3-1 model can behave as a reduced 3-3-1 model [22] while containing an inert scalar triplet responsible for dark matter.

The rest of this paper is organized as follows. In Sec. II we propose the new model. We first give a discussion on lepton number and its violation, and introduce the Z_2 symmetry and the inert scalar triplet. We then consider the gauge symmetry breaking and prove that the Z_2 is unbroken by the vacuum. The candidates for dark matter which lie in the scalar sector are identified, and their interactions are obtained. Section III is devoted to the dark matter relic density and dark matter constraints due to direct searches. In Sec. IV we point out why our work is necessary and unique, and describe its implications for other 3-3-1 models. Finally, we summarize our results and make conclusions in Sec. V.

II. THE MODEL

A. Lepton-number violation, Z_2 symmetry, and the inert scalar triplet

The model under consideration is based on the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (3-3-1) gauge symmetry. The fermion content is given by [12]

$$\psi_{a\mathrm{L}} \equiv \begin{pmatrix} \nu_{a\mathrm{L}} \\ e_{a\mathrm{L}} \\ (\nu_{a\mathrm{R}})^c \end{pmatrix} \sim (1,3,-1/3), \quad e_{a\mathrm{R}} \sim (1,1,-1), \tag{1}$$

$$Q_{1\mathrm{L}} \equiv \begin{pmatrix} u_{1\mathrm{L}} \\ d_{1\mathrm{L}} \\ U_{\mathrm{L}} \end{pmatrix} \sim (3,3,1/3), \quad Q_{\alpha\mathrm{L}} \equiv \begin{pmatrix} d_{\alpha\mathrm{L}} \\ -u_{\alpha\mathrm{L}} \\ D_{\alpha\mathrm{L}} \end{pmatrix} \sim (3,3^*,0), \tag{2}$$

$$u_{a\mathrm{R}} \sim (3,1,2/3), \qquad d_{a\mathrm{R}} \sim (3,1,-1/3), \tag{3}$$

$$U_{\mathrm{R}} \sim (3,1,2/3), \qquad D_{\alpha\mathrm{R}} \sim (3,1,-1/3), \tag{4}$$

where the quantum numbers defined in the parentheses are given by the $[SU(3)_C, SU(3)_L, U(1)_X]$ symmetries, respectively. The family indices are set as $a = 1, 2, 3$ and $\alpha = 2, 3$. The $\nu_{a\mathrm{R}}$ are the right-handed neutrinos, which are included to complete the lepton-triplet representations (and thus the model is named the 3-3-1 model with right-handed neutrinos). Similarly, the exotic quarks U, D_α take part of the respective quark multiplets. The last two families of quarks—which transform differently under $SU(3)_L$ than the first family—and the leptons are arranged in order to cancel the $SU(3)_L$ self-anomaly (i.e., the number of fermion triplets must be equal to the number of antitriplets). It is easily checked that the theory is free from all the other anomalies.

The electric charge operator, which is the only generator conserved after the gauge symmetry breaking, is given by

$$Q = T_3 - \frac{1}{\sqrt{3}}T_8 + X, \tag{5}$$

where $T_i (i = 1, 2, \ldots, 8)$ is the charge of $SU(3)_L$, while X is that of $U(1)_X$ [below, the $SU(3)_C$ charges will be denoted by t_i). Let us note that the exotic quarks U and D_α have electric charges like ordinary quarks, $Q(U) = 2/3$ and $Q(D_\alpha) = -1/3$, respectively.

The baryon number (B) as a global symmetry $U(1)_B$ commutes with the gauge symmetry and is always conserved by the general Lagrangian and vacuum [18]. However, the lepton number (L) of the lepton triplet components is given by $(+1, +1, -1)$, which does not commute with the gauge symmetry, similar to the case of the electric charge. In addition, the algebra of L and the 3-3-1 symmetry is nonclosed because in order for L to be some generator of $SU(3)_L \otimes U(1)_X$, i.e., $L = x_i T_i + yX$ with fixed x_i, y coefficients, we have $\mathrm{Tr}(L) = y\mathrm{Tr}(X)$ for every multiplet, which is generally incorrect [7]. For example, we have $y = -1$ for e_R, but $y = 0$ for u_R, which is a contradiction. Therefore, if the lepton number L is conserved, we can find in the resulting theory an extra $U(1)_\mathcal{L}$ group factor so that its Lagrangian is invariant under this group, and the combination obtained [18],

$$L = \frac{4}{\sqrt{3}} T_8 + \mathcal{L}, \tag{6}$$

is a residual charge of $SU(3)_L \otimes U(1)_\mathcal{L}$. The $\mathcal{L}$ charges for the fermion multiplets are given by

$$\begin{gathered} \mathcal{L}(\psi_{aL}) = 1/3, \quad \mathcal{L}(Q_{1L}) = -2/3, \quad \mathcal{L}(Q_{\alpha L}) = 2/3, \\ \mathcal{L}(e_{aR}) = 1, \quad \mathcal{L}(u_{aR}) = \mathcal{L}(d_{aR}) = 0, \quad \mathcal{L}(U_R) = -2, \\ \mathcal{L}(D_{\alpha R}) = 2. \end{gathered} \tag{7}$$

In addition, the exotic quarks satisfy $L(U) = -L(D) = -2$, and hence are called leptoquarks.

Notice that the above definition (6) is only true if the lepton number of the theory is conserved. Otherwise, the Yukawa Lagrangian and the scalar potential will take the most general forms, which have interactions that explicitly violate L [18]. Also, L is subsequently broken. There is no $U(1)_\mathcal{L}$ at all. The relation (6) disappears, which avoids the judgment of Ref. [7]. (There, the 3-3-1 model that conserves L was extended to the 3-3-1-1 model with a gauged B-L as a result of the gauged T_8.) This is a new observation of this work, which will be studied below (in other words, the 3-3-1 model is only self-consistent due to this lepton-number violation). Namely, the lepton number will not be regarded as an exact symmetry of the theory; however, we can consider it as an approximate symmetry to keep the model self-consistent. Therefore, Eq. (6) is only an approximate expression for calculating the lepton number of model particles (because the theory is obviously not constrained to be invariant under the approximate symmetry $U(1)_\mathcal{L}$, as supposed). Thus it is noteworthy that this charge should no longer be regarded as a gauge symmetry, as in Ref. [7]. All the above ingredients can also be applied to the minimal 3-3-1 model. One theory that does not satisfy the criteria of Ref. [7] is the economical 3-3-1 model [19]. In the present work we are going to realize a new 3-3-1 model of this kind.

As usual, the 3-3-1 model with right-handed neutrinos requires three scalar triplets [12],

$$\eta = \begin{pmatrix} \eta_1^0 \\ \eta_2^- \\ \eta_3^0 \end{pmatrix} \sim (1, 3, -1/3), \tag{8}$$

$$\phi = \begin{pmatrix} \phi_1^+ \\ \phi_2^0 \\ \phi_3^+ \end{pmatrix} \sim (1, 3, 2/3), \tag{9}$$

$$\chi = \begin{pmatrix} \chi_1^0 \\ \chi_2^- \\ \chi_3^0 \end{pmatrix} \sim (1, 3, -1/3), \tag{10}$$

to break the gauge symmetry and generate the masses. Hereafter, we use the notation ϕ instead of ρ mentioned in the Introduction so that it is similar to that of the economical 3-3-1 model. The $\mathcal{L}$ charges for the scalar triplets are obtained by [18]

$$\mathcal{L}(\chi) = 4/3, \quad \mathcal{L}(\phi) = -2/3, \quad \mathcal{L}(\eta) = -2/3. \tag{11}$$

The nonzero lepton numbers of the scalars are

$$L(\chi_1^0) = L(\chi_2^-) = -L(\phi_3^+) = -L(\eta_3^0) = 2. \tag{12}$$

Because the lepton number is an approximate symmetry, all the electrically neutral scalars, including the bileptons χ_1^0 and η_3^0, might develop VEVs, as shown in the next subsection. The electroweak gauge symmetry is broken in two stages. In the first stage, $SU(3)_L \otimes U(1)_X$ is broken down to that of the standard model, generating the masses of new particles. This is achieved by the VEV of χ_3^0 (and possibly that of η_3^0). In the second stage, the standard model electroweak symmetry is broken down to $U(1)_Q$, which is responsible for the masses of ordinary particles. This stage is achieved by the VEV of ϕ_2^0 and/or η_1^0 (and possibly that of χ_1^0).

Let us remind the reader that η and χ have the same gauge quantum numbers. They differ only in the $\mathcal{L}$ charge, as shown above. Since the $U(1)_\mathcal{L}$ symmetry is approximate, every interaction that violates it is allowed. These scalars equivalently act on the model. This gives the economical 3-3-1 model, which works with only two scalar triplets ϕ, χ by excluding the η [19]. In this paper, we will introduce another scenario in which we retain the η in the theory but impose a Z_2 symmetry so that the only η is odd,

$$\eta \to -\eta. \tag{13}$$

All the other multiplets, including ϕ and χ, are even, $\phi \to \phi$, $\chi \to \chi$, and so on.

Up to the gauge-fixing and ghost terms, the Lagrangian is given by

$$\mathcal{L} = \sum_{\text{Fermion multiplets}} \bar{F} i\gamma^\mu D_\mu F + \sum_{\text{Scalar multiplets}} (D^\mu S)^\dagger \times (D_\mu S) - \frac{1}{4} G_{i\mu\nu} G_i^{\mu\nu} - \frac{1}{4} A_{i\mu\nu} A_i^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \mathcal{L}_\mathrm{Y} - V, \tag{14}$$

with the covariant derivative $D_\mu = \partial_\mu + ig_s t_i G_{i\mu} + igT_i A_{i\mu} + ig_X(X/\sqrt{6})B_\mu$, and the field-strength tensors $G_{i\mu\nu} = \partial_\mu G_{i\nu} - \partial_\nu G_{i\mu} - g_s f_{ijk} G_{j\mu} G_{k\nu}$, $A_{i\mu\nu} = \partial_\mu A_{i\nu} - \partial_\nu A_{i\mu} - g f_{ijk} G_{j\mu} A_{k\nu}$, and $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$, which correspond to the $\mathrm{SU}(3)_C$, $\mathrm{SU}(3)_L$, and $\mathrm{U}(1)_X$ groups, respectively. The last two terms will be specified below.

The Yukawa Lagrangian is given by

$$\begin{aligned}\mathcal{L}_\mathrm{Y} = {} & h^e_{ab} \bar{\psi}_{a\mathrm{L}} \phi e_{b\mathrm{R}} + h^\nu_{ab} \bar{\psi}^c_{a\mathrm{L}} \psi_{b\mathrm{L}} \phi + h^U \bar{Q}_{1\mathrm{L}} \chi U_\mathrm{R} \\ & + h^D_{\alpha\beta} \bar{Q}_{\alpha\mathrm{L}} \chi^* D_{\beta\mathrm{R}} + h^d_a \bar{Q}_{1\mathrm{L}} \phi d_{a\mathrm{R}} + h^u_{\alpha a} \bar{Q}_{\alpha\mathrm{L}} \phi^* u_{a\mathrm{R}} \\ & + \bar{h}^u_a \bar{Q}_{1\mathrm{L}} \chi u_{a\mathrm{R}} + \bar{h}^d_{\alpha a} \bar{Q}_{\alpha\mathrm{L}} \chi^* d_{a\mathrm{R}} + \bar{h}^D_\alpha \bar{Q}_{1\mathrm{L}} \phi D_{\alpha\mathrm{R}} \\ & + \bar{h}^U_\alpha \bar{Q}_{\alpha\mathrm{L}} \phi^* U_\mathrm{R} + \mathrm{H.c.}\end{aligned} \tag{15}$$

Due to the Z_2 symmetry, the η does not interact with fermions. The Yukawa Lagrangian is achieved in a similar way as that of the economical 3-3-1 model [19]. The couplings $\bar{h}$ violate the lepton number, while the h's do not. The fermions get consistent masses at the one-loop level, or alternatively via the five-dimensional effective interactions [19]. Below, we will prove that $\langle \eta \rangle = 0$. Therefore, the gauge bosons get masses from the vacuum values of ϕ and χ, which is similar to the economical 3-3-1 model.

The scalar potential that is invariant under the gauge symmetry, the Z_2 symmetry and that is renormalizable is given by

$$\begin{aligned}V = {} & \mu_1^2 \phi^\dagger \phi + \mu_2^2 \chi^\dagger \chi + \mu_3^2 \eta^\dagger \eta + \lambda_1 (\phi^\dagger \phi)^2 \\ & + \lambda_2 (\chi^\dagger \chi)^2 + \lambda_3 (\eta^\dagger \eta)^2 + \lambda_4 (\phi^\dagger \phi)(\chi^\dagger \chi) \\ & + \lambda_5 (\phi^\dagger \phi)(\eta^\dagger \eta) + \lambda_6 (\chi^\dagger \chi)(\eta^\dagger \eta) \\ & + \lambda_7 (\phi^\dagger \chi)(\chi^\dagger \phi) + \lambda_8 (\phi^\dagger \eta)(\eta^\dagger \phi) \\ & + \lambda_9 (\chi^\dagger \eta)(\eta^\dagger \chi) + \frac{1}{2}[\lambda_{10} (\eta^\dagger \chi)^2 + \mathrm{H.c.}].\end{aligned} \tag{16}$$

Here, $\mu^2_{1,2,3}$ and $\lambda_{1,2,3,\ldots,9}$ are real, whereas λ_{10} can be complex. However, the phase of λ_{10} can be removed by redefining the relative phases of η and χ. Consequently, this potential conserves *CP* symmetry. But the *CP* symmetry can be broken spontaneously by the VEVs of the scalars. It is also noted that the coupling λ_{10} violates the lepton number [18].

We point out the fact that if the minimization of the above scalar potential conserves the Z_2 symmetry, i.e., $\langle \eta \rangle = 0$, the Z_2 is exact and unbroken. Consequently, the η is only coupled in pairs when interacting with the economical 3-3-1 model particles. This proposal already realizes a 3-3-1 model with an ''inert'' scalar triplet (η). The lightest particle contained in η is absolutely stabilized, which can be responsible for dark matter. The inert particles are naturally recognized by the original scalar sector of the 3-3-1 model with right-handed neutrinos [12]. By contrast, in the inert doublet model [21] one similar to that in the standard model should be introduced by hand. It is easily realized that ϕ and η contain two scalar doublets—the one in ϕ is similar to the standard model doublet, while that in η is the inert doublet. However, we note that due to the gauge symmetry η is not coupled to ϕ via a coupling that is similar to λ_{10}, which is unlike the inert doublet model. Hence, the dark matter phenomenology in our theory is completely unique, as shown below.

B. Gauge symmetry breaking and Z_2 conservation

Since the lepton number is violated, all the neutral scalars can develop VEVs. We assume that the scalar potential is minimized at

$$\langle \phi \rangle = (0, v_\phi, 0), \quad \langle \chi \rangle = (u_\chi, 0, \omega_\chi), \quad \langle \eta \rangle = (u_\eta, 0, \omega_\eta), \tag{17}$$

with its value given by

$$\begin{aligned}V_{\min} = {} & \mu_1^2 v^*_\phi v_\phi + \mu_2^2 (u^*_\chi u_\chi + \omega^*_\chi \omega_\chi) + \mu_3^2 (u^*_\eta u_\eta + \omega^*_\eta \omega_\eta) + \lambda_1 (v^*_\phi v_\phi)^2 + \lambda_2 (u^*_\chi u_\chi + \omega^*_\chi \omega_\chi)^2 \\ & + \lambda_3 (u^*_\eta u_\eta + \omega^*_\eta \omega_\eta)^2 + \lambda_4 (v^*_\phi v_\phi)(u^*_\chi u_\chi + \omega^*_\chi \omega_\chi) + \lambda_5 (v^*_\phi v_\phi)(u^*_\eta u_\eta + \omega^*_\eta \omega_\eta) \\ & + \lambda_6 (u^*_\eta u_\eta + \omega^*_\eta \omega_\eta)(u^*_\chi u_\chi + \omega^*_\chi \omega_\chi) + \lambda_9 (u^*_\eta u_\chi + \omega^*_\eta \omega_\chi)(u^*_\chi u_\eta + \omega^*_\chi \omega_\eta) \\ & + \frac{1}{2}[\lambda_{10} (u^*_\eta u_\chi + \omega^*_\eta \omega_\chi)^2 + \mathrm{H.c.}].\end{aligned} \tag{18}$$

The conditions of potential minimization are therefore given by

$$\frac{\partial V_{\min}}{\partial v^*_\phi} = v_\phi [\mu_1^2 + 2\lambda_1 (v^*_\phi v_\phi) + \lambda_4 (u^*_\chi u_\chi + \omega^*_\chi \omega_\chi) + \lambda_5 (u^*_\eta u_\eta + \omega^*_\eta \omega_\eta)] = 0, \tag{19}$$

$$\frac{\partial V_{\min}}{\partial u_\chi^*} = u_\chi[\mu_2^2 + 2\lambda_2(u_\chi^* u_\chi + \omega_\chi^*\omega_\chi) + \lambda_4(v_\phi^* v_\phi) + \lambda_6(u_\eta^* u_\eta + \omega_\eta^*\omega_\eta)] + u_\eta[\lambda_9(u_\eta^* u_\chi + \omega_\eta^*\omega_\chi) + \lambda_{10}^*(u_\chi^* u_\eta + \omega_\chi^*\omega_\eta)] = 0, \quad (20)$$

$$\frac{\partial V_{\min}}{\partial \omega_\chi^*} = \omega_\chi[\mu_2^2 + 2\lambda_2(u_\chi^* u_\chi + \omega_\chi^*\omega_\chi) + \lambda_4(v_\phi^* v_\phi) + \lambda_6(u_\eta^* u_\eta + \omega_\eta^*\omega_\eta)] + \omega_\eta[\lambda_9(u_\eta^* u_\chi + \omega_\eta^*\omega_\chi) + \lambda_{10}^*(u_\chi^* u_\eta + \omega_\chi^*\omega_\eta)] = 0, \quad (21)$$

$$\frac{\partial V_{\min}}{\partial u_\eta^*} = u_\eta[\mu_3^2 + 2\lambda_3(u_\eta^* u_\eta + \omega_\eta^*\omega_\eta) + \lambda_5(v_\phi^* v_\phi) + \lambda_6(u_\chi^* u_\chi + \omega_\chi^*\omega_\chi)] + u_\chi[\lambda_9(u_\chi^* u_\eta + \omega_\chi^*\omega_\eta) + \lambda_{10}(u_\eta^* u_\chi + \omega_\eta^*\omega_\chi)] = 0, \quad (22)$$

$$\frac{\partial V_{\min}}{\partial \omega_\eta^*} = \omega_\eta[\mu_3^2 + 2\lambda_3(u_\eta^* u_\eta + \omega_\eta^*\omega_\eta) + \lambda_5(v_\phi^* v_\phi) + \lambda_6(u_\chi^* u_\chi + \omega_\chi^*\omega_\chi)] + \omega_\chi[\lambda_9(u_\chi^* u_\eta + \omega_\chi^*\omega_\eta) + \lambda_{10}(u_\eta^* u_\chi + \omega_\eta^*\omega_\chi)] = 0. \quad (23)$$

Let us denote

$$A = \mu_2^2 + 2\lambda_2(u_\chi^* u_\chi + \omega_\chi^*\omega_\chi) + \lambda_4(v_\phi^* v_\phi) + \lambda_6(u_\eta^* u_\eta + \omega_\eta^*\omega_\eta),$$
$$A' = \mu_3^2 + 2\lambda_3(u_\eta^* u_\eta + \omega_\eta^*\omega_\eta) + \lambda_5(v_\phi^* v_\phi) + \lambda_6(u_\chi^* u_\chi + \omega_\chi^*\omega_\chi),$$
$$B = \lambda_9(u_\eta^* u_\chi + \omega_\eta^*\omega_\chi) + \lambda_{10}^*(u_\chi^* u_\eta + \omega_\chi^*\omega_\eta).$$

Equations (20)–(23) are rewritten as

$$\begin{pmatrix} u_\chi & u_\eta \\ \omega_\chi & \omega_\eta \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix} = 0, \quad (24)$$

$$\begin{pmatrix} u_\chi & u_\eta \\ \omega_\chi & \omega_\eta \end{pmatrix}\begin{pmatrix} B^* \\ A' \end{pmatrix} = 0. \quad (25)$$

First of all, we suppose that the scalar potential is bounded from below. The necessary conditions are given by

$$\lambda_1 > 0, \qquad \lambda_2 > 0, \qquad \lambda_3 > 0, \quad (26)$$

which can be obtained when ϕ, χ, or η separately tend to infinity, respectively. To have a desired vacuum structure, we assume $\mu_{1,2}^2 < 0$, $\mu_3^2 > 0$, $\lambda_5 > 0$, and $\lambda_6 > 0$. The last three conditions are given so that $A' > 0$. (This will rearrange the general vacuum of the 3-3-1 model with right-handed neutrinos into the new one where its Z_2-even part is similar to the economical 3-3-1 model, while its Z_2-odd part conserves the Z_2 symmetry.) Hence, from Eq. (25) we have $u_\eta/u_\chi = \omega_\eta/\omega_\chi \equiv t$, and Eqs. (24) and (25) are reduced to

$$A + tB = 0, \qquad B^* + tA' = 0. \quad (27)$$

The second equation is rewritten as

$$t[A' + \lambda_9(|u_\chi|^2 + |\omega_\chi|^2)] + t^*\lambda_{10}(|u_\chi|^2 + |\omega_\chi|^2) = 0, \quad (28)$$

which implies $t = 0$, and thus $u_\eta = \omega_\eta = 0$ provided that $\lambda_6 + \lambda_9 \pm \lambda_{10} > 0$ (to have such a unique solution). We have also that $B = 0$ and $A = 0$, with the help of Eq. (27). Combining this with Eq. (19), we have the solution for potential minimization,

$$|v_\phi|^2 = \frac{2\lambda_2\mu_1^2 - \lambda_4\mu_2^2}{\lambda_4^2 - 4\lambda_1\lambda_2} \neq 0, \quad (29)$$

$$|u_\chi|^2 + |\omega_\chi|^2 = \frac{2\lambda_1\mu_2^2 - \lambda_4\mu_1^2}{\lambda_4^2 - 4\lambda_1\lambda_2} \neq 0, \quad (30)$$

$$u_\eta = \omega_\eta = 0. \quad (31)$$

We need extra conditions for the couplings,

$$-2\sqrt{\lambda_1\lambda_2} < \lambda_4 < \mathrm{Min}\{2\lambda_1(\mu_2/\mu_1)^2, 2\lambda_2(\mu_1/\mu_2)^2\}, \quad \lambda_7 > 0, \quad (32)$$

These conditions have been designed to make sure that the right-hand sides of Eqs. (29) and (30) as well as the physical scalar masses given below are positive. They are also needed in order for the scalar potential to be bounded from below, when ϕ and χ simultaneously tend to infinity.

It is easily realized that some of the relations given above are similar to the economical 3-3-1 model [19]. Because $\langle\eta\rangle = 0$, the Z_2 symmetry is conserved by the vacuum as well. Therefore, this symmetry is exact and is not spontaneously broken, similarly to R parity in supersymmetry. Consequently, the so-called "inert" scalar triplet η—the only multiplet in the model that is charged under Z_2 (odd)—behaves like the superparticles in supersymmetry, which is distinguished from the remaining sector of Z_2-even normal matter. The lightest inert particle (LIP) contained in the η triplet, which cannot decay due to the Z_2 symmetry, may provide dark matter candidates. On the other hand, as already mentioned η does not couple to fermions in the Yukawa sector, but it can couple to gauge bosons and other scalars via the Z_2-conserving interactions. It does not give masses for the fermions or the gauge bosons since $\langle\eta\rangle = 0$. The identities and masses of the physical fermions and gauge bosons are exactly the same as in the economical 3-3-1 model [19]. However, the scalar sector will be changed, which is presented below. The interaction between the two sectors—Z_2-even and -odd—will also be obtained.

For convenience, we redefine $u \equiv u_\chi$, $\omega \equiv \omega_\chi$, and $v \equiv v_\phi$. Thus we have

$$\langle\phi\rangle = (0, v, 0), \quad \langle\chi\rangle = (u, 0, \omega), \quad \langle\eta\rangle = (0, 0, 0), \tag{33}$$

where v, u, and ω satisfy the potential minimization conditions (29) and (30), with the labels "ϕ" and "χ" removed. To be consistent with the standard model, we assume $u^2 \ll v^2 \ll \omega^2$, where $v = v_{\mathrm{weak}} = 174$ GeV, $u = \mathcal{O}(1)$ GeV, and $\omega = \mathcal{O}(1)$ TeV [19]. Also, the conditions for the scalar potential parameters can be summarized as follows:

$$\mu^2_{1,2} < 0 < \mu^2_3, \quad \lambda_{1,2,3,5,6,7} > 0, \quad \lambda_6 + \lambda_9 \pm \lambda_{10} > 0,$$
$$-2\sqrt{\lambda_1\lambda_2} < \lambda_4 < \mathrm{Min}\{2\lambda_1(\mu_2/\mu_1)^2, 2\lambda_2(\mu_1/\mu_2)^2\}. \tag{34}$$

As mentioned, this ensures that (i) the potential is bounded from below, (ii) the physical scalar masses are positive, (iii) the Z_2 symmetry is conserved by the vacuum, and (iv) the nonzero VEVs v, u, and ω, induce the gauge symmetry breaking and mass generation in the correct way, similarly to the economical 3-3-1 model [19].

C. Scalar identification, dark matter, and interactions

The mass terms of physical scalar fields are obtained from the scalar potential by shifting the vacuum values of the beginning scalars. They are given by

$$\begin{aligned} V_{\mathrm{mass}} = {} & M_3^2\eta^\dagger\eta + \lambda_8|v|^2\eta_2^-\eta_2^+ + \lambda_9|u^*\eta_1^0 + \omega^*\eta_3^0|^2 + \frac{1}{2}[\lambda_{10}^*(u^*\eta_1^0 + \omega^*\eta_3^0)^2 + \mathrm{H.c.}] + \lambda_1(v^*\phi_2^0 + v\phi_2^{0*})^2 \\ & + \lambda_2(u^*\chi_1^0 + \omega^*\chi_3^0 + u\chi_1^{0*} + \omega\chi_3^{0*})^2 + \lambda_4(v^*\phi_2^0 + v\phi_2^{0*})(u^*\chi_1^0 + \omega^*\chi_3^0 + u\chi_1^{0*} + \omega\chi_3^{0*}) \\ & + \lambda_7(v^*\chi_2^- + u\phi_1^- + \omega\phi_3^-)(v\chi_2^+ + u^*\phi_1^+ + \omega^*\phi_3^+), \end{aligned} \tag{35}$$

where $M_3^2 \equiv \mu_3^2 + \lambda_5|v|^2 + \lambda_6(|u|^2 + |\omega|^2)$, and the conditions of potential minimization as given above have been used. Also, the notations for physical scalar fields are the same as those used before.

Inert scalar sector (η): $h_\eta^\pm \equiv \eta_2^\pm$ is the physical charged inert scalar field by itself with its mass given by

$$m^2_{h_\eta^\pm} = M_3^2 + \lambda_8 v^2. \tag{36}$$

For the remaining inert fields let us define

$$\eta_1^0 = \frac{R_1 + iI_1}{\sqrt{2}}, \qquad \eta_3^0 = \frac{R_3 + iI_3}{\sqrt{2}}. \tag{37}$$

The mass Lagrangian for the neutral inert scalar fields is arranged as

$$\frac{1}{2}(R_1 I_1 R_3 I_3)M^2\begin{pmatrix} R_1 \\ I_1 \\ R_3 \\ I_3 \end{pmatrix}, \tag{38}$$

in which the mass matrix M^2 is given by

$$\begin{pmatrix} M_3^2 + \lambda_9|u|^2 + \mathrm{Re}(\lambda_{10}u^2) & \mathrm{Im}(\lambda_{10}u^2) & \mathrm{Re}[\omega(\lambda_9u^* + \lambda_{10}u)] & \mathrm{Im}[u(\lambda_{10}\omega - \lambda_9\omega^*)] \\ \mathrm{Im}(\lambda_{10}u^2) & M_3^2 + \lambda_9|u|^2 - \mathrm{Re}(\lambda_{10}u^2) & \mathrm{Im}[u(\lambda_{10}\omega + \lambda_9\omega^*)] & \mathrm{Re}[\omega(\lambda_9u^* - \lambda_{10}u)] \\ \mathrm{Re}[\omega(\lambda_9u^* + \lambda_{10}u)] & \mathrm{Im}[u(\lambda_{10}\omega + \lambda_9\omega^*)] & M_3^2 + \lambda_9|\omega|^2 + \mathrm{Re}(\lambda_{10}\omega^2) & \mathrm{Im}(\lambda_{10}\omega^2) \\ \mathrm{Im}[u(\lambda_{10}\omega - \lambda_9\omega^*)] & \mathrm{Re}[\omega(\lambda_9u^* - \lambda_{10}u)] & \mathrm{Im}(\lambda_{10}\omega^2) & M_3^2 + \lambda_9|\omega|^2 - \mathrm{Re}(\lambda_{10}\omega^2) \end{pmatrix}.$$

We recall that the scalar potential conserves CP, so we can assume that λ_{10} is real (otherwise its phase can be absorbed by redefining the relative phases of η and χ, as already mentioned). In addition, the vacuum structure as obtained does not support any spontaneous CP phase, i.e., the CP symmetry is not spontaneously broken by the VEVs in this case. Therefore, without loss of generality we can assume that u, ω, and v are all real. All the imaginary parts contained in the mass matrix vanish. Consequently, R_1 and R_3 mix, but are separate from $I_{1,3}$ and vice versa. We have the physical fields

$$h_\eta = c_\theta R_1 - s_\theta R_3, \qquad H_\eta = s_\theta R_1 + c_\theta R_3, \tag{39}$$

$$a_\eta = c_\theta I_1 - s_\theta I_3, \qquad A_\eta = s_\theta I_1 + c_\theta I_3, \tag{40}$$

with masses

$$m^2_{h_\eta} = M_3^2, \qquad m^2_{a_\eta} = M_3^2, \tag{41}$$

$$\begin{aligned} m^2_{H_\eta} &= M_3^2 + (\lambda_9 + \lambda_{10})(u^2+\omega^2), \\ m^2_{A_\eta} &= M_3^2 + (\lambda_9 - \lambda_{10})(u^2+\omega^2). \end{aligned} \tag{42}$$

Here, we have defined $s_\theta \equiv \sin(\theta)$, $c_\theta \equiv \cos(\theta)$, and so forth, with

$$t_\theta = \frac{u}{\omega}. \tag{43}$$

We notice that θ is the mixing angle of the charged gauge bosons W-Y, which must be small [19]. In the effective limit, we have $h_\eta \simeq R_1$, $a_\eta \simeq I_1$, $H_\eta \simeq R_3$, and $A_\eta \simeq I_3$. The degeneracy of the a_η and h_η masses is due to the fact that a coupling of η and ϕ that is similar to λ_{10} is suppressed by the gauge symmetry, which is unlike the case of the inert doublet model [21]. On the other hand, if the Z_2 symmetry was spontaneously broken, i.e., $\langle\eta\rangle \neq 0$ (which is not the case in the present work), the CP would be spontaneously broken too. In such a case, the degenerate masses of a_η and h_η would be separated.

Normal scalar sector (ϕ, χ): This section is identical to that of the economical 3-3-1 model, which can be adapted from Ref. [19] as given below for convenience. There are 12 real scalar fields in total for this sector, in which eight of them are Goldstone bosons eliminated by the corresponding eight massive gauge bosons associated with the broken gauge generators $[\mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X]/\mathrm{U}(1)_Q$. There remain four physical scalar fields, one charged (and its Hermitian conjugate) and two neutral, respectively obtained by

$$\begin{aligned} H^\pm &= s_\xi \chi_2^\pm + c_\xi(s_\theta \phi_1^\pm + c_\theta \phi_3^\pm), \\ m^2_{H^\pm} &= \lambda_7(u^2+v^2+\omega^2) \simeq \lambda_7 \omega^2, \\ h &= c_\zeta S_2 - s_\zeta(s_\theta S_1 + c_\theta S_3), \end{aligned} \tag{44}$$

$$\begin{aligned} m_h^2 &= 2\lambda_1 v^2 + 2\lambda_2(u^2+\omega^2) \\ &\quad - 2\sqrt{[\lambda_1 v^2 - \lambda_2(u^2+\omega^2)]^2 + \lambda_4^2 v^2(u^2+\omega^2)} \\ &\simeq \frac{4\lambda_1\lambda_2 - \lambda_4^2}{\lambda_2} v^2, \end{aligned} \tag{45}$$

$$\begin{aligned} H &= s_\zeta S_2 + c_\zeta(s_\theta S_1 + c_\theta S_3), \\ m_H^2 &= 2\lambda_1 v^2 + 2\lambda_2(u^2+\omega^2) \\ &\quad + 2\sqrt{[\lambda_1 v^2 - \lambda_2(u^2+\omega^2)]^2 + \lambda_4^2 v^2(u^2+\omega^2)} \\ &\simeq 4\lambda_2\omega^2, \end{aligned} \tag{46}$$

where we have defined

$$\phi_2^0 = \frac{S_2 + iA_2}{\sqrt{2}}, \qquad \chi_1^0 = \frac{S_1 + iA_1}{\sqrt{2}}, \qquad \chi_3^0 = \frac{S_3 + iA_3}{\sqrt{2}}, \tag{47}$$

and

$$\begin{aligned} t_\xi &= \frac{m_W}{m_X} = \frac{v}{\sqrt{u^2+\omega^2}} \simeq \frac{v}{\omega}, \\ t_{2\zeta} &= \frac{\lambda_4 t_\xi}{\lambda_2 - \lambda_1 t_\xi^2} \simeq (\lambda_4/\lambda_2) t_\xi. \end{aligned} \tag{48}$$

The mixing angles ξ and ζ must be small. The h is the standard model-like Higgs boson. The H and $H^\pm$ are the new Higgs bosons with masses at the ω scale. The Goldstone bosons are $G_Z = A_2$, $G_{Z'} = A_3$, $G_X^{0/0*} = (G_4 \pm iA_1)/\sqrt{2}$, with $G_4 = c_\theta S_1 - s_\theta S_3$, $G_W^\pm = c_\theta \phi_1^\pm - s_\theta \phi_3^\pm$, and $G_Y^\pm = c_\xi \chi_2^\pm - s_\xi(s_\theta \phi_1^\pm + c_\theta \phi_3^\pm)$. In the effective limit, we can summarize this as [19]

$$\phi \simeq \begin{pmatrix} G_W^+ \\ v + \frac{1}{\sqrt{2}}(h + iG_Z) \\ H^+ \end{pmatrix}, \qquad \chi \simeq \begin{pmatrix} u + G_X \\ G_Y^- \\ \omega + \frac{1}{\sqrt{2}}(H + iG_{Z'}) \end{pmatrix}. \tag{49}$$

We recall the fact that $M_3^2 \equiv \mu_3^2 + \lambda_5 v^2 + \lambda_6(u^2+\omega^2)$ is always at the scale of ω^2, independent of whether μ_3^2 is at the weak scale v^2 or at the 3-3-1 scale ω^2. Therefore, all the inert particles in this model are always heavy ($\sim\omega$). This is different from the inert doublet model, where the inert particles are naturally at the weak scale. Depending on the relations of λ_9 and λ_{10} and their signs, we can determine which inert particle is the LIP. There are three cases:

(1) H_η is the LIP: $\lambda_{10} < \mathrm{Min}\{0, -\lambda_9\}$
(2) A_η is the LIP: $\lambda_{10} > \mathrm{Max}\{0, \lambda_9\}$
(3) h_η and a_η are LIPs: $-\lambda_9 < \lambda_{10} < \lambda_9$

Because h_η and a_η are degenerate in mass, the third case may be ruled out by the direct detection experiments due to their scattering with nuclei via the Z exchange channel [23], which is unlike the inert doublet model. The first and second cases are realistic, which only exist in the 3-3-1 model. However, in the following we consider only the first case with the dark matter H_η. For the second case with A_η, the calculations can be done similarly.

To close this section, let us calculate the interactions between the two sectors, inert and normal. As mentioned, the inert scalars interact only with normal scalars and gauge bosons, and not with fermions. The effect of mixings, such as θ, ζ, and ξ, will be neglected in the present work since they give very small contributions due to the constraints $u \ll v \ll \omega$. The scalar interactions are obtained as follows:

$$\begin{aligned} V_{\text{normal-inert}} = & \left[(\lambda_5+\lambda_8)\left(\sqrt{2}vh+\frac{h^2}{2}\right)+\lambda_5 H^+H^- +\lambda_6\left(\sqrt{2}\omega H+\frac{H^2}{2}\right)\right]h^+_\eta h^-_\eta \\ & +\left[\lambda_5\left(\sqrt{2}vh+\frac{h^2}{2}\right)+\lambda_5 H^+H^- +\lambda_6\left(\sqrt{2}\omega H+\frac{H^2}{2}\right)\right]\frac{a^2_\eta+h^2_\eta}{2} \\ & +\left[\lambda_5\left(\sqrt{2}vh+\frac{h^2}{2}\right)+(\lambda_5+\lambda_8) H^+H^- +(\lambda_6+\lambda_9+\lambda_{10})\left(\sqrt{2}\omega H+\frac{H^2}{2}\right)\right]\frac{H^2_\eta}{2} \\ & +\left[\lambda_5\left(\sqrt{2}vh+\frac{h^2}{2}\right)+(\lambda_5+\lambda_8) H^+H^- +(\lambda_6+\lambda_9-\lambda_{10})\left(\sqrt{2}\omega H+\frac{H^2}{2}\right)\right]\frac{A^2_\eta}{2} \\ & +\frac{u(\lambda_9+\lambda_{10})}{\sqrt{2}}Hh_\eta H_\eta+\frac{u(\lambda_9-\lambda_{10})}{\sqrt{2}}Ha_\eta A_\eta+\left[\frac{\lambda_8}{2}(\sqrt{2}v+h)H^+h^-_\eta(H_\eta-iA_\eta)+\text{H.c.}\right]. \end{aligned} \tag{50}$$

The identity of the gauge bosons can be found in Ref. [19]. Hence, the interactions between the inert scalars and gauge bosons can be derived from the Lagrangian (14). The triple interactions of two inert scalars and one gauge boson are

$$\begin{aligned} \mathcal{L}^{\text{triple}}_{\text{gauge-inert}} = & \frac{g}{2}\left(\frac{1}{c_W}Z^\mu+\frac{c_{2W}}{c_W\sqrt{3-4s^2_W}}Z'^\mu\right)h_\eta \overleftrightarrow{\partial}_\mu a_\eta \\ & -g\frac{c_W}{\sqrt{3-4s^2_W}}Z'^\mu H_\eta \overleftrightarrow{\partial}_\mu A_\eta + i\frac{g}{2}\left(-2s_W A^\mu-\frac{c_{2W}}{c_W}Z^\mu+\frac{c_{2W}}{c_W\sqrt{3-4s^2_W}}Z'^\mu\right)h^-_\eta \overleftrightarrow{\partial}_\mu h^+_\eta \\ & +\frac{g}{\sqrt{2}}\left(iW^{+\mu}h^-_\eta \overleftrightarrow{\partial}_\mu \frac{h_\eta-ia_\eta}{\sqrt{2}}+iX^{0\mu}\frac{H_\eta+iA_\eta}{\sqrt{2}}\overleftrightarrow{\partial}_\mu \frac{h_\eta-ia_\eta}{\sqrt{2}}+iY^{-\mu}\frac{H_\eta+iA_\eta}{\sqrt{2}}\overleftrightarrow{\partial}_\mu h^+_\eta+\text{H.c.}\right), \end{aligned} \tag{51}$$

where we have denoted $A\overleftrightarrow{\partial}_\mu B = A(\partial_\mu B)-(\partial_\mu A)B$. The quartic interactions of two inert scalars and two gauge bosons are given by

$$\begin{aligned} \mathcal{L}^{\text{quartic}}_{\text{gauge-inert}} = & \frac{g^2}{2}\left[\frac{1}{2}\left(\frac{1}{c_W}Z_\mu+\frac{c_{2W}}{c_W\sqrt{3-4s^2_W}}Z'_\mu\right)^2+W^+_\mu W^{-\mu}+X^{0*}_\mu X^{0\mu}\right]\frac{h^2_\eta+a^2_\eta}{2} \\ & +\frac{g^2}{2}\left[\frac{1}{2}\left(-2s_W A_\mu-\frac{c_{2W}}{c_W}Z_\mu+\frac{c_{2W}}{c_W\sqrt{3-4s^2_W}}Z'_\mu\right)^2+W^+_\mu W^{-\mu}+Y^+_\mu Y^{-\mu}\right]h^+_\eta h^-_\eta \\ & +\frac{g^2}{2}\left[\frac{2c^2_W}{3-4s^2_W}Z'_\mu Z'^\mu+X^{0*}_\mu X^{0\mu}+Y^+_\mu Y^{-\mu}\right]\frac{H^2_\eta+A^2_\eta}{2} \\ & +\frac{g^2}{2\sqrt{2}}\left\{\left[2\left(-s_W A_\mu+\frac{s^2_W}{c_W}Z_\mu+\frac{c_{2W}}{c_W\sqrt{3-4s^2_W}}Z'_\mu\right)W^{+\mu}+\sqrt{2}X^0_\mu Y^{+\mu}\right]\frac{h_\eta-ia_\eta}{\sqrt{2}}h^-_\eta\right. \\ & +\left[\left(\frac{1}{c_W}Z_\mu-\frac{1}{c_W\sqrt{3-4s^2_W}}Z'_\mu\right)X^{0\mu}+\sqrt{2}W^+_\mu Y^{-\mu}\right]\frac{h_\eta-ia_\eta}{\sqrt{2}}\frac{H_\eta+iA_\eta}{\sqrt{2}} \\ & \left.+\left[-\left(2s_W A_\mu+\frac{c_{2W}}{c_W}Z_\mu+\frac{1}{c_W\sqrt{3-4s^2_W}}Z'_\mu\right)Y^{-\mu}+\sqrt{2}W^-_\mu X^{0\mu}\right]h^+_\eta\frac{H_\eta+iA_\eta}{\sqrt{2}}+\text{H.c.}\right\}. \end{aligned} \tag{52}$$

We recall that in Eqs. (51) and (52) A_μ is the photon field and Z_μ and $W^\pm_\mu$ are standard model-like, whereas Z'_μ is a new neutral gauge boson and $X^{0,0*}_\mu$, $Y^\pm_\mu$ are new non-Hermitian gauge bosons. From Eqs. (50)–(52), we explicitly see that the inert particles are only coupled in pairs in the interactions, as predicted. Also, the Feynman rules due to these interactions as used below are easily derived, which should be understood. The ordinary Feynman rules of the economical 3-3-1 model can be found in Ref. [19].

III. DARK MATTER CONSTRAINT

A. Relic density

We can discuss two cases: (i) H_η is lighter than every new particle of the economical 3-3-1 model, such as H, $H^\pm$, Z', X, Y, U, D, and ν_R; (ii) H_η is heavier than some or all these new particles. Which case is relevant depends on the parameter space (μ_3, ω, $\lambda_{2,6,7,9,10}$, and $h^{U,D}$) of the model. In the first case, the contribution to the dark matter relic density includes only the annihilation processes of dark matter into the standard model particles. In the second case, the dark matter can be annihilated into the new particles of the economical 3-3-1 model, which may dominate over the standard model productions. For our purpose, in this work it is sufficient to consider only the first case.

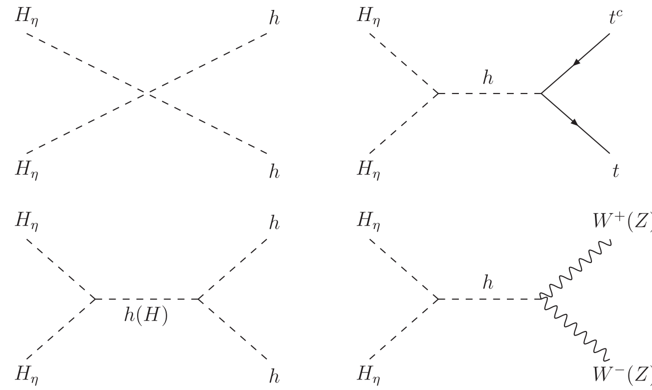

FIG. 1. Dominant contributions to H_η annihilation when it is lighter than the new particles of the economical 3-3-1 model.

Also, the coannihilation of H_η with A_η, h_η, a_η, or $h^\pm_\eta$ will be neglected. We leave a study of the second case to future work.

The dominant contributions to the relic density of dark matter H_η come from the diagrams given in Fig. 1. The thermal average on the cross section times the relative velocity between two incoming dark matter particles is

$$\begin{aligned}\langle\sigma v_{\mathrm{rel}}\rangle &= \frac{1}{64\pi m^2_{H_\eta}}\left(1-\frac{6}{x_F}-\frac{1}{2}\frac{m_h^2}{m^2_{H_\eta}}\right)\left[\lambda_5-3\lambda_5\frac{m_h^2}{4m^2_{H_\eta}}\left(1+\frac{m_h^2}{4m^2_{H_\eta}}\right)-2\lambda_4\frac{m^2_{H_\eta}-\mu_3^2}{4m^2_{H_\eta}-m_H^2}\right]^2\\
&\quad+\frac{3}{16\pi x_F m^2_{H_\eta}}\left(1-\frac{1}{2}\frac{m_h^2}{m^2_{H_\eta}}\right)\left[\lambda_5-3\lambda_5\frac{m_h^2}{4m^2_{H_\eta}}\left(1+\frac{m_h^2}{4m^2_{H_\eta}}\right)-2\lambda_4\frac{m^2_{H_\eta}-\mu_3^2}{4m^2_{H_\eta}-m_H^2}\right]\\
&\quad\times\left[3\lambda_5\frac{m_h^2}{4m^2_{H_\eta}}+8\lambda_4\frac{(m^2_{H_\eta}-\mu_3^2)m^2_{H_\eta}}{(4m^2_{H_\eta}-m_H^2)^2}\right]+\frac{\lambda_5^2}{64\pi}\left[\frac{m_W^4}{m^6_{H_\eta}}\left(1-\frac{18}{x_F}-\frac{1}{2}\frac{m_W^2-m_h^2}{m^2_{H_\eta}}\right)+\frac{2}{m^2_{H_\eta}}\left(1-\frac{6}{x_F}-\frac{3m_W^2-m_h^2}{2m^2_{H_\eta}}\right)\right]\\
&\quad+\frac{\lambda_5^2}{128\pi}\left[\frac{m_Z^4}{m^6_{H_\eta}}\left(1-\frac{18}{x_F}-\frac{1}{2}\frac{m_Z^2-m_h^2}{m^2_{H_\eta}}\right)+\frac{2}{m^2_{H_\eta}}\left(1-\frac{6}{x_F}-\frac{3m_Z^2-m_h^2}{2m^2_{H_\eta}}\right)\right]+\frac{3\lambda_5^2 m_t^2}{64\pi m^4_{H_\eta}}\left(1-\frac{12}{x_F}-\frac{3m_t^2}{2m^2_{H_\eta}}+\frac{m_h^2}{2m^2_{H_\eta}}\right).\end{aligned} \tag{53}$$

Here, we have used the fact that the H_η is nonrelativistic, and the result is given as an expansion up to the squared velocity of H_η with $\langle v^2\rangle = 6/x_F$ and $x_F = m_{H_\eta}/T_F \sim 20$ at the freeze-out temperature [24]. Also, we have utilized the approximation $m^2_{H_\eta} - \mu_3^2 = \lambda_5 v^2 + (\lambda_6 + \lambda_9 + \lambda_{10})(u^2 + \omega^2) \simeq (\lambda_6 + \lambda_9 + \lambda_{10})\omega^2$ due to $u^2, v^2 \ll \omega^2$.

Because H_η is lighter than the new particles of the economical 3-3-1 model (with the masses $\sim\omega$), it strongly imposes $\mu_3^2 \ll \omega^2$, i.e., $(\lambda_6 + \lambda_9 + \lambda_{10})\omega^2 \simeq m^2_{H_\eta} - \mu_3^2 \simeq m^2_{H_\eta}$. Therefore, the parameter space in the first case is given by appropriate conditions on the coupling $\lambda_6 + \lambda_9 + \lambda_{10}$. For example, for H with mass $m_H^2 \simeq 4\lambda_2\omega^2$, the condition is $\lambda_6 + \lambda_9 + \lambda_{10} < 4\lambda_2$. However, it is noticed that the following discussions are unchanged for any size of μ_3^2 that satisfies the present case. Because the dark matter H_η is naturally heavy at the ω scale, the ratios $\frac{m_W^2}{m^2_{H_\eta}}$, $\frac{m_Z^2}{m^2_{H_\eta}}$, $\frac{m_h^2}{m^2_{H_\eta}}$, and $\frac{m_t^2}{m^2_{H_\eta}}$ are negligible and can be terminated in the effective limit. Hence, the result (53) can be approximated as

$$\langle\sigma v_{\mathrm{rel}}\rangle \simeq \frac{\alpha^2}{(150\ \mathrm{GeV})^2}\left(\frac{\lambda_5\times 1.92\mathrm{TeV}}{m_{H_\eta}}\right)^2 \times (1.04 + 0.35a^2 + 2.39ab), \tag{54}$$

where $\alpha \simeq 1/128$ is the fine structure constant, $x_F = 20$ has been used, and

$$a \equiv 1 - 2\frac{\lambda_4}{\lambda_5}\frac{m^2_{H_\eta}}{4m^2_{H_\eta} - m^2_H}, \quad b \equiv \frac{\lambda_4}{\lambda_5}\frac{m^4_{H_\eta}}{(4m^2_{H_\eta} - m^2_H)^2}. \tag{55}$$

The dark matter density can be evaluated as $\Omega_{H_\eta}h^2 \simeq 0.1\text{pb}/\langle\sigma v_{\text{rel}}\rangle$ [24], which depends on only four parameters, such as $m^2_{H_\eta}$, m^2_H, λ_5, and λ_4 because of Eq. (53), or alternatively m_{H_η}/λ_5, a, and b due to Eq. (54). Since $\frac{\alpha^2}{(150\ \text{GeV})^2} \simeq 1$ pb, the WMAP data $\Omega_{H_\eta}h^2 \simeq 0.11$ [1] imply

$$m_{H_\eta} \simeq \lambda_5 \times \sqrt{1.04 + 0.35a^2 + 2.39ab} \times 2\ \text{TeV}. \tag{56}$$

Because H and H_η have masses at the ω scale, a and b can naturally be of the order of unity [their correct values can be derived from Eq. (55), which depend on only the scalar coupling ratios λ_4/λ_5 and $(\lambda_6 + \lambda_9 + \lambda_{10})/\lambda_2$]. Moreover, the λ_5 coupling is constrained by $0 < \lambda_5 < 8\pi$ (the right inequality exists if we require the potential to be perturbative), which is of the order of unity as well. Consequently, the dark matter H_η has the right relic density, with its mass naturally at the TeV scale due to Eq. (56), $m_{H_\eta} = \mathcal{O}(1)$ TeV. To be concrete, let us give an estimation as follows. Since, in the present case, the H_η considered is the lightest among the new particles including H, we can suppose that m^2_H is large enough in comparison to that of H_η so that the squared-mass ratios in a and b are negligible, and thus $a \simeq 1$ and $b \simeq 0$ (this also applies when H does not couple to h, i.e., $\lambda_4 = 0$). Therefore, we have

$$m_{H_\eta} \simeq \lambda_5 \times 2\ \text{TeV}, \tag{57}$$

which is around 2 TeV if one takes λ_5 to be of the order of unity.

The inert doublet model provides a LIP dark matter candidate at either the weak or TeV scales. However, our model implies only the LIP dark matter at the TeV scale, behaving as a scalar singlet under the standard model symmetry. The TeV mass of dark matter in our model is a natural consequence of the 3-3-1 symmetry-breaking scale (ω). However, in the inert doublet model, since there is only a scale v the large mass is only enhanced by the large scalar coupling, which reaches the applicable limit of perturbative theory. In this case, the normal sector and the inert sector become strongly coupled, which contradicts our case with the usual scalar couplings such as $\lambda_5 \sim 1$, as explained above.

B. Direct searches

Direct dark matter searches measure the recoil energy deposited by the dark matter scattering off the nuclei in a large detector. This scattering is due to the interactions of dark matter with quarks confined in nucleons. Since the dark matter is very nonrelativistic, the process can be described by the effective Lagrangian [25]

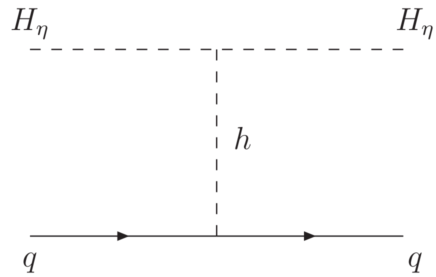

FIG. 2. Dominant contributions to H_η-quark scattering.

$$\mathcal{L}_S = 2\lambda_q m_{H_\eta} H_\eta H_\eta \bar{q}q. \tag{58}$$

Note that for the real scalar field only spin-independent and even interactions are possible. The effective interaction above can be obtained by the t-channel exchange of h, as depicted in Fig. 2. Therefore, we have

$$\lambda_q = \frac{\lambda_5 m_q}{2m_{H_\eta} m^2_h}. \tag{59}$$

The H_η-nucleon scattering amplitude can be given as a summation over the quark-level interactions with the respective nucleon form factors. The H_η-nucleon cross section is

$$\sigma_{H_\eta - N} = \frac{4m^2_r}{\pi}\lambda^2_N, \tag{60}$$

where $N = p, n$ denotes a nucleon, and

$$\begin{aligned} m_r &= \frac{m_{H_\eta} m_N}{m_{H_\eta} + m_N} \simeq m_N, \\ \frac{\lambda_N}{m_N} &= \sum_{u,d,s} f^N_{Tq}\frac{\lambda_q}{m_q} + \frac{2}{27}f^N_{TG}\sum_{c,b,t}\frac{\lambda_q}{m_q} \simeq 0.35\frac{\lambda_5}{2m_{H_\eta}m^2_h}, \end{aligned} \tag{61}$$

with $f^N_{TG} = 1 - \sum_{u,d,s} f^N_{Tq}$, and the f^N_{Tq} values have been taken from Ref. [26]. Let $m_N = 1$ GeV and $m_h = 125$ GeV [2,3]. We have

$$\sigma_{H_\eta - N} \simeq \left(\frac{\lambda_5 \times 2\ \text{TeV}}{m_{H_\eta}}\right)^2 \times 1.56 \times 10^{-44}\ \text{cm}^2. \tag{62}$$

Since the value in parenthesis is of the order of unity (as given above), the cross section is in good agreement with the XENON100 experimental data [27]. If the mass of H is much larger than H_η, the model predicts

$$\sigma_{H_\eta - N} = 1.56 \times 10^{-44}\ \text{cm}^2 \tag{63}$$

for the dark matter with a mass in the TeV range.

IV. THE NECESSITY OF THIS WORK AND ITS IMPLICATION

We have given a discussion on the dark matter search status in the 3-3-1 models in Ref. [7]. Here we will provide a detailed analysis in order to show explicitly why this work is needed. Its significance for solving the dark matter problem in typical 3-3-1 models is also given.

A. Why this work is needed

As a result of the 3-3-1 gauge symmetry and its particle content, the gauge interactions, minimal Yukawa Lagrangian, and the minimal scalar potential of the theory normally couple the new particles concerned in pairs when interacting with the standard model particles, similarly to superparticles in supersymmetry [7,17]. Therefore, the extended sectors in 3-3-1 models—such as the scalar, fermion, and gauge sectors—have usually been thought as providing some candidates for dark matter. However, the problem encountered is how to suppress the unwanted interactions and vacuums [7,18], which lead to the fast decay of dark matter. In the typical 3-3-1 models [12,13], the new particles concerned are bileptons and the unwanted interactions (other than the minimal interactions) are the ones that violate the lepton number [18]. In the 3-3-1 model with right-handed neutrinos, the unwanted vacuums are the ones where neutral scalar bileptons, such as χ_1^0 and η_3^0, develop nonzero VEVs.

The first three articles of Ref. [17] were the first works that identified dark matter candidates in 3-3-1 models. However, their stability mechanism was not given. The first article of Ref. [17] discussed dark matter in the minimal 3-3-1 model; however, it gave a incorrect identification of dark matter. In fact, the candidate obtained therein (which is similar to the imaginary part of χ_3^0 in this paper) is the Goldstone boson of the Z' gauge boson, which is an unphysical particle. Even if the corresponding Higgs scalar mentioned therein (which is similar to the real part of χ_3^0) was interpreted as dark matter, it will decay into the standard model particles via the tree-level coupling of the candidate to the standard model Higgs bosons $\sim\omega\mathrm{Re}(\chi_3^0)hh$ (since it has a VEV ω). As a matter of fact, the minimal 3-3-1 model in its current form may contain no dark matter.

The second and third articles of Ref. [17] gave a discussion of dark matter in the 3-3-1 model with right-handed neutrinos. The candidates identified were the real and/or imaginary parts of η_3, as in this paper. However, the mechanism that provides dark matter stability was not provided. Hence, there is no reason why η_3^0 (or even χ_1^0) cannot develop a VEV and have its lepton-number-violating interactions turned on, which leads to the tree-level couplings of dark matter with the standard model particles. For example, when η_3^0 develops a VEV, its real part will decay into two standard model Higgs bosons. Moreover, both the real and imaginary parts will decay into light quarks due to the mixing of ordinary and exotic quarks. The presence of lepton-number-violating Yukawa interactions will lead to the decay of the candidate into light quarks for the same reason as in the previous example. However, the lepton-number-violating scalar potential would lead to the tree-level coupling of the candidate to the two standard model Higgs bosons. In addition, the neutral scalar bileptons including the candidate might develop VEVs due to these violating scalar interactions.

To solve the above problems, the fourth article of Ref. [17] was the first one to introduce an extra symmetry for dark matter stability in 3-3-1 models. It studied the 3-3-1 model with right-handed neutrinos and regarded the lepton number symmetry as a mechanism for dark matter stability. It was intriguing that this symmetry would suppress all the unwanted interactions and vacuums, which violate or break the lepton number. There, the lightest bilepton particle (possibly η_3^0, as assumed in the fourth article of Ref. [17]) was predicted to be responsible for the stabilization of dark matter. However, the problem was to generate the mass for neutrinos. As Ref. [28] cited therein, the neutrinos would get masses from five-dimensional effective interactions which explicitly violate the lepton number (it was in contradiction to the postulate). In fact, these interactions will lead to the fast decay of dark matter into light neutrinos because there are mixings between right-handed and left-handed neutrinos.

To overcome the above difficulty, the fifth article of Ref. [17] introduced another lepton sector (the model was changed and called the 3-3-1 model with left-handed neutrinos) so that the bilepton character of the new particles is lost. The lepton number symmetry takes no role in stabilizing dark matter. Instead, a Z_2 symmetry or $\mathrm{U}(1)_G$ were included. The Z_2 must be broken by the Higgs vacuum. Therefore, there is no reason why the dark matter η_3^0 that carries no lepton number cannot develop a VEV and decay then. On the other hand, the $\mathrm{U}(1)_G$ must be broken due to its nontrivial dynamics, as was shown in Ref. [7]. It cannot prevent the dark matter from decaying. A suggestion in Ref. [7] was that G parity, $(-1)^G$, may be a mechanism for dark matter stability. In Ref. [7], we gave a mechanism for dark matter stability based on W parity that was similar to R parity in supersymmetry. However, the dark matter model works only with the fermion content of the 3-3-1 model with neutral fermions.

To conclude, the problems with dark matter identification and stability in the typical 3-3-1 models, the 3-3-1 model with right-handed neutrinos, and the minimal 3-3-1 model remain unsolved, which has drawn our attention.

Via our work given above, we see that the typical 3-3-1 models are only self-consistent if they contain interactions that explicitly violate the lepton number. If one scalar triplet of the 3-3-1 model with right-handed neutrinos is inert (Z_2-odd), the other two scalar triplets will result in an economical 3-3-1 model that is self-consistent. This model provides consistent masses for neutrinos [19]. [The neutrinos can get masses via two ways, similar to the economical 3-3-1 model: through radiative corrections (as given in the sixth article of Ref. [19]), or alternatively through effective interactions (as given in the sixth and eighth articles of Ref. [19]). In all these ways, the neutrino masses are generated due to the contributions of only χ and ϕ (Z_2-even scalars), while η does not contribute due to $\langle\eta\rangle = 0$ under the Z_2 symmetry. The generation of neutrino masses is also accompanied by the interaction of inert scalars (η) with leptons (ψ). But, since the theory conserves the Z_2 symmetry, the inert scalars (η) which are odd under Z_2 are only coupled in pairs in such interactions. For example, an effective interaction can be included as $\bar{\psi}^c_L \psi_L (\eta\eta)^*$ and its Hermitian conjugate, which leads to the interactions $\psi_i \psi_j \eta^*_i \eta^*_j$, where i, j are $SU(3)_L$ indices. Since all the components η_i are odd under Z_2, this may lead to the decay of an inert scalar (η_i) with a larger mass into another inert scalar (η_j) with a smaller mass (associated with two leptons $\psi_i \psi_j$). In other words, the transitions or decays ($\eta_i \leftrightarrow \eta_j$) happen only in the dark sector of inert particles. The LIP (H_η) cannot decay into other inert particles (which have larger masses) due to the fact that it is kinematically suppressed and that it cannot decay into the normal particles of the economical 3-3-1 model due to the Z_2 symmetry: it is absolutely stabilized. We recall that in the model of the fourth article of Ref. [17] such similar interactions did happen, by contrast, between the η_3 of the assumed dark sector (the bilepton particles) and the usual particles $\eta_{1,2}$ (which carry no lepton number and couple to the standard model particles; even η_1 develops the VEV) of the normal matter sector, which subsequently lead to the fast decay of η_3: the candidate is unstable.] The dark matter thus resides in the inert part of the model, as shown above. Although our candidates H_η and A_η are similar to those (η^0_3) studied in previously in the literature [7,17], its phenomenology is completely distinguished. This is due to the following.

(1) The masses of H_η and A_η are separated due to the lepton-number-violating coupling λ_{10}. They are two distinct particles. In the previous studies their masses were degenerate [7,17]. In fact, they are different components of a complex field η^0_3.

(2) H_η and A_η do not couple to fermions. However, those in Refs. [7,17] did.

(3) H_η and A_η work in the economical 3-3-1 model with lepton-number violations and the neutrino masses are naturally generated [19]. Those in Refs. [7,17] worked in different 3-3-1 models. In addition, for the 3-3-1 model with right-handed neutrinos we cannot simultaneously understand the physics of the assumed dark matter η^0_3 and the neutrino masses. The model is in fact unrealistic, as indicated above.

Finally, we can have other cases of inert scalar triplets, as given below. In these cases, the dark matter candidates completely differ from η^0_3.

B. Implication of this work

For the 3-3-1 model with right-handed neutrinos, we can introduce another scalar sector which can provide dark matter. That is, ϕ and χ are the same as in the model proposed above, but the inert triplet is changed to $\eta = (\eta^+_1, \eta^0_2, \eta^+_3) \sim (1, 3, 2/3)$, which is a replication of ϕ. In this case, we can have a doublet dark matter particle similar to that in the inert doublet model.

For the minimal 3-3-1 model, the scalar sector is $\rho = (\rho^+_1, \rho^0_2, \rho^{++}_3) \sim (1,3,1)$, $\eta = (\eta^0_1, \eta^-_2, \eta^+_3) \sim (1, 3, 0)$, and $\chi = (\chi^-_1, \chi^{--}_2, \chi^0_3) \sim (1, 3, -1)$. The reduced 3-3-1 model works with only ρ and χ by removing η, whereas either model works with η and χ by removing ρ [22]. Therefore, we have the following cases for dark matter in the minimal 3-3-1 model.

(1) η is an inert scalar triplet. We may have a doublet dark matter particle, similarly to the inert doublet model.

(2) ρ is an inert scalar triplet. A doublet dark matter particle may result, similarly to the previous case.

(3) Removing η (ρ), we introduce instead the inert scalar triplet as a replication of ρ (η).

(4) Removing η or ρ, we include instead the inert triplet as a replication of χ. These cases will yield a singlet dark matter particle.

All the cases above are worth exploring [29]. Therefore, as an example, in the present work we have presented only one case of the 3-3-1 model with right-handed neutrinos, as given in Secs. II and III.

To summarize, the mechanism given in this work responsible for dark matter stability is a solution to the dark matter problem of the typical 3-3-1 models, the 3-3-1 model with right-handed neutrinos, and the minimal 3-3-1 model. The dark matter candidates obtained and their phenomenologies are rich and unlike those in the previous studies [7,17]. The resulting 3-3-1 models with this mechanism are self-consistent and the neutrinos get desirable masses.

V. CONCLUSION

As is the nature of typical 3-3-1 models, the lepton number appears to be a residual charge that is not commuted with the gauge symmetry. If the lepton number is conserved, it will behave as a local charge, and the 3-3-1 gauge symmetry should be extended. One way to keep the 3-3-1 models self-consistent (which avoids an extension) is to have the lepton number belong to an approximate symmetry, and the 3-3-1 models must contain interactions that explicitly violate the lepton number. Looking into the other variants of the 3-3-1 models, we observe that the economical 3-3-1 model is a natural realization of the above criteria, while the reduced 3-3-1 model [22] at the renormalizable level is not. However, the reduced 3-3-1 model will be viable when the effective interactions responsible for fermion masses are included.

We have proved that the 3-3-1 model with right-handed neutrinos can by itself contain an inert scalar triplet (η) responsible for dark matter, while its remaining part with other multiplets works as in the economical 3-3-1 model. Formerly, the η triplet was neglected when one considered the economical 3-3-1 model [19]. The stability of the dark matter candidate (H_η) as contained in η is ensured by a Z_2 symmetry (assigned so that only η is odd; all other multiplets are even), which (as has been shown) is not broken by the vacuum. In contradiction to the inert doublet model, our dark matter candidate behaves as a singlet under the standard model symmetry and this particle is naturally heavy at the ω scale of 3-3-1 symmetry breaking. The interaction between the inert particles and the economical 3-3-1 model's particles has also been given at the effective limit.

We have calculated the relic density of dark matter for the case that this particle is the lightest among the new particles. The relic density will get the correct value in comparison to WMAP data provided that our dark matter candidate is in the TeV range, as is expected for the new physics of 3-3-1 models. In such a range of dark matter masses, the dark matter-nucleon scattering cross section also gets safe values in the bound of the strongest experimental data such as that of XENON100. If the new neutral scalar mass (H) is larger than the dark matter mass, i.e., $m^2_{H_\eta}/m^2_H$ is negligible, our model predicts the dark matter mass $m_{H_\eta} = \lambda_5 \times 2$ TeV and the nucleon scattering cross section $\sigma_{H_\eta - N} = 1.56 \times 10^{-44}\ \mathrm{cm}^2$, which remarkably coincide with the current bound of direct detection experiments such as XENON100 in the TeV range.

If the dark matter is heavier than some new particles of the economical 3-3-1 model, it will also annihilate into these new particles for the thermal process, which can dominate. Also, the coannihilation phenomenology of dark matter with other inert particles is interesting. In addition, the inert scalar triplet can be a replication of ϕ instead of the current one, which results in a doublet dark matter particle. All of these results call for further studies. It is well-known that the minimal 3-3-1 model in its current form does not contain any dark matter candidate. By our proposal, the model can similarly be modified to work as a reduced 3-3-1 model [22] while containing an inert scalar triplet responsible for dark matter. The dark matter candidate in such a model is either a scalar doublet under the standard model symmetry (similarly to the inert doublet model) or a scalar singlet (similarly to our model given in the text). However, its phenomenology is very unique [29].

Finally, our work is a solution to the long-standing problem of dark matter in the typical 3-3-1 models, the 3-3-1 model with right-handed neutrinos, and the minimal 3-3-1 model.

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[1] J. Beringer *et al.* (Particle Data Group), Phys. Rev. D **86**, 010001 (2012).
[2] G. Aad *et al.* (ATLAS Collaboration), Phys. Lett. B **716**, 1 (2012).
[3] S. Chatrchyan *et al.* (CMS Collaboration), Phys. Lett. B **716**, 30 (2012).
[4] See for example S. P. Martin, in *Perspectives on Supersymmetry II*, edited by G. L. Kane (World Scientific, Singapore, 2010), p. 1.
[5] See for example D. Hooper and S. Profumo, Phys. Rep. **453**, 29 (2007).
[6] See for example J. Hubisz and P. Meade, Phys. Rev. D **71**, 035016 (2005).
[7] P. V. Dong, T. D. Tham, and H. T. Hung, Phys. Rev. D **87**, 115003 (2013).
[8] See P. H. Frampton in Ref. [13].
[9] H. N. Long and V. T. Van, J. Phys. G **25**, 2319 (1999).
[10] P. B. Pal, Phys. Rev. D **52**, 1659 (1995).
[11] F. Pisano, Mod. Phys. Lett. A **11**, 2639 (1996); A. Doff and F. Pisano, Mod. Phys. Lett. A **14**, 1133 (1999); C. A. de S. Pires and O. P. Ravinez, Phys. Rev. D **58**, 035008 (1998); C. A. de S. Pires, Phys. Rev. D **60**, 075013 (1999); P. V. Dong and H. N. Long, Int. J. Mod. Phys. A **21**, 6677 (2006).
[12] M. Singer, J. W. F. Valle, and J. Schechter, Phys. Rev. D **22**, 738 (1980); J. C. Montero, F. Pisano, and V. Pleitez, Phys. Rev. D **47**, 2918 (1993); R. Foot, H. N. Long, and T. A. Tran, Phys. Rev. D **50**, R34 (1994); H. N. Long, Phys. Rev. D **53**, 437 (1996); **54**, 4691 (1996).
[13] F. Pisano and V. Pleitez, Phys. Rev. D **46**, 410 (1992); P. H. Frampton, Phys. Rev. Lett. **69**, 2889 (1992); R. Foot, O. F. Hernandez, F. Pisano, and V. Pleitez, Phys. Rev. D **47**, 4158 (1993).
[14] P. V. Dong, L. T. Hue, H. N. Long, and D. V. Soa, Phys. Rev. D **81**, 053004 (2010); P. V. Dong, H. N. Long, D. V. Soa, and V. V. Vien, Eur. Phys. J. C **71**, 1544 (2011); P. V. Dong, H. N. Long, C. H. Nam, and V. V. Vien, Phys. Rev. D **85**, 053001 (2012).
[15] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32**, 438 (1974); H. Georgi, H. R. Quinn, and S. Weinberg, Phys. Rev. Lett. **33**, 451 (1974).
[16] H. Georgi, in *Particles and Fields*, edited by C. E. Carlson (A. I. P., New York, 1975); H. Fritzsch and P. Minkowski, Ann. Phys. (N.Y.) **93**, 193 (1975).
[17] D. Fregolente and M. D. Tonasse, Phys. Lett. B **555**, 7 (2003); H. N. Long and N. Q. Lan, Europhys. Lett. **64**, 571 (2003); S. Filippi, W. A. Ponce, and L. A. Sanches, Europhys. Lett. **73**, 142 (2006); C. A. de S. Pires and P. S. Rodrigues da Silva, J. Cosmol. Astropart. Phys. 12 (2007) 012; J. K. Mizukoshi, C. A. de S. Pires, F. S. Queiroz, and P. S. Rodrigues da Silva, Phys. Rev. D **83**, 065024 (2011); J. D. Ruiz-Alvarez, C. A. de S. Pires, F. S. Queiroz, D. Restrepo, and P. S. Rodrigues da

Silva, Phys. Rev. D **86**, 075011 (2012); S. Profumo and F. S. Queiroz, arXiv:1307.7802.

[18] M. B. Tully and G. C. Joshi, Phys. Rev. D **64**, 011301 (2001); D. Chang and H. N. Long, Phys. Rev. D **73**, 053006 (2006).

[19] W. A. Ponce, Y. Giraldo, and L. A. Sanchez, Phys. Rev. D **67**, 075001 (2003); P. V. Dong, H. N. Long, D. T. Nhung, and D. V. Soa, Phys. Rev. D **73**, 035004 (2006); Adv. High Energy Phys. **2008**, 739492 (2008); P. V. Dong, T. T. Huong, D. T. Huong, and H. N. Long, Phys. Rev. D **74**, 053003 (2006); P. V. Dong, H. N. Long, and D. V. Soa, Phys. Rev. D **73**, 075005 (2006); **75**, 073006 (2007); P. V. Dong, D. T. Huong, M. C. Rodriguez, and H. N. Long, Nucl. Phys. **B772**, 150 (2007); P. V. Dong, H. T. Hung, and H. N. Long, Phys. Rev. D **86**, 033002 (2012)

[20] D. T. Huong, C. S. Kim, H. N. Long, and N. T. Thuy, arXiv:1110.1482.

[21] N. G. Deshpande and E. Ma, Phys. Rev. D **18**, 2574 (1978).

[22] J. G. Ferreira, Jr., P. R. D. Pinheiro, C. A. de S. Pires, and P. S. Rodrigues da Silva, Phys. Rev. D **84**, 095019 (2011); V. T. N. Huyen, T. T. Lam, H. N. Long, and V. Q. Phong, arXiv:1210.5833.

[23] R. Barbieri, L. J. Hall, and V. S. Rychkov, Phys. Rev. D **74**, 015007 (2006).

[24] G. Bertone, D. Hooper, and J. Silk, Phys. Rep. **405**, 279 (2005); G. Jungman, M. Kamionkowski, and K. Griest, Phys. Rep. **267**, 195 (1996).

[25] G. Belanger, F. Boudjema, A. Pukhov, and A. Semenov, Comput. Phys. Commun. **180**, 747 (2009).

[26] J. Ellis, A. Ferstl, and K. A. Olive, Phys. Lett. B **481**, 304 (2000).

[27] E. Aprile *et al.* (XENON100 Collaboration), Phys. Rev. Lett. **109**, 181301 (2012).

[28] A. G. Dias, C. A. de S. Pires, and P. S. Rodrigues da Silva, Phys. Lett. B **628**, 85 (2005).

[29] P. V. Dong and D. V. Soa (to be published).