

# Lepton flavor violating $\tau$ decays in TeV scale type I see-saw and Higgs triplet models

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**ABSTRACT:** The lepton flavour violating (LFV)  $\tau$  decays  $\tau \rightarrow (e, \mu)\gamma$  and  $\tau \rightarrow 3\mu$  are investigated in the frameworks of the TeV scale type I see-saw and Higgs Triplet (or type II see-saw) models. Predictions for the rates of these processes are obtained. The implications of the existing stringent experimental upper bounds on the  $\mu \rightarrow e + \gamma$  and  $\mu \rightarrow 3e$  decay branching ratios for the predictions of the  $\tau \rightarrow (e, \mu)\gamma$  and  $\tau \rightarrow 3\mu$  decay rates are studied in detail. The possibilities to observe the indicated LFV  $\tau$  decays in present and future experiments are analysed.

**KEYWORDS:** Rare Decays, Beyond Standard Model, Neutrino Physics, CP violation

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## 1 Introduction

It is well established at present that the flavour neutrino oscillations observed in the experiments with solar, atmospheric, reactor and accelerator neutrinos (see [1] and the references quoted therein) are caused by the existence of nontrivial neutrino mixing in the weak charged current interaction Lagrangian:

$$\mathcal{L}_{\text{CC}} = -\frac{g}{\sqrt{2}} \sum_{l=e,\mu,\tau} \bar{l}_L(x) \gamma_\alpha \nu_{lL}(x) W^{\alpha\dagger}(x) + h.c., \quad \nu_{lL}(x) = \sum_{j=1}^n U_{lj} \nu_{jL}(x), \quad (1.1)$$

where  $\nu_{lL}(x)$  are the flavour neutrino fields,  $\nu_{jL}(x)$  is the left-handed (LH) component of the field of the neutrino  $\nu_j$  having a mass  $m_j$ , and  $U$  is a unitary matrix - the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix [2-5],  $U \equiv U_{\text{PMNS}}$ . The data imply that among the neutrinos with definite mass at least three, say  $\nu_1$ ,  $\nu_2$  and  $\nu_3$ , have masses  $m_{1,2,3} \lesssim 1 \text{ eV}$ , i.e., are much lighter than the charged leptons and quarks.

The mixing of the three light neutrinos is described to a good approximation by  $3 \times 3$  unitary PMNS matrix. In the widely used standard parametrisation [1],  $U_{\text{PMNS}}$  is expressed in terms of the solar, atmospheric and reactor neutrino mixing angles  $\theta_{12}$ ,  $\theta_{23}$  and  $\theta_{13}$ , respectively, and one Dirac —  $\delta$ , and two Majorana [6] —  $\alpha_{21}$  and  $\alpha_{31}$ , CP violation (CPV) phases:

$$U_{\text{PMNS}} = \tilde{U} P, \quad P = \text{diag} \left( 1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}} \right), \quad (1.2)$$

where  $\tilde{U}$  is a CKM-like matrix containing the Dirac CPV phase  $\delta$ ,

$$\tilde{U} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}. \quad (1.3)$$

In eq. (1.3) we have used the standard notations  $c_{ij} = \cos \theta_{ij}$ ,  $s_{ij} = \sin \theta_{ij}$ , the angles  $\theta_{ij} = [0, \pi/2]$ ,  $\delta = [0, 2\pi]$  and, in general,  $0 \leq \alpha_{j1}/2 \leq 2\pi$ ,  $j = 2, 3$  [7]. If CP invariance holds, we have  $\delta = 0, \pi$ , and [8–10]  $\alpha_{21(31)} = k^{(\prime)} \pi$ ,  $k^{(\prime)} = 0, 1, 2, 3, 4$ .

The neutrino oscillation data, accumulated over many years, allowed to determine the parameters which drive the observed solar, reactor, atmospheric and accelerator neutrino oscillations,  $\Delta m_{21}^2 > 0$ ,  $\theta_{12}$ ,  $|\Delta m_{31}^2| \cong |\Delta m_{32}^2|$ ,  $\theta_{23}$  and  $\theta_{13}$  with a relatively high precision (see, e.g., [1]). The best fit values of these parameters, obtained in [11] from fitting the global neutrino oscillation data, read:

$$\Delta m_{21}^2 = 7.54 \times 10^{-5} \text{ eV}^2, \quad |\Delta m_{31(32)}^2| = 2.47 (2.46) \times 10^{-3} \text{ eV}^2, \quad (1.4)$$

$$\sin^2 \theta_{12} = 0.307, \quad \sin^2 \theta_{13} = 0.0241 (0.0244), \quad \sin^2 \theta_{23} = 0.386 (0.392), \quad (1.5)$$

where the values (values in brackets) correspond to  $\Delta m_{31(32)}^2 > 0$  ( $\Delta m_{31(32)}^2 < 0$ ), i.e., to neutrino mass spectrum with normal ordering (NO),  $m_1 < m_2 < m_3$  (inverted ordering (IO),  $m_3 < m_1 < m_2$ )<sup>1</sup> (see, e.g., [1]). We will use these values in our numerical analyses. Similar results have been obtained also in the global analysis of the neutrino oscillation data performed in [12].

In spite of the compelling evidence for nonconservation of the leptonic flavour in neutrino oscillations, reflected in the neutrino mixing present in eq. (1.1), all searches for lepton flavour violation (LFV) in the charged lepton sector have produced negative results so far. The most stringent upper limits follow from the experimental searches for the LFV muon decays  $\mu^+ \rightarrow e^+ \gamma$  and  $\mu^+ \rightarrow e^+ e^- e^+$ ,

$$\text{BR}(\mu^+ \rightarrow e^+ \gamma) < 5.7 \times 10^{-13} \quad [13, 14], \quad (1.6)$$

$$\text{BR}(\mu^+ \rightarrow e^+ e^- e^+) < 1.0 \times 10^{-12} \quad [15], \quad (1.7)$$

and from the non-observation of conversion of muons into electrons in Titanium,

$$\text{CR}(\mu^- + \text{Ti} \rightarrow e^- + \text{Ti}) < 4.3 \times 10^{-12} \quad [16]. \quad (1.8)$$

Besides, there are stringent constraints on the tau-muon and tau-electron flavour violation as well from the non-observation of LFV tau decays [17]:

$$\text{BR}(\tau \rightarrow \mu \gamma) < 4.4 \times 10^{-8}, \quad (1.9)$$

$$\text{BR}(\tau \rightarrow e \gamma) < 3.3 \times 10^{-8}, \quad (1.10)$$

$$\text{BR}(\tau \rightarrow 3\mu) < 2.1 \times 10^{-8}. \quad (1.11)$$

In the minimal extension of the Standard Model with massive neutrinos [18], in which the total lepton charge  $L$  is conserved ( $L = L_e + L_\mu + L_\tau$ ,  $L_l$ ,  $l = e, \mu, \tau$ , being the individual lepton charges) and the neutrinos with definite mass are Dirac particles, the rates of the LFV violating processes involving the charged leptons are extremely strongly suppressed,

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<sup>1</sup>As is well known, depending on the value of the lightest neutrino mass, the spectrum can also be normal hierarchical (NH),  $m_1 \ll m_2 < m_3$ ; inverted hierarchical (IH):  $m_3 \ll m_1 < m_2$ ; or quasi-degenerate (QD):  $m_1 \cong m_2 \cong m_3$ ,  $m_j^2 \gg |\Delta m_{31(32)}^2|$ . The QD spectrum corresponds to  $m_j \gtrsim 0.10 \text{ eV}$ ,  $j = 1, 2, 3$ .

which makes them unobservable in practice. Indeed, the  $\mu \rightarrow e + \gamma$  decay branching ratio, for instance, is given by [18]:

$$BR(\mu \rightarrow e + \gamma) = \frac{3\alpha}{32\pi} \left| U_{ej} U_{\mu j}^* \frac{m_j^2}{M_W^2} \right|^2 \cong (2.5 - 3.9) \times 10^{-55}, \quad (1.12)$$

where we have used the best fit values of the neutrino oscillation parameters given in eqs. (1.4) and (1.5) and the two values of  $BR(\mu \rightarrow e + \gamma)$  correspond to  $\delta = \pi$  and 0. The predicted branching ratio should be compared with the current experimental upper limit quoted in eq. (1.6).

The minimal extension of the Standard Model with massive Dirac neutrinos and conservation of the total lepton charge  $L$  [18] does not give us, however, any insight of why the neutrino masses are so much smaller than the masses of the charged leptons and quarks. The enormous disparity between the magnitude of the neutrino masses and the masses of the charged fermions suggests that the neutrino masses are related to the existence of new mass scale  $\Lambda$  in physics, i.e., to new physics beyond the Standard Model (SM). A natural explanation of the indicated disparity is provided by the see-saw models of neutrino mass generation. In the present study we will be primarily interested in the type I seesaw [19–22] and the Higgs Triplet (HT) [23–25] scenarios. In these models the scale  $\Lambda$  is set by the scale of masses of the new degrees of freedom present in the models. In the case of the type I see-saw scenario, these are the masses of the heavy (right-handed (RH)) Majorana neutrinos. In the Higgs Triplet Model (HTM), which is often called also “type II see-saw model”, the scale  $\Lambda$  is related to the masses of the new physical neutral, singly and doubly charged Higgs particles.

The scale  $\Lambda$  at which the new physics, associated with the existence of neutrino masses and mixing, manifests itself, in principle, can have in the cases of type I seesaw and HT models relevant for the present study an arbitrary large value (see, e.g., [19–26]), up to the GUT scale of  $2 \times 10^{16}$  GeV and even beyond, up to the Planck scale. An interesting possibility which can also be theoretically and phenomenologically well motivated both for the type I seesaw and HT models is to have the new physics at the TeV scale, i.e.,  $\Lambda \sim (100 - 1000)$  GeV (for the type I seesaw scenario see the discussions in, e.g., [27–30]). In the TeV scale class of type I see-saw models of interest, the flavour structure of the couplings of the new particles to the charged leptons is basically determined by the requirement of reproducing the data on the neutrino oscillation parameters [26–29]. In HTM these couplings are proportional to the Majorana mass matrix of the left-handed flavour neutrinos.

As a consequence, the rates of the LFV processes in the charged lepton sector can be calculated in terms of a few parameters. In the TeV scale type I seesaw scenario, for instance, these parameters are constrained by different sets of data such as, e.g., data on neutrino oscillations, from EW precision tests and on the LFV violating processes  $\mu \rightarrow e + \gamma$ ,  $\mu \rightarrow 3e$ ,  $\mu^- - e^-$  conversion nuclei, etc. Nevertheless, the predicted rates of the LFV charged lepton decays  $\mu \rightarrow e + \gamma$ ,  $\mu^+ \rightarrow e^+ e^- e^+$  and of the  $\mu^- - e^-$  conversion in both the TeV scale type I seesaw and HT models of interest are within the reach of the

future experiments searching for lepton flavour violation<sup>2</sup> even when the parameters of the model do not allow production of the new particles with observable rates at the LHC [29].

The role of the experiments searching for lepton flavour violation to test and constrain low scale see-saw models will be significantly strengthened in the next years. Searches for  $\mu - e$  conversion at the planned COMET experiment at KEK [35] and Mu2e experiment at Fermilab [36] aim to reach sensitivity to  $\text{CR}(\mu \text{ Al} \rightarrow e \text{ Al}) \approx 10^{-16}$ , while, in the longer run, the PRISM/PRIME experiment in KEK [37] and the project-X experiment in Fermilab [38] are being designed to probe values of the  $\mu - e$  conversion rate on Ti, which are by 2 orders of magnitude smaller,  $\text{CR}(\mu \text{ Ti} \rightarrow e \text{ Ti}) \approx 10^{-18}$  [37]. There are also plans to perform a new search for the  $\mu^+ \rightarrow e^+ e^- e^+$  decay [39, 40], which will probe values of the corresponding branching ratio down to  $\text{BR}(\mu^+ \rightarrow e^+ e^- e^+) \approx 10^{-15}$ , i.e., by 3 orders of magnitude smaller than the best current upper limit, eq. (1.7). Furthermore, searches for tau lepton flavour violation at superB factories aim to reach a sensitivity to  $\text{BR}(\tau \rightarrow (\mu, e)\gamma) \approx 10^{-9}$ , while a next generation experiment on the  $\tau \rightarrow 3\mu$  decay is expected to reach sensitivity to  $\text{BR}(\tau \rightarrow 3\mu) = 10^{-10}$  [41].

In the present article we investigate the LFV  $\tau$  decays  $\tau \rightarrow (e, \mu)\gamma$  and  $\tau \rightarrow 3\mu$  in the frameworks of the TeV scale type I see-saw and HT models of neutrino mass generation. We derive predictions for the rates of the indicated  $\tau$  LFV decays in the two models and analyse the possibilities of observation of these decays in present and planned future experiments.

Studies of the LFV  $\tau$  decays  $\tau \rightarrow \mu\gamma$  and  $\tau \rightarrow e\gamma$ , but not of the  $\tau \rightarrow 3\mu$  decay, in the TeV scale type I seesaw model were performed in [28]. In [43] the authors investigated the  $\tau \rightarrow 3\mu$  decay in the Higgs Triplet model. Comments about the  $\tau \rightarrow \mu\gamma$ ,  $\tau \rightarrow e\gamma$  and  $\tau \rightarrow 3\mu$  decays in the Higgs Triplet model were made in ref. [42]. We have included detailed discussions of the differences between our study and those performed in [28] and in [42, 43] at the end of sections 2 and 3, respectively.

## 2 TeV scale type I see-saw model

### 2.1 Brief review of the model

We denote the light and heavy Majorana neutrino mass eigenstates of the type I see-saw model [19–22] as  $\chi_i$  and  $N_k$ , respectively.<sup>3</sup> The charged and neutral current weak interactions involving the light and heavy Majorana neutrinos have the form:

$$\mathcal{L}_{CC}^\nu = -\frac{g}{\sqrt{2}} \bar{\ell} \gamma_\alpha \nu_{\ell L} W^\alpha + \text{h.c.} = -\frac{g}{\sqrt{2}} \bar{\ell} \gamma_\alpha ((1 + \eta)U)_{\ell i} \chi_{iL} W^\alpha + \text{h.c.}, \quad (2.1)$$

$$\mathcal{L}_{NC}^\nu = -\frac{g}{2c_w} \bar{\nu}_{\ell L} \gamma_\alpha \nu_{\ell L} Z^\alpha = -\frac{g}{2c_w} \bar{\chi}_{iL} \gamma_\alpha \left( U^\dagger (1 + 2\eta) U \right)_{ij} \chi_{jL} Z^\alpha, \quad (2.2)$$

$$\mathcal{L}_{CC}^N = -\frac{g}{2\sqrt{2}} \bar{\ell} \gamma_\alpha (RV)_{\ell k} (1 - \gamma_5) N_k W^\alpha + \text{h.c.}, \quad (2.3)$$

$$\mathcal{L}_{NC}^N = -\frac{g}{4c_w} \bar{\nu}_{\ell L} \gamma_\alpha (RV)_{\ell k} (1 - \gamma_5) N_k Z^\alpha + \text{h.c.}. \quad (2.4)$$

<sup>2</sup>Using the result given in eq. (1.12) as a starting point, it was shown in [31–34] that the rates of the LFV muon decays  $\mu \rightarrow e + \gamma$  and  $\mu \rightarrow 3e$ , can be close to the existing upper limits in theories with heavy neutral leptons (or heavy neutrinos, for that matter) which have charged current weak interaction type couplings to the electron and the muon. Although the specific model considered in [31–34] is not viable, the general conclusion of these studies remains valid.

<sup>3</sup>We use the same notations as in [29, 44].

Here  $(1 + \eta)U = U_{\text{PMNS}}$  is the PMNS neutrino mixing matrix [3–5],  $U$  is a  $3 \times 3$  unitary matrix which diagonalises the Majorana mass matrix of the left-handed (LH) flavour neutrinos  $\nu_{\ell L}$ ,  $m_\nu$ , generated by the see-saw mechanism,  $V$  is the unitary matrix which diagonalises the Majorana mass matrix of the heavy RH neutrinos and the matrix  $R$  is determined by (see [44])  $R^* \cong M_D M_N^{-1}$ ,  $M_D$  and  $M_N$  being the neutrino Dirac and the RH neutrino Majorana mass matrices, respectively,  $|M_D| \ll |M_N|$ . The matrix  $\eta$  characterises the deviations from unitarity of the PMNS matrix:

$$\eta \equiv -\frac{1}{2}RR^\dagger = -\frac{1}{2}(RV)(RV)^\dagger = \eta^\dagger. \quad (2.5)$$

In the TeV scale type I see-saw model on interest, the masses of the heavy Majorana neutrinos  $N_k$ ,  $M_k$ , are supposed to lie in the interval  $M_k \sim (100 - 1000)$  GeV. The couplings  $(RV)_{\ell k}$  are bounded, in particular, by their relation to the elements of the Majorana mass matrix of LH flavour neutrinos  $(m_\nu)_{\ell\ell'}$ , all of which have to be smaller than approximately 1 eV:

$$\left| \sum_k (RV)_{\ell'k}^* M_k (RV)_{k\ell}^\dagger \right| \cong |(m_\nu)_{\ell\ell'}| \lesssim 1 \text{ eV}, \quad \ell', \ell = e, \mu, \tau. \quad (2.6)$$

These constraints can be satisfied for sizeable values of the couplings  $|(RV)_{\ell k}|$  in a model with two heavy Majorana neutrinos  $N_{1,2}$ , in which  $N_1$  and  $N_2$  have close masses forming a pseudo-Dirac state [45–47],  $M_2 = M_1(1 + z)$ ,  $M_{1,2}, z > 0$ ,  $z \ll 1$ , and their couplings satisfy [29]

$$(RV)_{\ell 2} = \pm i (RV)_{\ell 1} \sqrt{\frac{M_1}{M_2}}, \quad \ell = e, \mu, \tau. \quad (2.7)$$

In this scenario with sizeable CC and NC couplings of  $N_{1,2}$ , the requirement of reproducing the correct low energy neutrino oscillation parameters constrains significantly [27, 30] and in certain cases determines the neutrino Yukawa couplings [26, 28, 29]. Correspondingly, the flavour dependence of the couplings  $(RV)_{\ell 1}$  and  $(RV)_{\ell 2}$  in eqs. (2.3) and (2.4) is also determined and in the case of interest takes the form [29]:

$$|(RV)_{\ell 1}|^2 = \frac{1}{2} \frac{y^2 v^2}{M_1^2} \frac{m_3}{m_2 + m_3} \left| U_{\ell 3} + i \sqrt{m_2/m_3} U_{\ell 2} \right|^2, \quad \text{NH}, \quad (2.8)$$

$$|(RV)_{\ell 1}|^2 = \frac{1}{2} \frac{y^2 v^2}{M_1^2} \frac{m_2}{m_1 + m_2} \left| U_{\ell 2} + i \sqrt{m_1/m_2} U_{\ell 1} \right|^2 \cong \frac{1}{4} \frac{y^2 v^2}{M_1^2} |U_{\ell 2} + i U_{\ell 1}|^2, \quad \text{IH}, \quad (2.9)$$

where  $y$  represents the maximum eigenvalue of the neutrino Yukawa matrix and  $v \simeq 174$  GeV. In the last equation we have neglected the  $N_1 - N_2$  mass difference setting  $z = 0$  and used the fact that for the IH spectrum one has  $m_1 \cong m_2$ . For  $(RV)_{\ell 1,2}$  satisfying eqs. (2.7)–(2.9), eq. (2.6) is automatically fulfilled.

The low energy electroweak precision data on processes involving light neutrinos imply the following upper limits on the couplings [48, 49] (see also [50]):

$$|(RV)_{e1}|^2 \lesssim 2 \times 10^{-3}, \quad (2.10)$$

$$|(RV)_{\mu 1}|^2 \lesssim 0.8 \times 10^{-3}, \quad (2.11)$$

$$|(RV)_{\tau 1}|^2 \lesssim 2.6 \times 10^{-3}. \quad (2.12)$$

Let us add finally that in the class of type I see-saw models with two heavy Majorana neutrinos we are considering (see, e.g., [51–59]), one of the three light (Majorana) neutrinos is massless and hence the neutrino mass spectrum is hierarchical (see, e.g., [1]). In the case of normal hierarchical (NH) spectrum we have  $m_1 = 0$ ,  $m_2 = \sqrt{\Delta m_{21}^2}$  and  $m_3 = \sqrt{\Delta m_{31}^2}$ , while if the spectrum is inverted hierarchical (IH),  $m_3 = 0$ ,  $m_2 = \sqrt{|\Delta m_{32}^2|}$  and  $m_1 = \sqrt{|\Delta m_{32}^2| - \Delta m_{21}^2} \cong \sqrt{|\Delta m_{32}^2|}$ , with  $\Delta m_{32}^2 = m_3^2 - m_2^2 < 0$ . In both cases we have:  $\Delta m_{21}^2 / \Delta m_{31(23)}^2 \cong 0.03 \ll 1$ .

## 2.2 The $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow e\gamma$ decays

In the type I see-saw scheme of interest with two heavy Majorana neutrinos, the ratio of the decay rates  $\Gamma(l_\alpha \rightarrow l_\beta\gamma)$  and  $\Gamma(l_\alpha \rightarrow \nu_\alpha l_\beta \bar{\nu}_\beta)$  can be written as [29, 31–33]:

$$\frac{\Gamma(l_\alpha \rightarrow l_\beta\gamma)}{\Gamma(l_\alpha \rightarrow \nu_\alpha l_\beta \bar{\nu}_\beta)} = \frac{3\alpha_{\text{em}}}{32\pi} |T|^2, \quad (2.13)$$

where

$$T \approx 2|(RV)_{\beta 1}^*(RV)_{\alpha 1}| |G(x) - G(0)|, \quad (2.14)$$

$$G(x) = \frac{10 - 43x + 78x^2 - 49x^3 + 4x^4 + 18x^3 \ln x}{3(x-1)^4}, \quad (2.15)$$

with  $x = M_1^2/M_W^2$ . In deriving eq. (2.15) we have used the relation (2.7) and have neglected the  $N_1 - N_2$  mass difference. The  $l_\alpha \rightarrow l_\beta\gamma$  decay branching ratio is given by:

$$\text{BR}(l_\alpha \rightarrow l_\beta\gamma) = \frac{\Gamma(l_\alpha \rightarrow l_\beta\gamma)}{\Gamma(l_\alpha \rightarrow \nu_\alpha l_\beta \bar{\nu}_\beta)} \text{Br}(l_\alpha \rightarrow \nu_\alpha l_\beta \bar{\nu}_\beta), \quad (2.16)$$

with  $\text{BR}(\mu \rightarrow \nu_\mu e \bar{\nu}_e) \approx 1$ ,  $\text{BR}(\tau \rightarrow \nu_\tau \mu \bar{\nu}_\mu) = 0.1739$ , and  $\text{BR}(\tau \rightarrow \nu_\tau e \bar{\nu}_e) = 0.1782$  [1].

The predictions of the model under discussion for  $\text{BR}(\mu \rightarrow e\gamma)$  and the constraints on the product of couplings  $|(RV)_{e1}^*(RV)_{\mu 1}|$ , as well as on the Yukawa coupling  $y$ , following from the experimental upper limit on  $\text{BR}(\mu \rightarrow e\gamma)$ , were discussed in detail in [29, 60]. Here we concentrate on the phenomenology of the  $\tau \rightarrow \mu\gamma$  and  $\tau \rightarrow e\gamma$  decays. Using the current upper limits on  $\text{BR}(\tau \rightarrow \mu\gamma)$  and  $\text{Br}(\tau \rightarrow e\gamma)$  quoted in eqs. (1.9) and (1.10), we obtain the following upper bounds:

$$\tau \rightarrow \mu\gamma: \quad |(RV)_{\mu 1}^*(RV)_{\tau 1}| \leq 2.7 \times 10^{-2} \quad (0.9 \times 10^{-2}) \quad M_1 = 100 \text{ (1000) GeV}, \quad (2.17)$$

$$\tau \rightarrow e\gamma: \quad |(RV)_{e1}^*(RV)_{\tau 1}| \leq 2.3 \times 10^{-2} \quad (0.8 \times 10^{-2}) \quad M_1 = 100 \text{ (1000) GeV}. \quad (2.18)$$

These constraints are weaker than those implied by the limits quoted in eqs. (2.10)–(2.12). The planned experiments at the SuperB factory, which are expected to probe values of  $\text{BR}(\tau \rightarrow (\mu, e)\gamma) \geq 10^{-9}$ , will be sensitive to

$$\tau \rightarrow (\mu, e)\gamma: \quad |(RV)_{(\mu, e)1}^*(RV)_{\tau 1}| \geq 4.0 \times 10^{-3} \quad (1.4 \times 10^{-3}) \quad M_1 = 100 \text{ (1000) GeV}. \quad (2.19)$$

The minimal values quoted above are of the same order as the upper limits following from the constraints (2.10)–(2.12).

The  $\tau$  decay branching ratios of interest depend on the neutrino mixing parameters via the quantity  $|(RV)_{*l1} (RV)_{\tau 1}|$ ,  $l = e, \mu$ . In the case of NH neutrino mass spectrum,  $|(RV)_{l1}| \propto |U_{l3} + i\sqrt{m_2/m_3} U_{l2}|$  is different from zero for any values of the neutrino mixing parameters from their  $3\sigma$  experimentally determined allowed ranges and for any  $l = e, \mu, \tau$ . This implies that there cannot be further suppression of the  $\tau \rightarrow (\mu, e)\gamma$  decay rates due to a cancellation between the terms in the expressions for  $|(RV)_{l1}|$ .

In contrast, depending on the values of the Dirac and Majorana CPV phases  $\delta$  and  $\alpha_{21}$  of the PMNS matrix, we can have strong suppression of the couplings  $|(RV)_{l1}|$ ,  $l = e, \mu$ , which enter into the expressions for  $\text{BR}(\tau \rightarrow (\mu, e)\gamma)$  if the neutrino mass spectrum is of the IH type [29, 60]. Indeed, in this case we have  $|(RV)_{l1}| \propto |U_{l3} + iU_{l2}|$ ,  $l = e, \mu, \tau$ . For  $\alpha_{21} = -\pi$ ,  $|U_{e3} + iU_{e2}|$  can be rather small:  $|U_{e2} + iU_{e1}|^2 = c_{13}^2(1 - \sin 2\theta_{12}) \cong 0.0765$ , where we have used the best fit values of  $\sin^2 \theta_{12} = 0.307$  and  $\sin^2 \theta_{13} = 0.0236$ . As was shown in [60], we can have  $|U_{\mu 2} + iU_{\mu 1}|^2 = 0$  for specific values of  $\delta$  lying the interval  $0 \leq \delta \lesssim 0.7$ . In this case the value of the phases  $\alpha_{21}$  is determined by the values of  $\delta$  and  $\theta_{12}$  (for further details see [60]).

We analyse next the possibility of having strongly suppressed coupling  $|(RV)_{\tau 1}|^2$ , i.e., to have  $|(RV)_{\tau 1}|^2 \propto |U_{\tau 2} + iU_{\tau 1}|^2 = 0$ , in the case of IH spectrum. The suppression in question can take place if

$$\sin \theta_{13} = \frac{s_{12} - c_{12} \sin \frac{\alpha_{21}}{2}}{c_{12} \cos \delta + s_{12} \sin(\delta + \frac{\alpha_{21}}{2})} \tan \theta_{23}, \tag{2.20}$$

and if in addition the values of the phases  $\delta$  and  $\alpha_{21}$  are related via the equation:

$$c_{12} s_{23} \cos \frac{\alpha_{21}}{2} - c_{23} s_{13} \left[ c_{12} \sin \delta - s_{12} \cos(\delta + \frac{\alpha_{21}}{2}) \right] = 0. \tag{2.21}$$

One simple solution to eq. (2.21) obviously is  $\delta = \alpha_{21} = \pi$ . For these values of  $\delta$  and  $\alpha_{21}$ , eq. (2.20) becomes:

$$\sin \theta_{13} = \frac{c_{12} - s_{12}}{c_{12} + s_{12}} \tan \theta_{23}. \tag{2.22}$$

Using the the best fit values of  $\sin^2 \theta_{12}$  and  $\sin^2 \theta_{23}$  quoted in eq. (1.5), we get from eq. (2.22):  $\sin \theta_{13} = 0.162$ , which is very close to the best fit value of 0.155 (0.156) quoted in eq. (1.5). For  $|U_{\tau 2} + iU_{\tau 1}|^2 \cong 0$ , all LFV decays of the  $\tau$  charged lepton, including  $\tau^- \rightarrow \mu^- + \mu^+ + \mu^-$ ,  $\tau^- \rightarrow \mu^- + e^+ + e^-$ , etc., in the TeV scale type I seesaw model we are considering will be strongly suppressed.

### 2.3 The $\tau \rightarrow 3\mu$ decay

We have obtained the  $\tau \rightarrow 3\mu$  decay rate by adapting the result of the calculation of the  $\mu \rightarrow 3e$  decay rate performed in [61] in a scheme with heavy RH neutrinos and type I seesaw mechanism of neutrino mass generation. After recalculating the form factors and neglecting the corrections  $\sim m_\mu/m_\tau \cong 0.06$  and the effects of the difference between the masses of  $N_1$



and  $N_2$ , we find in the model of interest to leading order in the small parameters  $|(RV)_{l1}|$ :

$$\text{BR}(\tau \rightarrow 3\mu) = \frac{\alpha_{em}^2}{16\pi^2 \sin^4 \theta_W} |(RV)_{\tau 1}^*(RV)_{\mu 1}|^2 |C_{\tau 3\mu}(x)|^2 \times \text{BR}(\tau \rightarrow \mu \bar{\nu}_\mu \nu_\tau), \quad (2.23)$$

$$\begin{aligned} |C_{\tau 3\mu}(x)|^2 &= 2 \left| \frac{1}{2} F_B^{\tau 3\mu} + F_z^{\tau 3\mu} - 2 \sin^2 \theta_W (F_z^{\tau 3\mu} - F_\gamma) \right|^2 + 4 \sin^4 \theta_W |F_z^{\tau 3\mu} - F_\gamma|^2 \\ &\quad + 16 \sin^2 \theta_W \left[ \left( F_z^{\tau 3\mu} + \frac{1}{2} F_B^{\tau 3\mu} \right) G_\gamma \right] - 48 \sin^4 \theta_W [(F_z^{\tau 3\mu} - F_\gamma) G_\gamma] \\ &\quad + 32 \sin^4 \theta_W |G_\gamma|^2 \left( \log \frac{m_\tau^2}{m_\mu^2} - \frac{11}{4} \right). \end{aligned} \quad (2.24)$$

Here

$$F_z^{\tau 3\mu}(x) = F_z(x) + 2G_z(0, x), \quad F_B^{\tau 3\mu}(x) = -2(F_{XB_{\text{Box}}}(0, x) - F_{XB_{\text{Box}}}(0, 0)), \quad (2.25)$$

$$F_\gamma(x) = \frac{x(7x^2 - x - 12)}{12(1-x)^3} - \frac{x^2(12 - 10x + x^2)}{6(1-x)^4} \log x, \quad (2.26)$$

$$G_\gamma(x) = -\frac{x(2x^2 + 5x - 1)}{4(1-x)^3} - \frac{3x^3}{2(1-x)^4} \log x, \quad (2.27)$$

$$F_z(x) = -\frac{5x}{2(1-x)} - \frac{5x^2}{2(1-x)^2} \log x, \quad (2.28)$$

$$G_z(x, y) = -\frac{1}{2(x-y)} \left[ \frac{x^2(1-y)}{(1-x)} \log x - \frac{y^2(1-x)}{(1-y)} \log y \right], \quad (2.29)$$

$$\begin{aligned} F_{XB_{\text{Box}}}(x, y) &= -\frac{1}{x-y} \left\{ \left( 1 + \frac{xy}{4} \right) \left[ \frac{1}{1-x} + \frac{x^2}{(1-x)^2} \log x - \frac{1}{1-y} - \frac{y^2}{(1-y)^2} \log y \right] \right. \\ &\quad \left. - 2xy \left[ \frac{1}{1-x} + \frac{x}{(1-x)^2} \log x - \frac{1}{1-y} - \frac{y}{(1-y)^2} \log y \right] \right\}. \end{aligned} \quad (2.30)$$

In writing the expression for  $\text{BR}(\tau \rightarrow 3\mu)$  in eq. (2.23) we have used for the decay rate  $\Gamma(\tau \rightarrow \mu \bar{\nu}_\mu \nu_\tau) = G_F^2 m_\tau^5 / (192\pi^3)$ .

The factor  $|C_{\tau 3\mu}(x)|^2$  in the expression for  $\text{BR}(\mu \rightarrow 3\mu)$  is a monotonically increasing function of the heavy Majorana neutrino mass  $M_1$ . The dependence of  $|C_{\tau 3\mu}(x)|^2$  on  $M_1$  is shown in figure 1. At  $M_1 = 100$  (1000) GeV, the function  $|C_{\tau 3\mu}(x)|^2$  has values 1.53 (36.85).

The present experimental limit on  $\text{BR}(\tau \rightarrow 3\mu)$ , eq. (1.11), leads to a weaker constraint than that following from the upper limits quoted in eqs. (2.11) and (2.12):

$$|(RV)_{\tau 1}^*(RV)_{\mu 1}| < 1.1 \times 10^{-1} \quad (2.3 \times 10^{-2}) \text{ for } M_1 = 100 \text{ (1000) GeV}. \quad (2.31)$$

The next generation of experiments will be sensitive to  $\text{BR}(\tau \rightarrow 3\mu) \geq 10^{-10}$ , and thus to:

$$|(RV)_{\tau 1}^*(RV)_{\mu 1}| \geq 7.7 \times 10^{-3} \quad (1.6 \times 10^{-3}) \text{ for } M_1 = 100 \text{ (1000) GeV}. \quad (2.32)$$

As we see, in the case of  $M_1 = 1000$  GeV, the minimal value of  $|(RV)_{\tau 1}^*(RV)_{\mu 1}|$  to which the future planned experiments will be sensitive is of the order of the upper bound on  $|(RV)_{\tau 1}^*(RV)_{\mu 1}|$  following from the limits (2.11) and (2.12).

Consider next the dependence of the decay rate on the CPV phases and the neutrino oscillation parameters. In the case of NH mass spectrum we have:

$$\text{BR}(\tau \rightarrow 3\mu) \propto |(RV)_{\tau 1}^*(RV)_{\mu 1}|^2 \propto |U_{\tau 3} + i\sqrt{\frac{m_2}{m_3}}U_{\tau 2}|^2 |U_{\mu 3} + i\sqrt{\frac{m_2}{m_3}}U_{\mu 2}|^2. \quad (2.33)$$

Using the best fit values of the neutrino mixing angles and mass squared differences, quoted in eqs. (1.4) and (1.5) and varying the Dirac and Majorana CPV phases in the interval of  $[0, 2\pi]$ , we find that  $|U_{\mu 3} + i\sqrt{m_2/m_3}U_{\mu 2}| |U_{\tau 3} + i\sqrt{m_2/m_3}U_{\tau 2}|$  takes values in the interval  $(0.31 - 0.59)$ . It follows from this result and the inequality (2.32) that the future experiments on the  $\tau \rightarrow 3\mu$  decay will be sensitive to values of the Yukawa coupling  $y \geq 0.10$  (0.46) for  $M_1 = 100$  (1000) GeV. The minimal values in these lower limits are larger than the upper limits on  $y$  following from the current upper bound (1.6) on  $\text{BR}(\mu \rightarrow e + \gamma)$  [60].

A suppression of the  $\tau \rightarrow 3\mu$  decay rate might occur in the case of IH mass due to possible cancellations between the terms in the factors  $|(RV)_{\mu 1}|$  and  $|(RV)_{\tau 1}|$ , as was discussed in the previous subsection. Using again the best fit values of the neutrino oscillation parameters and varying the leptonic CPV phases in the interval  $[0, 2\pi]$ , we find  $0.003 \leq |U_{\mu 2} + iU_{\mu 1}| |U_{\tau 2} + iU_{\tau 1}| \leq 0.51$ . Thus, in the case of IH spectrum, the future experiments with sensitivity to  $\text{BR}(\tau \rightarrow 3\mu) \geq 10^{-10}$  will probe values of  $y \geq 0.14$  (0.64) for  $M_1 = 100$  (1000) GeV. Again the minimal values in these lower limits are larger than the upper limits on  $y$  following from the current upper bound (1.6) on  $\text{BR}(\mu \rightarrow e + \gamma)$  [60].

For specific values of, e.g., the CPV phases of the neutrino mixing matrix one can obtain more stringent upper bounds than those already discussed on the branching ratios of the  $\tau \rightarrow \mu + \gamma$ ,  $\tau \rightarrow e + \gamma$  and  $\tau \rightarrow 3\mu$  decays due to their relation to the  $\mu \rightarrow e + \gamma$  decay branching ratio and the fact that the latter is severely constrained. Indeed, it follows from eqs. (2.16), (2.15) and (2.23) that we have:

$$\frac{\text{BR}(\tau \rightarrow e + \gamma)}{\text{BR}(\mu \rightarrow e + \gamma)} = \frac{|(RV)_{\tau 1}|^2}{|(RV)_{\mu 1}|^2} \text{BR}(\tau \rightarrow e\bar{\nu}_e\nu_\tau), \quad (2.34)$$

$$\frac{\text{BR}(\tau \rightarrow \mu + \gamma)}{\text{BR}(\mu \rightarrow e + \gamma)} = \frac{|(RV)_{\tau 1}|^2}{|(RV)_{e1}|^2} \text{BR}(\tau \rightarrow \mu\bar{\nu}_\mu\nu_\tau), \quad (2.35)$$

$$\frac{\text{BR}(\tau \rightarrow 3\mu)}{\text{BR}(\mu \rightarrow 3e)} = \frac{|(RV)_{\tau 1}|^2}{|(RV)_{e1}|^2} \text{BR}(\tau \rightarrow \mu\bar{\nu}_\mu\nu_\tau) = \frac{\text{BR}(\tau \rightarrow \mu + \gamma)}{\text{BR}(\mu \rightarrow e + \gamma)}, \quad (2.36)$$

$$\frac{\text{BR}(\tau \rightarrow 3\mu)}{\text{BR}(\mu \rightarrow e + \gamma)} = \frac{\alpha_{em}}{6\pi \sin^4 \theta_W} \frac{|C_{\tau 3\mu}(x)|^2}{|G(x) - G(0)|^2} \frac{|(RV)_{\tau 1}|^2}{|(RV)_{e1}|^2} \text{BR}(\tau \rightarrow \mu\bar{\nu}_\mu\nu_\tau). \quad (2.37)$$

The explicit expressions for  $|(RV)_{l1}|^2$ , eqs. (2.8) and (2.9), imply that the ratios of interest in eqs. (2.34)–(2.36) do not depend on the heavy Majorana neutrino mass  $M_1$  and on the Yukawa coupling  $y$  and are determined by the values of the neutrino oscillation parameters and of the CPV phases in the neutrino mixing matrix, as was noticed also in ref. [28]. Using the best fit values quoted in eqs. (1.4) and (1.5) and varying the Dirac and Majorana phases

in the interval  $[0, 2\pi]$  we obtain in the case of NH neutrino mass spectrum:

$$0.37 \leq \frac{|(RV)_{\tau 1}|^2}{|(RV)_{\mu 1}|^2} \leq 9.06, \quad (2.38)$$

$$1.90 \leq \frac{|(RV)_{\tau 1}|^2}{|(RV)_{e 1}|^2} \leq 191.82, \quad (2.39)$$

In a similar way, we get in the case of IH neutrino mass spectrum:

$$4.84 \times 10^{-4} \leq \frac{|(RV)_{\tau 1}|^2}{|(RV)_{\mu 1}|^2} \leq 15.13, \quad (2.40)$$

$$3.25 \times 10^{-4} \leq \frac{|(RV)_{\tau 1}|^2}{|(RV)_{e 1}|^2} \leq 0.56. \quad (2.41)$$

Thus, in the case of the best fit values of the neutrino oscillation parameters we always have

$$\text{BR}(\tau \rightarrow e + \gamma) \lesssim 2.67 \times \text{BR}(\mu \rightarrow e + \gamma) < 1.52 \times 10^{-12}, \quad (2.42)$$

$$\text{BR}(\tau \rightarrow \mu + \gamma) \lesssim 33.36 \times \text{BR}(\mu \rightarrow e + \gamma) < 1.90 \times 10^{-11}, \quad (2.43)$$

where we have used the current upper bound on  $\text{BR}(\mu \rightarrow e + \gamma)$ , eq. (1.6). The limits in eqs. (2.42) and (2.43) correspond respectively to the IH and NH spectra. These values are beyond the expected sensitivity reach of the planned future experiments.

Using the  $2\sigma$  ( $3\sigma$ ) allowed ranges of the neutrino oscillations parameters in the case of NH neutrino mass spectrum we obtain larger intervals of allowed values of the ratios of interest:

$$NH : \quad 0.26 \text{ (0.08)} \leq \frac{|(RV)_{\tau 1}|^2}{|(RV)_{\mu 1}|^2} \leq 14.06 \text{ (16.73)}, \quad (2.44)$$

$$NH : \quad 1.39 \text{ (0.53)} \leq \frac{|(RV)_{\tau 1}|^2}{|(RV)_{e 1}|^2} \leq 497.74 \text{ (980.32)}. \quad (2.45)$$

The maximal value of  $|(RV)_{\tau 1}|^2/|(RV)_{e 1}|^2$  correspond to  $\sin^2 \theta_{12} = 0.275$  (0.259),  $\sin^2 \theta_{23} = 0.359$  (0.348),  $\sin^2 \theta_{13} = 0.0298$  (0.0312),  $\delta = 0.203$  (0.234),  $\alpha_{21} = 6.199$  (3.560) and  $\alpha_{31} = 3.420$  (0.919). At these values of the neutrino mixing parameters we have  $|(RV)_{\mu 1}|^2|(RV)_{e 1}|^2 \cong 6.98 \times 10^{-4}$  ( $3.41 \times 10^{-4}$ )  $y^4 v^4/(16M_1^4)$ ,  $|(RV)_{\tau 1}|^2|(RV)_{\mu 1}|^2 \cong 0.347$  (0.335)  $y^4 v^4/(16M_1^4)$ . Thus, the bound on  $\text{BR}(\mu \rightarrow e + \gamma)$ , eq. (1.6), is satisfied for  $M_1 = 100$  GeV if  $y^4 v^4/(16M_1^4) \lesssim 2.29$  ( $4.69$ )  $\times 10^{-6}$ , and for  $M_1 = 1000$  GeV provided  $y^4 v^4/(16M_1^4) \lesssim 2.68$  ( $5.48$ )  $\times 10^{-7}$ . This implies that  $|(RV)_{\tau 1}|^2|(RV)_{\mu 1}|^2 \lesssim 7.95$  ( $15.7$ )  $\times 10^{-7}$  if  $M_1 = 100$  GeV, and  $|(RV)_{\tau 1}|^2|(RV)_{\mu 1}|^2 \lesssim 9.30$  ( $18.4$ )  $\times 10^{-8}$  for  $M_1 = 1000$  GeV. The bound for  $M_1 = 1000$  GeV is a stronger constraint than that following from the limits (2.11) and (2.12).

Using the inequalities in eqs. (2.44) and (2.45) we obtain:

$$\text{BR}(\tau \rightarrow e + \gamma) \lesssim 2.50 \text{ (2.98)} \times \text{BR}(\mu \rightarrow e + \gamma) < 1.43 \text{ (1.70)} \times 10^{-12}, \quad (2.46)$$

$$\text{BR}(\tau \rightarrow \mu + \gamma) \lesssim 86.56 \text{ (170.48)} \times \text{BR}(\mu \rightarrow e + \gamma) < 4.93 \text{ (9.72)} \times 10^{-11}. \quad (2.47)$$

These are the maximal values of  $\text{BR}(\tau \rightarrow e + \gamma)$  and  $\text{BR}(\tau \rightarrow \mu + \gamma)$ , allowed by the current upper bound on the  $\mu \rightarrow e + \gamma$  decay rate in the TeV scale type I seesaw model considered

and in the case of NH neutrino mass spectrum. If the  $\tau \rightarrow e + \gamma$  and/or  $\tau \rightarrow \mu + \gamma$  decays are observed to proceed with branching ratios which are larger than the bounds quoted above and it is established that the neutrino mass spectrum is of the NH type, the model under discussion will be strongly disfavored, if not ruled out.

Performing a similar analysis in the case of IH spectrum by employing the  $2\sigma$  ( $3\sigma$ ) allowed ranges of the neutrino oscillations parameters we get:

$$IH : \quad 0.0 \ (0.0) \leq \frac{|(RV)_{\tau 1}|^2}{|(RV)_{\mu 1}|^2} < \infty \ (\infty), \quad (2.48)$$

$$IH : \quad 0.0 \ (0.0) \leq \frac{|(RV)_{\tau 1}|^2}{|(RV)_{e 1}|^2} \leq 0.64 \ (0.83). \quad (2.49)$$

The infinity in eq. (2.48) corresponds to  $|(RV)_{\mu 1}| = 0$ ,  $|(RV)_{\tau 1}| \neq 0$ , i.e., to very strongly suppressed  $\text{BR}(\mu \rightarrow e + \gamma)$  and  $\text{BR}(\tau \rightarrow \mu + \gamma)$ . One obtains  $|(RV)_{\mu 1}| = 0$  for the following values of the neutrino mixing angles from the  $2\sigma$  allowed intervals, and of the CPV phases:  $\sin^2 \theta_{12} = 0.340$ ,  $\sin^2 \theta_{23} = 0.547$ ,  $\sin^2 \theta_{13} = 0.0239$ ,  $\delta = 6.185$ ,  $\alpha_{21} = 3.077$  and  $\alpha_{31} = 4.184$  (i.e.,  $\delta \cong 2\pi$ ,  $\alpha_{21} \cong \pi$  and  $\alpha_{31} \cong 1.3\pi$ ). For  $|(RV)_{\mu 1}| = 0$ , the branching ratios  $\text{BR}(\tau \rightarrow e + \gamma)$  and  $\text{BR}(\mu \rightarrow e + \gamma)$  are “decoupled”. Correspondingly, the upper bound on  $\text{BR}(\tau \rightarrow e + \gamma)$  is determined in this case by the limits quoted in eqs. (2.10) and (2.12) and has already been discussed by us.

Using the same strategy and eq. (2.36), we obtain the constraint on  $\text{BR}(\tau \rightarrow 3\mu)$  following from the upper bound on  $\text{BR}(\mu \rightarrow 3e)$  at the best fit values,  $2\sigma$  ( $3\sigma$ ) allowed ranges of the neutrino oscillation parameters:

$$\text{BR}(\tau \rightarrow 3\mu) \lesssim 33.36 \times \text{BR}(\mu \rightarrow 3e) < 3.34 \times 10^{-11}, \quad (2.50)$$

$$\text{BR}(\tau \rightarrow 3\mu) \lesssim 86.56 \ (170.48) \times \text{BR}(\mu \rightarrow 3e) < 8.66 \ (17.0) \times 10^{-11}. \quad (2.51)$$

The relation between  $\text{BR}(\tau \rightarrow 3\mu)$  and  $\text{BR}(\mu \rightarrow e\gamma)$  is somewhat less straightforward, since it involves the  $M_1$  dependent factor  $C_0(x)$ :

$$C_0(x) = \frac{\alpha_{em}}{6\pi \sin^4 \theta_W} \frac{|C_{\tau 3\mu}(x)|^2}{|G(x) - G(0)|^2}. \quad (2.52)$$

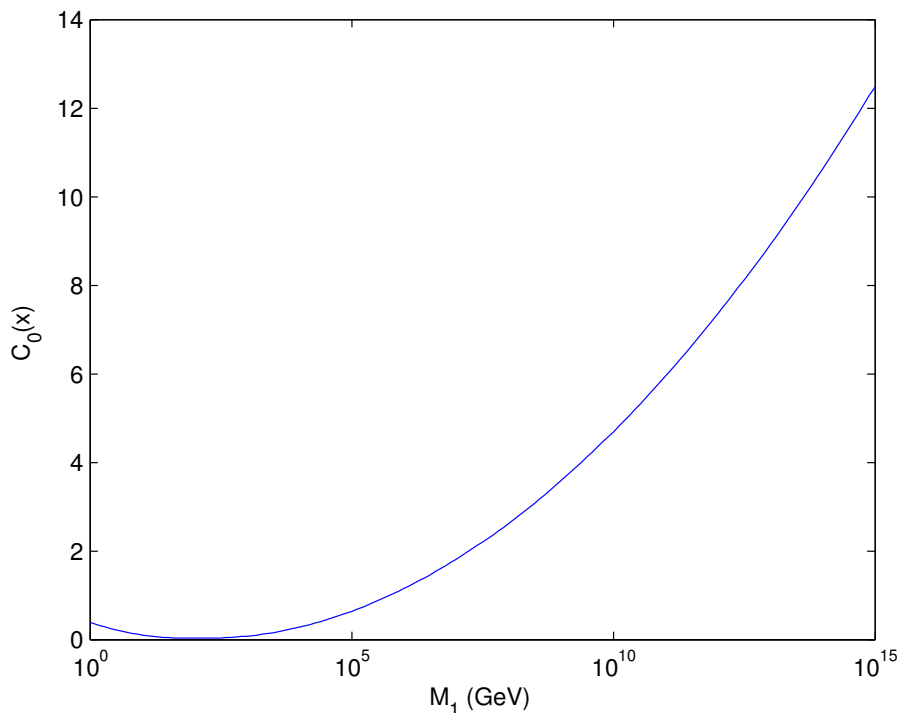
For  $50 \text{ GeV} \leq M_1 \leq 1000 \text{ GeV}$ ,  $C_0(x)$  has its maximum of 0.0764 at  $M_1 = 1000 \text{ GeV}$ . This leads to

$$\text{BR}(\tau \rightarrow 3\mu) \lesssim 2.55 \times \text{BR}(\mu \rightarrow e + \gamma) < 1.45 \times 10^{-12}, \quad (2.53)$$

$$\text{BR}(\tau \rightarrow 3\mu) \lesssim 6.61 \ (13.02) \times \text{BR}(\mu \rightarrow e + \gamma) < 3.77 \ (7.42) \times 10^{-12}. \quad (2.54)$$

Thus, for  $M_1$  having a value in the interval  $[50, 1000] \text{ GeV}$ , the branching ratio  $\text{BR}(\tau \rightarrow 3\mu)$  is predicted to be beyond the sensitivity reach of  $\sim 10^{-10}$  of the planned next generation experiment. The observation of the  $\tau \rightarrow 3\mu$  decay with a branching ratio  $\text{BR}(\tau \rightarrow 3\mu)$  which is definitely larger than the upper bounds quoted in eq. (2.54) would strongly disfavor (if not rule out) the TeV scale type I seesaw model under discussion with  $M_1 \sim (50 - 1000) \text{ GeV}$ .

It should be added that for  $M_1 \geq 10^3 \text{ GeV}$ , the factor  $C_0(x)$  is a monotonically (slowly) increasing function of  $M_1$  (see figure 1). The upper bound on  $\text{BR}(\tau \rightarrow 3\mu)$  following from



**Figure 1.** The dependence of  $C_0(x)$  as a function of the see-saw mass scale  $M_1$ .

the upper bound on  $BR(\mu \rightarrow e + \gamma)$  and the  $3\sigma$  ranges of the neutrino oscillation parameters, can be bigger than  $10^{-10}$  if  $C_0(x) \geq 1.8$ , which requires  $M_1 \geq 8.5 \times 10^6$  GeV. However, the rates of the processes of interest scale as  $\propto (v/M_1)^4$  and at values of  $M_1 \geq 8.5 \times 10^6$  GeV are too small to be observed in the currently planned experiments.

As we have already indicated, studies of the LFV  $\tau$  decays  $\tau \rightarrow \mu\gamma$  and  $\tau \rightarrow e\gamma$ , but not of the  $\tau \rightarrow 3\mu$  decay, in the TeV scale type I seesaw model were performed in [28]. The general difference between the studies performed in [28] and by us is that we were interested to get predictions for the absolute values of the branching ratios  $BR(\tau \rightarrow e\gamma)$ ,  $BR(\tau \rightarrow \mu\gamma)$  and  $BR(\tau \rightarrow 3\mu)$  to answer the question whether, given the various existing experimental constraints, these branching ratios can have values within the sensitivities of the future planned searches for the corresponding tau decays. The authors of ref. [28] were interested in predictions for the ratios  $B_{e\mu}/B_{e\tau}$  and  $B_{e\mu}/B_{\mu\tau}$ , where  $B_{e\mu} = BR(\mu \rightarrow e\gamma)$ ,  $B_{e\tau} = BR(\tau \rightarrow e\gamma)/BR(\tau \rightarrow e\bar{\nu}_e\nu_\tau)$  and  $B_{\mu\tau} = BR(\tau \rightarrow \mu\gamma)/BR(\tau \rightarrow \mu\bar{\nu}_\mu\nu_\tau)$ , and the dependence of these predictions on the values of the CPV phases and of  $\theta_{13}$ . In our study we have answered the question quoted above and thus obtained quantitative indications about the precision that should be reached in order to probe the predictions for the  $\tau$  LFV decay rates of the TeV scale type I seesaw model. We obtained also the constraints on the heavy Majorana neutrino CC couplings that can potentially be obtained in the future planned searches for the tau LFV decays  $\tau \rightarrow e\gamma$ ,  $\tau \rightarrow \mu\gamma$  and  $\tau \rightarrow 3\mu$ . We pointed out as well to the possibility of a strong suppression of the rates of these decays in the case of IH neutrino

mass spectrum due to a cancellation in the heavy Majorana neutrino CC coupling to the tau lepton and W-bosons and derived the general conditions for such a cancellation (eqs. (2.46) and (2.47)). Our results show, in particular, that  $BR(\tau \rightarrow \mu\gamma)$  and  $BR(\tau \rightarrow 3\mu)$ , predicted within the TeV scale type I seesaw, are at least by one order of magnitude smaller than the sensitivity of the future planned experiments. Thus, one of the conclusions of our study is that if  $\tau \rightarrow \mu\gamma$  and/or  $\tau \rightarrow 3\mu$  will be observed in the planned experiments, this will rule out the TeV scale type I seesaw model of neutrino mass generation.

None of the above was done in ref. [28] since the authors were focused on answering different questions, as we have indicated above. In particular, the  $\tau \rightarrow 3\mu$  decay is not considered at all in [28]. What is the overlap of our work with that done in ref. [28]? It is only in the formal fact that we consider the ratios  $(B_{e\mu}/B_{e\tau})^{-1}$  and  $(B_{e\mu}/B_{\mu\tau})^{-1}$ . However, we have done it in order to use the experimental upper limit on  $BR(\mu \rightarrow e\gamma)$  to constrain  $BR(\tau \rightarrow e\gamma)$  and  $BR(\tau \rightarrow \mu\gamma)$ . We derived the ranges in which  $(B_{e\mu}/B_{e\tau})^{-1}$  and  $(B_{e\mu}/B_{\mu\tau})^{-1}$  can vary by scanning the neutrino oscillation parameter space determined by the best fit, and the  $2\sigma$  and  $3\sigma$  allowed ranges of, values of the three neutrino mixing angles and two neutrino mass squared differences as well as by varying the CP violation Dirac and Majorana phases in their entire defining intervals. We were interested primarily in the absolute maxima of the ratios  $(B_{e\mu}/B_{e\tau})^{-1}$  and  $(B_{e\mu}/B_{\mu\tau})^{-1}$ , which we used to obtain absolute upper limits on  $BR(\tau \rightarrow e\gamma)$  and  $BR(\tau \rightarrow \mu\gamma)$  from the current experimental upper limit on  $BR(\mu \rightarrow e\gamma)$ . This was not done in ref. [28] in which the authors concentrated on the study of the dependence of  $B_{e\mu}/B_{e\tau}$  and  $B_{e\mu}/B_{\mu\tau}$  on the values of  $\sin\theta_{13}$  and on the CPV phases using the best fit values of the other 3-neutrino oscillation parameters. We found, in particular, using the  $2\sigma$  ( $3\sigma$ ) ranges of the neutrino oscillation parameters, the maximal values of  $BR(\tau \rightarrow e + \gamma)$  and  $BR(\tau \rightarrow \mu + \gamma)$  in the case of NH spectrum, eqs. (2.46) and (2.47). Similar results were obtained also in the case of IH spectrum. Our results for the absolute upper limits on  $BR(\tau \rightarrow e + \gamma)$  and  $BR(\tau \rightarrow \mu + \gamma)$  quoted above cannot be obtained from the results for the ratios  $B_{e\mu}/B_{e\tau}$  and  $B_{e\mu}/B_{\mu\tau}$  presented in ref. [28]. What we get from ref. [28] in the case of NH spectrum (figures 1 and 2 and the text), for instance, is upper limits on  $BR(\tau \rightarrow e + \gamma)$  and  $BR(\tau \rightarrow \mu + \gamma)$  which are smaller, respectively, by the factors  $\sim 25$  and  $\sim 50$  than the upper limits we have obtained. Similar observations are valid in the case of IH neutrino mass spectrum.

We have used the same method for the ratios  $BR(\tau \rightarrow 3\mu)/BR(\mu \rightarrow e + \gamma)$  and  $BR(\tau \rightarrow 3\mu)/BR(\mu \rightarrow 3e)$  to get an absolute upper bound on  $BR(\tau \rightarrow 3\mu)$  from the existing experimental upper limits on  $BR(\mu \rightarrow e + \gamma)$  and  $BR(\mu \rightarrow 3e)$ . The  $\tau \rightarrow 3\mu$  decay was not discussed in ref. [28].

### 3 The TeV scale Higgs triplet (type II see-saw) model

#### 3.1 Brief review of the TeV scale Higgs triplet model

In its simplest version the Higgs Triplet Model (HTM) [23–25] is an extension of the SM, which contains one additional  $SU(2)_L$  triplet scalar field  $\Delta$  carrying two units of the weak

hypercharge  $Y_W$ . The Lagrangian of the HTM has the form:<sup>4</sup>

$$\mathcal{L}_{\text{seesaw}}^{\text{II}} = -M_{\Delta}^2 \text{Tr} \left( \Delta^{\dagger} \Delta \right) - \left( h_{\ell\ell'} \overline{\psi}_{\ell L}^C i\tau_2 \Delta \psi_{\ell' L} + \mu_{\Delta} H^T i\tau_2 \Delta^{\dagger} H + \text{h.c.} \right), \quad (3.1)$$

where  $(\psi_{\ell L})^T \equiv (\nu_{\ell L}^T \ \ell_L^T)$ ,  $\overline{\psi}_{\ell L}^C \equiv (-\nu_{\ell L}^T C^{-1} \ -\ell_L^T C^{-1})$ , and  $H$  are, respectively, the SM lepton and Higgs doublets,  $C$  being the charge conjugation matrix, and

$$\Delta = \begin{pmatrix} \Delta^{+/\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\Delta^{+/\sqrt{2}} \end{pmatrix}. \quad (3.2)$$

In eq. (3.1),  $\mu_{\Delta}$  is a real parameter characterising the soft explicit breaking of the total lepton charge conservation. We will consider the TeV scale version of HTM, where the new physics scale  $M_{\Delta}$ , associated with the mass of  $\Delta$ , takes values  $100 \text{ GeV} \lesssim M_{\Delta} \lesssim 1 \text{ TeV}$ , which, in principle, can be probed by LHC [64–67].

The light neutrino mass matrix  $m_{\nu}$  is generated when the neutral component of  $\Delta$  develops a “small” vev  $v_{\Delta} \propto \mu_{\Delta}$ :

$$(m_{\nu})_{\ell\ell'} \equiv m_{\ell\ell'} \simeq 2 h_{\ell\ell'} v_{\Delta}. \quad (3.3)$$

Here  $h_{\ell\ell'}$  is the matrix of Yukawa couplings, which is directly related to the PMNS neutrino mixing matrix  $U_{\text{PMNS}} \equiv U$ :

$$h_{\ell\ell'} \equiv \frac{1}{2v_{\Delta}} \left( U^* \text{diag}(m_1, m_2, m_3) U^{\dagger} \right)_{\ell\ell'}. \quad (3.4)$$

It follows from the current data on the parameter  $\rho = M_W^2/M_Z^2 \cos^2 \theta_W$  that (see, e.g., [68])  $v_{\Delta}/v \leq 0.03$ , or  $v_{\Delta} < 5 \text{ GeV}$ ,  $v = 174 \text{ GeV}$  being the SM Higgs doublet v.e.v. We will consider in what follows values of  $v_{\Delta}$  lying roughly in the interval  $v_{\Delta} \sim (1 - 100) \text{ eV}$ . For  $M_{\Delta} \sim (100 - 1000) \text{ GeV}$  and the indicated values of  $v_{\Delta}$ , the rates of LFV processes involving the  $\mu^{\pm}$  can have values close to the existing upper limits (see [60] and references quoted therein). A small value of  $v_{\Delta}$  implies that that  $\mu_{\Delta}$  has also to be small: for  $M_{\Delta} \sim v = 174 \text{ GeV}$  we have  $v_{\Delta} \cong \mu_{\Delta}$ , while if  $M_{\Delta}^2 \gg v^2$ , then  $v_{\Delta} \cong \mu_{\Delta} v^2 / (2M_{\Delta}^2)$  (see, e.g., [62, 63, 68]). The requisite small value of  $\mu_{\Delta}$ , and thus of  $v_{\Delta}$ , can be generated, e.g., at higher orders in perturbation theory [69] or in the context of theories with extra dimensions (see, e.g., [70]).

The physical singly-charged Higgs scalar field practically coincides with the triplet scalar field  $\Delta^+$ , the admixture of the doublet charged scalar field being suppressed by the factor  $v_{\Delta}/v$ . The singly- and doubly- charged Higgs scalars  $\Delta^+$  and  $\Delta^{++}$  have, in general, different masses [69, 71–73]:  $m_{\Delta^+} \neq m_{\Delta^{++}}$ . Both possibilities  $m_{\Delta^+} > m_{\Delta^{++}}$  and  $m_{\Delta^+} < m_{\Delta^{++}}$  are allowed. In what follows, for simplicity, we will present numerical results for  $m_{\Delta^+} \cong m_{\Delta^{++}} \equiv M_{\Delta}$ .

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<sup>4</sup>We do not give here, for simplicity, all the quadratic and quartic terms present in the scalar potential (see, e.g., [62, 63]).

### 3.2 The $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow e\gamma$ decays

In the Higgs triplet model considered, the  $\ell \rightarrow \ell' + \gamma$  decay amplitude receives at leading order contributions from one loop diagrams with exchange of virtual singly and doubly-charged Higgs scalars. A detailed calculation of these contributions leads to the result [42, 43, 60, 74]:

$$\text{BR}(\ell \rightarrow \ell' + \gamma) = \frac{\alpha_{\text{em}}}{192\pi} \frac{|(h^\dagger h)_{\ell\ell'}|^2}{G_F^2} \left( \frac{1}{m_{\Delta^+}^2} + \frac{8}{m_{\Delta^{++}}^2} \right)^2 \text{BR}(\ell \rightarrow \nu_\ell \ell' \bar{\nu}_{\ell'}), \quad (3.5)$$

where  $\ell = \mu$  and  $\ell' = e$ , or  $\ell = \tau$  and  $\ell' = \mu, e$ . For  $m_{\Delta^+} \approx m_{\Delta^{++}} = M_\Delta$ , the expression in eq. (3.5) can be cast in the form:

$$\text{BR}(\ell \rightarrow \ell' + \gamma) = \frac{27\alpha_{\text{em}}}{64\pi} \frac{|(m^\dagger m)_{\ell\ell'}|^2}{16v_\Delta^4 G_F^2 M_\Delta^4} \text{BR}(\ell \rightarrow \nu_\ell \ell' \bar{\nu}_{\ell'}). \quad (3.6)$$

The factor  $|(m^\dagger m)_{\ell\ell'}|$ , as it is not difficult to show, is given by:

$$|(m^\dagger m)_{\ell\ell'}| = |U_{\ell 2} U_{\ell' 2}^* \Delta m_{21}^2 + U_{\ell 3} U_{\ell' 3}^* \Delta m_{31}^2|, \quad (3.7)$$

where we have used eqs. (3.3) and (3.4) and the unitarity of the PMNS matrix. The expression in eq. (3.7) is exact. Obviously,  $|(m^\dagger m)_{\ell\ell'}|$  does not depend on the Majorana phases present in the PMNS matrix  $U$ .

The branching ratios,  $\text{BR}(\ell \rightarrow \ell' + \gamma)$ , are inversely proportional to  $(v_\Delta M_\Delta)^4$ . From the the current upper bound on  $\text{BR}(\mu \rightarrow e + \gamma)$ , eq. (1.6), and the expression for  $|(m^\dagger m)_{\mu e}|$  in terms of the neutrino oscillation parameters, one can obtain a lower limit on  $v_\Delta M_\Delta$  [60]:

$$v_\Delta > 2.98 \times 10^2 |s_{13} s_{23} \Delta m_{31}^2|^{\frac{1}{2}} \left( \frac{100 \text{ GeV}}{M_\Delta} \right). \quad (3.8)$$

Using the the best fit values ( $3\sigma$  allowed ranges) of  $\sin \theta_{13}$ ,  $\sin \theta_{23}$  and  $\Delta m_{31}^2$ , obtained in the global analysis [11] we find:

$$v_\Delta M_\Delta > 4.60 (3.77) \times 10^{-7} \text{ GeV}^2. \quad (3.9)$$

As in the case of type I seesaw model, we can obtain an upper bounds on the branching ratios  $\text{BR}(\tau \rightarrow \mu + \gamma)$  and  $\text{BR}(\tau \rightarrow e + \gamma)$  of interest using their relation with  $\text{BR}(\mu \rightarrow e + \gamma)$  and the current experimental upper bound on  $\text{BR}(\mu \rightarrow e + \gamma)$ . We have:

$$\frac{\text{BR}(\tau \rightarrow \mu(e) + \gamma)}{\text{BR}(\mu \rightarrow e + \gamma)} = \frac{|(m^\dagger m)_{\tau\mu(e)}|^2}{|(m^\dagger m)_{\mu e}|^2} \text{BR}(\tau \rightarrow \nu_\tau \mu(e) \bar{\nu}_{\mu(e)}). \quad (3.10)$$

Using again the expressions for  $|(m^\dagger m)_{\ell\ell'}|$  in terms of neutrino oscillation parameters and the best fit values quoted in eqs. (1.4) and (1.5) we get in the case of NO (IO) neutrino mass spectrum:

$$4.41 (4.47) \leq \frac{|(m^\dagger m)_{\tau\mu}|}{|(m^\dagger m)_{\mu e}|} \leq 5.57 (5.64), \quad \text{NO (IO) b.f.} \quad (3.11)$$

$$1.05 (1.03) \leq \frac{|(m^\dagger m)_{\tau e}|}{|(m^\dagger m)_{\mu e}|} \leq 1.53 (1.51) \quad \text{NO (IO) b.f.} \quad (3.12)$$



Employing the  $3\sigma$  allowed ranges of the neutrino oscillation parameters derived in [11] we obtain:

$$0.87 \text{ (0.57)} \leq \frac{|(m^+m)_{\tau e}|}{|(m^+m)_{\mu e}|} \leq 1.79 \text{ (1.78)} \quad \text{NO (IO) } 2\sigma; \quad (3.13)$$

$$3.07 \text{ (3.04)} \leq \frac{|(m^+m)_{\tau \mu}|}{|(m^+m)_{\mu e}|} \leq 7.72 \text{ (7.85)} \quad \text{NO (IO) } 3\sigma; \quad (3.14)$$

$$0.55 \text{ (0.52)} \leq \frac{|(m^+m)_{\tau e}|}{|(m^+m)_{\mu e}|} \leq 1.95 \text{ (1.95)} \quad \text{NO (IO) } 3\sigma. \quad (3.15)$$

From eqs. (1.6), (3.10), (3.14) and (3.15) it follows that

$$\text{BR}(\tau \rightarrow \mu + \gamma) < 5.9 \text{ (6.1)} \times 10^{-12}, \quad \text{BR}(\tau \rightarrow e + \gamma) < 3.9 \times 10^{-13}, \quad \text{NO (IO)}. \quad (3.16)$$

These values are significantly below the planned sensitivity of the future experiments on the  $\tau \rightarrow \mu + \gamma$  and  $\tau \rightarrow e + \gamma$  decays. The observation of the any of the two decays having a branching ratio definitely larger than that quoted in eq. (3.16) would rule out the TeV scale Higgs triplet model under discussion.

### 3.3 The $\tau \rightarrow 3\mu$ decay

The leading contribution in the  $\tau \rightarrow 3\mu$  decay amplitude in the TeV scale HTM is due to a tree level diagram with exchange of the virtual doubly-charged Higgs scalar  $\Delta^{++}$ . The corresponding  $\tau \rightarrow 3\mu$  decay branching ratio is given by [75, 76] (see also, e.g., [43, 77]):

$$\text{BR}(\tau \rightarrow 3\mu) = \frac{|h_{\mu\mu}^* h_{\tau\mu}|^2}{G_F^2 M_\Delta^4} \text{BR}(\tau \rightarrow \mu \bar{\nu}_\mu \nu_\tau) = \frac{1}{G_F^2 M_\Delta^4} \frac{|m_{\mu\mu}^* m_{\tau\mu}|^2}{16v_\Delta^4} \text{BR}(\tau \rightarrow \mu \bar{\nu}_\mu \nu_\tau), \quad (3.17)$$

where  $M_\Delta \equiv m_{\Delta^{++}}$  is the  $\Delta^{++}$  mass and we have neglected corrections  $\sim m_\mu/m_\tau \cong 0.06$ .

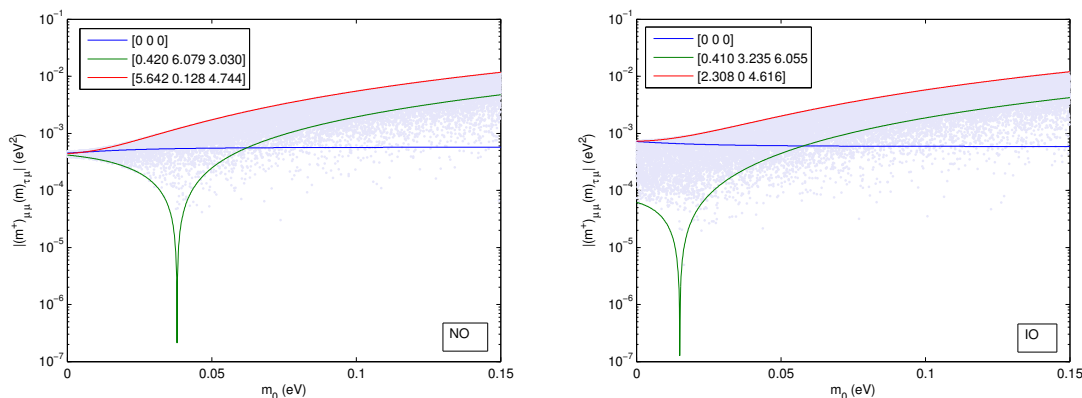
Using the current upper bound on  $\text{BR}(\tau \rightarrow 3\mu)$ , eq. (1.11), and eq. (3.17), we get the following constraint:

$$|h_{\mu\mu}^* h_{\tau\mu}| < 4.1 \times 10^{-5} \left( \frac{M_\Delta}{100 \text{ GeV}} \right)^2. \quad (3.18)$$

Further, the lower limit on the product of  $v_\Delta$  and  $M_\Delta$ , eq. (3.9), implies the following upper limit on  $\text{BR}(\tau \rightarrow 3\mu)$ :

$$\text{BR}(\tau \rightarrow 3\mu) < 1.88 \text{ (4.17)} \times 10^{-3} \frac{|m_{\mu\mu}^* m_{\tau\mu}|^2}{(1 \text{ eV})^4}. \quad (3.19)$$

The factor  $|m_{\mu\mu}^* m_{\tau\mu}|$ , as can be shown using eqs. (3.3) and (3.4), depends not only on the neutrino oscillation parameters, but also on the type of the neutrino mass spectrum, the lightest neutrino mass  $m_0 \equiv \min(m_j)$ ,  $j = 1, 2, 3$  (i.e., on the absolute neutrino mass scale), and on the Majorana CPV phases  $\alpha_{21}$  and  $\alpha_{31}$ , present in the PMNS matrix. The dependence of  $|m_{\mu\mu}^* m_{\tau\mu}|$  on  $m_0$  for three sets of values of the CPV Dirac and Majorana phases  $\delta$ ,  $\alpha_{21}$  and  $\alpha_{31}$  in the cases of NO and IO neutrino mass spectra is illustrated in figure 2. The neutrino oscillation parameters were set to their best fit values quoted in



**Figure 2.** The dependence of  $|m_{\mu\mu}^* m_{\tau\mu}|$  on the lightest neutrino mass  $m_0$  in the cases of NO (left panel) and IO (right panel) neutrino mass spectra, for three sets of values of the Dirac and Majorana CPV phases,  $[\delta, \alpha_{21}, \alpha_{31}]$ . The neutrino oscillation parameters  $\sin \theta_{12}$ ,  $\sin \theta_{23}$ ,  $\sin \theta_{13}$ ,  $\Delta m_{21}^2$  and  $\Delta m_{31}^2$  have been set to their best fit values, eqs. (1.4) and (1.5). The scattered points are obtained by varying Dirac and Majorana CPV phases randomly in the interval  $[0, 2\pi]$ .

eqs. (1.4) and (1.5). As figure 2 indicates, both for the NO and IO spectra, the maximal allowed value of  $|m_{\mu\mu}^* m_{\tau\mu}|$  is a monotonically increasing function of  $m_0$ .

The intervals of possible values of  $|m_{\mu\mu}^* m_{\tau\mu}|$  in the cases of NO and IO neutrino mass spectra determine the ranges of allowed values of  $\text{BR}(\tau \rightarrow 3\mu)$  in the TeV scale HTM. Varying the three CPV phases independently in the interval  $[0, 2\pi]$  and using the best fit, the  $2\sigma$  and the  $3\sigma$  allowed ranges of values of  $\sin \theta_{12}$ ,  $\sin \theta_{23}$ ,  $\sin \theta_{13}$ ,  $\Delta m_{21}^2$  and  $\Delta m_{31}^2$  derived in [11], we get for  $m_0 = 0; 0.01; 0, 10 \text{ eV}$ :

- $m_0 = 0 \text{ eV}$ , NO (IO)

$$38.0 (5.35) \times 10^{-5} \text{ eV}^2 \leq |(m^*)_{\mu\mu}(m)_{\tau\mu}| \leq 4.82 (7.38) \times 10^{-4} \text{ eV}^2 \quad \text{b.f.}; \quad (3.20)$$

$$2.77 (0.00) \times 10^{-4} \text{ eV}^2 \leq |(m^*)_{\mu\mu}(m)_{\tau\mu}| \leq 5.89 (8.11) \times 10^{-4} \text{ eV}^2 \quad 2\sigma; \quad (3.21)$$

$$2.33 (0.00) \times 10^{-4} \text{ eV}^2 \leq |(m^*)_{\mu\mu}(m)_{\tau\mu}| \leq 8.35 (8.45) \times 10^{-4} \text{ eV}^2 \quad 3\sigma. \quad (3.22)$$

- $m_0 = 0.01 \text{ eV}$ , NO (IO)

$$33.6 (1.66) \times 10^{-5} \text{ eV}^2 \leq |(m^*)_{\mu\mu}(m)_{\tau\mu}| \leq 5.34 (8.06) \times 10^{-4} \text{ eV}^2 \quad \text{b.f.}; \quad (3.23)$$

$$2.24 (0.00) \times 10^{-4} \text{ eV}^2 \leq |(m^*)_{\mu\mu}(m)_{\tau\mu}| \leq 6.41 (8.99) \times 10^{-4} \text{ eV}^2 \quad 2\sigma; \quad (3.24)$$

$$1.76 (0.00) \times 10^{-4} \text{ eV}^2 \leq |(m^*)_{\mu\mu}(m)_{\tau\mu}| \leq 8.96 (9.41) \times 10^{-4} \text{ eV}^2 \quad 3\sigma. \quad (3.25)$$

- $m_0 = 0.1 \text{ eV}$ , NO (IO)

$$0.00 (0.00) \text{ eV}^2 \leq |(m^*)_{\mu\mu}(m)_{\tau\mu}| \leq 5.48 (5.76) \times 10^{-3} \text{ eV}^2 \quad \text{b.f.}; \quad (3.26)$$

$$0.00 (0.00) \text{ eV}^2 \leq |(m^*)_{\mu\mu}(m)_{\tau\mu}| \leq 5.57 (5.85) \times 10^{-3} \text{ eV}^2 \quad 2\sigma; \quad (3.27)$$

$$0.00 (0.00) \text{ eV}^2 \leq |(m^*)_{\mu\mu}(m)_{\tau\mu}| \leq 5.85 (5.88) \times 10^{-3} \text{ eV}^2 \quad 3\sigma. \quad (3.28)$$

We would like to determine next whether  $\text{BR}(\tau \rightarrow 3\mu)$  predicted by the TeV scale HTM considered can be bigger than the sensitivity limit of  $\sim 10^{-10}$  of the future planned experiment on  $\tau \rightarrow 3\mu$  decay, given the stringent upper bounds on the  $\mu \rightarrow e + \gamma$  and  $\mu \rightarrow 3e$  decay branching ratios, eqs. (1.6) and (1.7). As we have seen, the current upper bound on  $\text{BR}(\mu \rightarrow e + \gamma)$  leads to the lower limit eq. (3.9) of  $v_\Delta M_\Delta$ . We have to take into account also the important constraint on  $\text{BR}(\tau \rightarrow 3\mu)$  following from the current upper bound on  $\mu \rightarrow 3e$  decay branching ratio  $\text{BR}(\mu \rightarrow 3e)$ , eq. (1.7). In the case of  $\text{BR}(\mu \rightarrow 3e)$  we have  $\text{BR}(\mu \rightarrow 3e) \propto |m_{\mu e}^* m_{ee}|^2$ . The quantity  $|m_{\mu e}^* m_{ee}|$ , and thus  $\text{BR}(\mu \rightarrow 3e)$ , depends on the same set of neutrino mass and mixing parameters as  $|(m^*)_{\mu\mu}(m)_{\tau\mu}|$ , and thus  $\text{BR}(\tau \rightarrow 3\mu)$ . We have performed a numerical analysis in order to determine the regions of values of the neutrino oscillation parameters and of the three CPV phases  $\delta$ ,  $\alpha_{21}$  and  $\alpha_{31}$ , in which the experimental upper bounds on  $\text{BR}(\mu \rightarrow e + \gamma)$  and  $\text{BR}(\mu \rightarrow 3e)$ , eqs. (1.6) and (1.7), and the following requirement,

$$10^{-10} \leq \text{BR}(\tau \rightarrow 3\mu) \leq 10^{-8}, \quad (3.29)$$

are simultaneously satisfied. The analysis is performed for three values of  $m_0 = 0; 0.01 \text{ eV}; 0.10 \text{ eV}$ . The neutrino oscillation parameters  $\sin \theta_{12}$ ,  $\sin \theta_{23}$ ,  $\sin \theta_{13}$ ,  $\Delta m_{21}^2$  and  $\Delta m_{31}^2$  were varied in their respective  $3\sigma$  allowed ranges taken from [11]. The CPV phases  $\delta$ ,  $\alpha_{21}$  and  $\alpha_{31}$  were varied independently in the interval  $[0, 2\pi]$ . The results of this analysis are presented graphically in figure 3, in which we show the regions of values of the quantities  $|m_{\mu\mu}^* m_{\tau\mu}|$  and  $v_\Delta M_\Delta$  where the three conditions (1.6), (1.7) and (3.29) are simultaneously fulfilled in the cases of  $m_0 = 0; 0.01 \text{ eV}; 0.10 \text{ eV}$  for the NO and IO spectra. For  $m_0 = 0$  and NO spectrum, the results depend weakly on the CPV phases; they are independent of the phase  $\alpha_{31}$  if  $m_0 = 0$  and the spectrum is of the IO type. The analysis performed by us shows that the maximal values  $\text{BR}(\tau \rightarrow 3\mu)$  can have are the following:

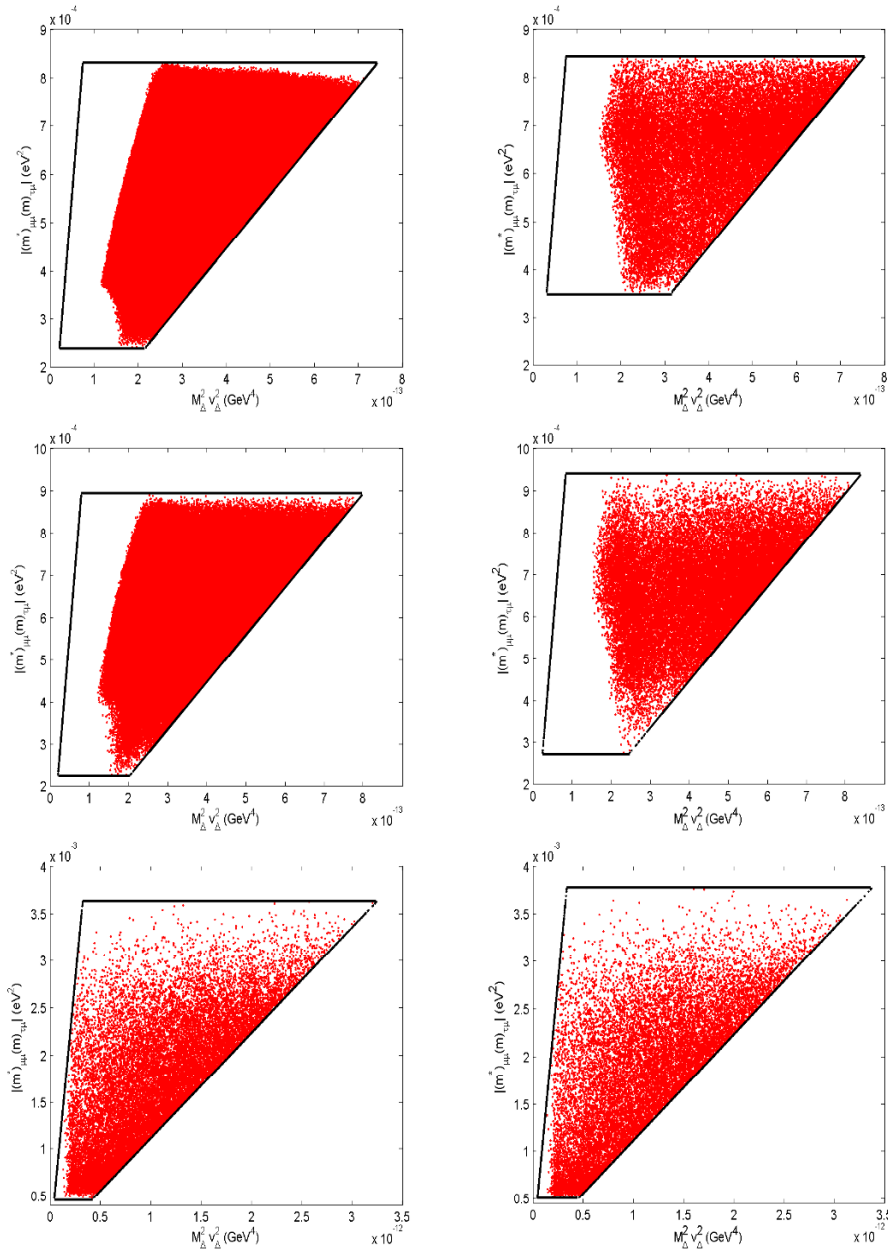
$$\text{BR}(\tau \rightarrow 3\mu) \leq 1.02 (1.68) \times 10^{-9}, \quad m_0 = 0 \text{ eV}, \quad \text{NO (IO)}, \quad (3.30)$$

$$\text{BR}(\tau \rightarrow 3\mu) \leq 1.24 (2.05) \times 10^{-9}, \quad m_0 = 0.01 \text{ eV}, \quad \text{NO (IO)}, \quad (3.31)$$

$$\text{BR}(\tau \rightarrow 3\mu) \leq 8.64 (9.11) \times 10^{-9}, \quad m_0 = 0.10 \text{ eV}, \quad \text{NO (IO)}. \quad (3.32)$$

Thus, for all the three values of  $m_0$  considered, which span essentially the whole interval of possible values of  $m_0$ , the maximal allowed values of  $\text{BR}(\tau \rightarrow 3\mu)$  is by a factor of  $\sim 10$  to  $\sim 90$  bigger than the projected sensitivity limit of  $10^{-10}$  of the future experiment on the  $\tau \rightarrow 3\mu$  decay. The regions on the  $|m_{\mu\mu}^* m_{\tau\mu}| - v_\Delta M_\Delta$  plane, where the three conditions of interest are satisfied, are sizable. The maximal value of  $\text{BR}(\tau \rightarrow 3\mu)$  for, e.g.,  $m_0 = 0.01 \text{ eV}$  and NO (IO) spectrum, quoted in eq. (3.31), is reached for  $\sin^2 \theta_{12} = 0.269 (0.308)$ ,  $\sin^2 \theta_{23} = 0.527 (0.438)$ ,  $\sin^2 \theta_{13} = 0.0268 (0.0203)$ ,  $\Delta m_{21}^2 = 7.38 (7.56) \times 10^{-5} \text{ eV}^2$ ,  $\Delta m_{31}^2 = 2.14 (2.40) \times 10^{-3} \text{ eV}^2$  and  $[\delta, \alpha_{21}, \alpha_{31}] = [2.300, 5.098, 3.437] ([1.577, 0.161, 3.436])$ .

As it follows from figure 2 and the results quoted in eqs. (3.20)–(3.28), for certain values of the absolute neutrino mass scale  $m_0$  and the CPV phases,  $|m_{\mu\mu}^* m_{\tau\mu}|$  can be strongly suppressed; we can have even  $|m_{\mu\mu}^* m_{\tau\mu}| = 0$ . For NO (IO) neutrino mass spectrum, such a strong suppression can happen for  $m_0 \gtrsim 38 \text{ meV}$  ( $m_0 \gtrsim 15 \text{ meV}$ ). The strong suppression of  $|m_{\mu\mu}^* m_{\tau\mu}|$  seen in figure 2 takes place in the case of NO (IO) spectrum at  $m_0 = 38 \text{ meV}$  and



**Figure 3.** The regions in the  $v_{\Delta}^2 M_{\Delta}^2 - |(m_{\mu\mu}^*)(m_{\tau\mu})|$  plane where  $10^{-10} \leq BR(\tau \rightarrow 3\mu) \leq 10^{-8}$  (the areas delimited by the black lines) and the the upper limits  $BR(\tau \rightarrow 3e) < 10^{-12}$  and  $BR(\tau \rightarrow e\gamma) < 5.7 \times 10^{-13}$  are satisfied (the colored areas), for  $m_0 = 0$  (upper panels), 0.01 eV (middle panels), 0.10 eV (lower panels) and NO (left panels) and IO (right panels) neutrino mass spectra. The figures are obtained by varying the neutrino oscillation parameters in their  $3\sigma$  allowed ranges [11]; the CPV Dirac and Majorana phases were varied in the interval  $[0, 2\pi]$ .

$[\delta, \alpha_{21}, \alpha_{31}] = [0.420, 6.079, 3.030]$  ( $m_0 = 15$  meV and  $[\delta, \alpha_{21}, \alpha_{31}] = [0.410, 3.235, 6.055]$ ). For  $m_0 = 0.10$  eV, for instance, we have  $|m_{\mu\mu}^* m_{\tau\mu}| = 0$  in the case of NO mass spectrum at  $\delta = 2.633$ ,  $\alpha_{21} = 2.533$  and  $\alpha_{31} = 5.349$ , while for the IO spectrum  $|m_{\mu\mu}^* m_{\tau\mu}|$  goes through zero for  $\delta = 4.078$ ,  $\alpha_{21} = 2.161$  and  $\alpha_{31} = 5.212$ . The above examples of the vanishing of

$|m_{\mu\mu}^* m_{\tau\mu}|$  when  $m_0 = 0.10 \text{ eV}$  are not unique, it can happen also at other specific sets of values of the Dirac and Majorana CPV phases.

If in the planned experiment on the  $\tau \rightarrow 3\mu$  decay the limit  $BR(\tau \rightarrow 3\mu) < 10^{-10}$  will be obtained, this will imply the following upper limit on the product  $|h_{\mu\mu}^* h_{\tau\mu}|$  of Yukawa couplings:

$$|h_{\mu\mu}^* h_{\tau\mu}| < 2.83 \times 10^{-6} \left( \frac{M_\Delta}{100 \text{ GeV}} \right)^2. \quad (3.33)$$

As we have mentioned in the Introduction, comments about the  $\tau \rightarrow \mu\gamma$ ,  $\tau \rightarrow e\gamma$  and  $\tau \rightarrow 3\mu$  decays in the Higgs Triplet model were made in ref. [42], while in [43] the authors investigated the predictions of the Higgs Triplet model for the  $\tau \rightarrow 3\mu$  decay.

A general comment concerning the investigations of the  $\tau$  LFV decays performed in the Higgs Triplet Model in in refs. [42, 43] is that these studies were made before the value of the reactor mixing angle  $\theta_{13}$ , which can play an important role in the phenomenology of the LFV  $\tau$  decays in the model, was known. In the period after refs. [42, 43] were published, the experimental upper bound on  $BR(\mu \rightarrow e + \gamma)$  improved (i.e., decreased) from  $1.2 \times 10^{-11}$  to  $5.7 \times 10^{-13}$ . The new bound plays important role in obtaining absolute upper bounds on the predicted branching ratios  $BR(\tau \rightarrow e\gamma)$ ,  $BR(\tau \rightarrow \mu\gamma)$  and  $BR(\tau \rightarrow 3\mu)$  both in the TeV scale type I seesaw and HT models. This new constraint could not be taken into account by the authors of refs. [42, 43]. As a consequence, the main quantitative results obtained and conclusions reached in [42, 43] are no longer valid.

To be more specific, the study performed in ref. [42] is devoted primarily to the investigation of the  $\mu \rightarrow e + \gamma$  and  $\mu \rightarrow 3e$  decays in the Higgs Triplet model. The authors of ref. [42] include comments about the  $\tau$  LFV decays, which take 6 lines in the arXiv version of the article. Obviously, none of the quantitative results obtained by us can be found in ref. [42]. In [42] the authors claim, in particular, that  $\tau \rightarrow 3\mu$  decay would be unobservable in the future planned experiments as a consequence of the experimental upper bound on  $BR(\mu \rightarrow 3e)$  which severely constrains also  $BR(\tau \rightarrow 3\mu)$ . As we have shown above, this conclusion is not correct.

The  $\tau \rightarrow 3\mu$  decay was considered in ref. [43]. In order to avoid the constraints on  $BR(\tau \rightarrow 3\mu)$  from the existing stringent experimental upper bound on  $BR(\mu \rightarrow 3e)$ , the authors of [43] exploit the possibility of a strong suppression of  $BR(\mu \rightarrow e\gamma)$  and/or  $BR(\mu \rightarrow 3e)$ , which, however, does not extend to  $BR(\tau \rightarrow 3\mu)$ . In the special cases of specific CP conserving values of the Dirac and Majorana CPV phases they consider they find that the suppression in question can be realised for values  $\sin\theta_{13}$ , most of which are ruled out by subsequent high precision measurements of  $\sin\theta_{13}$  in the Daya Bay and RENO experiments. In the case of  $BR(\tau \rightarrow 3\mu)$ , for instance, we have found sizable regions in the parameter space of HTM both in the cases of NO and IO neutrino mass spectrum where the current experimental limits on  $BR(\mu \rightarrow e\gamma)$  and  $BR(\mu \rightarrow 3e)$  are satisfied and at the same time  $BR(\tau \rightarrow 3\mu)$  can have a value in the range of sensitivity of the future planned experiments at the SuperB factory. In contrast to [43], this is done in the present article by performing a scan of the full parameter space of the model for three fixed values of the lightest neutrino mass  $\min(m_j) \equiv m_0$ , which effectively span the entire

interval of allowed values of  $m_0$ . The authors of [43] present their results in terms of the ratio  $BR(\tau \rightarrow 3\mu)/BR(\mu \rightarrow 3e)$  and use as criterion of observability of  $\tau \rightarrow 3\mu$  decay the inequality  $BR(\tau \rightarrow 3\mu)/BR(\mu \rightarrow 3e) > 10^3$ , given the upper limit  $BR(\mu \rightarrow 3e) < 10^{-12}$ . However, the fact that  $BR(\tau \rightarrow 3\mu)/BR(\mu \rightarrow 3e) > 10^3$  does not necessarily guarantee that  $BR(\tau \rightarrow 3\mu)$  falls in the range of sensitivity of planned experiments: if, for instance, for the chosen values of the parameters for which, e.g.,  $BR(\tau \rightarrow 3\mu)/BR(\mu \rightarrow 3e) = 2 \times 10^3$  one has also  $BR(\mu \rightarrow 3e) \leq 5 \times 10^{-14}$ , one finds  $BR(\tau \rightarrow 3\mu) \leq 10^{-10}$ , which is below the sensitivity of the planned experiments. We could not find an indication in ref. [43] that such possibilities are accounted for. Cases like the one just described are taken into account in the analysis we perform by the way we implement the constraints from the upper limits on  $BR(\mu \rightarrow e\gamma)$  and  $BR(\mu \rightarrow 3e)$ . The authors of ref. [43] could not and did not take into account the current severe upper limit on  $BR(\mu \rightarrow e\gamma)$ , which further restricts the parameter space in which  $BR(\tau \rightarrow 3\mu)$  has a value in the sensitivity range of the future planned experiments. Thus, our quantitative results on  $BR(\tau \rightarrow 3\mu)$ , illustrated in figure 2 and summarised in figure 3, were derived using a more comprehensive approach than those obtained in [43]. As a consequence, our results differ from the results reported in ref. [43].

## 4 Conclusions

In the present article we have investigated in detail the  $\tau \rightarrow (e, \mu) + \gamma$  and  $\tau \rightarrow 3\mu$  decays in the TeV scale type I see-saw and Higgs Triplet models of neutrino mass generation. Future experiments at the SuperB factory are planned to have sensitivity to the branching ratios of these decays  $BR(\tau \rightarrow (e, \mu) + \gamma) \gtrsim 10^{-9}$  and  $BR(\tau \rightarrow 3\mu) \gtrsim 10^{-10}$ , which is an improvement by one and two orders of magnitude with respect to that reached so far in the searches for the  $\tau \rightarrow (e, \mu) + \gamma$  and  $\tau \rightarrow 3\mu$  decays, respectively. In the models we have considered the scale of new physics associated with the existence of nonzero neutrino masses and neutrino mixing is assumed to be in the range of  $\sim (100 - 1000)$  GeV. In the type I see-saw scenario this scale is determined by the masses of the heavy Majorana neutrinos, while in the Higgs Triplet model it corresponds to the masses of the new singly charged, doubly charged and neutral physical Higgs particles. In the type I see-saw class of models of interest, the flavour structure of the couplings of the new particles — the heavy Majorana neutrinos  $N_j$  — to the charged leptons and  $W^\pm$ -boson and to the flavour neutrino fields and the  $Z^0$ -boson,  $(RV)_{lj}$ ,  $l = e, \mu, \tau$ , are basically determined by the requirement of reproducing the data on the neutrino oscillation parameters (see, e.g., [29]). In the Higgs Triplet model the Yukawa couplings of the new scalar particles to the charged leptons and neutrinos are proportional to the Majorana mass matrix of the LH active flavour neutrinos. As a consequence, the rates of the LFV processes in the charged lepton sector can be calculated in both models in terms of a few unknown parameters. These parameters are constrained by different sets of data such as, e.g., data on neutrino oscillations, from EW precision tests, on the LFV violating processes  $\mu \rightarrow e + \gamma$ ,  $\mu \rightarrow 3e$ , etc. In the TeV scale type I see-saw scenario considered all the constraints can be satisfied for sizable values of the couplings  $|(RV)_{lj}|$  in a model [29] with two heavy Majorana neutrinos  $N_{1,2}$ , in which the latter have close masses forming a pseudo-Dirac state,  $M_2 = M_1(1 + z)$ ,  $M_{1,2}, z > 0$ ,

$z \ll 1$ , and their charged and neutral current couplings (see eqs. (2.3) and (2.4)),  $(RV)_{lj}$ ,  $j = 1, 2$ , satisfy eq. (2.7). In this scheme the lightest neutrino mass  $m_0 = 0$  and the neutrino mass spectrum is either normal hierarchical (NH) or inverted hierarchical (IH),

We find using the constraints on the couplings  $(RV)_{lj}$ ,  $j = 1, 2$ , from the low energy electroweak precision data, eqs. (2.10)–(2.12), that the branching ratios of the decays  $\tau \rightarrow (e, \mu) + \gamma$  and  $\tau \rightarrow 3\mu$  predicted in the TeV scale type I see-saw model can at most be of the order of the sensitivity of the planned future experiments,  $\text{BR}(\tau \rightarrow (e, \mu) + \gamma) \lesssim 10^{-9}$  and  $\text{BR}(\tau \rightarrow 3\mu) \lesssim 10^{-10}$ . Taking into account the stringent experimental upper bounds on the  $\mu \rightarrow e + \gamma$  and  $\mu \rightarrow 3e$  decay rates has the effect of constraining further the maximal values of  $\text{BR}(\tau \rightarrow (e, \mu) + \gamma)$  and  $\text{BR}(\tau \rightarrow 3\mu)$  compatible with the data. In the case of NH spectrum, for instance, we get using the  $2\sigma$  ( $3\sigma$ ) ranges of the neutrino oscillations parameters from [11] and varying the CPV Dirac and Majorana phases  $\delta$ ,  $\alpha_{21}$  and  $\alpha_{31}$  independently in the interval  $[0, 2\pi]$ :  $\text{BR}(\tau \rightarrow e + \gamma) \lesssim 1.4$  ( $1.7$ )  $\times 10^{-12}$ ,  $\text{BR}(\tau \rightarrow \mu + \gamma) \lesssim 4.9$  ( $9.7$ )  $\times 10^{-11}$ , and  $\text{BR}(\tau \rightarrow 3\mu) \lesssim 3.8$  ( $7.4$ )  $\times 10^{-12}$ . For specific values of the neutrino mixing parameters in the case of the IH spectrum, the predicted rates of the  $\mu \rightarrow e + \gamma$  and  $\mu \rightarrow 3e$  decays are strongly suppressed and the experimental upper bounds on these rates are automatically satisfied. In this special case the  $\tau \rightarrow \mu + \gamma$  and the  $\tau \rightarrow 3\mu$  decay rates are also predicted to be strongly suppressed and significantly smaller than the planned sensitivity of the future experiments, while for the  $\tau \rightarrow e + \gamma$  decay we have  $\text{BR}(\tau \rightarrow e + \gamma) \lesssim 10^{-9}$ . Clearly, if any of the three  $\tau$  decays under discussion is observed in the planned experiments, the TeV scale type I see-saw model we have considered will be strongly disfavored if not ruled out.

The predicted rates of the  $\mu \rightarrow e + \gamma$  and of the  $\tau \rightarrow (e, \mu) + \gamma$  decays in the Higgs Triplet model are also correlated. Using the existing experimental upper bound on  $\text{BR}(\mu \rightarrow e + \gamma)$  we find the following upper limits on the  $\tau \rightarrow \mu + \gamma$  and  $\tau \rightarrow e + \gamma$  decay branching ratios for the NO (IO) neutrino mass spectrum:  $\text{BR}(\tau \rightarrow \mu + \gamma) \lesssim 5.9$  ( $6.1$ )  $\times 10^{-12}$ ,  $\text{BR}(\tau \rightarrow e + \gamma) \lesssim 3.9 \times 10^{-12}$ . These values are significantly below the planned sensitivity of the future experiments on the  $\tau \rightarrow \mu + \gamma$  and  $\tau \rightarrow e + \gamma$  decays. The observation of the any of the two decays having a branching ratio definitely larger than that quoted above would rule out the TeV scale Higgs triplet model under discussion. In contrast, we find that in a sizable region of the parameter space of the Higgs Triplet model, the  $\tau \rightarrow 3\mu$  decay branching ratio  $\text{BR}(\tau \rightarrow 3\mu)$  can have a value in the interval  $(10^{-10} - 10^{-8})$  and the predicted values of  $\text{BR}(\mu \rightarrow e + \gamma)$  and  $\text{BR}(\mu \rightarrow 3e)$  satisfy the existing stringent experimental upper bounds. Thus, the observation of the  $\tau \rightarrow 3\mu$  decay with  $\text{BR}(\tau \rightarrow 3\mu) \gtrsim 10^{-10}$  and the non-observation of the  $\tau \rightarrow \mu + \gamma$  and  $\tau \rightarrow e + \gamma$  decays in the planned experiments having a sensitivity to  $\text{BR}(\tau \rightarrow (e, \mu) + \gamma) \geq 10^{-9}$ , would constitute an evidence in favor of the Higgs Triplet model.

To conclude, the planned searches for the  $\tau \rightarrow \mu + \gamma$ ,  $\tau \rightarrow e + \gamma$  and  $\tau \rightarrow 3\mu$  decays with sensitivity to  $\text{BR}(\tau \rightarrow (e, \mu) + \gamma) \gtrsim 10^{-9}$  and to  $\text{BR}(\tau \rightarrow 3\mu) \gtrsim 10^{-10}$  will provide additional important test of the TeV scale see-saw type I and Higgs Triplet models of neutrino mass generation.

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