Physics Letters A 377 (2013) 2520–2523

Contents lists available at SciVerse ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Protecting entanglement of atoms stored in a common nonperfect cavity without measurements

Nguyen Ba An

Center for Theoretical Physics, Institute of Physics, 10 Dao Tan, Ba Dinh, Hanoi, Viet Nam

ARTICLE INFO

Article history:
Received 19 June 2013
Accepted 17 July 2013
Available online 22 July 2013
Communicated by V.M. Agranovich

Keywords:
EPR-type entanglement
W-type entanglement
Entanglement protection
Common cavity

ABSTRACT

We study entanglement dynamics of two and three atoms stored in a common nonperfect cavity together with some other nonentangled atoms. It is guessed at first thought that the presence of nonentangled atoms would favor the decoherence process of the interested entangled atoms. We show, on the contrary, that it is not so. Namely, as results of a rigorous nonperturbative analysis, disentanglement rate of the interested atoms decreases with the increase of the number of nonentangled atoms. If the number of nonentangled atoms is sufficiently large, the entanglement of interested atoms could be protected efficiently.

1. Introduction

The 'spooky-action-at-distance' feature of special quantum states was disliked by Einstein who, however, exploited it in an attempt to convince the incompleteness of quantum mechanics in 1935 [1]. Later in the same year, in a gedanken experiment aiming at a possible demonstration of how the microscopic and macroscopic worlds can directly be coupled to each other [2], Schrödinger coined that 'spooky' feature entanglement (verschränkung in German) and the states possessing it entangled states. Nowadays, it is ubiquitous that entanglement offers a vital shared resource allowing to perform various quantum network protocols only by means of local operations and classical communication (see, e.g., [3]). After production and before distribution to remote authorized parties, entangled states are often stored for some time in a register. But during the storage process entangled states are degrading and tend to be separable for a sufficiently long time. Therefore, protecting entanglement for a later use is of paramount importance.

In this Letter we are interested in bipartite and tripartite entangled states of two-level atoms (served as qubits) stored in a common nonperfect cavity together with some $\mathcal{N}$ additional atoms which are not entangled with the interested atoms as well as with each other. At first thought one might guess that the presence of such additional atoms would cause a negative affect on the quality of the atomic entangled states when all the atoms evolve due to interaction with the cavity modes. We shall develop exact theory of the dynamics of a general class of multiatom states and then base on it to explicitly show that the entanglement degradation of EPR-type and W-type states turns out to slow down as the number of the additional atoms increases. Theoretically, the entanglement quality could be invariant in the large-$\mathcal{N}$ limit.

2. EPR-type entanglement

In Ref. [4] the authors analyzed the exact entanglement dynamics of two two-level atoms initially prepared in an EPR-type state

$$\left|epr(0)\right\rangle_{12} = \left(a_1(0)|10\rangle + a_2(0)|01\rangle\right)_{12}, \tag{1}$$

with $|a_1(0)|^2 + |a_2(0)|^2 = 1$ (which reduces to an EPR state [1] when $|a_1(0)| = |a_2(0)|$) and $|0\rangle$ ($|1\rangle$) the atom ground (excited) state, inside a common lossy (i.e., nonperfect) cavity at zero-temperature with the Lorentzian spectral density

$$J(\omega) = \frac{R^2}{\pi}\frac{\Gamma}{(\omega-\omega_c)^2+\Gamma^2}, \tag{2}$$

where ω_c is the frequency of the mode supported by the cavity, Γ^2 describes the leakage probability of the cavity mode through the nonideal walls and R measures the atom–cavity coupling strength. The result obtained is that the entanglement degree measured by concurrence [6] decreases quickly with time in both the bad (i.e., weak coupling or Markovian regime) and good (i.e., strong coupling or non-Markovian regime) cavity limits. To fight against the entanglement deterioration the authors proposed a method by using the quantum Zeno effect, that requires a series of frequent specific nonselective measurements on the collective atomic system. Alternatively, atomic entanglement can also be protected by

E-mail address: nban@iop.vast.ac.vn.

0375-9601/$ – see front matter
http://dx.doi.org/10.1016/j.physleta.2013.07.032

weak measurements combined with quantum measurement reversals [5]. Here, we propose another method that does not need any measurements. Instead, we add some $\mathcal{N}$ auxiliary atoms in the ground state and let all the atoms evolve inside the cavity. That is, at $t=0$ the total atom–cavity system state is

$$\big|\psi(0)\big\rangle_s = \big|epr(0)\big\rangle_{12}|00\ldots0\rangle_{34\ldots N}|\bar{\mathbf{0}}\rangle_c, \tag{3}$$

where $N=\mathcal{N}+2$ and $|\bar{\mathbf{0}}\rangle_c=\bigotimes_j|0_j\rangle_c$ with $|0_j\rangle_c$ the cavity state containing zero photon in mode j.

To see how the total system state (3) evolves in time, let us study a more general problem as follows. Consider a nonperfect cavity at zero-temperature with the spectral density (2) that contains in it an N-atom ($N\geqslant 3$) state of the form

$$\begin{aligned}\big|\Phi(0)\big\rangle_{12\ldots N} = {}&\big(a_1(0)|10\ldots0\rangle + a_2(0)|01\ldots0\rangle+\cdots\\ &+a_N(0)|00\ldots1\rangle\big)_{12\ldots N},\end{aligned} \tag{4}$$

with $\sum_{n=1}^{N}|a_n(0)|^2=1$. Extending the case of $N=2$ in Ref. [4] to an arbitrary N, the total atom–cavity system Hamiltonian is ($\hbar=1$)

$$\hat{H}=\hat{H}_0+\hat{H}_{int}, \tag{5}$$

with

$$\hat{H}_0=\sum_{n=1}^{N}\Omega\hat{\sigma}_n^+\hat{\sigma}_n+\sum_j\omega_j\hat{a}_j^+\hat{a}_j \tag{6}$$

and

$$\hat{H}_{int}=\sum_{n=1}^{N}\sum_j\alpha_n\big(g_j\hat{\sigma}_n^+\hat{a}_j+g_j^*\hat{\sigma}_n\hat{a}_j^+\big). \tag{7}$$

In the above equations $\hat{\sigma}_n=|0\rangle_n\langle1|$, Ω is the atomic transition frequency, while ω_j and $\hat{a}_j$ ($\hat{a}_j^+$) are the frequency and the annihilation (creation) operator of the cavity mode-j photon. The interaction between the nth atom and the mode-j photon is described by $\alpha_n g_j$, with α_n a real positive dimensionless constant whose value depends on the value of the cavity field at the atom position and can be effectively adjusted, say, by tuning the atomic transition thanks to dc Stark effect [4]. In what follows, as in Ref. [4], $\alpha=\sqrt{\sum_{n=1}^{N}\alpha_n^2}$ and $r_n=\alpha_n/\alpha$ (so that $\sum_{n=1}^{N}r_n^2=1$) are introduced for convenience.

The total system state at $t=0$ is then

$$\big|\Psi(0)\big\rangle_s=\big|\Phi(0)\big\rangle_{12\ldots N}|\bar{\mathbf{0}}\rangle_c. \tag{8}$$

Since $\sum_j\hat{a}_j^+\hat{a}_j+\sum_{n=1}^{N}\hat{\sigma}_n^+\hat{\sigma}_n$ commutes with $\hat{H}$, at time $t>0$ the state (8) evolves into

$$\begin{aligned}\big|\Psi(t)\big\rangle_s = {}&e^{-i\Omega t}\big|\Phi(t)\big\rangle_{12\ldots N}|\bar{\mathbf{0}}\rangle_c\\ &+\sum_j b_j(t)e^{-i\omega_j t}|00\ldots0\rangle_{12\ldots N}|1_j\rangle_c,\end{aligned} \tag{9}$$

with $|1_j\rangle_c$ the cavity state containing only one photon in mode j. The time-dependent coefficients $a_n(t)$ and $b_j(t)$ can be derived from the equation of motion governed by $\hat{H}$:

$$i\frac{da_n(t)}{dt}=\alpha_n\sum_j g_j e^{-i(\omega_j-\Omega)t}b_j(t), \tag{10}$$

$$i\frac{db_j(t)}{dt}=g_j^*e^{i(\omega_j-\Omega)t}\sum_{n=1}^{N}\alpha_n a_n(t). \tag{11}$$

Solving Eq. (11) for $b_j(t)$ with the initial condition $b_j(0)=0$ then substituting it into Eq. (10) yields

$$\frac{da_n(t)}{dt}=-\int_0^t\int d\omega\, J(\omega)e^{-i(\omega-\Omega)(t-t')}\alpha_n\sum_{m=1}^{N}\alpha_m a_m\big(t'\big)\,dt'. \tag{12}$$

By Laplace transforming both sides of Eq. (12) we obtain an algebraic closed set of equations for $\{\tilde{a}_n(z)=\mathcal{L}[a_n(t)]\}$. Solving this equation set and inverse Laplace transforming the solutions gives

$$a_n(t)=a_n(0)-r_nF(t)\sum_{m=1}^{N}r_m a_m(0) \tag{13}$$

where

$$\begin{aligned}F(t)=1-e^{-(\Gamma+i\Delta)t/2}\Bigg[&\cosh\left(\frac{t}{2}\sqrt{(\Gamma+i\Delta)^2-4\alpha^2R^2}\right)\\ &+\frac{\Gamma+i\Delta}{\sqrt{(\Gamma+i\Delta)^2-4\alpha^2R^2}}\\ &\times\sinh\left(\frac{t}{2}\sqrt{(\Gamma+i\Delta)^2-4\alpha^2R^2}\right)\Bigg],\end{aligned} \tag{14}$$

with $\Delta=\omega_c-\Omega$. Defining the single-photon collective normalized state of the cavity field as

$$|\bar{\mathbf{1}}\rangle_c=\frac{e^{i\omega_c t}}{b(t)}\sum_j b_j(t)e^{-i\omega_j t}|1_j\rangle_c, \tag{15}$$

with

$$b(t)=\sqrt{1-\sum_{n=1}^{N}\big|a_n(t)\big|^2}, \tag{16}$$

we obtain the explicit expression for $|\Psi(t)\rangle_s$:

$$\begin{aligned}\big|\Psi(t)\big\rangle_s = {}&e^{-i\Omega t}\big[a_1(t)|10\ldots0\rangle+a_2(t)|01\ldots0\rangle+\cdots\\ &+a_N(t)|00\ldots1\rangle\big]_{12\ldots N}|\bar{\mathbf{0}}\rangle_c\\ &+e^{-i\omega_c t}b(t)|00\ldots0\rangle_{12\ldots N}|\bar{\mathbf{1}}\rangle_c,\end{aligned} \tag{17}$$

with $a_n(t)$ and $b(t)$ determined by Eqs. (13) and (16), respectively.

Returning now to the atoms' state of interest, i.e., state $|epr(0)\rangle_{12}|00\ldots0\rangle_{34\ldots N}$ with $|epr(0)\rangle_{12}$ given by Eq. (1). It can be verified that state $|epr(0)\rangle_{12}|00\ldots0\rangle_{34\ldots N}$ is a particular case of the states $|\Phi(0)\rangle_{12\ldots N}$ in Eq. (4) with $a_3(0)=a_4(0)=\cdots=a_N(0)=0$. So the above analytical results for the general case apply to the dynamics of EPR-type endamagement which we are interested in in this section. Concretely, according to the general results derived above, at any $t>0$ state (3) evolves into state (17) with

$$a_n(t)=a_n(0)-r_nF(t)\big(r_1a_1(0)+r_2a_2(0)\big). \tag{18}$$

The atomic state $\rho^{12\ldots N}(t)$ is obtained by tracing out over the field modes:

$$\begin{aligned}\rho^{12\ldots N}(t)&=\mathrm{Tr}_c\big|\Psi(t)\big\rangle_s\langle\Psi(t)|\\ &=\big|\Phi(t)\big\rangle_{12\ldots N}\langle\Phi(t)|+\big|b(t)\big|^2|00\ldots0\rangle_{12\ldots N}\langle00\ldots0|.\end{aligned} \tag{19}$$

The state of the two interested atoms 1 and 2 is then described by the reduced density matrix $\rho^{12}(t)=\mathrm{Tr}_{34\ldots N}\,\rho^{12\ldots N}(t)$. After the necessary tracing procedure we arrive at

$$\begin{aligned}\rho^{12}(t)=\left(\big|b(t)\big|^2+\sum_{m=3}^{N}\big|a_m(t)\big|^2\right)&|00\rangle_{12}\langle00|\\ &+\big(a_1(t)|10\rangle+a_2(t)|01\rangle\big)_{12}\big(a_1^*(t)\langle10|+a_2^*(t)\langle01|\big).\end{aligned} \tag{20}$$

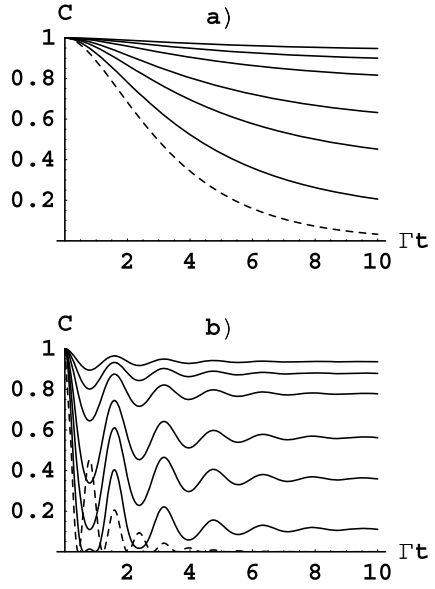

Fig. 1. Concurrence C, Eq. (21), of the two-atom EPR-type entangled state (1) versus the scaled time Γt for $a_1(0) = a_2(0) = 1/\sqrt{2}$, $\Delta = 0$, $r_n = 1/\sqrt{N}$ ($N = \mathcal{N} + 2$) and different values of $\mathcal{N}$ in (a) the Markovian regime with $2R\alpha/\Gamma = 0.8$ and (b) the non-Markovian regime with $2R\alpha/\Gamma = 8.0$. In both (a) and (b) the curves from bottom to top correspond to $\mathcal{N} = 0$ (dashed) and $\mathcal{N} = 1$, 3, 6, 15, 30 and 60 (solid).

As is clear from Eq. (20), $\rho^{12}(t)$ is of X form in which the only nonzero elements are $\rho^{12}_{00,00}(t)$, $\rho^{12}_{10,10}(t)$, $\rho^{12}_{10,01}(t)$, $\rho^{12}_{01,10}(t)$ and $\rho^{12}_{01,01}(t)$. Hence, the concurrence $C(t)$ [6] of $\rho^{12}(t)$ is easy to calculate. Explicitly in terms of the parameters involved it reads

$$C(t) = 2\big|a_1(0)a_2(0) - \big(r_1a_2(0) + r_2a_1(0)\big) \times \big(r_1a_1(0) + r_2a_2(0)\big)F(t) + r_1r_2\big(r_1a_1(0) + r_2a_2(0)\big)^2F^2(t)\big|, \tag{21}$$

with $F(t)$ defined in Eq. (14).

Fig. 1 plots the concurrence C, Eq. (21), as a function of Γt for different numbers $\mathcal{N}$ of the nonentangled atoms that are put into the cavity in addition to the two interested entangled atoms 1 and 2. The dashed curve is for $\mathcal{N} = 0$, while the solid ones are for $\mathcal{N} \geqslant 1$. We see that independent of the atom–cavity coupling strength (i.e., in both the Markovian and non-Markovian regimes) the concurrence decreases with time slower for $\mathcal{N} \geqslant 1$ than for $\mathcal{N} = 0$. The oscillatory behavior of C in the non-Markovian regime (Fig. 1b) is due to the well-known memory effect. It is worth emphasizing that for $\mathcal{N} = 0$ the dead and revival phenomenon of entanglement is well pronounced at the initial stage of evolution but then the entanglement is rapidly vanishing. It is in clear contrast to the case of $\mathcal{N} > 0$ for which the entanglement may survive all the time (i.e., never be dead); it just suffers some oscillation at the beginning and then saturates to a finite value, as confirmed in Fig. 1b for $\mathcal{N} \geqslant 3$. Of general significance is the better protection of entanglement for a greater value of $\mathcal{N}$. The decay of entanglement, in theory, could be suppressed in the limit of large $\mathcal{N}$.

3. W-type entanglement

A symmetric three-qubit entangled state called W state was originally introduced in Ref. [7] which is more robust than others in the sense that if either of the three qubits is lost the two remaining ones are still entangled. W states have found useful applications in quantum information processing [8]. Useful are also asymmetric versions of W state of the form

$$|w\rangle_{123} = \big(a_1|100\rangle + a_2|010\rangle + a_3|001\rangle\big)_{123}, \quad \sum_{n=1}^{3}|a_n|^2 = 1, \tag{22}$$

which are referred to as W-type states [9]. In Ref. [10] entanglement dynamics of W-type state (22) of three two-level atoms embedded in a common structured cavity was examined in detail and several methods to control the atoms' entanglement were suggested. In this section we shall study the influence of $\mathcal{N} \geqslant 1$ nonentangled atoms on the evolution of the atomic W-type state. These $\mathcal{N}$ auxiliary atoms are put into the cavity and evolve together with the three entangled atoms of interest. That is, at $t = 0$ we have the following state for the whole atom–cavity system

$$\big|\chi(0)\big\rangle_s = \big(a_1(0)|100\rangle + a_2(0)|010\rangle + a_3(0)|001\rangle\big)_{123} \times |00\ldots0\rangle_{45\ldots N}|\bar{\mathbf{0}}\rangle_c. \tag{23}$$

Note that the N-atom state $(a_1(0)|100\rangle + a_2(0)|010\rangle + a_3(0)|001\rangle)_{123}|00\ldots0\rangle_{45\ldots N}$ is nothing else but the state (4) when $a_4(0) = a_5(0) = \cdots = a_N(0) = 0$. This allows us to employ the general theory developed in the previous section. Actually, at $t > 0$ state (23) evolves into state (17) in which now the coefficients $a_n(t)$ are determined by

$$a_n(t) = a_n(0) - r_nF(t)\big(r_1a_1(0) + r_2a_2(0) + r_3a_3(0)\big). \tag{24}$$

The three interested atoms 1, 2 and 3, after a time duration t of evolution together with the $\mathcal{N}$ atoms $4, 5, \ldots, N$ are described by the reduced density matrix $\rho^{123}(t) = \mathrm{Tr}_{c45\ldots N}|\Psi(t)\rangle_s\langle\Psi(t)|$. As a result of calculations we obtain

$$\rho^{123}(t) = \left(\big|b(t)\big|^2 + \sum_{m=4}^{N}\big|a_m(t)\big|^2\right)|000\rangle_{123}\langle000| + \big(a_1(t)|100\rangle + a_2(t)|010\rangle + a_3(t)|001\rangle\big)_{123} \times \big(a_1^*(t)\langle100| + a_2^*(t)\langle010| + a_3^*(t)\langle001|\big). \tag{25}$$

To assess entanglement degree analytically the so-called lower bound of concurrence (*LBC*) [11,12] can be used. Although vanishing *LBC* does not necessarily imply separability, a positive *LBC* reveal entanglement with certainty. Thus, use of *LBC* for the tendency of entanglement dynamics is acceptable. The *LBC* of a three-qubit mixed state ρ^{123} is defined by

$$LBC\big(\rho^{123}\big) = \sqrt{\frac{1}{3}\sum_{j=1}^{6}\big[\big(C_j^{12|3}\big(\rho^{123}\big)\big)^2 + \big(C_j^{31|2}\big(\rho^{123}\big)\big)^2 + \big(C_j^{23|1}\big(\rho^{123}\big)\big)^2\big]}, \tag{26}$$

where

$$C_j^{kl|m}\big(\rho^{123}\big) = \max\left\{0, \sqrt{\lambda_{j,1}^{kl|m}} - \sum_{n>1}\sqrt{\lambda_{j,n}^{kl|m}}\right\} \tag{27}$$

and $\lambda_{j,n}^{kl|m}$ are the eigenvalues, in decreasing order, of the non-Hermitian matrix $\rho(L_j^{kl} \otimes \sigma_y^m)\rho^*(L_j^{kl} \otimes \sigma_y^m)$ with $\{L_j^{kl};\ j = 1, 2, \ldots, 6\}$ the six generators of group $SO(4)$ [13] acting on qubits k, l and σ_y^m the y-Pauli matrix acting on qubit m. For the density matrix (25) we have explicitly

$$LBC = \left\{\frac{8}{3}\big[\big|a_1(0)a_2(0) - \big(r_1a_2(0) + r_2a_1(0)\big) \times \big(r_1a_1(0) + r_2a_2(0) + r_3a_3(0)\big)F(t)\right.$$

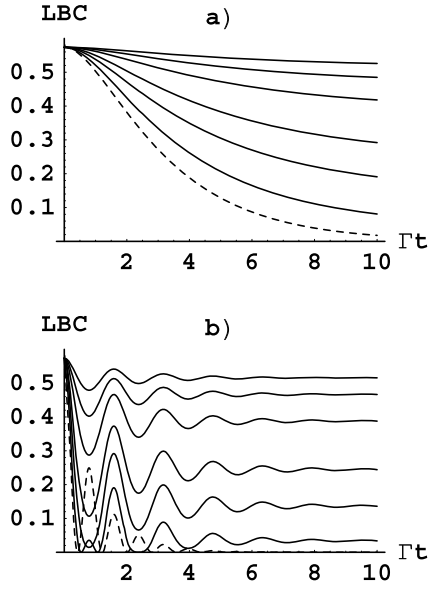

Fig. 2. Lower bound concurrence *LBC*, Eq. (28), of the three-atom W-type entangled state (22) versus the scaled time Γt for $a_1(0) = a_2(0) = a_3(0) = 1/\sqrt{3}$, $\Delta = 0$, $r_n = 1/\sqrt{N}$ ($N = \mathcal{N} + 3$) and different values of $\mathcal{N}$ in (a) the Markovian regime with $2R\alpha/\Gamma = 0.8$ and (b) the non-Markovian regime with $2R\alpha/\Gamma = 8.0$. In both (a) and (b) the curves from bottom to top correspond to $\mathcal{N} = 0$ (dashed) and $\mathcal{N} = 1$, 3, 6, 15, 30 and 60 (solid).

$$
\begin{aligned}
&+ r_1 r_2 \big(r_1 a_1(0) + r_2 a_2(0) + r_3 a_3(0)\big)^2 F^2(t)\big|^2\big] \\
&+ \big[\big|a_1(0)a_3(0) - \big(r_1 a_3(0) + r_3 a_1(0)\big) \\
&\times \big(r_1 a_1(0) + r_2 a_2(0) + r_3 a_3(0)\big) F(t) \\
&+ r_1 r_3 \big(r_1 a_1(0) + r_2 a_2(0) + r_3 a_3(0)\big)^2 F^2(t)\big|^2\big] \\
&+ \big[\big|a_2(0)a_3(0) - \big(r_2 a_3(0) + r_3 a_2(0)\big) \\
&\times \big(r_1 a_1(0) + r_2 a_2(0) + r_3 a_3(0)\big) F(t) \\
&+ r_2 r_3 \big(r_1 a_1(0) + r_2 a_2(0) + r_3 a_3(0)\big)^2 F^2(t)\big|^2\big]\Big\}^{1/2}. \qquad (28)
\end{aligned}
$$

The dynamics of W-type entanglement displayed in Fig. 2 is similar to that of EPR-type entanglement, consolidating the efficiency of our proposed method to protect atoms' entanglement in a common lossy environment. Generally, as visual from Fig. 2, $LBC(\mathcal{N}') > LBC(\mathcal{N})$ for $\mathcal{N}' > \mathcal{N}$ and $LBC(t) \to LBC(0)$ in the large-$\mathcal{N}$ limit.

4. Conclusion

To summarize, we have suggested a method to cope with degradation of EPR-type and W-type entanglement of atoms immersed in a common nonperfect cavity by adding a number of auxiliary atoms prepared in their ground states. All the atoms interact with the common cavity modes and evolve together. We have analytically solved the problem in a rigorous nonperturbative manner so that the results are valid in both the Markovian and non-Markovian regimes. Although during their evolution all the atoms get entangled with each other, we have shown that the entanglement of the interested atoms' state obtained by tracing out over the added atoms decays slower than it would do without the added atoms. And, the entanglement protection is more efficient when the number of added atoms increases. So the entanglement degradation of the interested atoms could be remarkably suppressed by adding sufficiently many auxiliary atoms. Our method requires neither cavity detuning nor repeated projective measurements using the quantum Zeno effect as proposed previously in Refs. [4,5,10]. Similar idea was touched upon in Ref. [14] to provide more entanglement of a special class of state of two qubits within the same environment.

Acknowledgements

This work is devoted to my respected teacher, Professor Nguyen Van Hieu, on the occasion of his 75th birthday.

References

[1] A. Einstein, B. Podolsky, N. Rosen, Phys. Rev. 47 (1935) 777.
[2] E. Schrödinger, Naturwissenschaften 23 (1935) 812.
[3] A.K. Ekert, Phys. Rev. Lett. 67 (1991) 661;
C. Bennett, S.J. Wiesner, Phys. Rev. Lett. 69 (1992) 2881;
C.H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, W.K. Wootters, Phys. Rev. Lett. 70 (1993) 1895;
A. Karlsson, M. Bourennane, Phys. Rev. A 58 (1998) 4394;
M. Hillery, V. Buzek, A. Berthiaume, Phys. Rev. A 59 (1999) 1829;
B.A. Nguyen, Phys. Lett. A (2004) 328;
B.A. Nguyen, J. Kor. Phys. Soc. 47 (2005) 562.
[4] S. Maniscalco, F. Francica, R.L. Zaffino, N.L. Gullo, F. Plastina, Phys. Rev. Lett. 100 (2008) 090503.
[5] Z.X. Man, Y.J. Xia, N.B. An, Phys. Rev. A 86 (2012) 012325.
[6] W.K. Wootters, Phys. Rev. Lett. 80 (1998) 2245.
[7] W. Dur, G. Vidal, J.I. Cirac, Phys. Rev. A 62 (2000) 062314.
[8] B.S. Shi, A. Tomita, Phys. Lett. A 296 (2002) 161;
J. Joo, Y.J. Park, S. Oh, J. Kim, New J. Phys. 5 (2003) 136;
J. Joo, Y.J. Park, J. Kor. Phys. Soc. 46 (2005) 763;
M.Y. Wang, F.L. Yan, Chin. Phys. Lett. 28 (2011) 060301.
[9] A.K. Pati, Pramana J. Phys. 59 (2002) 217;
P. Agrawal, A. Pati, Phys. Rev. A 74 (2006) 062320;
S.B. Zheng, Phys. Rev. A 74 (2006) 054303;
W. Cui, E. Chitambar, H.K. Lo, Phys. Rev. A 82 (2010) 062314.
[10] N.B. An, J. Kim, K. Kim, Phys. Rev. A 82 (2010) 032316.
[11] X.H. Gao, S.M. Fei, K. Wu, Phys. Rev. A 74 (2006) 050303(R);
X.H. Gao, S.M. Fei, Eur. Phys. J. Special Topics 159 (2008) 71.
[12] M. Li, S.M. Fei, Z.X. Wang, J. Phys. A: Math. Theor. 42 (2009) 145303.
[13] Z.Q. Ma, X.Y. Gu, Problems and Solutions in Group Theory for Physicists, World Scientific Publishing Co. Pte. Ltd., 2004.
[14] F. Benatti, A. Nagy, Annals Phys. 326 (2011) 740.