Scheme for the generation of freely traveling optical trio coherent states

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Trio coherent states (TCSs) are non-Gaussian three-mode entangled states which can serve as a useful resource for continuous-variable quantum tasks, so their generation is of primary importance. Schemes exist to generate stable TCSs in terms of vibrational motion of a trapped ion inside a crystal. However, to perform quantum communication and distributed quantum computation the states should be shared beforehand among distant parties. That is, their modes should be able to be directed to different desired locations in space. In this work, we propose an experimental setup to generate such free-traveling TCSs in terms of optical fields. Our scheme uses standard physical resources, such as coherent states, balanced beam splitters, phase shifters, nonideal on-off photodetectors, and realistic weak cross-Kerr nonlinearities, without the need of single photons or homodyne or heterodyne measurements. We study the dependences of the fidelity of the state generated by our scheme with respect to the target TCS and the corresponding generation probability for the parameters involved. In theory, the fidelity could be nearly perfect for whatever weak nonlinearities τ and low photodetector efficiency η, provided that the amplitude $|\alpha|$ of an input coherent state is large enough, namely, $|\alpha| \geqslant 5/(\sqrt{\eta}\tau)$.

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I. INTRODUCTION

Coherent states introduced by Glauber [1] and Sudarshan [2] as minimum uncertainty states or eigenstates of the field annihilation operator play a very significant role in mathematical physics, quantum optics, and quantum information sciences, from both theoretical and experimental points of view, especially after the invention of lasers. Although coherent states are themselves the most classical states, their superposition may exhibit various interesting nonclassical effects that have no counterparts in the classical world. Thus, coherent-state superpositions quickly attracted a lot of intensive attention [3]. The simplest single-mode superposition of two distinguishable coherent states is perhaps the cat state, a name coined from Schrödinger's [4] paradox where a cat could simultaneously be alive and dead. Information can be encoded in cat states. However, for the encoding to be useful, one should be capable of manipulating that kind of quantum information; that is, one should be able to teleport or to remotely prepare such cat states for the purpose of quantum communication as well as to perform quantum logic gates on them for the purpose of quantum computation. These require special nonlocal resources by means of superpositions of multimode coherent states, which are referred to as entangled coherent states (ECSs) [5]. In fact, many types of ECSs exist, such as Einstein-Podolsky-Rosen [6], Greenberger-Horne-Zeilinger [7], W [8,9], cluster [10,11], and so on. Among other types of ECSs worthy of studying are the so-called pair coherent state (PCS) [12–17] and trio coherent state (TCS) [18,19], which provide attractive examples for non-Gaussian nonclassical states of two- and three-mode continuous-variable fields. PCSs have found applications in testing quantum mechanics versus local realism [13], in quantum teleportation [14], in entanglement swapping [15], in Heisenberg-limited interferometry [16], etc. As for TCSs, they promise to be useful in quantum tasks involving three parties, such as controlled teleportation [20], quantum secret sharing [21], and joint remote state preparation [22] in the discrete-variable context. Before proposing possible quantum protocols using TCSs, one needs to generate them. Actually, stable TCSs can be produced for the vibrational motion of an ion trapped in a three-dimensional isotropic harmonic potential [19]. However, such vibrational TCSs are confined inside a crystal, giving no benefit to performing global quantum information processing and distributed quantum computing when the participating parties are far apart from each other.

In this paper, we propose a feasible scheme to generate optical TCSs whose modes can freely travel in space so that any three distant parties are able to share them for a subsequent desired cooperation. The actual physical inputs are very simple: only four coherent states are needed, without any single-photon sources. Furthermore, our scheme does not require photon-number-resolving detectors or homodyne or heterodyne measurements. As for linear optical devices, four balanced beam splitters and several ordinary phase shifters suffice. Also, four cross-Kerr media are necessary,

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but strong nonlinearities are not at all a prerequisite for implementing the generation process. Taken altogether, our scheme is within the reach of current technologies. In Sec. II, we first recapitulate the definition of the TCS and describe the experimental setup, then proceed to explain in detail how our scheme works. We derive explicit analytic expressions for the fidelity of the generated state with respect to the desired TCS and the corresponding probability. The dependence of these characteristics on the parameters involved is also studied. Finally, we conclude in Sec. III.

II. OPTICAL TRIO COHERENT STATES AND GENERATION SCHEME

A TCS $|\Psi_{p,q}(\xi)\rangle_{123}$, with ξ being complex and p,q being integers, is a three-mode entangled state which can be represented by a continuous superposition of three coherent states as

$$|\Psi_{p,q}(\xi)\rangle_{123} = \mathcal{N}_{p,q}(r)e^{3r^2/2}\int_0^{2\pi}\frac{d\vartheta}{2\pi}\int_0^{2\pi}\frac{d\vartheta'}{2\pi}e^{-i[q\vartheta+(q+p)\vartheta']} \times |\xi e^{i\vartheta}\rangle_1|\xi e^{i\vartheta'}\rangle_2|\xi e^{-i(\vartheta+\vartheta')}\rangle_3, \quad (1)$$

where $|\xi e^{i\vartheta}\rangle_1$, $|\xi e^{i\vartheta'}\rangle_2$, and $|\xi e^{-i(\vartheta+\vartheta')}\rangle_3$ are three ordinary coherent states, $r = |\xi|$, and

$$\mathcal{N}_{p,q}(r) = \left(\sum_{n=0}^{\infty}\frac{r^{2(3n+2q+p)}}{n!(n+q)!(n+q+p)!}\right)^{-1/2}. \quad (2)$$

By definition [18] the TCS is the simultaneous eigenstate of the three operators $\widehat{A}_{123} = \hat{a}_1\hat{a}_2\hat{a}_3$, $\widehat{N}_{21} = \hat{a}_2^\dagger\hat{a}_2 - \hat{a}_1^\dagger\hat{a}_1$, and $\widehat{N}_{32} = \hat{a}_3^\dagger\hat{a}_3 - \hat{a}_2^\dagger\hat{a}_2$, with $\hat{a}_i^\dagger$ ($\hat{a}_i$) being the bosonic creation (annihilation) operator of mode $i = 1,2,3$ of an optical field. That is, $|\Psi_{p,q}(\xi)\rangle_{123}$ is the solution of the following three equations:

$$\widehat{A}_{123}|\Psi_{p,q}(\xi)\rangle_{123} = \xi^3|\Psi_{p,q}(\xi)\rangle_{123}, \quad (3)$$

$$\widehat{N}_{21}|\Psi_{p,q}(\xi)\rangle_{123} = q|\Psi_{p,q}(\xi)\rangle_{123}, \quad (4)$$

$$\widehat{N}_{32}|\Psi_{p,q}(\xi)\rangle_{123} = p|\Psi_{p,q}(\xi)\rangle_{123}. \quad (5)$$

Represented in Fock space, $|\Psi_{p,q}(\xi)\rangle_{123}$ reads [18]

$$|\Psi_{p,q}(\xi)\rangle_{123} = \mathcal{N}_{p,q}(r)\sum_{n=0}^{\infty}\frac{\xi^{3n+2q+p}}{\sqrt{n!(n+q)!(n+q+p)!}} \times |n\rangle_1\,|n+q\rangle_2\,|n+q+p\rangle_3\,, \quad (6)$$

where $|m\rangle_i$ denotes a number state of mode i containing m photons. Nonclassical properties of TCSs, such as sub-Poissonian statistics, violation of Cauchy-Schwarz inequalities, antibunching, and squeezing, as well as extensions to higher-order and multidimensional effects, were considered in great detail [18]. As is evident from Eq. (6), if we detect either one of the three modes and find a concrete number of photons, then we immediately know with certainty how many photons there are in the other two modes, without any ways to "touch" those modes and independent of how far apart the modes are. Such a spooky influence at distance signifies that $|\Psi_{p,q}(\xi)\rangle_{123}$ are entangled states and could serve as nonlocal resources for quantum communication and distributed quantum computing, provided that they can be generated in practice and shared among remote parties. The schemes proposed in Ref. [19] indeed generate TCSs which are, however, associated with the bosonic field of the vibrational motion of a trapped ion inside a crystal that cannot be distributed for global quantum tasks. Here, we design an experimental setup to produce optical TCSs whose modes, although entangled with each other, can be guided to propagate to any location in open space.

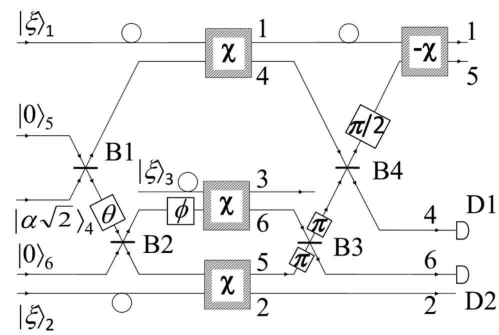

FIG. 1. Schematic experimental setup for generation of free-traveling optical TCSs. B1 to B4 are balanced beam splitters, a square with, e.g., θ is a phase shifter $\hat{P}(\theta)$, a shaded rectangle with $\pm\chi$ is a cross-Kerr medium with nonlinearities $\pm\chi$, and D1 and D2 are nonideal on-off photodetectors. Inputs 1, 2, and 3 are coherent states whose amplitude $|\xi|$ is the same as that of the TCS to be prepared, but the amplitude $|\alpha|\sqrt{2}$ of input 4 must be large enough (see text), while nothing is injected into the inputs 5 and 6. The circles on paths 1, 2, and 3 denote devices that synchronize the modes' incoming to the cross-Kerr media.

Our experimental setup is schematically sketched in Fig. 1. The linear optical devices we use are balanced (or 50:50) beam splitters and phase shifters. A balanced beam splitter is described by a unitary operator $\hat{B}_{ij}$ that transforms two separable coherent states $|\alpha\rangle_i|\beta\rangle_j$ to two other separable ones as follows:

$$\hat{B}_{ij}|\alpha\rangle_i|\beta\rangle_j = \left|\frac{\alpha+i\beta}{\sqrt{2}}\right\rangle_i\left|\frac{\beta+i\alpha}{\sqrt{2}}\right\rangle_j. \quad (7)$$

A phase shifter is described by a unitary operator $\hat{P}_i(\phi)$ that induces a shift of a coherent state $|\alpha\rangle_i$ in phase space as

$$\widehat{P}_i(\phi)|\alpha\rangle_i = |\alpha e^{i\phi}\rangle_i. \quad (8)$$

Because, as is well known, it is impossible to produce entanglement through linear optics alone, a kind of nonlinearity should be demanded. We employ the cross-Kerr nonlinearity (CKN) for our purpose. CKN has proven to be a powerful tool to implement various interesting tasks in quantum information processing and quantum computing. Namely, CKN was extensively exploited for preparing Schrödinger's cat states [23], W-type ECSs [9], cluster-type ECSs [11], and hybrid entanglement [25], for quantum nondemolition measurement of photon numbers [24], for constructing quantum logic gates [26], for entanglement distillation [27], for a Bell-state analyzer [28], etc. As two optical beams do not talk to each other in the vacuum, CKN plays a role. Its effect is to bring about an interaction between the two beams when they propagate at the same time through a nonlinear (Kerr) medium. The self-Kerr effect of each beam can be ignored by appropriately choosing resonances of the Kerr medium [29].

Then, the action of CKN amounts to an evolution operator

$$\widehat{K}_{ij}(\chi) = e^{-i\chi t \hat{a}_i^\dagger \hat{a}_i \hat{a}_j^\dagger \hat{a}_j}, \tag{9}$$

with χ being the strength of nonlinearity and t being the time the two beams, i and j, interact with each other inside the medium. If the two optical beams are in coherent states $|\alpha\rangle_i|\beta\rangle_j$, then $\widehat{K}_{ij}(\chi)|\alpha\rangle_i|\beta\rangle_j = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\alpha^n|n\rangle_i|\beta e^{-i\chi t n}\rangle_j/\sqrt{n!}$. Of interest is the case when the first beam is in a Fock state and the second beam is in a coherent one, in which case we have a simple evolution:

$$\widehat{K}_{ij}(\chi)\,|n\rangle_i\,|\beta\rangle_j = |n\rangle_i\,|\beta e^{-i\chi t n}\rangle_j; \tag{10}$$

that is, the Fock state is uninfluenced but the coherent one gains a phase, $-\chi t n$, after leaving the medium.

Usually, as in various quantum tasks, input states are single photons, thus facing delicate problems. First, on-demand single-photon sources are still challenging. Second, even more serious, CKN at the single-photon level may not be helpful if the causal, noninstantaneous behavior of the nonlinearities are accurately taken into account [30]. Our scheme does not confront us with those problems since we employ four coherent states as the inputs: three of them, $|\xi\rangle_1$, $|\xi\rangle_2$, and $|\xi\rangle_3$, are the same size as the TCS, $|\Psi_{p,q}(\xi)\rangle_{123}$, to be generated, while the fourth one, $|\sqrt{2}\alpha\rangle_4$, has a large size, $|\alpha| \gg 1$. Hence, our overall input state is (see Fig. 1)

$$|\Psi_{in}\rangle = |\xi,\xi,\xi\rangle_{123}|\sqrt{2}\alpha\rangle_4|0\rangle_5|0\rangle_6, \tag{11}$$

with $|\xi,\xi,\xi\rangle_{123} \equiv |\xi\rangle_1\,|\xi\rangle_2\,|\xi\rangle_3$ and $|0\rangle_i$ being the vacuum state. Since it is convenient to use formula (10), we reexpress Eq. (11) as

$$|\Psi_{in}\rangle = \sum_{n,m,k=0}^{\infty} a_{nmk}|n,m,k\rangle_{123}|\sqrt{2}\alpha\rangle_4|0\rangle_5|0\rangle_6, \tag{12}$$

where $|n,m,k\rangle_{123} \equiv |n\rangle_1|m\rangle_2|k\rangle_3$ and $a_{nmk} = a_n a_m a_k$, with

$$a_l = e^{-r^2/2}\frac{\xi^l}{\sqrt{l!}} \tag{13}$$

coming from the Fock representation of a coherent state, $|\xi\rangle_i = \sum_{l=0}^{\infty} a_l|l\rangle_i$.

After passing the first balanced beam splitter B1 and phase shifter $\hat{P}_5(\theta)$, $|\Psi_{in}\rangle$ is transformed to

$$|\Psi_1\rangle = \sum_{n,m,k=0}^{\infty} a_{nmk}|n,m,k\rangle_{123}|\alpha\rangle_4|i\alpha e^{i\theta}\rangle_5|0\rangle_6, \tag{14}$$

which, after the second balanced beam splitter B2 and phase shifter $\hat{P}_6(\phi)$, becomes

$$|\Psi_2\rangle = \sum_{n,m,k=0}^{\infty} a_{nmk}|n,m,k\rangle_{123}|\alpha\rangle_4\left|\frac{i\alpha e^{i\theta}}{\sqrt{2}}\right\rangle_5\left|-\frac{\alpha e^{i(\theta+\phi)}}{\sqrt{2}}\right\rangle_6. \tag{15}$$

The lengths of the optical path of beams 1, 2, and 3 should be controlled so that they enter the Kerr media with the nonlinearity χ at the same time as beams 4, 5, and 6 do. Then, after passing through the three Kerr media $\widehat{K}_{14}(\chi)$, $\widehat{K}_{36}(\chi)$, and $\widehat{K}_{25}(\chi)$, the total system state emerges in the form

$$|\Psi_3\rangle = \sum_{n,m,k=0}^{\infty} a_{nmk}\,|n,m,k\rangle_{123}\,|\alpha e^{-i\chi t n}\rangle_4\left|\frac{i\alpha e^{-i(\chi t m-\theta)}}{\sqrt{2}}\right\rangle_5 \times\left|-\frac{\alpha e^{-i(\chi t k-\theta-\phi)}}{\sqrt{2}}\right\rangle_6. \tag{16}$$

Setting

$$\chi t = \tau \ll 1, \quad \theta = \tau q, \quad \phi = \tau p \tag{17}$$

in Eq. (16) yields

$$|\Psi_3\rangle = \sum_{n,m,k=0}^{\infty} a_{nmk}\,|n,m,k\rangle_{123}\,|\alpha_n\rangle_4\left|\frac{i\alpha_{m-q}}{\sqrt{2}}\right\rangle_5\left|-\frac{\alpha_{k-q-p}}{\sqrt{2}}\right\rangle_6, \tag{18}$$

with

$$\alpha_l = \alpha e^{-i\tau l}. \tag{19}$$

Next, mode 5 is superposed first with mode 6 on the third balanced beam splitter B3 and then with mode 4 on the fourth balanced beam splitter B4. Note that mode 5 should go through a sequence of three phase shifters, $\hat{P}_5(\pi) \to \hat{P}_5(\pi) \to \hat{P}_5(\pi/2)$, as shown in Fig. 1. As a result, state $|\Psi_3\rangle$ is brought to

$$|\Psi_4\rangle = \sum_{n,m,k=0}^{\infty} a_{nmk}\,|n,m,k\rangle_{123}\left|\frac{2\alpha_n-\alpha_{m-q}-\alpha_{k-q-p}}{2\sqrt{2}}\right\rangle_4 \times\left|-\frac{2\alpha_n+\alpha_{m-q}+\alpha_{k-q-p}}{2\sqrt{2}}\right\rangle_5\left|\frac{\alpha_{m-q}-\alpha_{k-q-p}}{2}\right\rangle_6. \tag{20}$$

Afterwards, modes 1 and 5 are guided to simultaneously propagate through the fourth Kerr medium with the nonlinearity $-\chi$, which outputs the overall state

$$|\Psi_{\rm out}\rangle = \sum_{n,m,k=0}^{\infty} a_{nmk}\,|n,m,k\rangle_{123}\,|\beta_{nmk}\rangle_4\,|\gamma_{nmk}\rangle_5\,|\lambda_{nmk}\rangle_6, \tag{21}$$

with β_{nmk}, γ_{nmk}, and λ_{nmk} given explicitly by

$$\beta_{nmk} = \frac{\alpha e^{-i\tau n}}{2\sqrt{2}}\left(2-e^{i\tau(n-m+q)}-e^{i\tau(n-k+q+p)}\right), \tag{22}$$

$$\gamma_{nmk} = -\frac{\alpha}{2\sqrt{2}}\left(2+e^{i\tau(n-m+q)}+e^{i\tau(n-k+q+p)}\right), \tag{23}$$

$$\lambda_{nmk} = \frac{\alpha e^{-i\tau n}}{2}\left(e^{i\tau(n-m+q)}-e^{i\tau(n-k+q+p)}\right). \tag{24}$$

At the final stage, we arrange two photodetectors, D1 and D2, to detect modes 4 and 6, respectively. Assuming ideal photodetectors (i.e., they are able to resolve photon numbers), the detection of mode j amounts to performing a complete set of measurement operators,

$$\widehat{M}_j^{n_j} = |n_j\rangle_j\langle n_j|, \quad n_j = 0,1,2,\ldots \tag{25}$$

Because $\widehat{M}_j^{n_j+}\widehat{M}_j^{n_j} = (\widehat{M}_j^{n_j})^2 = \widehat{M}_j^{n_j}$, this is a projective measurement projecting the mode onto a certain number state in Fock space. Let the measurement outcomes be $\{n_4,n_6\}$ (i.e., n_4 photons of mode 4 and n_6 photons of mode 6 are detected), which occurs with a probability $P_{n_4n_6}$. Then $|\Psi_{\rm out}\rangle$ in Eq. (21) is collapsed into

$$|\Psi_{n_4n_6}\rangle = \frac{|\Phi_{n_4n_6}\rangle}{\sqrt{P_{n_4n_6}}}, \tag{26}$$

where

$$|\Phi_{n_4n_6}\rangle = \widehat{M}_4^{n_4} \otimes \widehat{M}_6^{n_6}\,|\Psi_{\rm out}\rangle = |\phi_{n_4n_6}\rangle_{1235}\,|n_4\rangle_4\,|n_6\rangle_6\,, \tag{27}$$

with

$$|\phi_{n_4n_6}\rangle_{1235} = \sum_{n,m,k=0}^{\infty} a_{nmk}\langle n_4|\beta_{nmk}\rangle \times \langle n_6|\lambda_{nmk}\rangle|n,m,k\rangle_{123}\,|\gamma_{nmk}\rangle_5\,, \tag{28}$$

and

$$P_{n_4n_6} = \langle\Phi_{n_4n_6}|\Phi_{n_4n_6}\rangle = \sum_{n,m,k=0}^{\infty} |a_{nmk}\langle n_4|\beta_{nmk}\rangle\langle n_6|\lambda_{nmk}\rangle|^2. \tag{29}$$

We are concerned with modes 1, 2, and 3, whose state is characterized by a reduced density matrix of the form

$$\begin{aligned}\rho_{123}^{n_4n_6} &= \frac{1}{P_{n_4n_6}}{\rm Tr}_5|\phi_{n_4n_6}\rangle_{1235}\langle\phi_{n_4n_6}|\\ &= \frac{1}{P_{n_4n_6}}\sum_{n,m,k,n',m',k'=0}^{\infty} a^*_{n'm'k'}a_{nmk}\langle n_4|\beta_{nmk}\rangle\langle\beta_{n'm'k'}|n_4\rangle\\ &\quad\times\langle\gamma_{n'm'k'}|\gamma_{nmk}\rangle\langle n_6|\lambda_{nmk}\rangle\\ &\quad\times\langle\lambda_{n'm'k'}|n_6\rangle|n,m,k\rangle_{123}\langle n',m',k'|,\end{aligned} \tag{30}$$

where ${\rm Tr}_5$ denotes the trace over mode 5. The fidelity of state $\rho_{123}^{n_4n_6}$ with respect to the target state $|\Psi_{p,q}(\xi)\rangle_{123}$, $F_{n_4n_6} = {}_{123}\langle\Psi_{p,q}(\xi)|\rho_{123}^{n_4n_6}|\Psi_{p,q}(\xi)\rangle_{123}$, can be calculated as

$$F_{n_4n_6} = \frac{e^{-3r^2}}{P_{n_4n_6}\mathcal{N}^2_{p,q}(r)}\,|\langle n_4\,|0\rangle\,\langle n_6\,|0\rangle|^2\,. \tag{31}$$

Due to the factors $|\langle n_4\,|0\rangle\,\langle n_6\,|0\rangle|^2$ in Eq. (31), it is transparent that a nonzero fidelity is obtained only for $n_4 = n_6 = 0$. If so, the fidelity of the so-generated state ρ_{123}^{00} is

$$F = F_{00} = \frac{e^{-3r^2}}{P_{00}\mathcal{N}^2_{p,q}(r)}. \tag{32}$$

As for the corresponding generation probability P_{00}, we put $n_4 = n_6 = 0$ in Eq. (29) and then use the equality $\langle 0\,|\zeta\rangle = \exp(-|\zeta|^2/2)$ to obtain

$$P = P_{00} = \sum_{n,m,k=0}^{\infty} |a_{nmk}|^2 e^{-(|\beta_{nmk}|^2+|\lambda_{nmk}|^2)}, \tag{33}$$

which, after some transformations using Eqs. (13), (22), (23), and (24), has the explicit expression

$$P = e^{-3r^2}\sum_{n,m,k=0}^{\infty}\frac{r^{2(n+m+k)}}{n!m!k!} \times e^{-\frac{|\alpha|^2}{4}\{5-2\cos[(m-n-q)\tau]-2\cos[(k-n-p-q)\tau]-\cos[(k-m-p)\tau]\}}. \tag{34}$$

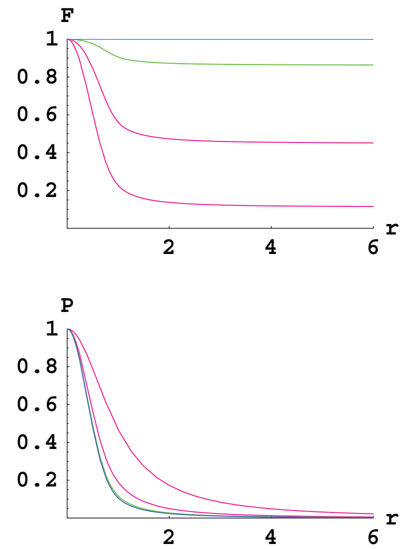

FIG. 2. (Color online) (top) The fidelity F, Eq. (32), of the generated state ρ_{123}^{00} with respect to the target TCS, Eq. (6), and (bottom) the corresponding generation probability P, Eq. (34), vs $r = |\xi|$ for $p = q = 0$, $\tau = 10^{-3}$, and $|\alpha| = 10^3$, 2×10^3, 3×10^3, and 5×10^3.

The above analysis indicates that we fail whenever any finite number of photons is detected by D1 and/or D2. This means that photon-number resolving turns out to not be necessary, and what we actually need is just on-off photodetectors that are silent for zero photons and click otherwise.

We plot in Fig. 2 the fidelity F, Eq. (32), and the probability P, Eq. (34), as functions of $r = |\xi|$ for $p = q = 0$, $\tau = 10^{-3}$, and several values of $|\alpha|$. The fidelity improves as the size of the coherent state in input mode 4 increases, at the expense of reduced success probability. As a numerical illustration, for $\tau = 10^{-3}$, $r = 0.5$, and $|\alpha| = 10^3$ (3×10^3, 5×10^3), the fidelity is $F \simeq 60.74\%$ (97.59%, 99.99%), and the corresponding probability is $P \simeq 0.7898$ (0.4916, 0.4798).

Recall that we have assumed photodetectors D1 and D2 to be ideal, and in order to succeed we have to have no clicks at either of them; that is, no photons at all should be detected. Nevertheless, in realistic circumstances photodetectors are not ideal in the sense that they may still be silent (i.e., not produce a click) even when there are some photons coming in. To account for such effects we introduce the detector's efficiency η $(0 < \eta < 1)$ and describe nonideal on-off photodetection by the two following measurement operators:

$$\widehat{\Pi}_j^{\rm on} = \sum_{n_j=0}^{\infty}\sqrt{f_{n_j}^{\rm on}(\eta)}|n_j\rangle_j\langle n_j|, \quad f_{n_j}^{\rm on}(\eta) = 1-(1-\eta)^{n_j} \tag{35}$$

and

$$\widehat{\Pi}_j^{\rm off} = \sum_{n_j=0}^{\infty}\sqrt{f_{n_j}^{\rm off}(\eta)}|n_j\rangle_j\langle n_j|, \quad f_{n_j}^{\rm off}(\eta) = (1-\eta)^{n_j}, \tag{36}$$

which correspond to a "click" (on) and "no clicks" (off) of a nonideal photodetector, respectively. It is simple to verify that $f_m^{\rm on}(\eta)$ and $f_m^{\rm off}(\eta)$ are probabilities of the outcomes on and off for a state containing m photons. Note, in particular, that $f_m^{\text{off}}(\eta) > 0$ even for $m > 0$. Although $\widehat{\Pi}_j^{\text{on}+}\widehat{\Pi}_j^{\text{on}} + \widehat{\Pi}_j^{\text{off}+}\widehat{\Pi}_j^{\text{off}} = \widehat{I}_j$ ($\widehat{I}_j$ is the identity operator for mode j), $\widehat{\Pi}_j^{\text{on}+}\widehat{\Pi}_j^{\text{on}} = (\widehat{\Pi}_j^{\text{on}})^2$, and $\widehat{\Pi}_j^{\text{off}+}\widehat{\Pi}_j^{\text{off}} = (\widehat{\Pi}_j^{\text{off}})^2$, as it should, the following inequalities hold: $\widehat{\Pi}_j^{\text{on}}\widehat{\Pi}_j^{\text{off}} \neq 0$, $(\widehat{\Pi}_j^{\text{on}})^2 \neq \widehat{\Pi}_j^{\text{on}}$, and $(\widehat{\Pi}_j^{\text{off}})^2 \neq \widehat{\Pi}_j^{\text{off}}$. Thus, this is not a projective measurement but in fact belongs to a two-element positive operator-valued measure (POVM) specified by $\{\widehat{E}_j^{\text{on}}, \widehat{E}_j^{\text{off}}\}$, with $\widehat{E}_j^{\text{on}} = \widehat{\Pi}_j^{\text{on}+}\widehat{\Pi}_j^{\text{on}}$ and $\widehat{E}_j^{\text{off}} = \widehat{I}_j - \widehat{E}_j^{\text{on}}$.

Let the outcome of measuring mode 4 be x = on or off and the outcome of measuring mode 6 be y = on or off. Then, the possible outcomes of measuring modes 4 and 6 by two nonideal photodetectors D1 and D2 with the same efficiency η are $\{x,y\}$ = {on,on}, {on, off}, {off, on}, or {off, off}, for which $|\Psi_{\text{out}}\rangle$ in Eq. (21) becomes

$$|\Psi_{x,y}(\eta)\rangle = \frac{|\Phi_{x,y}(\eta)\rangle}{\sqrt{P_{x,y}(\eta)}}, \tag{37}$$

where

$$\begin{aligned} |\Phi_{x,y}(\eta)\rangle &= \widehat{\Pi}_4^x \otimes \widehat{\Pi}_6^y |\Psi_{\text{out}}\rangle \\ &= \sum_{n,m,k,n_4,n_6=0}^{\infty} a_{nmk}\sqrt{f_{n_4}^x(\eta) f_{n_6}^y(\eta)}\langle n_4|\beta_{nmk}\rangle \\ &\quad \times \langle n_6|\lambda_{nmk}\rangle |n,m,k\rangle_{123}|n_4\rangle_4|\gamma_{nmk}\rangle_5|n_6\rangle_6 \end{aligned} \tag{38}$$

and

$$\begin{aligned} P_{x,y}(\eta) &= \langle\Phi_{x,y}(\eta)|\Phi_{x,y}(\eta)\rangle \\ &= \sum_{n,m,k,n_4,n_6=0}^{\infty} f_{n_4}^x(\eta) f_{n_6}^y(\eta)|a_{nmk}\langle n_4|\beta_{nmk}\rangle\langle n_6|\lambda_{nmk}\rangle|^2. \end{aligned} \tag{39}$$

After obtaining the outcomes $\{x,y\}$ the reduced density matrix characterizing modes 1, 2, and 3 takes the form

$$\begin{aligned} \rho_{123}^{x,y}(\eta) &= \text{Tr}_{456}|\Psi_{x,y}(\eta)\rangle\langle\Psi_{x,y}(\eta)| \\ &= \frac{1}{P_{x,y}(\eta)} \sum_{n',m',k',n,m,k,n_4,n_6=0}^{\infty} f_{n_4}^x(\eta) f_{n_6}^y(\eta) a_{n'm'k'}^* a_{nmk} \\ &\quad \times \langle n_4|\beta_{nmk}\rangle\langle\beta_{n'm'k'}|n_4\rangle\langle\gamma_{n'm'k'}|\gamma_{nmk}\rangle \\ &\quad \times \langle n_6|\lambda_{nmk}\rangle\langle\lambda_{n'm'k'}|n_6\rangle|n,m,k\rangle_{123}\langle n',m',k'|, \end{aligned} \tag{40}$$

where Tr_{456} denotes the trace over modes 4, 5, and 6. Now we can calculate the fidelity of $\rho_{123}^{x,y}(\eta)$ with respect to $|\Psi_{p,q}(\xi)\rangle_{123}$, which after some transformations can be expressed explicitly as

$$\begin{aligned} F_{x,y}(\eta) &= \frac{e^{-3r^2}}{P_{x,y}(\eta)\mathcal{N}_{p,q}^2(r)} \sum_{n_4,n_6=0}^{\infty} f_{n_4}^x(\eta) f_{n_6}^y(\eta)|\langle n_4|0\rangle\langle n_6|0\rangle|^2 \\ &= \frac{e^{-3r^2}}{P_{x,y}(\eta)\mathcal{N}_{p,q}^2(r)} f_0^x(\eta) f_0^y(\eta). \end{aligned} \tag{41}$$

Returning to Eqs. (35) and (36), we realize that $f_0^x(\eta) f_0^y(\eta)$ vanishes unless $x = y$ = off, in which case $f_0^{\text{off}}(\eta) f_0^{\text{off}}(\eta) = 1$. Thus, we would succeed iff both of the nonideal photodetectors, D1 and D2, are silent. Then, the fidelity of the so-generated state $\rho_{123}^{\text{off,off}}(\eta)$ is

$$F(\eta) = F_{\text{off,off}}(\eta) = \frac{e^{-3r^2}}{P_{\text{off,off}}(\eta)\mathcal{N}_{p,q}^2(r)}. \tag{42}$$

As for the corresponding generation probability $P_{\text{off,off}}(\eta)$, we put $x = y$ = off in Eq. (39) and then use the equality $\langle m|\zeta\rangle = \exp(-|\zeta|^2/2)\zeta^m/\sqrt{m!}$ to obtain

$$\begin{aligned} P(\eta) = P_{\text{off,off}}(\eta) &= \sum_{n,m,k,n_4,n_6=0}^{\infty} f_{n_4}^{\text{off}}(\eta) f_{n_6}^{\text{off}}(\eta)\frac{|\beta_{nmk}|^{2n_4}}{n_4!} \\ &\quad \times \frac{|\lambda_{nmk}|^{2n_6}}{n_6!}|a_{nmk}|^2 e^{-(|\beta_{nmk}|^2+|\lambda_{nmk}|^2)}, \end{aligned} \tag{43}$$

which, after some transformations using Eqs. (13), (22), (23), (24), and (36), has the explicit expression

$$\begin{aligned} P(\eta) = e^{-3r^2} &\sum_{n,m,k=0}^{\infty} \frac{r^{2(n+m+k)}}{n!m!k!} \\ &\times e^{-\frac{\eta|\alpha|^2}{4}\{5-2\cos[(m-n-q)\tau]-2\cos[(k-n-p-q)\tau]-\cos[(k-m-p)\tau]\}}. \end{aligned} \tag{44}$$

We plot in Fig. 3 the fidelity $F(\eta)$, Eq. (42), and the corresponding probability $P(\eta)$, Eq. (44), as functions of $r = |\xi|$ for $p = q = 0$, fixed values of τ and $|\alpha|$, and different values of η. As the photodetector efficiency increases, the fidelity gets better while the probability remains almost unchanged. For τ as small as 10^{-3} and $|\alpha|$ as large as 5×10^3, the fidelity $F(\eta)$ already exceeds 99.9% for $\eta \geqslant 0.7$. As a numerical illustration, for the data used in Fig. 3, when $\eta = 0.2$, 0.3,0.5, and 0.7, we have, at $r = 0.5$, $F(\eta) = 89.23\%, 95.74\%, 99.37\%$, and 99.91%, while $P(\eta) = 53.76\%$, 50.11%, 48.28%, and 48.02%, respectively.

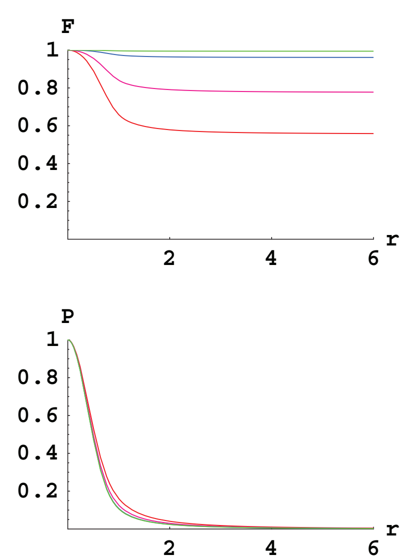

FIG. 3. (Color online) (top) The fidelity F, Eq. (42), of the generated state ρ_{123}^{00} with respect to the target TCS, Eq. (6), and (bottom) the corresponding generation probability P, Eq. (44), vs $r = |\xi|$ for $p = q = 0$, $\tau = 10^{-3}$, $|\alpha| = 5 \times 10^3$, and $\eta = 0.2$, 0.3, 0.5, and 0.7.

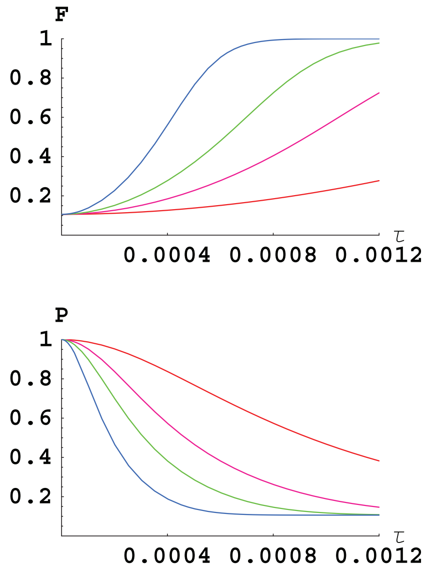

FIG. 4. (Color online) (top) The fidelity F, Eq. (32), of the generated state ρ_{123}^{00} with respect to the target TCS, Eq. (6), and (bottom) the corresponding generation probability P, Eq. (34), vs τ for $p = q = 0$, $r = |\xi| = 1$, and $|\alpha| = 10^3$, 2×10^3, 3×10^3, and 5×10^3.

The influence of the CKN's strength can be clearly seen from Fig. 4, which plots F, Eq. (32), and P, Eq. (34), as functions of τ for a fixed value of $r = |\xi|$ and different values of $|\alpha|$. As expected, for given values of $|\alpha|$ and r, the fidelity F (the probability P) increases (decreases) with the CKN's strength τ, which is reasonable. In this context a comment on Ref. [17], which dealt with the generation of PCS, is worth mentioning. According to that reference, the generated state (given by Eq. (19) in [17]) gets closer to the target PCS (given by Eq. (2) in [17]) for a bigger N. However, as seen from the setting preceding Eq. (9) in [17], the strength of nonlinearity decreases with increasing N. This means that their scheme would have worked better for a smaller strength of nonlinearity. Particularly, the result claimed by the authors of [17], that "in the limit of $K \to \infty$, which implies $N \to \infty$, the state Eq. (19) returns to the Eq. (2)," i.e., the fidelity tends to 100% in the limit $N \to \infty$ ($\tau \to 0$), is implausible. The unreasonable point rests in their setting $\chi t = \pi/N$, which renders CKN an N dependence.

Before concluding we would like to establish an interesting combined dependence on the parameters involved. Making use of the fact that the CKN is generally very weak (i.e., $\tau \ll 1$), we can, to a good approximation, express the probability $P(\eta)$ of Eq. (44) in the form

$$P(\eta) \simeq e^{-3r^2} \sum_{n,m,k=0}^{\infty} \frac{r^{2(n+m+k)}}{n!m!k!} \times e^{-\frac{\eta|\alpha|^2\tau^2}{4}[\frac{1}{2}(m-k+p)^2+(n-m+q)^2+(n-k+p+q)^2]}. \quad (45)$$

This formula reveals that the probability $P(\eta)$ and thus the fidelity $F(\eta)$ depend collectively on the product $\eta|\alpha|^2\tau^2$ rather than individually on each of η, $|\alpha|$, and τ. From Fig. 5, which shows the dependence of the fidelity and the corresponding probability on the collective parameter $Z = \sqrt{\eta}|\alpha|\tau$ for various values of r, we can recognize that the fidelity is almost 100% when $Z \geqslant 5$. The inequality $Z = \sqrt{\eta}|\alpha|\tau \geqslant 5$ is meaningful. It signifies that our scheme still works well even for weak CKN and low photodetector efficiency if the coherent beam injected into mode 4 is intense enough. For instance, for η as low as 0.8, a weak CKN of the order of $\tau = 10^{-4}$ would require a coherent state with $|\alpha| \geqslant 5.59 \times 10^4$. Hence, a realistic pump having on average 10^{12} photons per pulse (corresponding to $|\alpha| \sim 10^6$) would compensate for CKNs as weak as with $\tau \sim 1.1 \times 10^{-5}$ (7.9×10^{-6},6.4×10^{-6}) if $\eta = 0.2$ (0.4, 0.6).

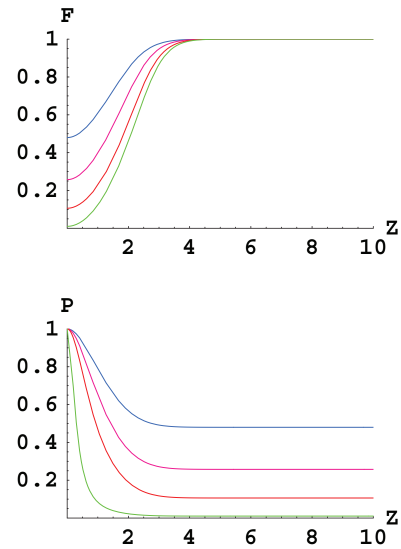

FIG. 5. (Color online) (top) The fidelity F, Eq. (42), of the generated state ρ_{123}^{00} with respect to the target TCS, Eq. (6), and (bottom) the corresponding generation probability P, Eq. (45), vs $Z = \sqrt{\eta}|\alpha|\tau$ for $p = q = 0$ and $r = 0.5$, 0.7, 1.0, and 3.0.

III. CONCLUSION

In conclusion, we have designed an experimental setup for generating TCSs in terms of three light fields that can freely be directed to any intended locations in open space. The set of physical resources comprises four coherent states, standard linear optical elements, and four cross-Kerr media. Among the four input coherent states, three are of the size of $O(1)$, the same as that of the target TCS, and one is of a much larger size. The optical elements include ordinary balanced beam splitters, phase shifters, and nonideal on-off photodetectors. As for CKNs, their natural strength is tiny (of the order of $\tau \sim 10^{-18}$ [31]), while direct use of them would demand $\tau \sim \pi$, which is extremely difficult to achieve. Here the use of CKNs is combined with a large-size coherent state supplied by an intense laser source. The fidelity of the state generated by our scheme with respect to the desired TCS depends approximately upon the product $\eta|\alpha|^2\tau^2$ and reaches almost 100% for $\sqrt{\eta}|\alpha|\tau \geqslant 5$. Hence, even with an inefficient on-off photodetector with $\eta \sim 0.8$, a laser pulse with about a 10^6 mean photon number [i.e., $|\alpha| \sim O(10^3)$] would offset the smallness of CKNs with $\tau \sim O(10^{-3})$. Such weak (but not tiny) CKNs can potentially be engineered in practice within present technologies using various means, such as doped optical fibers, cavity quantum electrodynamics, electromagnetically induced transparency, etc. (see, e.g., [32]). Therefore, strong CKNs are not at all a compulsory precondition, and weak CKNs are thought to promise a bright perspective for quantum information processing and quantum computing [33]. The fact that our scheme is nondeterministic causes no problems, for the TCSs we generate will be supplied off-line for a subsequent given quantum task. Furthermore, unlike in many other schemes and protocols for quantum-state engineering, we require neither homodyne or heterodyne measurements nor single-photon sources, which would raise the delicate issues associated with single-photon-level Kerr nonlinearities [30]. In our scheme we instead used coherent states as inputs to cross-Kerr media. An advantage of using coherent states over single photons is obvious since the rate of single-photon generation is very small, which through parametric down conversion is about one in a million. However, whether coherent-state inputs can overcome the problems pointed out in Ref. [30] remains an open question. If they can, our scheme would be feasible.

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